

A simple analytical method to obtain achromatic waveplate retarders

This content has been downloaded from IOPscience. Please scroll down to see the full text.

2017 J. Opt. 19 045701

(<http://iopscience.iop.org/2040-8986/19/4/045701>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 132.239.1.231

This content was downloaded on 23/03/2017 at 11:25

Please note that [terms and conditions apply](#).

You may also be interested in:

[Error analysis and calibration of a spectroscopic Mueller matrix polarimeter](#)

B Boulbry, B Le Jeune, B Bousquet et al.

[Optimal broadband Mueller matrix ellipsometer using multi-waveplates with flexibly oriented axes](#)

Honggang Gu, Xiuguo Chen, Hao Jiang et al.

[Broad-band superachromatic retarders](#)

P Hariharan

[Spectral manipulation and complementary spectra with birefringence polarization control](#)

Pan-Feng Ding and Pin Han

[Achromatic retarders using quartz and mica](#)

P Hariharan

[Spectral correlation functions of the sum of two independent complex Wishart matrices with unequal covariances](#)

Gernot Akemann, Tomasz Checinski and Mario Kieburg

[Tunable spectral singularities: coherent perfect absorber and laser in an atomic medium](#)

Chao Hang, Guoxiang Huang and Vladimir V Konotop

[Classification of spin and multipolar squeezing](#)

Emi Yukawa and Kae Nemoto

[Error parameters and calibration of a wave plate](#)

B Boulbry, B Le Jeune, F Pellen et al.

A simple analytical method to obtain achromatic waveplate retarders

Jose Luis Vilas and Aleksandra Lazarova-Lazarova

BioComputing Unit, National Center for Biotechnology (CNB-CSIC) Darwin 3, Campus Universidad Autonoma de Madrid, E-28049 Cantoblanco, Madrid, Spain

E-mail: jlvilas@cnb.csic.es

Received 2 November 2016, revised 31 January 2017

Accepted for publication 3 February 2017

Published 28 February 2017



Abstract

A new linear and analytical method to design achromatic retarders using waveplates is proposed. The root of this procedure is a generalization of the Hariharan method, which supposes a set of waveplates with fast axes aligned. Hence, it imposes a set of contour conditions over the overall retardation with the aim of determining the thicknesses of the waveplates. Our method proposes a polynomial approximation of the birefringences, thus removing the contour condition. Analytic expressions for calculating the thicknesses of the waveplates are then derived, showing a non-explicit dependence on the wavelength. Moreover, the overall retardation obtained by this method is close to the optimal retardation curve achieved by minimizing the merit function of the achromatism degree.

Keywords: polarization, optical design, physical optics, birefringence, waveplate, retarder

(Some figures may appear in colour only in the online journal)

1. Introduction

Since its inception, polarimetry has been established as a technique with special interest in optics and applied physics and countless applications in other fields such as biology [1], astronomy [2] or materials [3], among others. The cornerstone in all of them is the phase control and its measurement. Much research has been focused on developing devices to generate, transform or measure polarization states. In particular, retarders allow us to modify the phase, introducing a shift without affecting the amplitude of the vector light. To do this there are several devices, such as waveplates, liquid crystals or Fresnel rhombs, among others. In general, these elements work at specific wavelengths, thus the design of achromatic retarders has a deep impact on optics.

The use of waveplates confers an advantage with respect to other devices due to their small size and easy setup, high accuracy in retardation and low cost. Pancharatnam was one of the pioneers, with his design of an achromatic retarder using three quartz waveplates [4]. From then on, several design methodologies have been undertaken. They can be coarsely classified in two groups. The first group tries to obtain an achromatization of the overall retardation by optimizing a merit function inside a spectral range [5–7]; the

second group makes use of contour conditions with the goal of achieving a global achromatization through local conditions in the overall retardation [4, 8, 9]. Global methods usually give flatter retardation curves, and therefore a better achromatism.

Controlled changes in the phase of the light are obtained under very specific incidence conditions. The directions of the eigenstates of a waveplate are called the neutral axis, and also present a chromatic dependence. Thus, when a waveplate is illuminated at two wavelengths with the same polarization state, the output state might be different. In this sense, an achromatic system can be defined as that configuration that maintains constant its overall retardation and axis orientation inside an interval of wavelengths. The Hariharan method [9] allows us to design achromatic retarders in this sense; nevertheless, the overall retardation obtained is far from the methods that make use of global achromatizations [5, 7]. The method supposes two waveplates of different materials with aligned axes, then imposes a boundary condition over the retardation at two different wavelengths. As a consequence, the thicknesses of the waveplates can be determined by solving a linear equation system; the result is the thickness of an achromatic configuration.

In this work a generalization and extension of the Hariharan method is presented. Hence, retarders of different materials with aligned axes are supposed and the linear equation system will define the contour condition. In our proposal, the generalized Hariharan method performs a polynomial approximation of the birefringence of the waveplates inside the spectral interval where the retarder is being designed. The approximation removes the boundary condition, leaving only implicit a dependence on the wavelength in the polynomial coefficients. Then, it is only necessary to solve a linear system as in the original Hariharan method. The result of this approach gives analytical expressions for thicknesses; as a consequence, the system is achromatized in the desired spectral range.

2. Theoretical frame

Consider a configuration composed by an arbitrary number N of waveplates parallel to each other. All waveplates are characterized by a retardation, δ_j , and the orientation of their fast axes, ϕ_j ; the subindex $j = 1, \dots, N$ determines the position of the waveplate in the configuration. A beam of light with spectrum $g(\lambda)$ is incident orthogonally to the waveplates along the positive direction of the z -axis. Without loss of generality, the fast axis of the first waveplate is fixed as the reference x -axis, $\phi_1 = 0$, thus the azimuths ϕ_j are defined as the angle between the x -axis and the fast axis of the waveplate j . Using Jones formalism each waveplate is then represented by the matrix

$$C_j = \begin{pmatrix} \cos^2 \frac{\delta_j}{2} + i \sin^2 \frac{\delta_j}{2} \cos 2\phi_j & i \sin \frac{\delta_j}{2} \sin 2\phi_j \\ i \sin \frac{\delta_j}{2} \sin 2\phi_j & \cos^2 \frac{\delta_j}{2} - i \sin^2 \frac{\delta_j}{2} \cos 2\phi_j \end{pmatrix}, \quad (1)$$

where the retardation $\delta_j = 2\pi\Delta n_j(\lambda)d_j/\lambda$ depends on the wavelength, λ , the thickness of each waveplate, d_j , and the birefringence of the material at that wavelength, $\Delta n_j(\lambda) = n_e(\lambda) - n_o(\lambda)$, defined as the difference of extraordinary and ordinary refractive indexes. Note that the retardation presents a strong wavelength dependence, so that the use of waveplates is restricted to specific wavelengths or narrow spectral ranges. The aim of this work is to achieve achromatic retarders in broad spectral ranges. The characteristic Jones matrix of the system, J , will be

$$J = \prod_{j=1}^N C_j(\delta_j, \phi_j) = \begin{pmatrix} A & B \\ -B^* & A^* \end{pmatrix}, \quad (2)$$

where A and B are complex expressions and the symbol $*$ denotes the complex conjugate operation. Because $C_j(\delta_j, \phi_j) \in SU(2)$, $J \in SU(2)$ therefore can be written in terms of A and B as equation (2). The Jones–Hurwitz equivalence theorem [10] establishes that ‘a configuration constituted by an arbitrary number of linear retarders is equivalent to a rotor and a linear retarder, $J = R(\omega)C(\Delta, \Psi)$ ’; the rotor effect is represented by a rotation matrix $R(\omega)$. As a consequence, the expressions for

the overall retardation, Δ , azimuth, Ψ , and rotation angle, ω , are obtained:

$$\cos \frac{\Delta}{2} = \frac{\text{Tr}(J)}{2 \cos(\omega)},$$

$$\tan 2\Psi = \frac{\text{Re}(A)\text{Im}(B) + \text{Re}(B)\text{Im}(A)}{\text{Re}(A)\text{Im}(A) - \text{Re}(B)\text{Im}(B)}, \quad (3)$$

$$\tan \omega = -\frac{\text{Re}(B)}{\text{Re}(A)}. \quad (4)$$

The rotation and the azimuth can be removed, $\omega = \Psi = 0$, when the axes of the waveplates are parallel, $\phi_j = \xi_j \pi/2$, with $\xi_j = 0, 1$. This condition gives an overall retardation,

$$\Delta = \sum_{j=1}^N (-1)^{\xi_j} \delta_j = \frac{2\pi}{\lambda} \sum_{j=1}^N \Delta n_j(\lambda) d_j. \quad (5)$$

Looking at equation (1), it is observed that $C(\delta, 0) = C(-\delta, \pi/2)$, i.e. when the fast axis is rotated through $\pi/2$ the retardation changes its sign. The thickness, d_j , can absorb the sign defined by the factor $(-1)^{\xi_j} = -1$; therefore, a negative thicknesses in a waveplate represents a rotation of $\pi/2$ in the orientation of its fast axis when $\Delta n_j(\lambda) > 0$, while if the birefringence is negative, $\Delta n_j(\lambda) < 0$, the fast axis is not rotated. This fact justifies the second equality in equation (5) and the calculation is considerably simplified.

Using equation (5), Hariharan proposed a linear method for designing achromatic half-wave retarders by imposing contour conditions at two wavelengths [9] λ_1, λ_2 ,

$$\Delta_1(\lambda_1)d_1 + \Delta_2(\lambda_1)d_2 = \lambda_1/2$$

$$\Delta_1(\lambda_2)d_1 + \Delta_2(\lambda_2)d_2 = \lambda_2/2$$

and then solving the thicknesses d_1, d_2 . Here a generalization of the Hariharan method is presented for designing achromatic λ/m -wave retarders composed by an arbitrary number N of waveplates. This is achieved by imposing the λ/m -wave retarder condition in the overall retardation, equation (5), at N wavelengths, λ_k , with $k = 1, \dots, N$. Next the linear system is formed as a contour condition,

$$B \cdot \mathbf{d} = \mathbf{l},$$

with B the birefringence matrix, with elements $b_{kj} = \Delta n_j(\lambda_k)$,

$$B = \begin{pmatrix} \Delta n_1(\lambda_1) & \Delta n_2(\lambda_1) & \cdots & \Delta n_N(\lambda_1) \\ \Delta n_1(\lambda_2) & \Delta n_2(\lambda_2) & \cdots & \Delta n_N(\lambda_2) \\ \vdots & \vdots & \ddots & \vdots \\ \Delta n_1(\lambda_N) & \Delta n_2(\lambda_N) & \cdots & \Delta n_N(\lambda_N) \end{pmatrix}, \quad (6)$$

$\mathbf{d} = (d_1, \dots, d_N)^T$ and $\mathbf{l} = (\lambda_1/m, \dots, \lambda_N/m)^T$ two column vectors with thicknesses of the waveplates d_j , and wavelengths (contour condition) where the system fulfill exactly the λ/m -wave retarder condition. The thicknesses can be determined by the inverse, B^{-1} , as $\mathbf{d} = B^{-1} \cdot \mathbf{l}$; however, this requires a non-singular matrix, $|B| \neq 0$, where the notation $|B|$ denotes the determinant of the matrix B . The last condition can only be verified when all waveplates are made of different materials, $\Delta n_i \neq \Delta n_j$ for all $i \neq j$. The refractive indexes are given by Sellmeier relations [11, 12], and their analytical

dependence on the wavelength is a complicated expression. Nevertheless, these expressions can be simplified if they are approximated by a polynomial in the working spectral range,

$$\Delta \tilde{n}_j(\lambda_k) = \sum_{i=1}^N a_{ij} \lambda_k^{i-1} \approx \Delta n_j(\lambda_k). \quad (7)$$

By introducing equation (7) into equation (6), an approximated matrix, \hat{B} , with elements $\hat{b}_{kj} = \Delta \tilde{n}_j(\lambda_k)$ is obtained and verifies the following properties: if the degree of the polynomials is less than $N - 1$, where N is the number of waveplates, then the matrix \hat{B} is singular, $|\hat{B}| = 0$. If the polynomial in equation (7) has exactly degree $N - 1$, then

$$|\hat{B}| = |V| \cdot |A|, \quad (8)$$

where $|V|$ is the Vandermonde determinant of the matrix formed by the row vectors $(1, \lambda_k, \dots, \lambda_k^{N-1})$, i.e. their matrix elements are $v_{ij} = \lambda_k^{j-1}$ with $i, j = 1, \dots, N$;

$$|V| = \begin{vmatrix} 1 & \lambda_1 & \lambda_1^2 & \dots & \lambda_1^{N-1} \\ 1 & \lambda_2 & \lambda_2^2 & \dots & \lambda_2^{N-1} \\ 1 & \lambda_3 & \lambda_3^2 & \dots & \lambda_3^{N-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \lambda_N & \lambda_N^2 & \dots & \lambda_N^{N-1} \end{vmatrix}, \quad (9)$$

and $|A|$ is the determinant of the matrix A , defined by the fitting coefficient birefringence matrix,

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1N} \\ a_{21} & a_{22} & \dots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \dots & a_{NN} \end{pmatrix}. \quad (10)$$

Thus, the A columns contain the fitting coefficients of the birefringence of the same waveplate, the A rows show the coefficients with the same degree, and the last row the independent terms. The rigorous proof of the Vandermonde decomposition given in equation (8) is given in the [appendix](#); however, intuitively the decomposition $\hat{B} = VA$ gives $|\hat{B}| = |V| \cdot |A|$. The thicknesses of the waveplates can be then obtained by means of Cramer's rule as

$$d_j = |\hat{B}_j|/|\hat{B}|, \quad (11)$$

where \hat{B}_j is the result of substituting the column j of independent terms, $\mathbf{1} = (\lambda_1/m, \dots, \lambda_N/m)^T$, in the matrix B . However, equation (11) can be simplified using the A matrix as follows:

$$d_j = \frac{|\hat{A}_j|}{m|A|}. \quad (12)$$

The matrix \hat{A}_j is therefore the result of removing the second row and the column j in the matrix A ,

$$\hat{A}_j = \begin{pmatrix} a_{11} & \dots & a_{1j-1} & a_{1j+1} & \dots & a_{1N} \\ a_{31} & \dots & a_{3j-1} & a_{3j+1} & \dots & a_{3N} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{N1} & \dots & a_{Nj-1} & a_{Nj+1} & \dots & a_{NN} \end{pmatrix}. \quad (13)$$

Hence the thicknesses given in equation (12) depend implicitly on the wavelength through the fitting coefficients of the

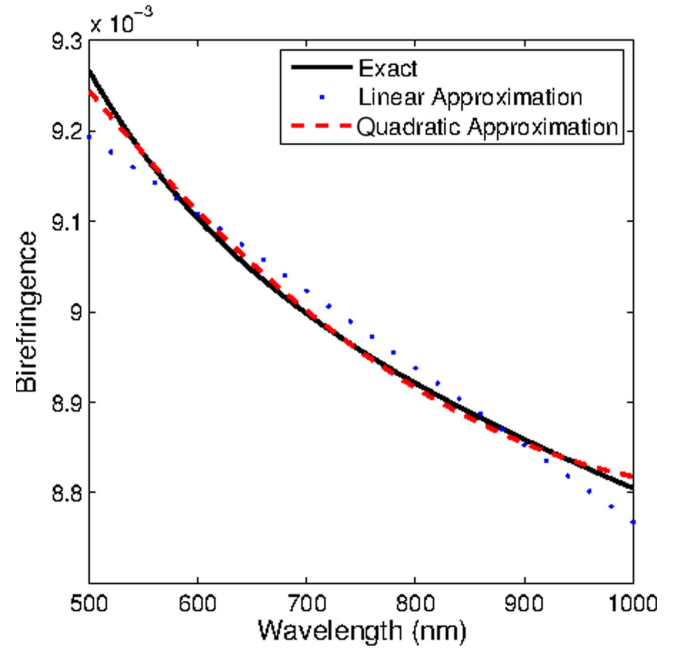


Figure 1. Continuous black, birefringence curve of quartz in the spectral range [500, 1000] nm; dashed red, linear approximation of the birefringence; dotted blue, quadratic approximation of the birefringence.

birefringence. In the [appendix](#), the calculation of $|A|$ and $|B|$, the equality $|\hat{B}_j| = 1/m |V| \cdot |\hat{A}_j|$ and the explicit form of \hat{A}_j is shown.

3. Results

Using this technique achromatic half and quarter wave retarders in the spectral ranges of [500 – 700]nm and [500 – 1000]nm respectively have been designed. However, other type λ/m retarders can be designed. The chosen materials were quartz, MgF_2 and calcite because of the abundant literature about them [11, 12] and their wide use as retarders [8]. The birefringence of these materials was approximated by least squares to polynomials of degree $N - 1$, in particular to first and second degree polynomials. The use of these polynomial approximations is justified by plotting the birefringence in terms of the wavelength; the quartz case can be observed in figure 1. It is noteworthy that their birefringences are always positive. As a design condition we assume aligned axes; therefore, $\phi_j = \xi_j \pi/2$ with $\xi \in \{0, 1\}$ for $j = 1, 2, 3$. As a consequence, the overall azimuth and the rotation defined by the Jones equivalence theorem are $\omega = \Psi = 0$ and independent of the wavelength. This condition represents an advantage due to implication of an achromatism in ω and Ψ , and the chromatic dependence of equations (3) and (4) is removed. Moreover, equation (5) demonstrates that the overall retardation, Δ , is invariant under permutations of the order of the waveplates. Thereby, without loss of generality, the first waveplate will be composed of quartz, the second of MgF_2 and the third of calcite. The thicknesses for the cases

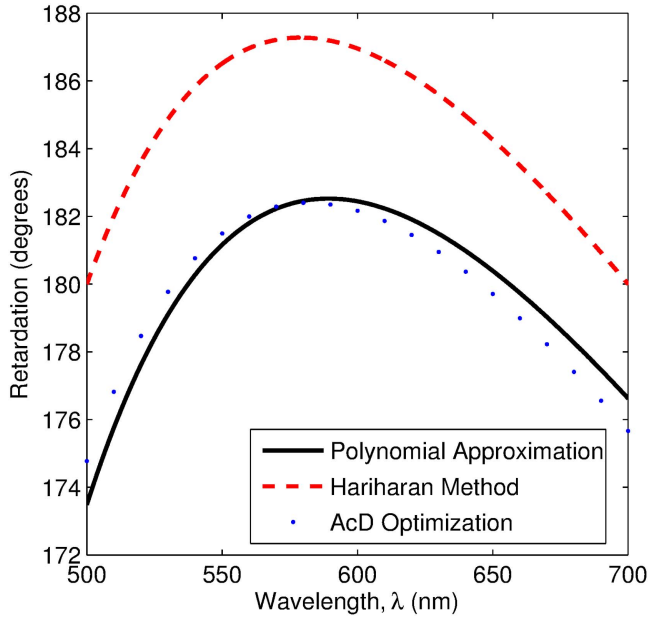


Figure 2. Overall retardation curve for plane spectrum in the bandwidth [500, 700] nm: continuous black, polynomial approximation; dashed red, Hariharan method; dotted blue, AcD minimization with fixed axis. See thickness in table 1.

with $N = 2$ are

$$d_1 = -\frac{a_{12}}{m \cdot |A|}, \quad d_2 = \frac{a_{11}}{m \cdot |A|},$$

and if $N = 3$ then

$$d_1 = \frac{a_{13}a_{32} - a_{12}a_{33}}{m \cdot |A|}, \quad d_2 = \frac{a_{11}a_{33} - a_{13}a_{31}}{m \cdot |A|},$$

$$d_3 = \frac{a_{12}a_{31} - a_{11}a_{32}}{m \cdot |A|}.$$

In order to establish a comparison among the polynomial approximation (PA), the Hariharan method (HM) [9] and the achromatism degree optimization (AcDOp) [5, 6], the retardation curves obtained with these methods were plotted; see figures 2 and 3. In table 1, the calculated thicknesses are shown. We would like to analyze the achromatism in the whole spectral range; the achromatism degree metric allows this task. Thus, the achromatism degree (AcD metric) is a distance measurement between a target retardation Δ_0 and the overall retardation curve, Δ , of the system. It is defined as

$$\text{AcD} = \frac{\sqrt{\int_{\Omega} |\Delta(\lambda) - \Delta_0|^2 g(\lambda) d\lambda}}{\sqrt{\int_{\Omega} g(\lambda) d\lambda}} \quad (14)$$

where the function $g(\lambda)$ represents the normalized spectrum of the incident light (in our case $g(\lambda) = 1$) and Ω the spectral range to be analyzed. As a contour condition in the HM, a retardation λ/m was imposed at the extremes of the spectral range; i.e., if only two waveplates are considered, $N = 2$ and the bandwidth is [500, 700] nm, we impose $\Delta(500 \text{ nm}) = \Delta(700 \text{ nm}) = \lambda/2$ (see figure 2). In contrast, for the spectral region of [500, 1000] nm three waveplates were used; in this case, the contour condition at the extremes

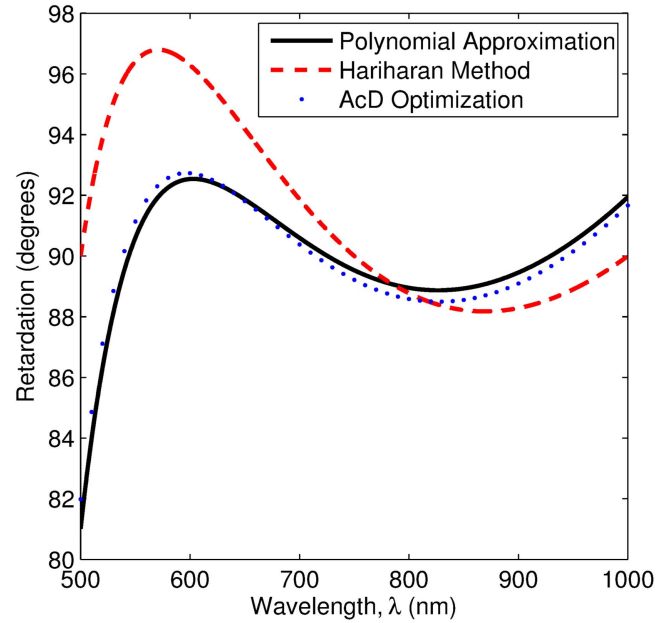


Figure 3. Overall retardation curve for plane spectrum in the bandwidth [500, 1000] nm: continuous black, polynomial approximation; dashed red, Hariharan method; dotted blue, AcD minimization with fixed axis. See thickness in table 1.

Table 1. Material thicknesses for quartz (d_1), MgF_2 (d_2) and calcite (d_3), with $N = 2$ and $N = 3$ waveplates in the spectral ranges of 500–700 nm and 500–1000 nm for retarders of $\lambda/2$ and $\lambda/4$ respectively.

Ret.	Method	d_1 (μm)	d_2 (μm)	d_3 (μm)	AcD
$\lambda/2$	PA	−765.8	616.4	—	1.5×10^{-3}
	HM	−748.8	603.9	—	8.4×10^{-3}
	AcDOp	−739.6	596.1	—	1.4×10^{-4}
$\lambda/4$	PA	11 287.8	−3809.3	335.4	1.0×10^{-3}
	HM	11 000.7	−3736.1	325.2	3.4×10^{-3}
	AcDOp	11 580.0	−3918.6	343.4	9.7×10^{-4}

is kept as $\Delta(500 \text{ nm}) = \Delta(1000 \text{ nm}) = \lambda/4$, but a new constraint is added in the middle, $\Delta(750 \text{ nm}) = \lambda/4$. The AcD optimization was performed in the case of a flat spectrum and aligned axis as in the original article [5]. The results were also compared using the AcD metric [6]; see table 1. Thus, the PA gives a closer behavior to the AcDOp than the HM. This proximity is validated when the retardations are plotted (figures 2 and 3). It should be remembered as mentioned above that the minus sign of some thicknesses in table 1 indicates that the retarder is rotated through an angle of $\pi/2$.

4. Conclusion

In summary, Hariharan's method has been generalized to design achromatic λ/m retarders with an arbitrary number of waveplates. The cases with $N = 2$ and $N = 3$ in the spectral ranges of 500–700 nm and 500–1000 nm have been presented. The retardation curves indicate good behavior as

achromatic retarders. In particular, the polynomial approximation exhibits a similar retardation curve to the AcD method in both cases. Moreover, mathematically this method is particularly simple; it only requires us to fit the birefringence to a polynomial in the spectral range of interest and then solve a linear equation system.

Acknowledgments

The authors thank Alfredo Luis, Ana G Moreno, Jose Maria Herrera-Fernandez, Jorge Jimenez, Javier Vargas, Carlos Oscar Sorzano, Pablo Conesa and Jose Maria Carazo for their encouragement and many useful comments.

Appendix

The birefringence matrix is composed by column vectors (Δn_j , see equation (6)); each of them contains information about the birefringence of the waveplate in position j at specific wavelengths. The matrix, B , can be then expressed as $B = (\Delta \tilde{n}_1, \Delta \tilde{n}_2, \dots, \Delta \tilde{n}_N)$, where $\Delta \tilde{n}_j$ is defined by equation (7). Hence its determinant will be

$$|B| = \left| \sum_{i_1=1}^N a_{i_1,1} \lambda_k^{i_1-1} \dots \sum_{i_N=1}^N a_{i_N,N} \lambda_k^{i_N-1} \right|$$

$$= \sum_{i_1, \dots, i_N=1}^N \prod_{j=1}^N a_{i_j,j} |\lambda_k^{i_1-1} \lambda_k^{i_2-1} \dots \lambda_k^{i_N-1}|, \quad (15)$$

where α is an index, $\alpha = 1, \dots, N$. The last factor is different from zero when each element of the same column is a power of different degree, i.e. $i_\alpha \neq i_\beta$ for all $\alpha, \beta \in [1, N]$. Therefore, the sum is restricted to the N -tuples of non-repetitive indexes (i_1, \dots, i_N), in other words all permutations of the set $\{1, \dots, N\}$. As a consequence, equation (15) is rewritten as

$$|B| = \sum_{\sigma \in S_N} \prod_{j=1}^N a_{\sigma(j),j} |\lambda_k^{\sigma(1)} \lambda_k^{\sigma(2)} \dots \lambda_k^{\sigma(N)}|, \quad (16)$$

where S_n is the symmetric group, also called the permutation group, of N elements, σ being a permutation of the group. Then, $\sigma(h)$ represents the element located in position h in permutation σ . Note that the columns of the matrix $(\lambda_k^{\sigma(1)} \lambda_k^{\sigma(2)} \dots \lambda_k^{\sigma(N)})$ are always powers of degree $0 \dots N-1$; therefore, it is a Vandermonde determinant (see equation (9))

$$|V| = |1 \quad \lambda \quad \dots \quad \lambda^{N-1}|.$$

Thus, the general expression will be

$$|B| = |V| \sum_{\sigma \in S_N} \left[\text{sgn}(\sigma) \prod_{j=1}^N a_{\sigma(j),j} \right] = |V| |A|,$$

where $\text{sgn}(\sigma)$ is the sign function and A the matrix of coefficients of the birefringence fitting, $A_{ij} = a_{ij}$, defined in equation (10). The calculation of $|\hat{B}_j|$ is similar; indeed, substituting $\Delta n_j(\lambda_k) = \lambda_k/m$, we have $a_{i,j} = \lambda_k/m$, and thus

$$|B_j| = \sum_{\sigma \in S_N} \prod_{j \neq \alpha}^N a_{\sigma(j),j} \left| \lambda_k^{\sigma(1)} \dots \frac{\lambda_k}{n} \dots \lambda_k^{\sigma(N)} \right|. \quad (17)$$

$\sigma(j) = 2$ implies a zero factor, thus $|\hat{B}_j| = 1/m |V| |\hat{A}_j|$ as defined in equation (13).

References

- [1] Qi J and Elson D S 2016 A high definition Mueller polarimetric endoscope for tissue characterisation *Sci. Rep.* **6** 25953
- [2] Greaves J S, Holland W S, Jenness T and Hawarden T G 2000 Magnetic field surrounding the starburst nucleus of the Galaxy M82 from polarized dust emission *Nature* **404** 732–3
- [3] Copuroğlu O 2016 Revealing the dark side of portlandite clusters in cement paste by circular polarization microscopy *Materials* **9** 176
- [4] Pancharatnam S 1955 Achromatic combinations of birefringent plates *Proc. Indian Acad. Sci. A* **41** 137–44
- [5] Vilas J L, Sanchez-Brea L M and Bernabeu E 2013 Optimal achromatic wave retarders using two birefringent wave plates *Appl. Opt.* **52** 1892–6
- [6] Herrera-Fernandez J M, Vilas J L, Sanchez-Brea L M and Bernabeu E 2015 Design of superachromatic quarter-wave retarders in a broad spectral range *Appl. Opt.* **54** 9758–62
- [7] Mu T, Zhang C, Li Q and Liang R 2015 Achromatization of waveplate for broadband polarimetric system *Opt. Lett.* **40** 2485–8
- [8] Saha A, Bhattacharya K and Chakraborty A K 2012 Achromatic quarter-wave plate using crystalline quartz *Appl. Opt.* **51** 1976–80
- [9] Hariharan P 1995 Achromatic retarders using quartz and mica *Meas. Sci. Technol.* **6** 1078–9
- [10] Hurwitz J H and Jones R C 1941 A new calculus for the treatment of optical systems II. Proof of three general equivalence theorems *J. Opt. Soc. Am.* **31** 493–9
- [11] Bass M, DeCusatis C, Enoch J, Lakshminarayanan V, Li G, MacDonald C, Mahajan V and Van Stryland E 2009 *Handbook of Optics* Vol 4 (Optical Society of America)
- [12] Ghosh G 1999 Dispersion-equation coefficients for the refractive index and birefringence of calcite and quartz crystals *Opt. Commun.* **163** 95–102