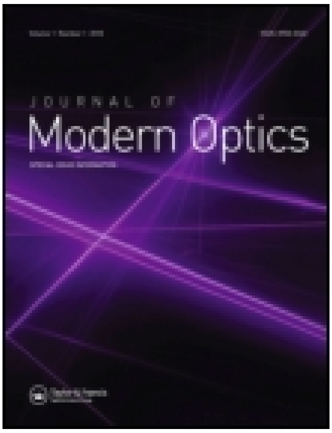

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A mixed Solc birefringent filter

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Abstract. In this article, the transmission characteristics of new mixed birefringent filter have been studied. The term 'mixed' is used because in this filter both linear and circular birefringence are utilized for shaping its transmission function. The filter has a so called 'lossless core', consisting of alternate rotators and retarders, and two linear polarisers at the two ends. Because of the similarity of the proposed mixed birefringent filter with the fan type Solc filter, it has been given the name 'Mixed Solc Birefringent Filter' (MSBF).

1. Introduction

The birefringent filter was first invented and designed by the French astronomer Lyot in 1933 [1]. About 20 years later, a Czechoslovakian physicist, Solc, invented a second type of birefringent filter which is today known as the Solc filter. These filters, although different in construction from the Lyot filter, have similar transmission characteristics [2–4]. In 1952, Hurlbut and Rosenfeld utilised the rotary disperson power of quartz for making a monochromator [5]. Apparently, though, their work did not draw the attention of physicists working in the field of polarization filters. Many articles have appeared on the theory and performance of birefringent filters since the publication of the paper by Hurlbut and Rosenfeld without any reference to their work. In 1958, Evans derived the transmission of Solc filters [6]. In his derivation, Evans followed the matrix diagonalisation technique used by Jones (for explaining Sohncke theory of optical activity) in terms of the new calculus invented by and named after him [7].

Although rotators have been used for fabricating polarization filters, nobody has yet studied the possibility of utilizing rotators and retarders together to form mixed birefringent filters. In this present work, we have studied the transmission characterics of such a filter. The mathematical approach followed by us for the derivation of the transmission characteristics is different from that followed by Evans. In fact, we have utilized a well-known property of the unimodular matrix for our purpose. We have already used this mathematical method for deriving the transmittance of Solc filters [8].

2. Construction of the proposed filter

The filter proposed consists of a lossless core having alternate rotators and retarders and two linear polarizer placed at the two ends, as shown in figure 1. In the figure, P(0) and P($\pi/2$) denote linear polarizers whose transmission axes are respectively along the x and y axis of the reference coordinate system. The first and the last rotators, denoted by R($\alpha/2$), rotate the direction of vibration of a linearly

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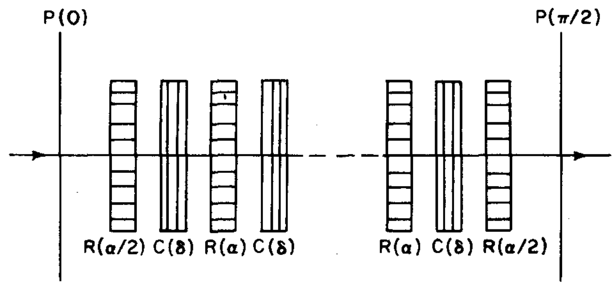

Figure 1. Schematic diagram of a Mixed Solc Birefringent Filter (MSBF).

polarized light through an angle of $\alpha/2$, where α is a function of wavelength. The other rotators, denoted by R(α), introduce a rotation of α. C(δ) represents a retarder that introduces a phase difference of δ between two orthogonal components of light along its two privileged directions. The optical axes of all the retarders are mutually parallel, but the presence of rotators in-between the retarders simulates the effect of introducing a tilt between the optical axes of two successive retarders. Since both linear and circular birefringence are utilized, the filter may be called a 'mixed birefringent filter'. Because of the essential similarity of the filter described here with the Solc filter, we have called it a Mixed Solc Birefringent Filter (MSBF).

3. Derivation of the transmission function

In order to obtain the expression for the transmission of the MSBF, let us assume that the input beam is given by the Jones vector

$$E_i = \begin{bmatrix} E_x \\ E_y \end{bmatrix}. \tag{1}$$

The Jones vector of the beam emerging from the filter, E_o, may, therefore, be written as

$$E_o = P(\pi/2)R(\alpha/2)[C(\delta)R(\alpha)]^{N-1}C(\delta)R(\alpha/2)P(0)E_i, \tag{2}$$

where $P(\pi/2)$, $P(0)$, $C(\delta)$ and $R(\alpha)$ are the relevant Jones matrices of the elements of the filter given by

$$P(\theta) = \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}, \tag{3}$$

$$C(\delta) = \begin{bmatrix} \exp(i\delta/2) & 0 \\ 0 & \exp(-i\delta/2) \end{bmatrix}, \tag{4}$$

and

$$R(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}. \tag{5}$$

Let us put

$$C(\delta)R(\alpha) = M(\alpha, \delta). \tag{6}$$

Using equation (1), the Jones vector of the outcoming beam may be written as

$$E_o = P(\pi/2)R(\alpha/2)[M(\alpha,\delta)]^{N-1}C(\delta)R(\alpha/2)P(0)E_i \tag{7}$$

Now, carrying out matrix multiplication, we get

$$M(\alpha,\delta)=\begin{bmatrix} \cos\alpha \exp(i\delta/2) & -\sin\alpha\exp(i\delta/2) \\ \sin\alpha\exp(-i\delta/2) & \cos\alpha\exp(-i\delta/2)\end{bmatrix}, \tag{8}$$

or rather,

$$M(\alpha,\delta)=\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22}\end{bmatrix}. \tag{9}$$

Obviously, the matrix $M(\alpha, \delta)$ is unimodular. Now, in order to find out the output wave-vector, we raise the matrix $M(\alpha, \delta)$ to $(N-1)$th power. For this purpose, we make use of a well known result from the theory of matrices, according to which the Kth power of a unimodular matrix M is

$$M^K=\begin{bmatrix} m_{11}P_{k-1}(x)-P_{k-2}(x) & m_{12}P_{k-1}(x) \\ m_{21}P_{k-1}(x) & m_{22}P_{k-1}(x)-P_{k-2}(x)\end{bmatrix}, \tag{10}$$

where

$$x=\tfrac{1}{2}(m_{11}+m_{22}) \tag{11}$$

and P_ks are the Chebyshev polynomial of the second kind, given by

$$P_k(x)=\frac{\sin[(K+1)\cos^{-1}x]}{\sqrt{[1-x^2]}} \tag{12}$$

A proof of the above relation, based on the theory of matrices, was given by Abelès [9]. Using the above result in equation (7), we get

$$E_0=\begin{bmatrix} 0 \\ p \end{bmatrix}E_x \tag{13}$$

where

$$p=P_{N-1}(x)\cos(\delta/2)\sin\alpha \tag{14}$$

Thus, apart from a photometric factor, the intensity transmittance of the filter is given by

$$T=|P|^2=[P_{N-1}(x)\cos(\delta/2)\sin\alpha]^2. \tag{15}$$

Here,

$$x=\tfrac{1}{2}(m_{11}+m_{22})=\cos\alpha\cos(\delta/2)=\cos\chi. \tag{16}$$

Substituting this expression of χ in equation (12), we get

$$P_{N-1}(x)=\frac{\sin N\chi}{\sin\chi}. \tag{17}$$

Using this equation, expression (15) for the transmission, T, may be rewritten as

$$\begin{aligned} T &= \left[\sin \alpha \cos (\delta/2) \frac{\sin N\chi}{\sin \chi} \right]^2, \\ &= \left[\tan \alpha \cos \chi \frac{\sin N\chi}{\sin \chi} \right]^2. \end{aligned} \tag{18}$$

The above expression is identical to the expression for the transmission of the Solc filters as seen from the reference [6]. This, however, does not imply that both the Solc filters and the MSBF proposed by us have the same transmission characteristics. In fact, the transmittance of the MSBF is essentially different from that of Solc filters. This is because of the fact that in the case of the MSBF, the parameter α represents the rotation of the plane of vibration of light by rotators and is function of wavelength; but in the case of the Solc filters α is a geometrical angle which does not depend on wavelength.

4. Computation and discussion

The rotation α of the plane of vibration produced by an optically active substance varies with the wavelength of light. For the purpose of computation of the transmission of the MSBF as a function of wavelength, we have made use of the approximation law of rotary dispersion given by Biot. According to this law,

$$\alpha = \frac{K_1}{\lambda_2} \tag{19}$$

where K_1 is a constant. Taking the values of α for different values of wavelengths from standard tables, we have observed that the above relation agrees reasonably well for quartz and calcite within the entire visible range.

The retardation δ introduced by the birefringent plate between the two orthogonal component of light is given by

$$\delta = \frac{2\pi}{\lambda}(n_e - n_o)d \tag{20}$$

where $(n_e - n_o)$ is the birefringence of the material of the retarders and d is the thickness of each plate. Birefringence is a function of frequency, but within the range of frequency under consideration, $(n_e - n_o)$ may be assumed to be constant. Under this assumption, we may write

$$\delta = \frac{K_2}{\lambda} \tag{21}$$

For the purpose of comparison we have calculated the transmittance of both the Solc filters and MSBF. For calculating the transmittance of the Solc filter, we have made use of the condition $n\alpha = \pi/2$, as stipulated by Solc [6]. In order to compute the transmittance of MSBF, the value of α is taken in such a way that the above condition holds for the wavelength at which the peak transmission is desired.

A typical transmission curve of the MSBF is shown in figure 2, which shows a peak at 6500 Å. The corresponding transmission curve for the Solc filter, whose retarders have the same thickness as those of the MSBF, is shown in figure 3. It is

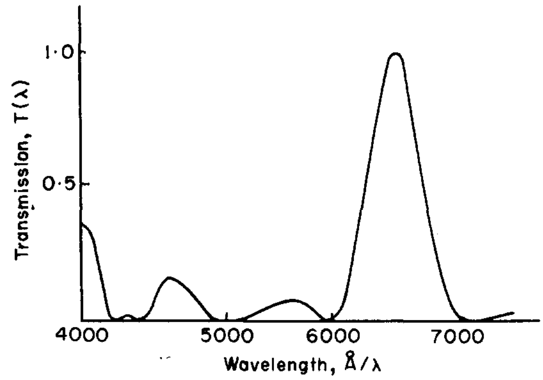

Figure 2. Typical transmission curve for the MSBF.

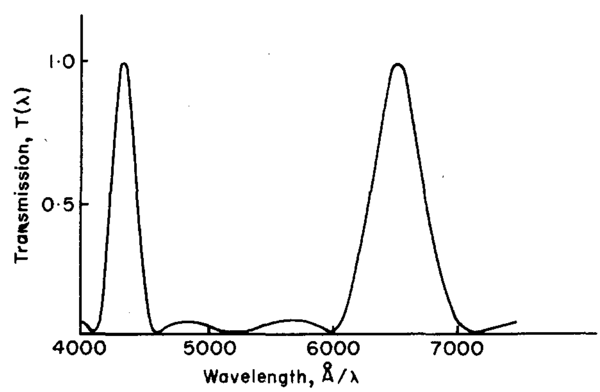

Figure 3. Typical transmission curve for the Solc filter.

seen that the transmission curve of the Solc filter has another peak at 4333 Å. In the case of the MSBF, the frequency dependence of the parameter α has suppressed this principal peak, but has given rise to side oscillations of higher amplitudes.

We may, therefore, conclude that the use of circular birefringent plates in polarization filters gives an additional dimension that can be effectively utilized for shaping their transmission function. Quite a number of interesting combinations of retarders and rotators forming mixed birefringent filters seem to be possible. In forthcoming articles, we shall study the characteristics of some of these mixed birefringent filters.

On étudie, dans cet article, les caractéristiques de transmission d'un nouveau filtre biréfringent mélangé. Le terme 'mélangé' est utilisé parce que, dans ce filtre, la biréfringence linéaire et la biréfringence circulaire sont à la fois utilisées pour mettre en forme sa fonction de transmission. Le filtre a un 'coeur sans perte' consistant en des rotateurs et des retardateurs alternés et deux polariseurs linéaires aux deux extrémités. A cause de la similarité du filtre biréfringent mélangé proposé avec le filtre de type Solc, le filtre est désigné par 'Filtre Biréfringent Solc Mélangé'.

In dieser Arbeit studieren wir die Übertragungseigenschaften eines neuen "gemischten" doppelbrechenden Filters. Der Ausdruck "gemischt" wird benutzt, weil in diesem Filter sowohl lineare als auch zirkulare Doppelbrechung benutzt wird, um die Übertragungsfunktion entsprechend zu formen. Der Filter hat einen sogenannten verlustfreien Kern, bestehend aus abwechselnd Rotatoren und Kompensatoren und zwei linearen Polarisatoren an den zwei Enden. Wegen der Ähnlichkeit des vorgeschlagenen gemischten doppelbrechenden Filters mit dem Solc-Filter haben wir diesen Filter "Mixed Solc Birefringent Filter" genannt.

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