Chapter 4: Probability

Chapter 4 Summary and Notes: Probability

4.1 The Concept of Probability

- **Experiment:** Any process of observation that has an uncertain outcome.
- **Sample Space:** The set of all possible outcomes of an experiment.
- **Probability:** A numerical measure (between 0 and 1) of the likelihood that a specific outcome will occur.

Methods of Assigning Probability:

1. **Classical Method:** Used when all experimental outcomes are equally likely. The probability of an event is the number of favorable outcomes divided by the total number of possible outcomes.
2. **Relative Frequency Method:** Based on historical data or experimentation. The probability is the proportion of times the event occurred in the long run.
3. **Subjective Method:** Based on personal belief, experience, or intuition when there is little to no direct data.

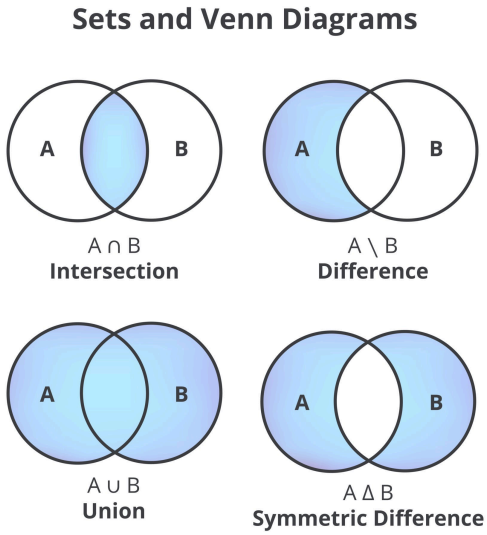
4.2 Sample Spaces and Events

- **Event:** A specific collection of sample space outcomes. The probability of an event is the sum of the probabilities of the individual outcomes that make up the event. A tree diagram is often used to visualize all possible outcomes.

4.3 Elementary Probability Rules

- **Complement of an Event** $(\bar{A})$ The set of all outcomes in the sample space that are *not* in event A.
  - **Rule of Complements:** $P(\bar{A}) = 1 - P(A)$

- **Intersection ($A \cap B$) and Union ($A \cup B$):** The intersection is when both A and B occur. The union is when either A or B (or both) occurs.



- **Addition Rule:** This rule is used to calculate the probability of a union. The intersection is subtracted to avoid double-counting the outcomes that are in both events.

4.4 Conditional Probability and Independence

- **Conditional Probability** $(P(A|B))$**:** The probability of event A occurring, *given that* event B has already occurred. It is calculated by focusing on the reduced sample space where event B is true.

Conditional Probability Formula

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

- **General Multiplication Rule:** Used to find the probability of an intersection.

$$P(A \cap B) = P(A) \times P(B|A)$$

- **Independent Events:** Two events are independent if the occurrence of one does not affect the probability of the other.
  - **Test for Independence:** A and B are independent if $P(A|B) = P(A)$
  - **Multiplication Rule for Independent Events:** If A and B are independent:

$$P(A \cap B) = P(A) \times P(B)$$

4.5 Bayes' Theorem (Optional (Not tested))

Bayes' Theorem provides a way to update the probability of an initial belief (the prior) based on new evidence, resulting in a revised probability (the posterior).

- **Prior Probability:** The initial probability of an event before new information is considered (e.g., $P(B)$).
- **Posterior Probability:** The revised probability of an event after taking new information into account (e.g., $P(B|A)$)
- **Bayes' Theorem Formula:** It is particularly useful in medical diagnostics, where we might know the probability of a positive test given a disease, but we want to find the more critical probability of having the disease given a positive test.
