

Calculus Derivatives Chain Rule

Consider the equation $y = x^2$.

Differentiating y with respect to (w.r.t.) x , we get $\frac{dy}{dx}$.

Consider the equation $s^2 = x^2 = y^2$.

Differentiating the equation w.r.t. t , we get $2s\left(\frac{ds}{dt}\right) = 2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right)$.

Why is this so?

We want to differentiate s^2 w.r.t. t , but we can only differentiate s^2 w.r.t. s because that is the only available term.

Thus, $\frac{d}{ds}(s^2) = 2s$.

Since what we want is $\frac{d}{dt}(s^2)$, we can get it using the chain rule, which states that $\frac{d}{dt}(s^2) = \frac{d}{ds}(s^2) \times \frac{ds}{dt} = 2s\left(\frac{ds}{dt}\right)$.

Therefore, applying the same rule to x^2 and y^2 , we get the equation $2s\left(\frac{ds}{dt}\right) = 2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right)$.