

# Calculus Derivatives Chain Rule

Consider the equation  $y=x$  .

Differentiating  $y$  with respect to (w.r.t.)  $x$  , we get  $\frac{dy}{dx}$  .

Consider the equation  $s^2=x^2+y^2$  .

Differentiating the equation w.r.t.  $t$  , we get  $2s(\frac{ds}{dt})=2x(\frac{dx}{dt})+2y(\frac{dy}{dt})$  .

Why is this so?

We want to differentiate  $s^2$  w.r.t.  $t$  , but we can only differentiate  $s^2$  w.r.t  $s$  because that is the only available term.

Thus,  $\frac{d}{ds}(s^2)=2s$  .

Since what we want is  $\frac{d}{dt}(s^2)$  , we can get it using the chain rule, which states that  $\frac{d}{dt}(s^2)=\frac{d}{ds}(s^2)\times\frac{ds}{dt}=2s(\frac{ds}{dt})$  .

Therefore, applying the same rule to  $x^2$  and  $y^2$  , we get the equation  $2s(\frac{ds}{dt})=2x(\frac{dx}{dt})+2y(\frac{dy}{dt})$  .