



Discussion

Broadband composite polarization rotator

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ABSTRACT

We describe a broadband optical device that is capable of rotating the polarization plane of a linearly polarized light at any desired angle over a wide range of wavelengths. The device is composed of a sequence of half-wave plates rotated at specific angles with respect to their fast-polarization axes. This design draws on an analogy with composite pulses, which is a well-known control technique from quantum physics. We derive the solutions for the rotation angles of the single half-wave plates depending on the desired polarization rotation angle. We show that the broadband polarization rotator is robust against variations of the parameters of both the crystal and the light field.

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1. Introduction

One of the fundamental properties of light is its polarization [1–5], and the ability to observe and manipulate the polarization state is highly desirable for practical applications. For example, there are many optical measurement techniques based on polarization which are used in stress measurements, ellipsometry, physics, chemistry, biology, astronomy and others [6–8]. Furthermore, the controlled rotation of the light polarization is the underlying principle on which display and telecommunications technologies are based [9].

Key optical elements for polarization state manipulation are optical polarization rotators, which rotate the light polarization plane at a desired angle [1–5]. The commercially available polarization rotators, which typically exploit the effect of circular birefringence, possess two main advantages. First, the angle of light polarization rotation is independent of rotation of the rotator around its own optical axis; and second, they are fairly inexpensive. However, they are useful only for a limited range of wavelengths. Alternatively, polarization plane rotation can be achieved through consecutive reflections, which is the underlying principle of Fresnel rhombs [1–5]. They possess the advantages of operating over a wide range of wavelengths but are quite expensive.

A scheme to rotate the light polarization plane for several fixed wavelengths was suggested by Koester [10]. The design presented in Ref. [10] uses up to four half-wave plates in a series to ensure

good rotation of the polarization plane for four different wavelengths. Another possible method to enlarge the spectral width of a light polarization plane rotation is to use two wave plates of the same material as shown by Kim and Chang in Ref. [11].

In this paper, we further extend the approach of Kim and Chang [11] in combination with the Koester idea [10] and design an arbitrary broadband polarization rotator which outperforms existing rotators for broadband operation. Our scheme consists of two crossed half-wave plates, where the angle between their fast axes is half the angle of polarization rotation. We design the two half-wave plates to be broadband using composite pulses approach [12–15]. That is, each composite half-wave plate consists of a number of individual wave plates arranged at specific angles with respect to their fast axes. This ensures that the proposed polarization rotator is broadband and stable with respect to wavelength variations. We provide the recipe for constructing a polarization rotator device such that, in principle, an arbitrary broadband polarization profile can be achieved.

2. Background

Any reversible polarization transformation can be represented as a composition of a retarder and a rotator [16]. A rotation at an angle θ is represented by the Jones matrix in the horizontal-vertical (HV) basis as

$$\mathcal{R}(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}. \quad (1)$$

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A retarder is expressed in the HV basis by the Jones matrix:

$$\mathfrak{J}(\varphi) = \begin{bmatrix} e^{i\varphi/2} & 0 \\ 0 & e^{-i\varphi/2} \end{bmatrix}. \quad (2)$$

where the phase shift is $\varphi = 2\pi L(n_f - n_s)/\lambda$, with λ being the vacuum wavelength, n_f and n_s the refractive indices along the fast and slow axes, correspondingly, and L the thickness of the retarder. The most widely used retarders are the half-wave plate ($\varphi = \pi$) and the quarter-wave plate ($\varphi = \pi/2$). The performance of such retarders is usually limited to a narrow range of wavelengths around λ due to their strong sensitivity to variations in the thickness and the rotary power of the plate.

Let us now consider a single polarizing birefringent plate of phase shift φ and let us present a system of HV polarization axes (HV basis), which are rotated at an angle θ with reference to the slow and the fast axes of the plate. The Jones matrix \mathfrak{J}_θ then has the form

$$\mathfrak{J}_\theta(\varphi) = \mathfrak{R}(-\theta)\mathfrak{J}(\varphi)\mathfrak{R}(\theta). \quad (3)$$

In the left-right circular polarization (LR) basis this matrix attains the form $\mathbf{J}_\theta(\varphi) = \mathbf{W}^{-1}\mathfrak{J}_\theta(\varphi)\mathbf{W}$, where \mathbf{W} connects the HV and LR polarization bases:

$$\mathbf{W} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -i & i \end{bmatrix}. \quad (4)$$

Explicitly, the Jones matrix for a retarder with a phase shift φ and rotated at an angle θ is given as (in the LR basis)

$$\mathbf{J}_\theta(\varphi) = \begin{bmatrix} \cos(\varphi/2) & i\sin(\varphi/2)e^{2i\theta} \\ i\sin(\varphi/2)e^{-2i\theta} & \cos(\varphi/2) \end{bmatrix}. \quad (5)$$

For example, half- and quarter-wave plates rotated at an angle θ , $(\lambda/2)_\theta$ and $(\lambda/4)_\theta$, are described by $\mathbf{J}_\theta(\pi)$ and $\mathbf{J}_\theta(\pi/2)$, respectively.

3. Composite broadband half-wave plate

Our first step is to design a half-wave plate that is robust to variations in the phase shift φ at $\varphi = m\pi$ ($m = 1, 2, 3, \dots$). Such half-wave plates allow for imperfect rotary power φ/L and deviations in the plate thickness L , and furthermore, operate over a wide range of wavelengths λ . To achieve this, we will follow an analogous approach to that of composite pulses [12–15], which is widely adopted for robust control in quantum physics [17–19]. In detail, we replace the single half-wave plate with an arrangement of an odd number $N = 2n + 1$ half-wave plates (shown schematically in Fig. 1). Each wave plate has a phase shift $\varphi = \pi$ and is rotated by an angle θ_k with the “anagram” condition $\theta_k = \theta_{N+1-k}$, ($k = 1, 2, \dots, n$). The composite Jones matrix of the above-described arrangement of wave plates in the LR basis is given by

$$\mathbf{J}^{(N)} = \mathbf{J}_{\theta_N}(\varphi)\mathbf{J}_{\theta_{N-1}}(\varphi)\cdots\mathbf{J}_{\theta_1}(\varphi). \quad (6)$$

Our objective is to implement an ideal half-wave plate propagator with Jones matrix \mathbf{J}_0 in the LR basis (up to a global phase factor):

$$\mathbf{J}_0 = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}, \quad (7)$$

by the product of half-wave plates $\mathbf{J}^{(N)}$ from Eq. (6). That is, we set $\mathbf{J}^{(N)} \equiv \mathbf{J}_0$ at $\varphi = \pi$ which leaves us with n independent angles θ_k to use as control parameters. We then nullify as many lowest order derivatives of $\mathbf{J}^{(N)}$ vs the phase shift φ at $\varphi = \pi$ as possible. We thus obtain a system of nonlinear algebraic equations for the rotation

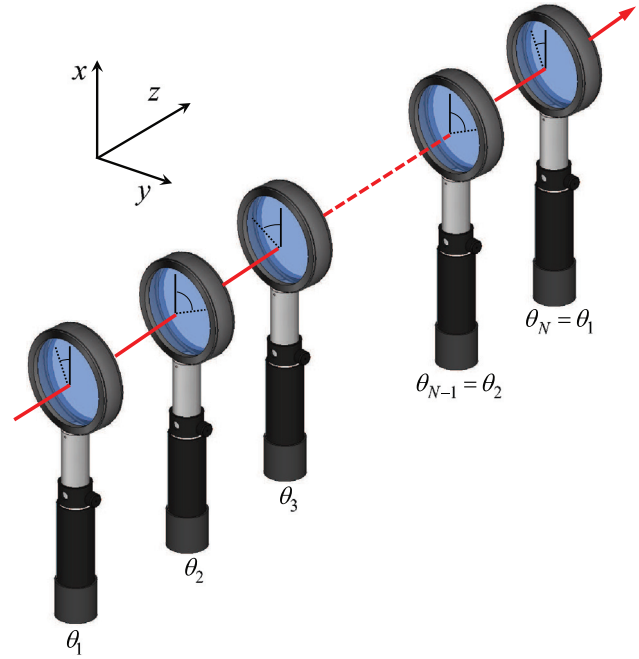


Fig. 1. Schematic structure of the composite broadband half-wave plate, which consists of a stack of N ordinary half-wave plates rotated at specific angles θ_k . The fast polarization axes of the wave plates are represented by dashed lines, while the solid lines represent the x direction of the coordinate system.

Table 1

Rotation angles θ_k (in degrees) for composite broadband half-wave plates with a different number N of constituent half-wave plates.

N	Rotation angles ($\theta_1; \theta_2; \dots; \theta_{N-1}; \theta_N$)
3	(60; 120; 60)
5	(51.0; 79.7; 147.3; 79.7; 51.0)
7	(68.0; 16.6; 98.4; 119.8; 98.4; 16.6; 68.0)
9	(99.4; 25.1; 64.7; 141.0; 93.8; 141.0; 64.7; 25.1; 99.4)
11	(31.2; 144.9; 107.8; 4.4; 44.7; 158.6; 44.7; 4.4; 107.8; 144.9; 31.2)
13	(59.6; 4.9; 82.5; 82.7; 42.7; 125.8; 147.2; 125.8; 42.7; 82.7; 82.5; 4.9; 59.6)

angles θ_k :

$$\left[\partial_\varphi^k \mathbf{J}_{11}^{(N)} \right]_{\varphi=\pi} = 0 \quad (k = 1, 2, \dots, n), \quad (8a)$$

$$\left[\partial_\varphi^k \mathbf{J}_{12}^{(N)} \right]_{\varphi=\pi} = 0 \quad (k = 1, 2, \dots, n). \quad (8b)$$

The anagram symmetry assumption for the angles, θ_k ($\theta_k = \theta_{N+1-k}$), ensures that all even-order derivatives of $\mathbf{J}_{11}^{(N)}$ (8a) and all odd-order derivatives of $\mathbf{J}_{12}^{(N)}$ (8b) vanish; hence, the n angles allow us to nullify the first n derivatives of the matrix $\mathbf{J}^{(N)}$ (6).

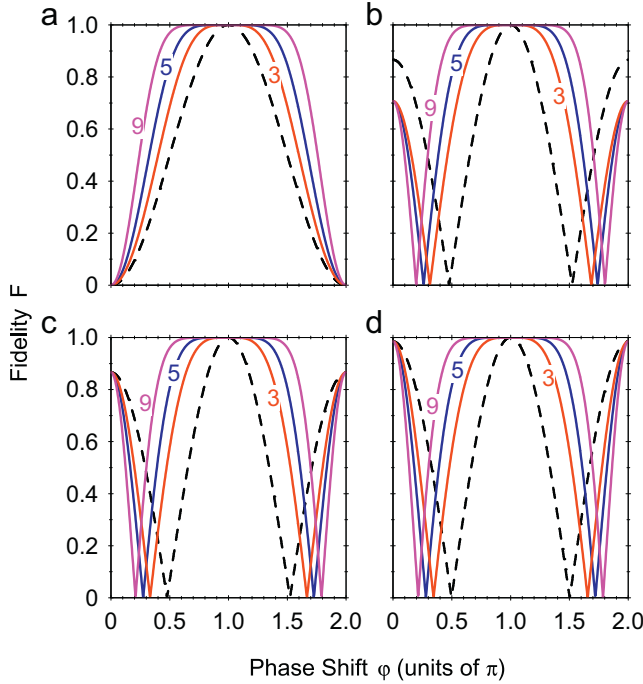
Solutions to Eqs. (8) provide the recipe to construct *arbitrary broadband* composite half-wave plates. A larger number N of ordinary half-wave plates provide a higher order of robustness against variations in the phase shift φ and thus, the light wavelength λ . We list several examples of broadband half-wave plates in Table 1.

4. Broadband rotator

We proceed to show how a broadband rotator can be constructed as a sequence of two broadband half-wave plates with an

Table 2Rotation angles θ_k (in degrees) for each half-wave plate in the composite broadband polarization rotator for arbitrary rotation angle α .

$N + N$	Rotation angles of composite half-wave plates ($\theta_1 - \alpha/4$; $\theta_2 - \alpha/4$; ...; $\theta_N - \alpha/4$)($\theta_1 + \alpha/4$; $\theta_2 + \alpha/4$; ...; $\theta_N + \alpha/4$)
3+3	($60 - \alpha/4$; $120 - \alpha/4$; $60 - \alpha/4$)($60 + \alpha/4$; $120 + \alpha/4$; $60 + \alpha/4$)
5+5	($51.0 - \alpha/4$; $79.7 - \alpha/4$; $147.3 - \alpha/4$; $79.7 - \alpha/4$; $51.0 - \alpha/4$)($51.0 + \alpha/4$; $79.7 + \alpha/4$; $147.3 + \alpha/4$; $79.7 + \alpha/4$; $51.0 + \alpha/4$)
7+7	($68.0 - \alpha/4$; $16.6 - \alpha/4$; $98.4 - \alpha/4$; $119.8 - \alpha/4$; $98.4 - \alpha/4$; $16.6 - \alpha/4$; $68.0 - \alpha/4$) ($68.0 + \alpha/4$; $16.6 + \alpha/4$; $98.4 + \alpha/4$; $119.8 + \alpha/4$; $98.4 + \alpha/4$; $16.6 + \alpha/4$; $68.0 + \alpha/4$)
9+9	($99.4 - \alpha/4$; $25.1 - \alpha/4$; $64.7 - \alpha/4$; $141.0 - \alpha/4$; $93.8 - \alpha/4$; $141.0 - \alpha/4$; $64.7 - \alpha/4$; $25.1 - \alpha/4$; $99.4 - \alpha/4$) ($99.4 + \alpha/4$; $25.1 + \alpha/4$; $64.7 + \alpha/4$; $141.0 + \alpha/4$; $93.8 + \alpha/4$; $141.0 + \alpha/4$; $64.7 + \alpha/4$; $25.1 + \alpha/4$; $99.4 + \alpha/4$)

**Fig. 2.** Fidelity F vs phase shift φ for broadband polarization rotator, for a different number of constituent plates N in the composite half-wave plates. The rotation angles of the two composite half-wave plates are those given in Table 2. Frame (a) for polarization rotation of 90° , frame (b) for polarization rotation of 45° , frame (c) for polarization rotation of 30° , and frame (d) for polarization rotation of 10° . The black dashed line is for a rotator composed of just two half-wave plates, given for easy reference.

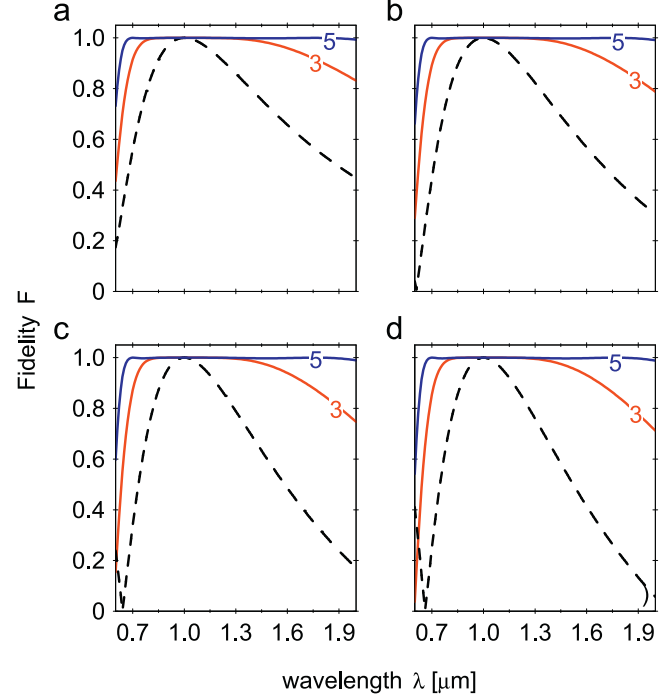
additional rotation between them. Let us first consider a simple sequence of two ordinary half-wave plates with a relative rotation angle $\alpha/2$ between them. We multiply the Jones matrices of the two half-wave plates ($\varphi = \pi$) given in Eq. (5), where one is rotated at $\alpha/4$ while the other is rotated at $-\alpha/4$. Thus, we obtain the total propagator

$$\mathbf{J}_{\alpha/4}(\pi)\mathbf{J}_{-\alpha/4}(\pi) = -\begin{bmatrix} e^{i\alpha} & 0 \\ 0 & e^{-i\alpha} \end{bmatrix}, \quad (9)$$

which represents a Jones matrix for a rotator in the LR basis (up to unimportant π phase),

$$\mathbf{R}(\alpha) = \begin{bmatrix} e^{i\alpha} & 0 \\ 0 & e^{-i\alpha} \end{bmatrix}. \quad (10)$$

However, a rotator constructed from two ordinary half-wave plates according to Eq. (9) is not broadband. We overcome this limitation and extend the range of operation of the rotator by replacing the two ordinary half-wave plates with two identical broadband composite half-wave plates as described in Section 3. In order to calculate the efficiency of our composite polarization

**Fig. 3.** Fidelity F vs wavelength λ for broadband polarization rotator, for a different number of constituent plates N in the composite half-wave plates. The rotation angles of the two composite half-wave plates are those given in Table 2. Frame (a) for polarization rotation of 90° , frame (b) for polarization rotation of 75° , frame (c) for polarization rotation of 60° , and frame (d) for polarization rotation of 45° . The black dashed line is for a rotator composed of just two half-wave plates, given for easy reference.

rotator we define the fidelity F [12,20] according to

$$F = \frac{1}{2} \text{Tr}(\mathbf{R}^{-1}(\alpha)\mathbf{J}_{\text{tot}}), \quad (11)$$

where $\mathbf{J}_{\text{tot}} = \mathbf{J}_{\alpha/4}^{(N)}\mathbf{J}_{-\alpha/4}^{(N)}$ with $\mathbf{J}_{\alpha/4}^{(N)}$ and $\mathbf{J}_{-\alpha/4}^{(N)}$ representing the N -composite broadband half-wave plates from Eq. (6) rotated additionally to $\alpha/4$ and $-\alpha/4$, respectively.

Several examples of broadband polarization rotators, using the composite broadband half-wave plates from Table 1, are given in Table 2. Their fidelity with respect to variations in the phase shift φ is illustrated in Fig. 2.

Furthermore, in Fig. 3 we illustrate the fidelity of the suggested broadband polarization rotator, using true zero order half-wave plates, which operate around $1 \mu\text{m}$, made from quartz (thickness L of each wave plate is $57 \mu\text{m}$). The retardance of the wave plates is calculated using the Sellmeier equations for ordinary and extraordinary refractive indexes [21].

5. Conclusion

We have presented an approach to construct an arbitrary broadband composite polarization rotator which can rotate the

polarization of a linearly polarized light at any desired angle α . The polarization rotator is composed of two composite broadband half-wave plates with a relative rotation of $\alpha/2$ between them. Furthermore, using a sequence of zero order quartz half-wave plates, we numerically showed that the polarization rotator is broadband and operates over a wide range of wavelengths. An experimental implementation of the suggested broadband polarization rotator with half-wave plates which are readily available in most laboratories should be straightforward. Finally, we note that this technique has an analogue in atomic physics in terms of composite phase gates as recently was demonstrated by Torosov and Vitanov [22].

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References

- [1] E. Hecht, *Optics*, Addison Wesley, San Francisco, 2002.
- [2] M. Born, E. Wolf, *Principles of Optics*, Cambridge University Press, Cambridge, 2005.
- [3] M.A. Azzam, N.M. Bashara, *Ellipsometry and Polarized Light*, North Holland, Amsterdam, 1977.
- [4] D. Goldstein, E. Collett, *Polarized Light*, Marcel Dekker, New York, 2003.
- [5] F.J. Duarte, *Tunable Laser Optics*, Elsevier Academic, New York, 2003.
- [6] J.D. Pye, *Polarised Light in Science and Nature*, Institute of Physics, Bristol, 2001.
- [7] J.N. Damask, *Polarization Optics in Telecommunications*, Springer, New York, 2005.
- [8] E.L. Degl'innocenti, M. Landolfi, *Polarization in Spectral Lines*, Kluwer Academic Publishers, Dordrecht, 2004.
- [9] E. Matioli, S. Brinkley, K.M. Kelchner, Y.L. Hu, S. Nakamura, S.P. DenBaars, J. S. Speck, C. Weisbuch, *Light: Sci. Appl.* 1 (2012) e22.
- [10] C.J. Koester, *J. Opt. Soc. Am.* 49 (1959) 405.
- [11] J.S. Kim, J.K. Chang, *J. Korean Phys. Soc.* 48 (2006) 51.
- [12] S.S. Ivanov, A.A. Rangelov, N.V. Vitanov, T. Peters, T. Halfmann, *J. Opt. Soc. Am. A* 29 (2012) 265.
- [13] T. Peters, S.S. Ivanov, D. Englisch, A.A. Rangelov, N.V. Vitanov, T. Halfmann, *Appl. Opt.* 51 (2012) 7466.
- [14] A.A. Rangelov, *Opt. Comm.* 285 (2012) 4157.
- [15] E. Kyoseva, N.V. Vitanov, *Phys. Rev. A* 88 (2013) 063410.
- [16] H. Hurvitz, R.C. Jones, *J. Opt. Soc. Am.* 31 (1941) 493.
- [17] B.T. Torosov, N.V. Vitanov, *Phys. Rev. A* 83 (2011) 053420.
- [18] G.T. Genov, B.T. Torosov, N.V. Vitanov, *Phys. Rev. A* 84 (2011) 063413.
- [19] B.T. Torosov, S. Guerin, N.V. Vitanov, *Phys. Rev. Lett.* 106 (2011) 233001.
- [20] A. Ardavan, *New J. Phys.* 9 (2007) 24.
- [21] G. Ghosh, *Opt. Commun.* 163 (1999) 95.
- [22] B.T. Torosov, N.V. Vitanov, *Phys. Rev. A* 90 (2014) 012341.