Achromatic quarter-wave plate using crystalline quartz

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Achromatic wave plates are ideal components for use with tunable and multiline laser systems, broadband sources, and in astronomical instrumentation. The present study deals with the design and characteristics of two different quarter-wave achromatic retarders in the 500–700 nm range, using a cascaded system of two birefringent plates. The first of these shows a variation of less than $\pm 0.5°$, whereas the second system shows a variation of $\pm 4°$ where the azimuth remains constant. Finally, a comparison between the two systems is made. The succinct and simple Jones matrix formalism has been used to derive the general expression for the equivalent retardation and azimuth of the combinations. It appears that the proposed arrangement has the promise of producing good achromatic combinations.

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1. Introduction

Quarter-wave plates take a major role in many experimental arrangements with tunable and multiline lasers and broadband sources, where the strong wavelength dependence of these devices poses a genuine problem. Design of achromatic retarders, over at least the wavelength range of interest, therefore assumes significance. Different techniques have been used to attain such achromatism using both passive and active devices. In some of the cases, the same birefringent material has been used to fabricate the retarder combination, whereas in other cases different materials are employed. Pancharatnam proposed one of the most successful combinations of three plates, which were produced of the same material [1]. In this configuration, the bounding retardation plates have identical retardations and orientations of the optical axes while the central plate has a different retardation and orientation. Subsequently, achromatic quarter-wave retarders have been produced using a combination of two or three plates of different birefringent materials [2–4] to minimize retardation dispersion across a broad wavelength range. A procedure to optimize the choice of materials for a broadband retarder [5] has been proposed by Hariharan. Achromatic phase retardation based on a subwavelength dielectric diffraction grating has been studied numerically in a total-internal-reflection configuration in both the visible and near-infrared regions [6]. Broadband retarders based on total internal reflections [7] and using subwavelength grating structures have also been proposed [8,9]. A combination of two quarter-wave plates and one half-wave plate of the same material that can be used as an achromatic half-wave retarder was proposed by Hariharan and Malacara [10]. Hariharan showed that two plates of different birefringent materials, when suitably configured, exhibit achromatic behavior over a limited range of wavelengths [11]. Moreover, several studies have been carried out on thin-film retarders [12] and liquid crystal retarders [13]. Samoylov and Samoylov [14] proposed the method of construction of achromatic and superachromatic zero-order wave plates, in

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which a combination of five wave plates was utilized to cover the intended wavelength range [14]. An achromatic quarter-wave plate for the visible spectrum using six sapphire plates, based on the synthesis procedure of Harris, Ammann, and Chang, was studied by McIntyre and Harris [15]. Recently, an achromatic quarter-wave retarder was designed for the terahertz range; it consists of six wave plates of varying thicknesses [16]. A reconfigurable achromatic half-wave and quarter-wave retarder in the near-infrared was proposed using three crystalline quartz plates [17]. However, one of the early works in this domain was carried out by Destriau and Prouteaul [18]; the authors showed that an almost achromatic quarter-wave retarder could be obtained by juxtaposing two retarders one half-wave and the other quarter-wave plate. Using the Poincaré sphere method to design this system, the authors, however, did not consider the chromatic variation of the azimuth.

The present work proposes a simple setup consisting of two retarders with crystalline quartz as the birefringent material. Two different combinations have been studied: the first is a combination of a half-wave plate and a quarter-wave plate, whereas the second is a combination of two quarter-wave plates. The fast axis of the second retarder is oriented in a particular orientation in both cases. The mathematical analysis, carried out using Jones calculus, shows that, at normal incidence, the proposed device behaves as an achromatic quarter-wave plate over the 500–700 nm wavelength range, with a maximum deviation of less than $\pm 0.5°$ for the first system. The second system shows a maximum deviation of less than $\pm 4°$, with the advantage that the azimuth remains constant throughout the wavelength range considered.

2. Formulation of the Problem

As illustrated in Fig. 1, we consider δ_1 to be the retardation of the first plate; δ_2 is that of the second plate, and ϕ is the angle between the fast vibration directions of the second plate and the first plate, as shown in Fig. 1.

The Jones matrix of a retarder whose fast axis coincides with the x axis is given by

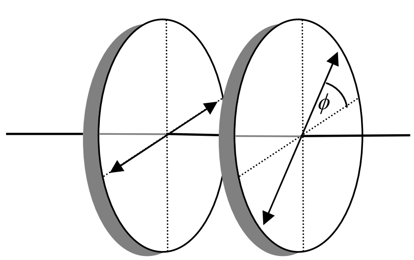

Fig. 1. Configuration of the two birefringent plates. The double arrow line indicates the direction of the fast axis.

$$C(\delta,0)=\begin{vmatrix} e^{i\delta/2} & 0 \\ 0 & e^{-i\delta/2}\end{vmatrix}, \tag{1}$$

where δ is the phase difference introduced by the retarder.

The characteristic Jones matrix of an oblique retarder whose fast axis makes an angle ϕ with the x axis is

$$C(\delta,\phi)=\begin{vmatrix} \cos\frac{\delta}{2}+i\sin\frac{\delta}{2}\cos 2\phi & i\sin\frac{\delta}{2}\sin 2\phi \\ i\sin\frac{\delta}{2}\sin 2\phi & \cos\frac{\delta}{2}-i\sin\frac{\delta}{2}\cos 2\phi \end{vmatrix}. \tag{2}$$

Therefore, the Jones matrix of the cascaded system of the configuration discussed above is

$$C(\Delta,\psi)=C(\delta_2,\phi)C(\delta_1,0), \tag{3}$$

where Δ is the equivalent retardation of the combination.

Using the appropriate Jones matrix, we may write

$$C(\Delta,\psi)=\begin{vmatrix} \cos\frac{\delta_2}{2}+i\sin\frac{\delta_2}{2}\cos 2\phi & i\sin\frac{\delta_2}{2}\sin 2\phi \\ i\sin\frac{\delta_2}{2}\sin 2\phi & \cos\frac{\delta_2}{2}-i\sin\frac{\delta_2}{2}\cos 2\phi \end{vmatrix} \times\begin{vmatrix} e^{i\delta_1/2} & 0 \\ 0 & e^{-i\delta_1/2}\end{vmatrix} \tag{4}$$

$$=\begin{vmatrix} A & B \\ -B^* & A^*\end{vmatrix}, \tag{5}$$

where

$$A=e^{i\delta_1/2}\left(\cos\frac{\delta_2}{2}+i\sin\frac{\delta_2}{2}\cos 2\phi\right),$$

$$B=ie^{-i\delta_1/2}\sin\frac{\delta_2}{2}\sin 2\phi.$$

The resulting retardation dephasing Δ [16] and azimuth Ψ is obtained by

$$\tan^2\frac{\Delta}{2}=\frac{|\mathrm{Im}A|^2+|\mathrm{Im}B|^2}{|\mathrm{Re}A|^2+|\mathrm{Re}B|^2}, \tag{6}$$

$$\tan 2\Psi=\frac{B-B^*}{A-A^*}. \tag{7}$$

3. Design Procedure

The aim of this study is to design an achromatic quarter-wave plate over the wavelength range

500 to 700 nm. To achieve this, two different combinations have been attempted, each consisting of two zero-order quartz retarders at 600 nm. In the first combination we consider the first retarder as a half-wave plate designed for 600 nm and, under this condition, we determine the required retardation of the second wave plate and calculate the orientation of the same. In the second system, both the wave plates are considered to be quarter-wave plates. In this case, by imposing the required conditions, the orientation of the second plate with respect to the first is calculated.

The retardation δ introduced by a birefringent plate between the two orthogonal components of light is given by

$$\delta = \frac{2\pi}{\lambda_0}(n_e - n_o)d, \tag{8}$$

where $(n_e - n_o)$ is the birefringence of the material of the retarders, d is the thickness of each plate, and λ_0 is the design wavelength. The values of birefringence for crystalline quartz for the wavelength range from 500 to 700 nm are calculated according to the formula given for n_o and n_e by Chandrasekhar [19]. The formulas for the extraordinary and ordinary indices are given as

$$\begin{aligned} n_e^2 - 1 = {} & \frac{0.665721\lambda^2}{\lambda^2 - (0.0600)^2} + \frac{0.503511\lambda^2}{\lambda^2 - (0.1060)^2} \\ & + \frac{0.214792\lambda^2}{\lambda^2 - (0.1190)^2} + \frac{0.539173\lambda^2}{\lambda^2 - (8.792)^2} \\ & + \frac{1.8076613\lambda^2}{\lambda^2 - (19.70)^2}, \end{aligned} \tag{9}$$

$$\begin{aligned} n_o^2 - 1 = {} & \frac{0.663044\lambda^2}{\lambda^2 - (0.0600)^2} + \frac{0.517852\lambda^2}{\lambda^2 - (0.1060)^2} \\ & + \frac{0.175912\lambda^2}{\lambda^2 - (0.1190)^2} + \frac{0.565380\lambda^2}{\lambda^2 - (8.844)^2} \\ & + \frac{1.675299\lambda^2}{\lambda^2 - (20.742)^2}. \end{aligned} \tag{10}$$

For a design wavelength of 600 nm, the thicknesses required for quarter- and half-wave plates are calculated to be 16.5 and 33 μm, respectively. The variation of the birefringence of crystalline quartz with wavelength is illustrated in Fig. 2. Crystalline quartz has more than 90% transmittance in the intended wavelength range.

In the first combination (Case I), the retardation value (i.e., δ_1) and the orientation of one of the birefringent plates is prespecified; the parameters that can be adjusted are the retardation value of the second plate (i.e., δ_2) and its orientation (i.e., ϕ). In the second system (Case II), where the orientation of the first plate and the retardation values of both the birefringent plates are prespecified, the only available degree of freedom is the orientation of one of the plates relative to the other.

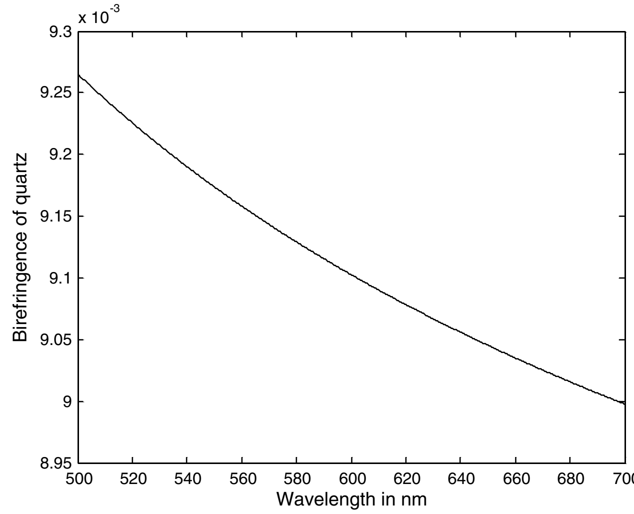

Fig. 2. Variation of birefringence of crystalline quartz with wavelength.

4. Computational Results

It will be shown that, although both the combinations proposed in this study can effectively result in a near-achromatic quarter-wave plate, each has its individual strength in terms of the effective birefringence and azimuthal dispersion exhibited. As stated previously, the stated magnitude of retardance for the individual retarders constituting the proposed combinations refer to the 600 nm wavelength.

Case I.

The first plate is considered to be a half-wave plate designed for 600 nm. Using Eq. (6) and imposing all the conditions specified, for the first combination, it is found that the retardation value of the second plate will be $\delta_2 = \pi/2$, which means the second plate has to be a quarter-wave plate designed for the same 600 nm. At the same time, the orientation of the second plate with respect to the first plate is calculated to be 58.7°. The overall variation of retardation and azimuth of the combined system with respect to wavelength is calculated from Eqs. (6) and (7), respectively. The overall retardation and azimuth of the first combination of a half-wave plate and a quarter-wave plates are shown in Figs. 3 and 4, respectively. The percentage variation of retardation is shown in Fig. 5.

Case II.

For the second combination, where both the wave plates are prespecified as quarter-wave plates at

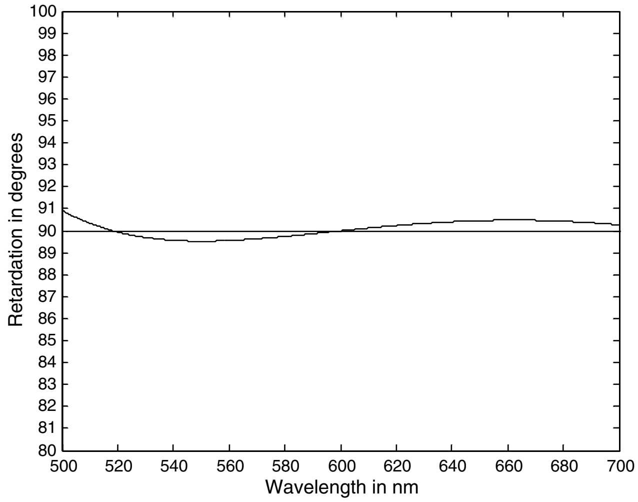

Fig. 3. Wavelength dependent retardation Δ for the combination of $\lambda/2$ and $\lambda/4$ plates.

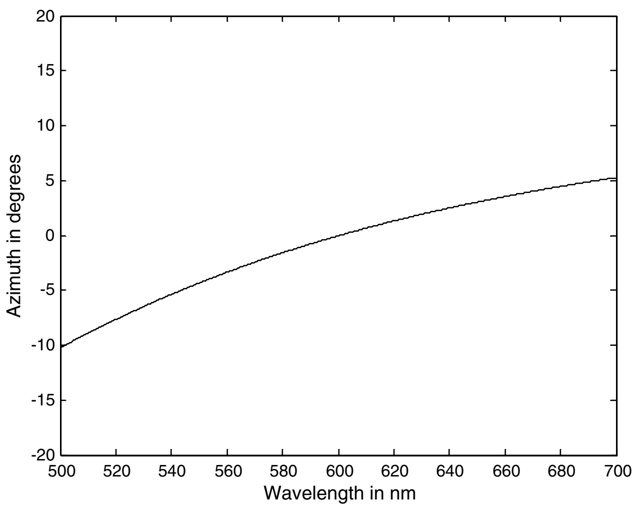

Fig. 4. Wavelength dependent azimuth Ψ for the combination of $\lambda/2$ and $\lambda/4$ plates.

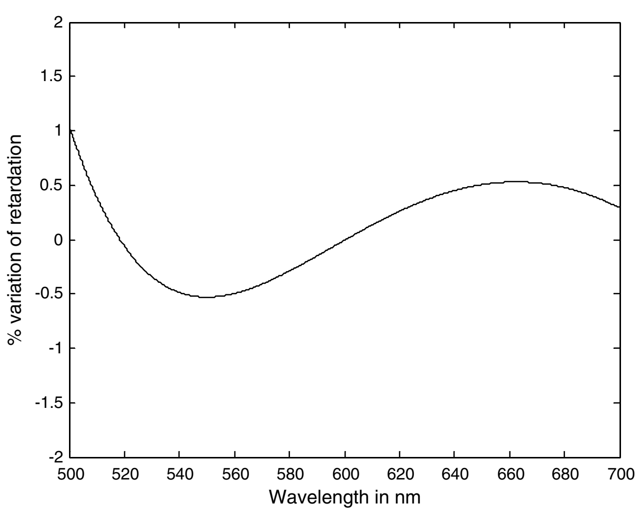

Fig. 5. Percentage variation of retardation Δ with wavelength for the combination of $\lambda/2$ and $\lambda/4$ plates.

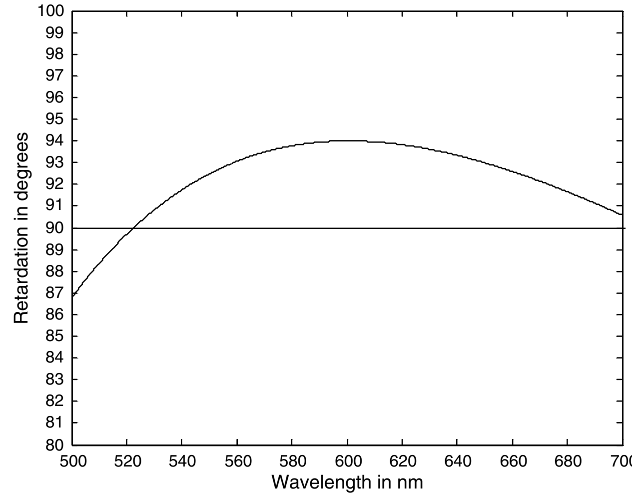

Fig. 6. Wavelength dependent retardation Δ for the combination of two $\lambda/4$ plates.

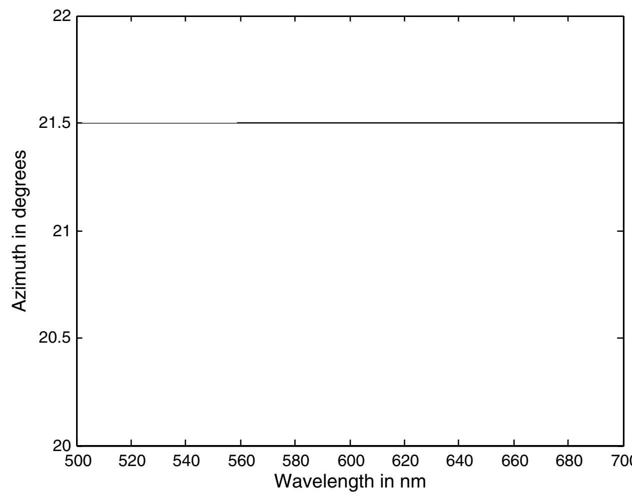

Fig. 7. Wavelength dependent azimuth Ψ for the combination of two $\lambda/4$ plates.

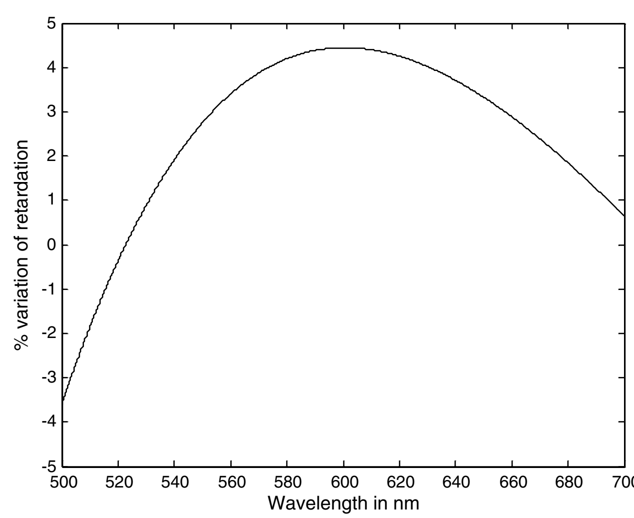

Fig. 8. Percentage variation of retardation Δ with wavelength for the combination of two $\lambda/4$ plates.

600 nm, it is observed that, for optimum achromatic behavior, the orientation of the second plate with respect to the first plate must be 43°. In this case, the overall retardation and azimuth of the second system of two quarter-wave plates are shown in Figs. 6 and 7, respectively. The percentage variation of the retardation is shown in Fig. 8.

5. Discussion

We show that both the combinations of two birefringent plates of crystalline quartz behave as a single achromatic birefringent quarter-wave plate, within the 500–700 nm wavelength spectrum range. For the combination of $\lambda/2$ and $\lambda/4$ plates, it is found that, in this spectral range, the retardation varies within $\pm 0.5°$. This region of spectrum may be shifted to some other wavelength region by a proper choice of the birefringent material and design wavelength.

In the second system having two quarter-wave plates, it is found that, to a tolerance of about 4° in retardation, this combination works well as an achromatic quarter-wave plate. But the advantage of the system is that the overall azimuth remains constant throughout the wavelength spectrum. The proposed design and characterization procedure is relatively simple. That all the wave plates used are of the same material is definitely an advantage over similar devices using wave plates of different birefringent materials. Moreover, the proposed achromatic combinations employ only two standard retarders that are commercially available.

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