Dot Product

Definition

Let $u=(u_1,u_2,\ldots,u_n), v=(v_1,v_2,\ldots,v_n)$ be vectors in $\mathbb{R}^n$.

$u\cdot v=u_1v_1+u_2v_2+\ldots+u_nv_n$

Basic Properties

Let u,v,w be vectors in $\mathbb{R}^n$ and c a scalar.

1. $u\cdot v=v\cdot u$
2. $(u+v)\cdot w=u\cdot w+v\cdot w$ and $w\cdot(u+v)=w\cdot u+w\cdot v$
3. $(cu)\cdot v=u\cdot(cv)=c(u\cdot v)$
4. $\|cu\|=|c|\|u\|$
5. $u\cdot u\geq 0$ and $u\cdot u=0$ if and only if $u=0$.