Chapter 5

Discrete Random Variables

Discrete Random Variables

We often use what we call **random variables** to describe the important aspects of the outcomes of experiments. In this chapter we introduce two important types of random variables—**discrete random variables** and **continuous random variables**—and learn how to find probabilities concerning discrete random variables. As one application, we will begin to see how to use probabilities concerning discrete random variables to make statistical inferences about populations.

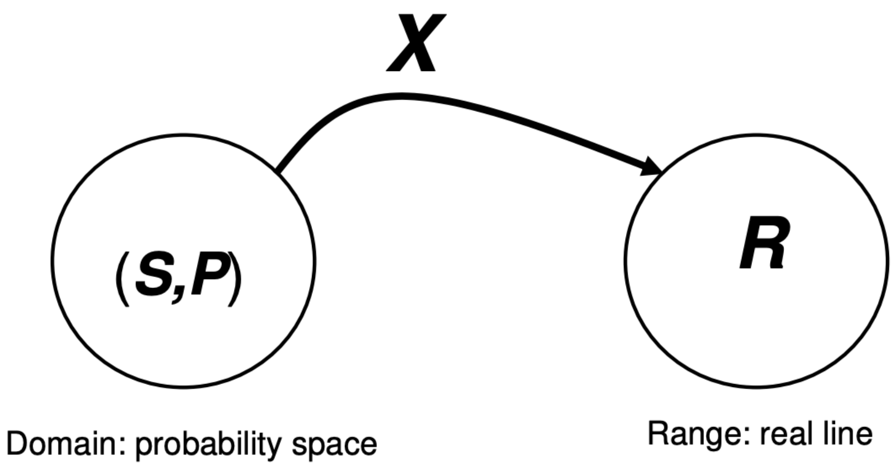
5.1 Two Types of Random Variables

- A **random variable** is a variable that assumes **numerical values** that are determined by the outcome of an experiment, where one and only one numerical value is assigned to each experimental outcome.

- Before an experiment is carried out, its outcome is **uncertain**. It follows that, because a random variable assigns a number to each experimental outcome, a random variable can be thought of as representing ***an uncertain numerical outcome.***

Random Variables

A random variable: a function



- A (real-valued) random variable, often denoted by X (or some other capital letter), is a function mapping a probability space (S, P) into the real line R.

Example

- Consider a random experiment in which a coin is tossed three times. Let X be the number of heads. Let H represent the outcome of a head and T the outcome of a tail.
- The possible outcomes for such an experiment:

TTT, TTH, THT, THH,
HTT, HTH, HHT, HHH

- Thus the possible values of X (number of heads) are 0,1,2,3.
- From the definition of a random variable, X as defined in this experiment, is a ***random variable***.

Discrete Random Variables

5.1 Two Types of Random Variables

Two Types of Random Variables

- **Discrete random variable: Possible values can be counted or listed**
  - For example, the number of TV sets sold at the store in one day. Here x could be 0, 1, 2, 3, 4 and so forth.

- **Continuous random variable: May assume any numerical value in one or more intervals**
  - For example, the waiting time for a credit card authorization, the interest rate charged on a business loan

Example: Two Types of Random Variables

Question	Random Variable x	Type
Family size	x = Number of people in family reported on tax return	
Distance from home to store	x = Distance in miles from home to a store	
Own dog or cat	x = 1 if own no pet; = 2 if own dog(s) only; = 3 if own cat(s) only; = 4 if own dog(s) and cat(s)	

Discrete Random Variables

5.2 Discrete Probability Distributions

5.2 Discrete Probability Distributions

The **probability distribution** of a discrete random variable is a ***table, graph, or formula*** that gives the probability associated with each possible value that the variable can assume

Notation: Denote the values of the random variable by x and the value's associated probability by $p(x)$

Properties

1. For any value x of the random variable, $p(x) \geq 0$
2. The probabilities of all the events in the sample space must sum to 1, that is,

$$\sum_{\text{all} x} p(x) = 1$$

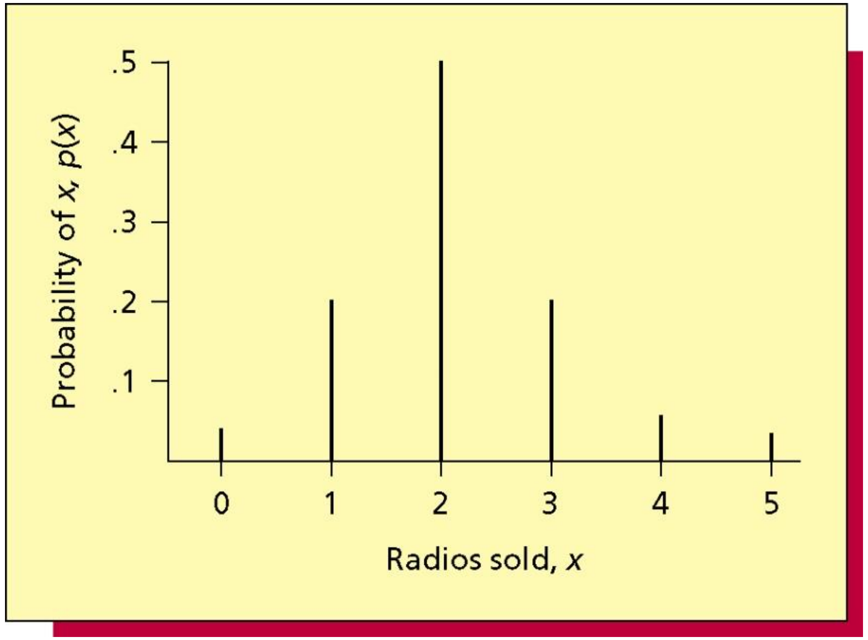
Example 5.3: Number of Radios （Sold at Sound City in a Week）

- Let x be the random variable of the number of radios sold per week
  - x has values x = 0, 1, 2, 3, 4, 5
- Given sales history over past 100 weeks
  - Let f be the number of weeks (of the past 100) during which x number of radios were sold
  - Records tell us that
    - f(0)=3 No radios have been sold in 3 of the weeks
    - f(1)=20 One radios has been sold in 20 of the weeks
    - f(2)=50 Two radios have been sold in 50 of the weeks
    - f(3)=20 Three radios have been sold in 20 of the weeks
    - f(4)=5 Four radios have been sold in 5 of the weeks
    - f(5)=2 Five radios have been sold in 2 of the weeks
    - No more than five radios were sold in any of the past 100 weeks

Frequency distribution of sales history over past 100 weeks

# Radios, x	Frequency	Relative Frequency	Probability, $p(x)$
0	f(0) =3	3/100 = 0.03	$p(0) = 0.03$
1	f(1) =20	20/100 = 0.20	$p(1) = 0.20$
2	f(2) =50	0.50	$p(2) = 0.50$
3	f(3) =20	0.20	$p(3) = 0.20$
4	f(4) = 5	0.05	$p(4) = 0.05$
5	f(5) = 2	0.02	$P(5) = 0.02$
	100	1.00	1.00

- Interpret the relative frequencies as probabilities
  - So for any value x, $f(x)/n = p(x)$
  - Assuming that sales remain stable over time



- **What is the chance that two radios will be sold in a week?**
  - $P(x = 2) = 0.50$
- **What is the chance that fewer than 2 radios will be sold in a week?**
  - $p(x < 2) = p(x = 0 \text{ or } x = 1)$
    $= p(x = 0) + p(x = 1)$
    $= 0.03 + 0.20 = 0.23$

Using the addition rule for the mutually exclusive values of the random variable.

- **What is the chance that three or more radios will be sold in a week?**
  - $p(x \geq 3) = p(x = 3, 4, \text{ or } 5)$
    $= p(x = 3) + p(x = 4) + p(x = 5)$
    $= 0.20 + 0.05 + 0.02 = 0.27$

Expected Value of a Discrete Random Variable

- Suppose that the experiment described by a random variable x is *repeated an indefinitely large number* of times.
- If the values of the random variable x observed on the repetitions are recorded, we would obtain *the population of all possible observed values* of the random variable x.
- This population has a **mean**, which we denote as μ_x and which we sometimes call the **expected value** of x. In order to calculate μ_x, we multiply each value of x by its probability $p(x)$ and then sum the resulting products over all possible values of x.

Expected Value of a Discrete Random Variable

- The **mean or expected value** of a discrete random variable x is:

$$\mu_x = \sum_{All\ x} xp(x)$$

- μ_x is the value expected to occur *in the long run* and on average

Example 5.3: Number of Radios Sold at Sound City in a Week

- How many radios should be expected to be sold in a week?
  - Calculate the **expected value** of the number of radios sold, m_X

Radios, x	*Probability, p(x)*	*x p(x)*
0	$p(0) = 0.03$	$0 \times 0.03 = 0.00$
1	$p(1) = 0.20$	$1 \times 0.20 = 0.20$
2	$p(2) = 0.50$	$2 \times 0.50 = 1.00$
3	$p(3) = 0.20$	$3 \times 0.20 = 0.60$
4	$p(4) = 0.05$	$4 \times 0.05 = 0.20$
5	$p(5) = 0.02$	$5 \times 0.02 = 0.10$
	1.00	**2.10**

  - On average, expect to sell 2.1 radios per week

Example 5.6

EXAMPLE 5.4 The Life Insurance Case: Setting a Policy Premium

An insurance company sells a $20,000 whole life insurance policy for an annual premium of $300. Actuarial tables show that a person who would be sold such a policy with this premium has a .001 probability of death during a year. Let x be a random variable representing the insurance company's profit made on one of these policies during a year. The probability distribution of x is

x, Profit	$p(x)$, Probability of x
$300 (if the policyholder lives)	.999
$300 − $20,000 = −$19,700 (a $19,700 loss if the policyholder dies)	.001

Example 5.6

The expected value of x (expected profit per year) is

$$\begin{aligned}\mu_x &= \$300(.999) + (-\$19{,}700)(.001)\\ &= \$280\end{aligned}$$

This says that if the insurance company sells a very large number of these policies, it will average a profit of \$280 per policy per year. Because insurance companies actually do sell large numbers of policies, it is reasonable for these companies to make profitability decisions based on expected values.

Next, suppose that we wish to find the premium that the insurance company must charge for a \$20,000 policy if the company wishes the average profit per policy per year to be greater than \$0. If we let *prem* denote the premium the company will charge, then the probability distribution of the company's yearly profit x is

x, Profit	$p(x)$, Probability of x
prem (if policyholder lives)	.999
prem − \$20,000 (if policyholder dies)	.001

The expected value of x (expected profit per year) is

$$\begin{aligned}\mu_x &= prem(.999) + (prem - 20{,}000)(.001) \\ &= prem - 20\end{aligned}$$

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In order for this expected profit to be greater than zero, the premium must be greater than \$20. If, as previously stated, the company charges \$300 for such a policy, the \$280 charged in excess of the needed \$20 compensates the company for commissions paid to salespeople, administrative costs, dividends paid to investors, and other expenses.

Variance and Standard Deviation

- The **variance** of a discrete random variable is:

$$\sigma_X^2 = \sum_{All x} (x - \mu_X)^2 p(x)$$

- The **variance** is the average of the squared deviations of the different values of the random variable from the expected value.
- The **standard deviation** is the square root of the variance

$$\sigma_X = \sqrt{\sigma_X^2}$$

- The variance and standard deviation measure the spread of the values of the random variable from their expected value

Example 5.7: Number of Radios

Sold at Sound City in a Week

Calculate the **variance and standard deviation** of the number of radios sold at Sound City in a week

Radios, x	*Probability, p(x)*	$(x - \mu_X)^2 \; p(x)$
0	$p(0) = 0.03$	$(0 - 2.1)^2\,(0.03) = 0.1323$
1	$p(1) = 0.20$	$(1 - 2.1)^2\,(0.20) = 0.2420$
2	$p(2) = 0.50$	$(2 - 2.1)^2\,(0.50) = 0.0050$
3	$p(3) = 0.20$	$(3 - 2.1)^2\,(0.20) = 0.1620$
4	$p(4) = 0.05$	$(4 - 2.1)^2\,(0.05) = 0.1805$
5	$p(5) = 0.02$	$(5 - 2.1)^2\,(0.02) = 0.1682$
	1.00	**0.8900**

Standard deviation

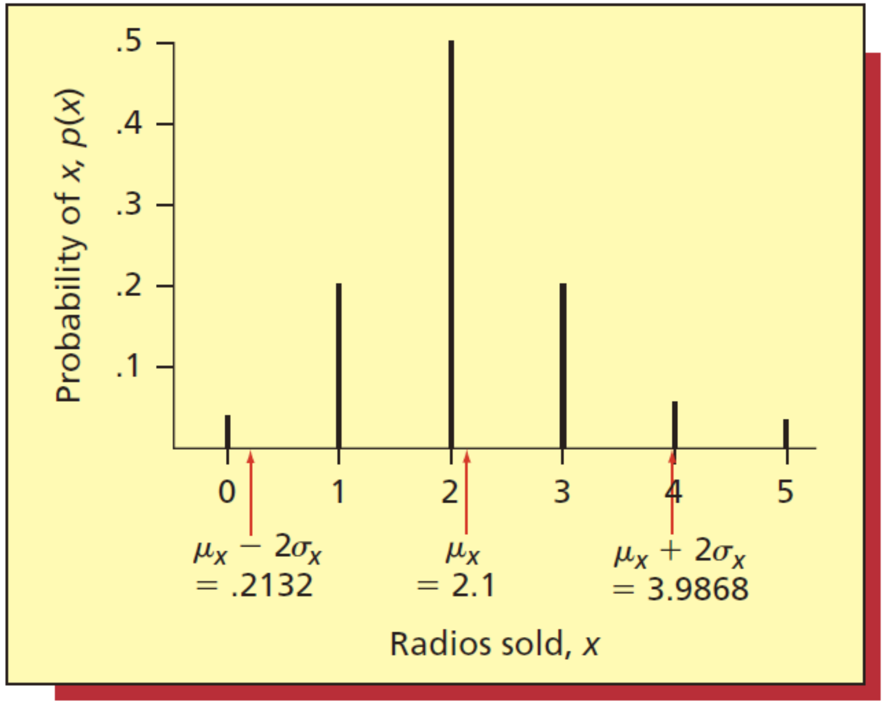
$$\sigma_X = \sqrt{0.89} = 0.9434$$

Variance

$$\sigma_X^2 = 0.89$$

Example 5.7: Number of Radios

Sold at Sound City in a Week



$\mu_x = 2.1$

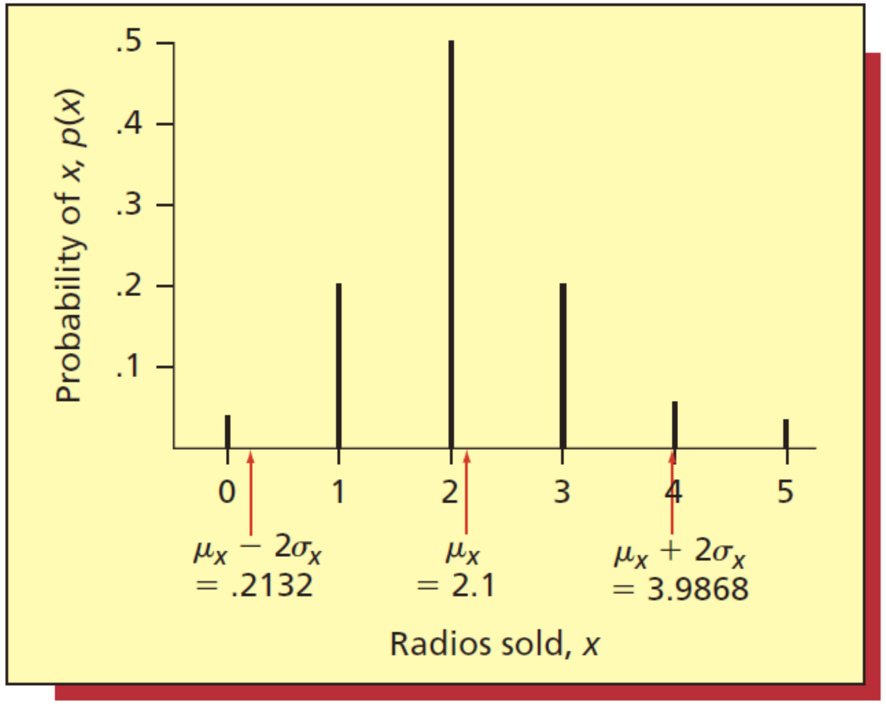
$\sigma_x = 0.9434$

$[\mu_x \pm 2\sigma_x] = [0.2132, 3.9868]$

- Three values of x (x=1, 2, or 3) that lie in the interval [.2132, 3.9868]
- P(1)+P(2)+P(3)=0.2+0.5+0.2=0.9
- 90 percent of all weeks, the number of TrueSound-XL radios sold at Sound City will be within (plus or minus) two standard deviations of the mean weekly sales of the TrueSound-XL radio at Sound City

Example 5.7: Number of Radios

Sold at Sound City in a Week



$\mu_x = 2.1$

$\sigma_x = 0.9434$

$[\mu_x \pm 2\sigma_x] = [0.2132, 3.9868]$

P($x = [\mu_x \pm 2\sigma_x]$)=0.9

- In Chebyshev's Theorem, for k=2, $P(x = [\mu_x \pm 2\sigma_x]) = 1 - 1/2^2 = 3/4$
- For k=3, $P(x = [\mu_x \pm 3\sigma_x]) = 1 - 1/3^2 = 8/9$
- Chebyshev's Theorem does not work for this distribution, which is closer to symmetrical.

Discrete Random Variables

5.3 The Binomial Distribution

5.3 The Binomial Distribution

The Binomial Experiment:

1. Experiment consists of *n* identical trials
2. Each trial results in either "success" or "failure"
3. Probability of success, *p*, is constant from trial to trial
4. Trials are independent

Note: The probability of failure, *q*, is 1 – *p* and is constant from trial to trial

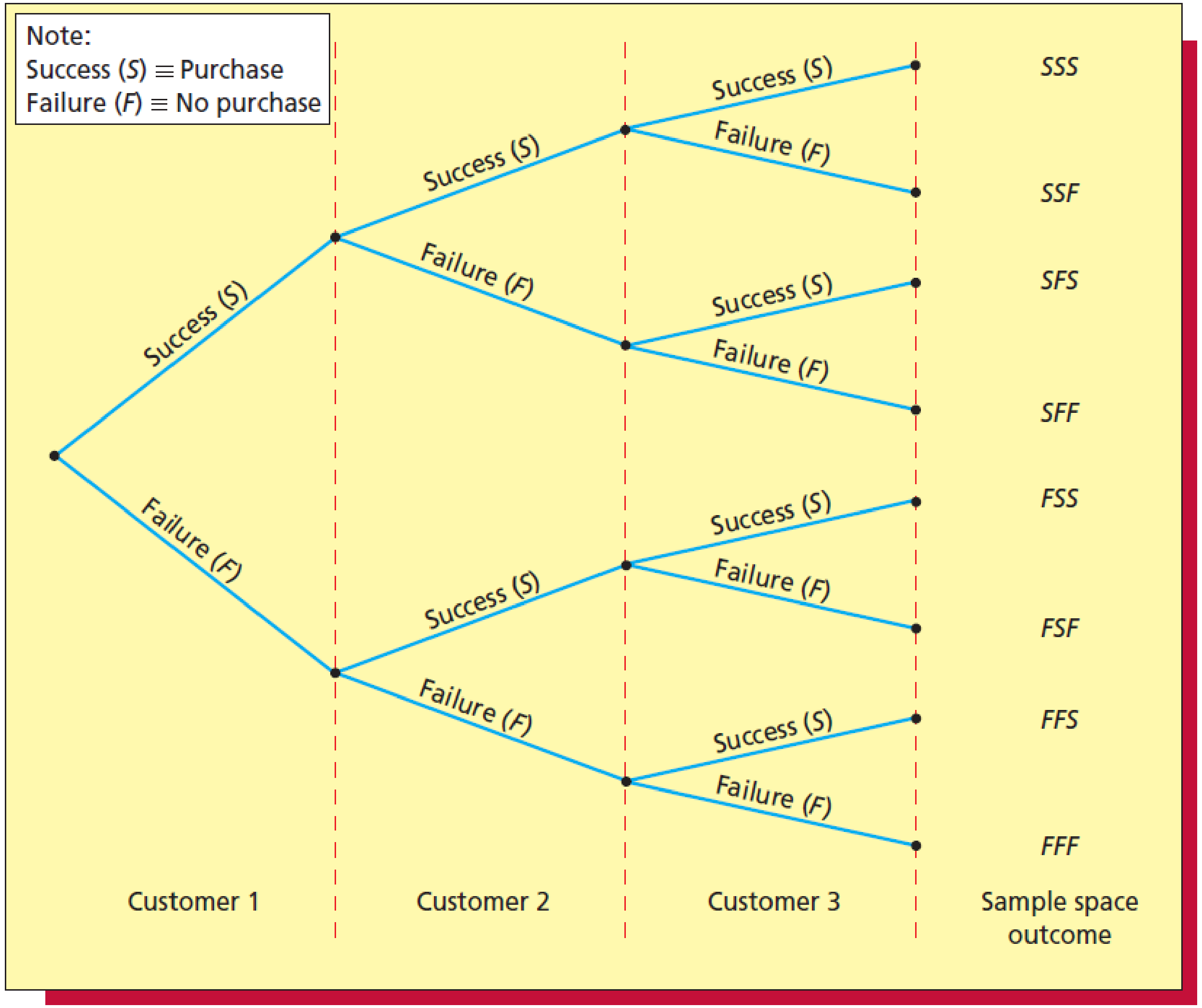
If *x* is the total number of successes in *n* trials of a binomial experiment, then *x* is a **binomial random variable**

Example: Three Customers Making a Purchase Decision

- Example: Suppose that historical sales records indicate that 40 percent of all customers who enter a discount department store make a purchase. What is the probability that two of the next three customers will make a purchase?

- S: the customer makes a purchase
- F: the customer does not make a purchase
- $p(S)=0.4$

Example: Three Customers Making a Purchase Decision



The Binomial Distribution

The sample space of the experiment:

SSS SSF SFS FSS FFS FSF SFF FFF

What is the probability that two of the next three customers will make a purchase?

$P(SSF)$=P(S)P(S)P(F)=(0.4)(0.4)(0.6)=$(0.4)^2(0.6)$
$P(SFS)$=P(S)P(F)P(S)=(0.4)(0.6)(0.4)=$(0.4)^2(0.6)$
$P(FSS)$=P(F)P(S)P(S)=(0.6)(0.4)(0.4)=$(0.4)^2(0.6)$

$$P(SSF)+P(SFS)+P(FSS)$$
$$=3(0.4)^2(0.6)=0.288$$

Example: The Binomial Distribution

The Binomial Distribution

A **binomial experiment** has the following characteristics:

1. The experiment consists of n *identical trials.*
2. Each trial results in a **success** or a **failure.**
3. The probability of a success on any trial is p and remains constant from trial to trial. This implies that the probability of failure, q, on any trial is $1 - p$ and remains constant from trial to trial.
4. The trials are **independent** (that is, the results of the trials have nothing to do with each other).

Furthermore, if we define the random variable

$$x = \text{the total number of successes in } n \text{ trials of a binomial experiment}$$

then we call x a **binomial random variable,** and the probability of obtaining x successes in n trials is

$$p(x) = \frac{n!}{x!\,(n-x)!} p^x q^{n-x}$$

The Binomial Distribution

For a binomial random variable *x*, the probability of *x* successes in *n* trials is given by the binomial distribution:

$$p(x) = \frac{n!}{x!\,(n-x)!} p^x q^{n-x}$$

- Note: $n!$ is read as "n factorial" and $n! = n \times (n-1) \times (n-2) \times \ldots \times 1$
  - For example, $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$
- Also, $0! = 1$
- Factorials are not defined for negative numbers or fractions

The Binomial Distribution

- What does the equation mean?
  - The equation for the binomial distribution consists of the product of two factors

$$p(x) = \boxed{\frac{n!}{x!\,(n-x)!}} \times \boxed{p^x q^{n-x}}$$

Number of ways to get *x* successes and (*n*–*x*) failures in *n* trials

The chance of getting *x* successes and (*n*–*x*) failures in a particular arrangement

Example Purchase at a Discount Store

- P(Purchase) = 0.4
- Probability that exactly 3 of next 5 customers make purchase

$$
\begin{aligned}
p(3) &= \frac{5!}{3!\,(5-3)!}(.4)^3(.6)^{5-3} = \frac{5!}{3!\,2!}(.4)^3(.6)^2 \\
&= \frac{5\cdot 4\cdot 3\cdot 2\cdot 1}{(3\cdot 2\cdot 1)(2\cdot 1)}(.4)^3(.6)^2 \\
&= 10(.064)(.36) \\
&= .2304
\end{aligned}
$$

Example: The Phe-Mycin Case: Drug Side Effects

- Antibiotics occasionally cause nausea as a side effect. A major drug company has developed a new antibiotic called Phe-Mycin. The company claims that, at most, 10 percent of all patients treated with Phe-Mycin would experience nausea as a side effect of taking the drug. We randomly select n=4 patients and treat them with Phe-Mycin.
- $p(\textit{side effects})=0.1$

Example: The Phe-Mycin Case: Drug Side Effects

TABLE 5.3 The Binomial Probability Distribution of x, the Number of Four Randomly Selected Patients Who Will Experience Nausea as a Side Effect of Being Treated with Phe-Mycin

x (Number Who Experience Nausea)	$p(x) = \frac{n!}{x!\,(n-x)!}p^x(1-p)^{n-x}$
0	$p(0) = P(x=0) = \frac{4!}{0!\,(4-0)!}(.1)^0(.9)^{4-0} = .6561$
1	$p(1) = P(x=1) = \frac{4!}{1!\,(4-1)!}(.1)^1(.9)^{4-1} = .2916$
2	$p(2) = P(x=2) = \frac{4!}{2!\,(4-2)!}(.1)^2(.9)^{4-2} = .0486$
3	$p(3) = P(x=3) = \frac{4!}{3!\,(4-3)!}(.1)^3(.9)^{4-3} = .0036$
4	$p(4) = P(x=4) = \frac{4!}{4!\,(4-4)!}(.1)^4(.9)^{4-4} = .0001$

Example: The Phe-Mycin Case: Drug Side Effects

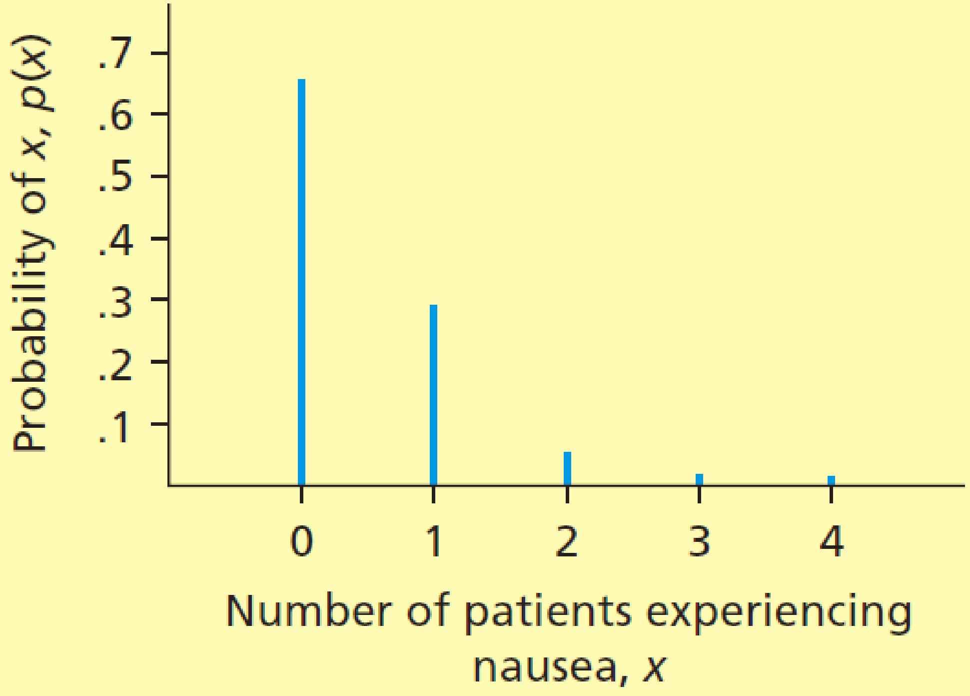
FIGURE 5.5 The Binomial Probability Distribution with $p = .10$ and $n = 4$

(a) Excel output of the binomial distribution

Binomial distribution with n = 4
and p = 0.10

x	P(X = x)
0	0.6561
1	0.2916
2	0.0486
3	0.0036
4	0.0001

(b) A graph of the distribution



Binomial Probability Table

(a) A Table for $n = 4$ Trials

Values of p (.05 to .50)

Number of Successes ↓	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	Number of Successes ↑
0	.8145	.6561	.5220	.4096	.3164	.2401	.1785	.1296	.0915	.0625	4
1	.1715	.2916	.3685	.4096	.4219	.4116	.3845	.3456	.2995	.2500	3
2	.0135	.0486	.0975	.1536	.2109	.2646	.3105	.3456	.3675	.3750	2
3	.0005	.0036	.0115	.0256	.0469	.0756	.1115	.1536	.2005	.2500	1
4	.0000	.0001	.0005	.0016	.0039	.0081	.0150	.0256	.0410	.0625	0
	.95	.90	.85	.80	.75	.70	.65	.60	.55	.50	

Values of p (.50 to .95)

What can we learn from this?

Do not gamble!!!

You will always loss everything at least once in many of the trials.

Rare event approach to making a statistical inference

Company claims that, at most, 10 percentage of all patients treated with Phe-Mycin would experience nausea. ***Is this claim right?***

- Suppose at least three of four sampled patients ***actually*** did experience nausea following treatment, the fraction of patients in the sample that experience nausea is 3/4 = 0.75, much larger than 0.1
  - If p = 0.1 is believed, then $P(x \geq 3) = P(x = 3) + P(x = 4) = 0.0037$, 37 in 10,000 of observing at least 3 out of 4 patients experiencing nausea
  - So this is very unlikely!
  - We have very strong evidence that p does not equal 0.1, but actually greater than 0.1

Rare event approach to making a statistical inference

- Suppose if only one of the four randomly selected patients had experienced nausea, the fraction of patients in the sample that experience nausea is 1/4 = 0.25, larger than 0.1
  - If p = 0.1 is believed, then $P(x \geq 1) = P(x = 1, 2, 3, or\ 4) = 0.2961 + 0.0486 + 0.0036 + 0.001 = 0.3439$, which means 34.39% chance of observing at least 1 of 4 patients experiencing nausea
  - So this could happen
  - We would not have much evidence against the claim that p equals .10

Rare event approach to making a statistical inference

- The idea of this approach is that if the probability of an observed sample result under a given assumption is small, then we have strong evidence that the assumption is false.
- Although there are no strict rules, many statisticians judge the probability of an observed sample result to be small if it is less than 0.05. The logic behind this will be explained more fully in Chapter 9.

EXAMPLE: The ColorSmart-5000 Case: TV Repairs

- The manufacturer of the ColorSmart-5000 television set claims that 95 percent of its sets last at least five years without requiring a single repair. Suppose that we will contact n=8 randomly selected ColorSmart-5000 purchasers five years after they purchased their sets.
- $p(5) = \frac{8!}{5!(8-5)!}(0.95)^5(0.05)^{8-5} = 0.0054 = 0.54\%$

EXAMPLE: The ColorSmart-5000 Case: TV Repairs

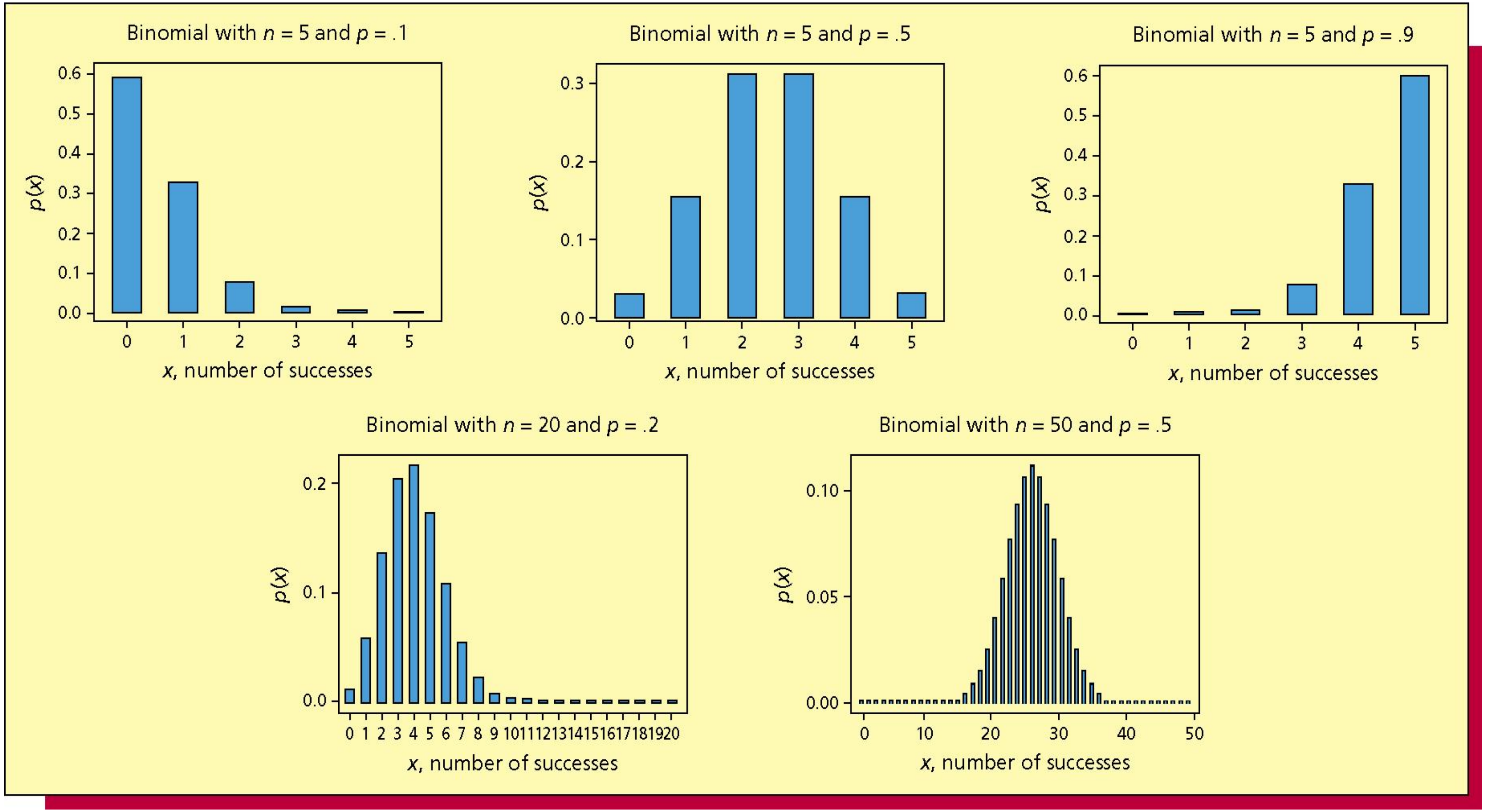
- Suppose that when we actually contact eight randomly selected purchasers, we find that five out of the eight television sets owned by these purchasers have lasted at least five years without a single repair. Because the sample fraction, 5/8=0.625, lower than 0.95 (contradicting the manufacturer's claim)
- $p(x \leq 5) = p(x = 0) + p(x = 1) \ldots + p(x = 5) = 0.0058 = 0.58\%$
- if p equals .95, then in only 0.58% of all possible samples of eight randomly selected televisions would five or fewer of the eight televisions last five years without a single repair. Because it is difficult to believe such a small chance has occurred. we have strong evidence that p does not equal .95.

*Binomial with n = 8 and p = 0.95

x	P (X = x)
3	0.0000
4	0.0004
5	0.0054
6	0.0515
7	0.2793
8	0.6634

*Probabilities for $x = 0,1$, and 2 are not listed because each has a probability that is approximately zero.

Several Binomial Distributions



- The values of n and p are often called the parameters of the binomial distribution.
- Depending on the parameters, a binomial distribution can be skewed to the right, skewed to the left, or symmetrical.

Mean, Variance, and Standard deviation of a Binomial Random Variable

If x is a binomial random variable with **parameters** n and p (so $q = 1 - p$), then

$$\text{mean } \mu_X = np$$

$$\text{variance } \sigma_X^2 = npq$$

$$\text{standard deviation } \sigma_X = \sqrt{npq}$$

where n is the number of trials, p is the probability of success on each trial, and q=1-p is the probability of failure on each trial.

Example: ColorSmart-5000 televisions

- p = 0.95, q = 0.05, n = 8

$$\mu_x = np = 8(.95) = 7.6$$
$$\sigma_x^2 = npq = 8(.95)(.05) = .38$$
$$\sigma_X = \sqrt{\sigma_x^2} = \sqrt{.38} = .6164$$

Suppose that we were to randomly select all possible samples of eight ColorSmart-5000 televisions and record the number of sets in each sample that last five years without a repair. If we averaged all of our results, we would find that the average number of sets per sample that last five years without a repair is equal to 7.6.

Binomial Distribution EXAMPLE:

- Pat Statsdud is registered in a statistics course and intends to rely on luck to pass the next quiz.
- The quiz consists on 10 multiple choice questions with 5 possible choices for each question, only one of which is the correct answer.
- Pat will guess the answer to each question
- Find the following probabilities
  - Pat gets no answer correct
  - Pat gets two answers correct
  - Pat fails the quiz (≤ 5 correct answers)
  - If all students in Pat's class intend to guess the answers to the quiz, what is the mean and the standard deviation of the quiz mark?

- Solution
  - Checking the conditions
    - An answer can be either correct or incorrect.
    - There is a fixed finite number of trials (n=10)
    - Each answer is independent of the others.
    - The probability p of a correct answer (.20) does not change from question to question.

Determining the binomial probabilities:

Let X = the number of correct answers

$$P(X=0)=\frac{10!}{0!\,(10-0)!}(.20)^{0}(.80)^{10-0}=.1074$$

$$P(X=2)=\frac{10!}{2!\,(10-2)!}(.20)^{2}(.80)^{10-2}=.3020$$

Determining the binomial probabilities:
Pat fails the test if the number of correct answers is less than 5, which means less than or equal to 4.

$$P(X \leq 4) = p(0) + p(1) + p(2) + p(3) + p(4)$$
$$= .1074 + .2684 + .3020 + .2013 + .0881$$
$$=.9672$$

The mean and the standard deviation of the quiz mark?

$\boldsymbol{\mu}$= np = 10(.2) = 2.

σ= $[np(1-p)]^{1/2}$ = $[10(.2)(.8)]^{1/2}$ = 1.26

Chapter Summary

- In this chapter, we learned that a random variable represents an uncertain numerical outcome.
- discrete random variable: values can be listed, while the values of a continuous random variable correspond to one or more intervals on the real number line.
- We saw that a probability distribution of a discrete random variable is a table, graph, or formula that gives the probability associated with each of the random variable’s possible values.
- Mean (or expected value), variance, and its standard deviation.
- One important commonly used discrete probability distribution—the binomial distribution—and we demonstrated how the distribution can be used to make statistical inferences.

Thank you!

5.4 The Poisson Distribution (Optional)

- Consider the number of times an event occurs over an interval of time and assume
  1. The probability of occurrence is the same for any intervals of equal length
  2. The occurrence in any interval is independent of an occurrence in any non-overlapping interval
- If x = the number of occurrences in an interval, then x is a Poisson random variable

$$p(x) = \frac{e^{-\mu}\mu^{x}}{x!}$$

Poisson Probability Table

x, Number of Occurrences	μ, Mean Number of Occurrences .1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
0	.9048	.8187	.7408	.6703	.6065	.5488	.4966	.4493	.4066	.3679
1	.0905	.1637	.2222	.2681	.3033	.3293	.3476	.3595	.3659	.3679
2	.0045	.0164	.0333	.0536	.0758	.0988	.1217	.1438	.1647	.1839
3	.0002	.0011	.0033	.0072	.0126	.0198	.0284	.0383	.0494	.0613
4	.0000	.0001	.0003	.0007	.0016	.0030	.0050	.0077	.0111	.0153
5	.0000	.0000	.0000	.0001	.0002	.0004	.0007	.0012	.0020	.0031
6	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0003	.0005

Poisson Probability Calculations

$$p(x) = \frac{e^{-\mu}\mu^{x}}{x!}$$

$$p(0) = \frac{e^{-.4}(.4)^{0}}{0!} = .6703$$

$$p(1) = \frac{e^{-.4}(.4)^{1}}{1!} = .2681$$

$$p(2) = \frac{e^{-.4}(.4)^{2}}{2!} = .0536$$

$$p(3) = \frac{e^{-.4}(.4)^{3}}{3!} = .0072$$

$$p(4) = \frac{e^{-.4}(.4)^{4}}{4!} = .0007$$

$$p(5) = \frac{e^{-.4}(.4)^{5}}{5!} = .0001$$

Mean and Variance of a Poisson Random Variable

- Mean $\mu_x = \mu$
- Variance $\sigma^2_x = \mu$
- Standard deviation σ_x is square root of variance σ^2_x

Several Poisson Distributions

Figure 5.9