Degree of polarization fading of light passing through birefringent medium with optical axis variation

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ABSTRACT

Numerical implementation of Mueller-Stokes matrix calculus for polychromatic light is used to analyze and plainly illustrate polarization properties of multi-section linearly birefringent systems illuminated by the light of any spectrum profile. Numerical investigations are preceded by a detailed review of known concepts for modeling the depolarization phenomenon in anisotropic media. The numerical study examines efficiency of the Lyot depolarizer system undergoing variations from the optimal configuration. In addition, the power spectrum density profile and intrinsic polarization state of light passing through the system are considered as interesting degrees of freedom. The comparative analysis makes use of the degree of polarization and the depolarization index diagrams.

Keywords: Mueller-Stokes calculus, complex degree of coherence, linear birefringence, Lyot depolarizer, depolarization index

1. INTRODUCTION

Degree of polarization (*DOP*) is one of most basic quantities describing partially coherent (polychromatic) light. While the state of polarization (*SOP*) determines the polarization ellipse of each monochromatic component in a cross-section of a light beam, *DOP* carries information on variations of all the *SOP*'s. These variations can take place in two domains independently: space (cross-section plane of the beam) and time (frequency). Therefore we should also distinguish two types of depolarization, i.e. the phenomenon of reducing the *DOP* of light by means of some interaction with matter. The first type of depolarization involves polarization state scrambling in the cross-section plane of the beam. This is the most widely discussed case in literature, occurring when the light interacts (by transmission or reflection) with scattering media[1] or birefringent media with thickness varying across the plane perpendicular to the propagation direction (the principle of wedge depolarizers). The latter mechanism of *DOP* fading, which is the issue of this paper, causes the electric vector oscillation curve (*SOP*) varying in time and making the polarization state of polychromatic beam indefinite, to some extent. Time variation of *SOP* can be achieved by active, modulated devices (the Billings monochromatic depolarizer[2]) or in the passive manner, by use of the Lyot depolarizer principle[2,3,4,5,6,7].

Depolarizers working in temporal domain play very important role in a wide variety of optical systems. Devices like integrated-optic modulators, switches, couplers, fiber optic gyroscopes or Raman amplifiers exhibit polarization sensitivity that affects light transmission, interference patterns or other parameters making these systems influenced by temperature fluctuations and other environmental distortions. Temporal randomization of *SOP* with a Lyot depolarizer is one of the most commonly employed solutions for effective noise reduction in these systems.

The Lyot depolarizer is a simple device consisting of two sections of linearly birefringent material with length ratio 1:2. Their optical axes (axes of anisotropy) are perpendicular to the propagation direction and oriented at 45^o to each other. However, the precise physical model of this system was an serious issue of theoretical research even several decades after the invention. In 1951 Billings[2] solved the problem of a two-section static crystal depolarizer in the case of rectangular-shaped spectrum of light. Making use of the Mueller-Stokes calculus he proved that the orientation angle equal to 45^o is optimal and the thickness ratio of the segments should be an integer, but not equal to one. In 1982 Loeber[4] performed similar calculations for a blackbody source and showed that the integer thickness ratio condition concerns only the special case rectangular-shaped source considered by Billings. Nevertheless, the rest of the Billings conclusions have been confirmed and from that moment the configuration of the Lyot depolarizer was analytically fully justified.

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Further works concentrate on developing different approaches that would enable to avoid performing integrals over the light spectrum in the theoretical analysis and numerical simulations of static depolarizers. In 1983 Burns[5] proposed a direct formula for *DOP* of light emerging the Lyot depolarizer by making use of the complex degree of coherence quantity under conditions of the quasi-monochromatic light approximation. In 1984 Mochizuki[6] extended this description for varying orientation angle between the birefringence axes of the two segments. The most recent works attempt to adjust the Lyot depolarizer description to some specific applications like the fiber-optic gyroscope[7] as well as the depolarizer's configuration itself, including addition of more sections to the depolarizer[8].

The objective of this paper is to clarify and illustrate the physical fundamentals of partially coherent light depolarization phenomenon in linearly birefringent structures. Considering the particular well known setup of the Lyot depolarizer we attempt to provide some intuitions for understanding the behavior of all similar devices with particular emphasis on distinction between the assumptions within the model of the light and the medium that are necessary to apply and these which can be added optionally. Presented simulation results were obtained by use of Maxima script implementing the Mueller-Stokes calculus for polychromatic light of any power spectrum density profile.

2. STATISTICAL OPTICS IN ANIZOTROPIC MEDIA

2.1 Optical birefringence

Optical birefringence is a property of anisotropic medium that differentiates propagation velocities of the two orthogonal components of the optical wave and make the refraction on the material surface not complied with the Snell's law in general. If the difference concerns linearly polarized orthogonal components of the field, then we deal with linear birefringence. In the in this case two quantities are distinguished: the linear phase birefringence and the group birefringence. If we neglect the optical dispersion of the medium the latter type disappears, whereas the definition of the linear phase birefringence depends on the type of the medium again. In general there are two types of linearly anisotropic materials: uniaxial and biaxial. Biaxial crystals are described by a tensor that consists of three (if expressed in a proper base) distinct refraction indexes assigned to three orthogonal linear polarization azimuths available in space. An uniaxial crystal (a crystal with one axis of anisotropy or optical axis) is a simpler case described by two quantities only: the ordinary and extraordinary refractive index (n_o, n_e). It is worth to mention here that highly birefringent (HB) fibers within the weak guiding approximation can be described analogically, as pairs of linearly polarized (LP) modes of the same order are equivalent to orthogonal linearly polarized components in free space[9]. Due to this remark it is justified to define the effective linear phase birefringence magnitude of a linear retarder in the fallowing way:

$$B = n_x^{eff} - n_y^{eff}\,, \tag{1}$$

where n_x^{eff} denotes alternatively the extraordinary refractive index n_e (in case of an uniaxial crystal) or the effective refractive index referring to the fast axis in the cross-section plane of an optical fiber. This convention relating the fast axis with the *x*-coordinate will be respected consistently in further studies.

In conclusion, in all further analysis we will consider polychromatic light passing through systems consisting of uniaxial anisotropic medium segments, whose optical axes are oriented perpendicularly to the propagation direction. Additionally, we assume no optical activity in the medium (circular birefringence), no losses, no scattering, and we neglect the optical dispersion as well as reflections at dielectric boundaries (Fresnel reflections). In the case of single mode highly birefringent fibers (SM HB) we also assume absence of the polarization modes coupling (power flow between the LP_{01}^{x} and LP_{01}^{y} modes).

2.2 Statistical description of light partially polarized in temporal domain

The only one important property of the considered optical system is that it affects the polarization state of the electromagnetic waves (the only influence of ideal optical retarders). Since we are not interested in diffraction of the beam, it is convenient to employ the plane wave approximation for each spectral component propagating along the z axis:

$$\mathbf{E}_\omega^{(r)}(z,t) = \left[E_{\omega,x}^{(r)}(z,t), E_{\omega,y}^{(r)}(z,t)\right]^T = \mathbf{A}_\omega \cdot \cos(\boldsymbol{\varphi}_\omega + \omega t - kz)\,, \tag{2}$$

where $E^{(r)}_{\omega,x}$, $E^{(r)}_{\omega,y}$ are real functions indicating the spatial components of the electric vector $\mathbf{E}(t)$ rotating in the polarization ellipse in a cross-section plane intersecting the propagation axis at the point z. In order to construct a broadband beam from these plain waves propagating in the same direction we shall utilize the complex calculus. The so-called analytical signals associated with the real functions $E^{(r)}_x(t)$ and $E^{(r)}_y(t)$ in the case of polychromatic light are obtained by the reverse Fourier transform[7,10]:

$$\mathbf{E}(t) = \left[E_x(t), E_y(t)\right]^T = \int_0^{+\infty} \mathbf{e}(\omega) \cdot e^{-i\omega t} d\omega . \tag{3}$$

This is a complex representation of the polychromatic optical signal in a specified point in space. The column vector $\mathbf{e}(\omega) = [e_x(\omega), e_y(\omega)]^T$ is called the Jones vector and contains complex spectral amplitudes of the two orthogonal components of the electric (or magnetic) field. The components $E_x(t)$ and $E_y(t)$ are also called the complex half-range functions as the integrals are performed over the positive frequencies only, which have clear physical meaning[10].

Additionally, since the spectral profile of the beam should be spatially invariant (ordinary light sources), we introduce the following condition:

$$\left|e_y(\omega)\right|^2 = a \cdot \left|e_x(\omega)\right|^2, \; a \in \Re , \tag{4}$$

where the real a coefficient allows the optical power being not equally distributed between the Cartesian axes in cross-section plane of the beam. Then we can introduce the spectral power density distribution of the radiation:

$$G(\omega) = \left|e_x(\omega)\right|^2 + \left|e_y(\omega)\right|^2, \qquad \hat{G}(\omega) = \frac{G(\omega)}{\int_0^{+\infty} G(\omega) d\omega}, \tag{5}$$

where $\hat{G}(\omega)$ is a profile normalized to the total intensity equal to unity.

Since the sufficient approximations on the nature of a light beam have been established, we shall focus on the description of its polarization properties. It is possible to build this description without invoking equations for electric field vector components, but by making use of second-order quantities within the framework of statistical optics instead. The statistical approach treats an optical signal in the form (3) as an realization of a stochastic process. Fortunately, most of real light sources obey the conditions of stationarity and ergodicity of a stochastic system, which are very helpful in the formalism simplification[10]. The full information on the polarization state as well as the degree of polarization of the stationary and ergodic optical perturbation is given be the Wolf's hermitian coherence matrix:

$$\mathbf{K} = \begin{bmatrix} \mathit{\Gamma}_{\mathrm{xx}} & \mathit{\Gamma}_{\mathrm{xy}} \\ \mathit{\Gamma}_{\mathrm{yx}} & \mathit{\Gamma}_{\mathrm{yy}} \end{bmatrix} = \begin{bmatrix} \left\langle E_x E_x^* \right\rangle & \left\langle E_x E_y^* \right\rangle \\ \left\langle E_y E_x^* \right\rangle & \left\langle E_y E_y^* \right\rangle \end{bmatrix}, \tag{6}$$

where the angle brackets denotes an ensemble average or a time average:

$$\Gamma_{ij} = \Gamma_{ij}(\tau = 0) = \left\langle E_i E_j^* \right\rangle = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} E_i(t) E_j^*(t) dt, \qquad i, j = x, y . \tag{7}$$

The statement (7) is also named as cross-correlation of the two optical waveforms in a specified point in space. Thus, it is a measure of temporal coherence of the radiation. What is worth noting here, this description is not restricted to monochromatic or quasi-monochromatic regime. Correlation coefficients have the same meaning with respect to an optical wave of any spectrum profile, since the electric field variations are considered as functions of time only. It is also important that these quantities are proportional to intensity of light and they are easily measurable.

It is possible to transform one coherence matrix to another one regarding the light beam after passing through an optical system of linear elements given by 2x2 so called Jones matrices[7]. However, there is a substantially more suitable way to do this by taking advantage of the Mueller-Stokes calculus. In this formalism the state of polarization of light partially coherent in the temporal domain is given by the real-numbered Stokes vector:

$$\mathbf{S}=\begin{bmatrix} \mathrm{S}_0 \\ \mathrm{S}_1 \\ \mathrm{S}_2 \\ \mathrm{S}_3 \end{bmatrix}=\begin{bmatrix} \Gamma_{\mathrm{xx}}+\Gamma_{\mathrm{yy}} \\ \Gamma_{\mathrm{xx}}-\Gamma_{\mathrm{yy}} \\ 2\,\mathrm{Re}\{\Gamma_{\mathrm{xy}}\} \\ 2\,\mathrm{Im}\{\Gamma_{\mathrm{xy}}\} \end{bmatrix}=S_0\cdot\begin{bmatrix} 1 \\ \cos(2\alpha)\cos(2\psi) \\ \sin(2\alpha)\cos(2\psi) \\ \sin(2\psi) \end{bmatrix}=S_0\cdot\mathbf{s}, \tag{8}$$

where S_0 is proportional to the total intensity: $I_0=\sqrt{\varepsilon_0/\mu_0}\cdot S_0$ (ε_0,μ_0 – electric and magnetic permittivity of the vacuum), while α and ψ are the azimuth and ellipticity angles determining the state of polarization of a spectral component of light (commonly visualized on the Poincare sphere using 2α and 2ψ as coordinates in spherical system). In the case of a polychromatic beam the Stokes vector is also valid and the α,ψ angles can be interpreted as parameters of the fully polarized part of the radiation, according to *DOP* definition. Another crucial property of the Stokes vector for the polychromatic light is that the contribution of particular monochromatic components has a form of simple summation of corresponding monochromatic Stokes vectors, which is a consequence of the assumption that the waves of different frequencies are totally uncorrelated within the time interval $2T$ in (7) :

$$\mathbf{S}_{total}=\mathbf{S}(\omega_1)+\mathbf{S}(\omega_2)+\ldots+\mathbf{S}(\omega_n). \tag{9}$$

The degree of polarization of the light emerging the setup, defined as the contribution of the totally polarized light power to the absolute intensity, can be obtained from the output Stokes vector (8) or directly from the coherence matrix elements[10] (6):

$$DOP=\frac{I_{pol}}{I_{tot}}=\frac{\sqrt{S_1^2+S_2^2+S_3^2}}{S_0}=\sqrt{1-\frac{4(\Gamma_{xx}\Gamma_{yy}-\left|\Gamma_{xy}\right|^2)}{(\Gamma_{xx}+\Gamma_{yy})^2}}\leq 1. \tag{10}$$

In the approach described above, the propagation of light is considered in one spatial dimension only. One can visualize the set of plain waves as a continuous, time infinite signal stretched over a line crossing the optical system. There is neither intensity, nor phase, nor of polarization state distribution of the EM field in the cross-section of the light beam. Nevertheless, it is worth mentioning there are some works reporting successful attempts of developing efficient algorithms calculating the propagation of *SOP* distribution over the cross-section plane of the partially coherent light beams[11]. This approach is a novel extension of the second order diffraction theory by Wolf[10].

3. DEPOLARIZATION IN A SINGLE LINEAR RETARDER

3.1 Mueller-Stokes matrix calculus for polychromatic light

When a polychromatic light beam represented by multiple parallel plane waves encounters a segment of an ideal birefringent medium (within the set of assumptions specified in subsection 2.1) with two birefringence axes perpendicular to the propagation direction (Fig. 1) the optical disturbance has to be decomposed into two orthogonal linearly polarized components: one parallel to the "fast" x axis and the other parallel to the "slow" y axis. Due to the fact that we are dealing with an uniaxial material with its axis of anisotropy oriented perpendicularly to the propagation direction, it arrives that these two components propagate without any change of direction. Therefore, the only influence of the medium on the light beam is that the plain wave has been split into two linearly polarized components of different refraction indexes causing a phase velocity difference between them and, in consequence, a time delay at the output of the system: τ. Eventually, increasing time delay causes evolution of polarization ellipse of each of the spectral components. After passing the distance equal to beat length of the medium: L_B (which, in considered configuration, can characterize anisotropic crystals as well, not only polarization maintaining fibers) the initial *SOP* of given spectral component gets reconstructed. There are two possible points of view for explaining the influence of the time shift τ and *SOP* evolution on the residual *DOP* of the considered system.

The first description (Billings[2] and Loeber[4]) is based on the remark that the beat length characterizing the medium is always a function of wavelength. A given time delay applied to waveforms of a shorter period leads to higher phase retardance. Thus, it arrives that in case of broadband light the output polarization state differs for particular spectral components. A strictly monochromatic wave cannot be depolarized, but superposition of all these polarization states at the output plane results in rapid variations of the electric vector and, in consequence, in *DOP* level being less then unity. Just to formalize this reasoning within the framework of Mueller-Stokes calculus we shall invoke the Mueller matrix of a linear retarder (with respect to the configuration presented in Fig. 1):

$$\mathbf{M}(\lambda)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & \cos\left(\frac{2\pi}{\lambda}B\cdot L\right) & \sin\left(\frac{2\pi}{\lambda}B\cdot L\right)\\ 0 & 0 & -\sin\left(\frac{2\pi}{\lambda}B\cdot L\right) & \cos\left(\frac{2\pi}{\lambda}B\cdot L\right)\end{bmatrix}, \tag{11}$$

where: λ – wavelength of a spectral component in vacuum, B – effective linear phase birefringence, L – length of the birefringent segment. Then the matrix equation for the output polarization state of polychromatic light passing through a single linearly birefringent section is:

$$\mathbf{s}^{out}=\int_0^{+\infty}\mathbf{M}(\lambda)\,\mathbf{s}^{in}(\lambda)\cdot\hat{G}(\lambda)d\lambda\,. \tag{12}$$

The initial normalized Stokes vector can be taken out of the integral by applying an assumption that the beam illuminating the system is fully polarized (*DOP* = 1), therefore *SOP* of each of the spectral components is described by the same normalized vector: $\mathbf{s}^{in}(\lambda)=\mathbf{s}^{in}$. One can notice, that the linear birefringence in the matrix (11) can also be considered as a function of wavelength, so it is very easy to introduce phase dispersion of the medium, if needed.

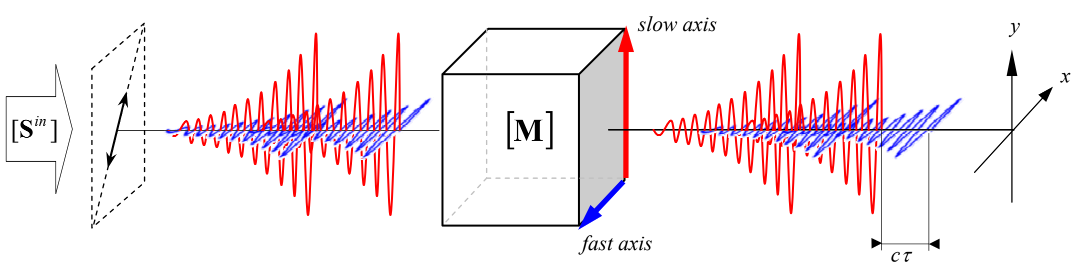

Figure 1. Depolarization of laser light in a linearly birefringent medium in the semi-classical regime.

3.2 Application of the complex degree of coherence

The competitive approach utilizes the concept of complex degree of coherence. It arrives that it is possible to connect the residual *DOP* level of the considered system with the cross-correlation of optical disturbances $E_x(t)$ and $E_y(t)$, without any spectral decomposition[9]. First, we shall invoke the so-called mutual coherence function, which is basic for theory of partial coherence, setting at once the assumption of stationarity of the field:

$$\Gamma_{12}(\tau)=\left\langle E_1(t+\tau)\cdot E_2^*(t)\right\rangle, \tag{13}$$

where the angle brackets indicate time average, as in the statement (7), returning a cross correlation function of two optical disturbances E_1 and E_2. Basically, the terms E_1 and E_2 are associated with wavelets emerging from the two pinholes in the Young interferometer setup, coming together at the observation point with a time delay τ due to different optical paths and having the same linear polarization. This is the most general case, when Γ_{12} is affected simultaneously by the temporal and spatial coherence of the of the two point sources. However, one can interpret E_1 and E_2 terms as the E_x and E_y components in the cross-section plane of a light beam as well; they cannot interfere, but it is not required, indeed. Additionally, if we assume that the $E_x(t)$ and $E_y(t)$ disturbances are obtained by a projection of one specified waveform illuminating our birefringent sample, the symbol τ in the formula (13) also receives a clear interpretation. Thus, from that moment we shall consider the mutual coherence function $\Gamma_{xy}(\tau)$ as a measure of decay of correlation between the x and y components due to the time delay induced by linear birefringence.

Consequently, we shall introduce the complex degree of coherence for the considered setup:

$$\gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{\sqrt{\Gamma_{11}(0)} \cdot \sqrt{\Gamma_{22}(0)}} = \frac{\varepsilon_0}{\mu_0} \frac{\Gamma_{12}(\tau)}{I_1 \cdot I_2}, \qquad 1,2 = x, y, \tag{14}$$

which should be understood as a measure of correlation of two optical disturbances invariant form their intensities: I_1 and I_2. Finally, according to the conventions above and (6):

$$\mathbf{K} = \frac{\mu_0}{\varepsilon_0} \begin{bmatrix} I_x^2 & I_x I_y \gamma_{xy}(\tau) \\ I_x I_y \gamma_{xy}^*(\tau) & I_y^2 \end{bmatrix}, \tag{15}$$

so the formula (10) transforms to:

$$DOP^{out} = \sqrt{1 - \frac{4 \cdot (1 - |\gamma_{xy}(\tau)|^2)}{\left(\sqrt{I_x / I_y} + \sqrt{I_y / I_x}\right)^2}}. \tag{16}$$

As we can see, the residual degree of polarization of our single-crystal (fiber) system depends only on the complex degree of mutual coherence of the polychromatic light components lying in birefringence axes and the power distribution between the axes. These dependencies have been fully confirmed experimentally in recent works with the use of a lithium niobate crystal and a laser diode[12,13].

The most important outcome of the reasoning above is that there is no need to perform any integral during *DOP* calculation. That is because the full information on spectral distribution of the light is included in the complex degree of coherence. In the simplest case, when the medium is illuminated by a totally polarized light, the gamma function in (16) can be replaced by the complex degree of self-coherence of light: $\gamma_{11}(\tau)$, which is obtained directly from the normalized power spectrum density by a reverse Fourier transform (optical equivalent of the Wiener–Khinchin theorem of stationary random processes):

$$\gamma_{11}(\tau) = \int_0^{+\infty} \hat{G}(\omega) \, e^{-i\omega\tau} d\omega, \tag{17}$$

leading to analytical solutions in case of ordinary spectrum profiles like Lorentzian or Gaussian curve. However, within this approach there is no simple way to introduce phase dispersion. In this one-section system, that is the only disadvantage of the complex coherence-based description in comparison to the Mueller-Stokes calculus.

4. DEPOLARIZATION IN A MULTI-SECTION LINEAR RETARDER

It fallows by the analysis provided in the previous section, that a single segment of a linearly birefringent medium can be considered as a type of static depolarizer for polychromatic light. However, as we see from (16), this simple device work efficiently only if the power of the radiation is distributed between the birefringence axes of the sample – we say the depolarizer is nonuniform. In extreme case, when only one of the axes is excited, we obtain no depolarization. Obviously, it is possible to insert properly a polarizer in front of the sample, but this would make the outgoing intensity polarization-sensitive, which is in contradiction with the general purpose of optical depolarizers employment. This is the circumstance that justifies developing multi-segment systems and determines the Lyot depolarizer scheme (Fig. 2).

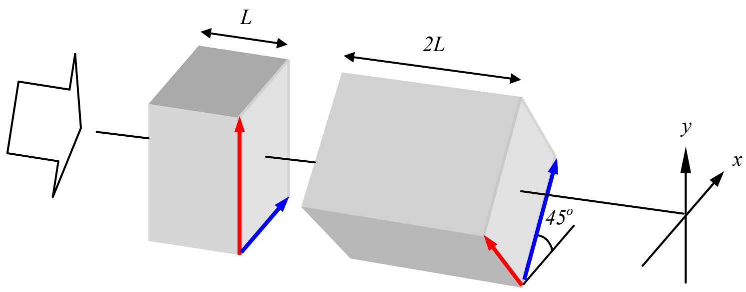

Figure 2. Example of a linearly birefringent system of varying axes orientation: the Lyot depolarizer.

Let us consider the simplest case of a two-section retarder according to the Lyot depolarizer scheme. As before, the Mueller-Stokes calculus can be utilized, this time with support of a Mueller matrix rotating a θ angle of the xy coordinate system counterclockwise:

$$\mathbf{T}(\theta)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos(2\theta) & -\sin(2\theta) & 0\\ 0 & \sin(2\theta) & \cos(2\theta) & 0\\ 0 & 0 & 0 & 1\end{bmatrix}, \tag{18}$$

Then the matrix equation for the normalized Stokes vector (in xy coordinates system) of polychromatic light emerging from the Lyot depolarizer is[2,4]:

$$\mathbf{s}^{out}=\left(\int_0^{+\infty}\mathbf{M}_{eff}(\lambda)\cdot\hat{G}(\lambda)d\lambda\right)\cdot\mathbf{s}^{in}, \qquad \mathbf{M}_{eff}(\lambda)=\mathbf{T}(-45^{\circ})\mathbf{M}_{2L}(\lambda)\,\mathbf{T}(45^{\circ})\,\mathbf{M}_{L}(\lambda)\ , \tag{19}$$

if assumed that every spectral component of the initial beam has the same *SOP* giving $DOP^{in}=1$. $\mathbf{M}_{eff}(\lambda)$ is an effective monochromatic Mueller matrix of the system. Apparently, there is no difficulty to expand this approach for systems of any number of segments and any axes orientation angles.

In order to omit troublesome integrals, the competitive coherence-based approach can be applied. In the particular case of Lyot depolarizer this problem was solved by Burns[5] and expanded for any-angle joints of the birefringent segments by Mochizuki[6]. However, what is important, these solutions operate in the quasi-monochromatic regime only, so there is a significant loss of generality in comparison to the M-S formalism. Those equations are not valid in case of incandescent, fluorescent or supercontinuum light sources. Moreover, just like in the single-segment case, there is also no simple way to take into account the phase dispersion of the medium, which can be an important issue when dealing with relatively wide-spectrum quasi-monochromatic sources like LEDs. Finally, this method is not scalable: description of more complex systems would require new, far more difficult derivations carried out from the very beginning.

It appears that presence of the 45° joint angle significantly reduces the nonuniformity of the system, but it cannot remove it completely for any light spectrum. One of popular metrics characterizing properties of nonuniform depolarizers is the depolarization index[14,15]:

$$DI(\mathbf{M}) = \frac{\sqrt{\sum_{i,j=1}^{4} \mathbf{M}_{ij}^2 - \mathbf{M}_{00}^2}}{\sqrt{3}\mathbf{M}_{00}}, \qquad \mathbf{M} = \int_0^{+\infty} \mathbf{M}_{eff}(\lambda) \cdot \hat{G}(\lambda) d\lambda, \tag{20}$$

where $\mathbf{M}$ denotes the effective polychromatic Mueller matrix of the system. This quantity can be interpreted as the Euclidean distance from the ideal depolarizer to the effective Mueller matrix of the considered system and constitutes an equivocal metric in terms of the initial polarization state. However, we should emphasize here that in case of Lyot-like multi-segment depolarizers the *DOP* of exiting light has an additional degree of freedom, which is the *SOP* at the joints between the birefringent sections. This is an often overlooked issue in the literature that the Lyot depolarizer, specially in case of narrow-band light sources, not only is nonuniform (initial *SOP*-sensitive) but also tend to exhibit significant output *DOP* variations under subtle environmental perturbations like temperature and instability of the central wavelength of the light, even if the initial *SOP* remains unchanged. Due to these reasons the Lyot depolarizer is used to be considered analytically almost only in the full-depolarization limit, which is not fully reachable in real applications. From the practical point of view it is essential to know when to expect the *DOP* noise and to predict its amplitude, just to find the optimal depolarizing system length for given light source, for example. On the other hand, one can carry an issue of a static depolarizer configuration itself that would ensure the best invariance to any perturbations with the minimum size of the depolarizer[8].

5. NUMERICAL EXPERIMENTS ON THE LYOT DEPOLARIZER

In order to illustrate the most significant properties of *DOP* evolution in static multi-segment linearly birefringent media of varying (along the propagation axis) birefringence axes orientation we perform a set of numerical tests for a two-segment system deriving from the Lyot depolarizer. As a starting point, we consider the standard Lyot's scheme (Fig. 2) with parameters shown in Tab. 1. The investigations are carried out for two power spectrum profiles of the illuminating light in parallel, which are depicted in Fig. 3a). Figure 3b) shows absolute values of complex degree of self-coherence: $|\gamma_{11}(\tau)|$ corresponding to the considered optical spectra. The black continuous line in Fig. 3b) is obtained by use of inverse Fourier series performed on the discrete spectrum profile:

$$\gamma_{11}(\tau) = \frac{\sum_{n=1}^{N} G(\omega_n) \cdot e^{-i\omega_n \tau}}{\sum_{n=1}^{N} G(\omega_n)}, \tag{21}$$

while the red line in Fig. 3b) is an exponential curve being a continuous Fourier transform of the Lorentzian peak.

Table 1. Parameters of the Lyot depolarizer.

Linear birefringence of the medium B [1]	First segment length L_1 [mm]	Second segment length L_2 [mm]	twist angle between optical axes of the segments θ [°]
0.085	10	20	45

The Lorentzian curve has been adjusted to the experimental spectrum in that way to fit their central wavelengths together and additionally to make their degrees of self-coherence equal after passing the first section (i.e., for the time delay τ induced by this section – first green square in Fig. 3B). The definition of central wavelength of the irregular spectrum is based on the condition that the normalized Stokes vectors of the polychromatic beam and a monochromatic wave with this central wavelength have to be identical after passing the first section L_1.

In the following analysis we will consider systems illuminated always by a linearly polarized beam with a certain azimuth α with respect to the x (fast) axis of the first section of length L_1. This implies $DOP^{in} = 1$.

a) b)

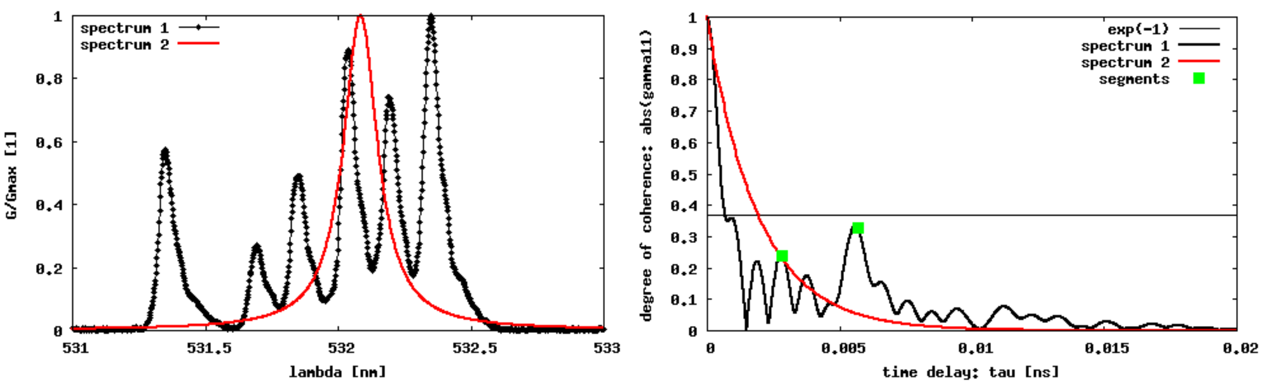

Figure 3. a) Normalized power spectrum density profiles used in simulations: discrete "spectrum 1" obtained experimentally and its Lorentzian analog, b) corresponding complex degree of self-coherence functions (absolute values are plotted); green squares mark time retardations induced by the two segments of the Lyot depolarizer when illuminated separately (with parameters shown in Tab. 1)

5.1 DOP distribution along the two-segment depolarizer in function of *SOP* at joint of the segments.

First, let us examine how the degree of polarization behaves inside the depolarizer along its propagation axis (or, equivalently, at the output of a system of varying total length) in a special case of $\alpha = 45^\circ$ (α is the azimuth of initial linear polarization) and $\theta = 45^\circ$. Figure 4a) depicts the result in a situation when the length L_1 was slightly adjusted in that way to ensure the polarization state at the joint of the sections (*SOP* of the fully polarized factor of the beam) being the same as at the input plane of the system, while Fig. 4b) relates to the system with a circular polarization at the joint. Several properties are worth noting here:

i) The DOP curve along the first segment reproduces the course of the $|\gamma_{11}(\tau)|$ function, which stays in agreement with the formula (16) in case of $I_x = I_y$.

ii) When $L_2 = L_1$ we observe the partial depolarization compensation, which means that the DOP fading process is being inverted after entrance to the segment with axes of birefringence oriented differently. For $\theta = \pm 90^\circ$ we would obtain full compensation.

iii) Full depolarization is not possible in case a), regardless of how long the second section would be. The other extreme case provides *DOP* falling asymptotically to zero with increasing L_2.

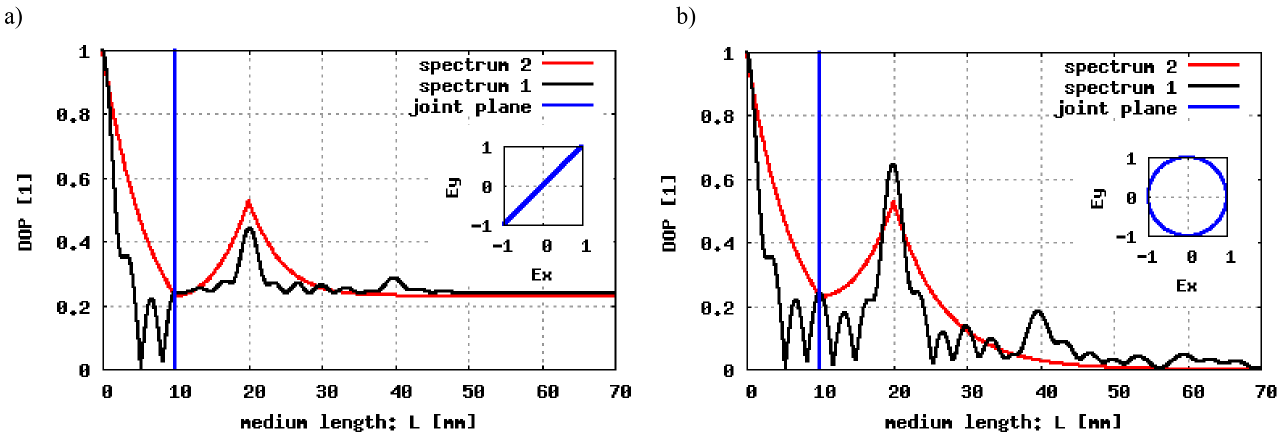

Figure 4. DOP evolution along a two-section system with $\theta = 45^\circ$ and initial azimuth $\alpha = 45^\circ$: a) in case when the *SOP* of polarized fraction of the beam reproduces itself at the joint plane, b) in case of circular polarization at the joint plane.

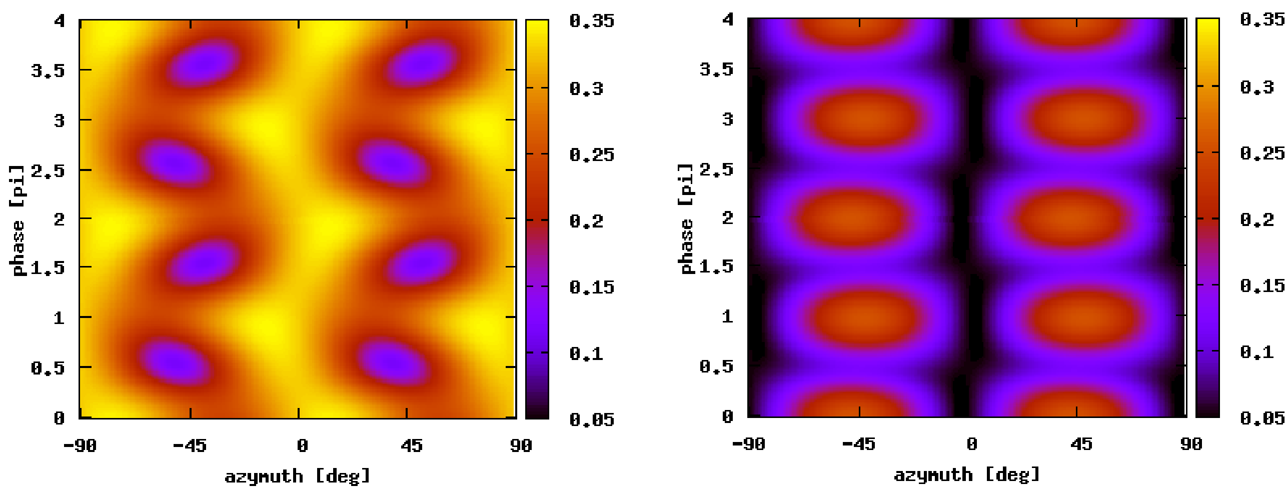

Figure 5. *DOP* diagrams describing the standard Lyot depolarizer for the "spectrum 1" (left) and "spectrum 2" (right). "Azymuth" refers to α, while "phase" determines polarization state of the fully polarized fraction of the beam at the joint plane.

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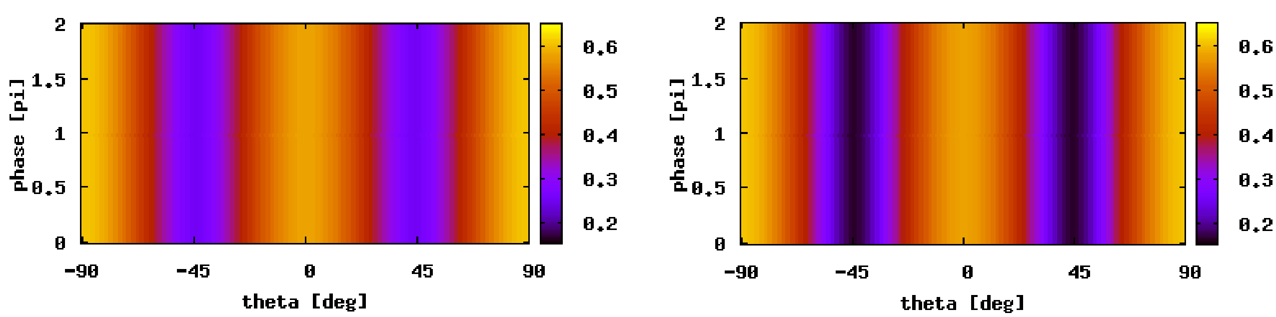

Figure 6. Depolarization index (*DI*) diagrams for the Lyot depolarizer configuration given by parameters in Tab. 1 for different spectra of the light: "spectrum 1" (left) and "spectrum 2" (right). "Theta" indicates the twist angle between optical axes of the two birefringent sections

5.2 DOP as a function of the input linear polarization azimuth and *SOP* at joint of the segments.

If we are interested in unequivocal description of a system with fixed parameters (Tab. 1) being disturbed by some external conditions we must take into account variations of the *SOP* of fully polarized fraction of the beam at the joint of the segments. This variations can be caused also by instability of the central wavelength of the illuminating light. Apparently, they have significant influence on the residual *DOP* of the Lyot depolarizer, if the first section does not provide almost full depolarization (remark 3 form the previous subsection). In Fig. 5 the intrinsic *SOP* at the joint of the sections is described by the "phase" parameter, which indicates an additional retardation applied to the central wavelength caused by variations of the time delay in the first segment. Phase equal to zero refers to the linear polarization at the joint, $\pi/2$ gives the most elliptical polarization possible for given α, π drives to the linear polarization again, but with different azimuth, and so on. We can see that the vertical cross-sections of the diagrams for $\alpha = \pm 45^\circ$ lead to the same courses $DOP(phase)$, which stays in agreement with relations plotted in Fig. 4. This is a consequence of the Lorentzian spectrum adjusting condition, ensuring values $|\gamma_{11}(\tau_1)|$ being equal for both spectra. However, this is the only similarity between the two diagrams. That is due to the irregular structure of "spectrum 1" and its special property making the longer section less depolarizing then the first one (second green square in Fig. 3b).

5.3 Depolarization index as a function of the twist angle between birefringence axes and *SOP* at the joint of the segments.

Finally, we shall examine which values of the axes misalignment angle at the sections joint: θ are optimal for the two-section depolarizer. For this purpose we utilize the depolarization index (*DI*) which characterizes the depolarizing capability of the system independently from the polarization state of light at the input plane. In Fig. 6 we see that *DI* takes minima in $\theta = \pm 45^\circ$, as expected. Additionally it arrives that in terms of *DI* parameter our Lyot depolarizer is absolutely insensitive to intrinsic *SOP* variations, regardless of the power spectral distribution of light (Fig. 6). However, it is clearly seen that the light with "spectrum 2" is depolarized stronger, which stays in agreement with the diagrams in Fig. 5.

6. CONCLUSIONS

Physical fundamentals of depolarization phenomenon in uniaxial anisotropic media have been provided in detail, invoking different points of view on the problem with particular emphasis on all the assumptions that must be applied in light modeling in individual theoretical approaches. In order to illustrate invoked relations concerning the multi-section linearly birefringent systems we performed a set of numerical experiments on the Lyot depolarizer illuminated by linearly polarized polychromatic beam of two different power spectrum distributions. Obtained results lead to the following conclusions:

i) The optimal configuration of the classic Lyot depolarizer has been confirmed.

ii) Residual *DOP* of systems similar to the Lyot depolarizer is sensitive to the intrinsic polarization state variations, which can be caused by temperature fluctuations or instability of the used light source. This property cannot be predicted with the use of depolarization index only.

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