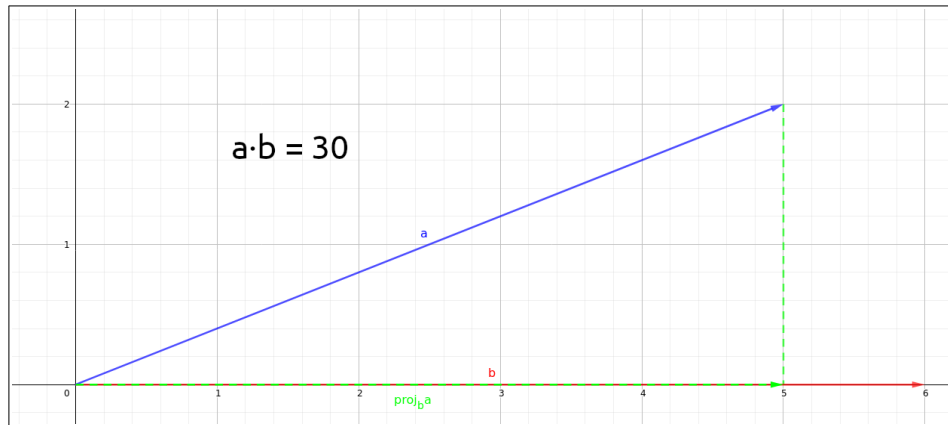


Vector Projection

The formula for projecting vector \vec{a} onto vector \vec{b} is as follows: $proj_{\vec{b}} \vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \vec{b}$.



But what does it really mean? We can break the formula down into simpler terms by rewriting it as $proj_{\vec{b}} \vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \times \frac{\vec{b}}{|\vec{b}|}$.

To find the projection of \vec{a} onto \vec{b} , we need the length and direction of the projected vector.

Firstly, let's find the length of $proj_{\vec{b}} \vec{a}$. $\vec{a} \cdot \vec{b}$ gives us the length of $proj_{\vec{b}} \vec{a}$ multiplied by the length of \vec{b} . To get the length of $proj_{\vec{b}} \vec{a}$, we divide $\vec{a} \cdot \vec{b}$ by the length of \vec{b} which gives us $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$. And there we have it, the length of $proj_{\vec{b}} \vec{a}$ has been found.

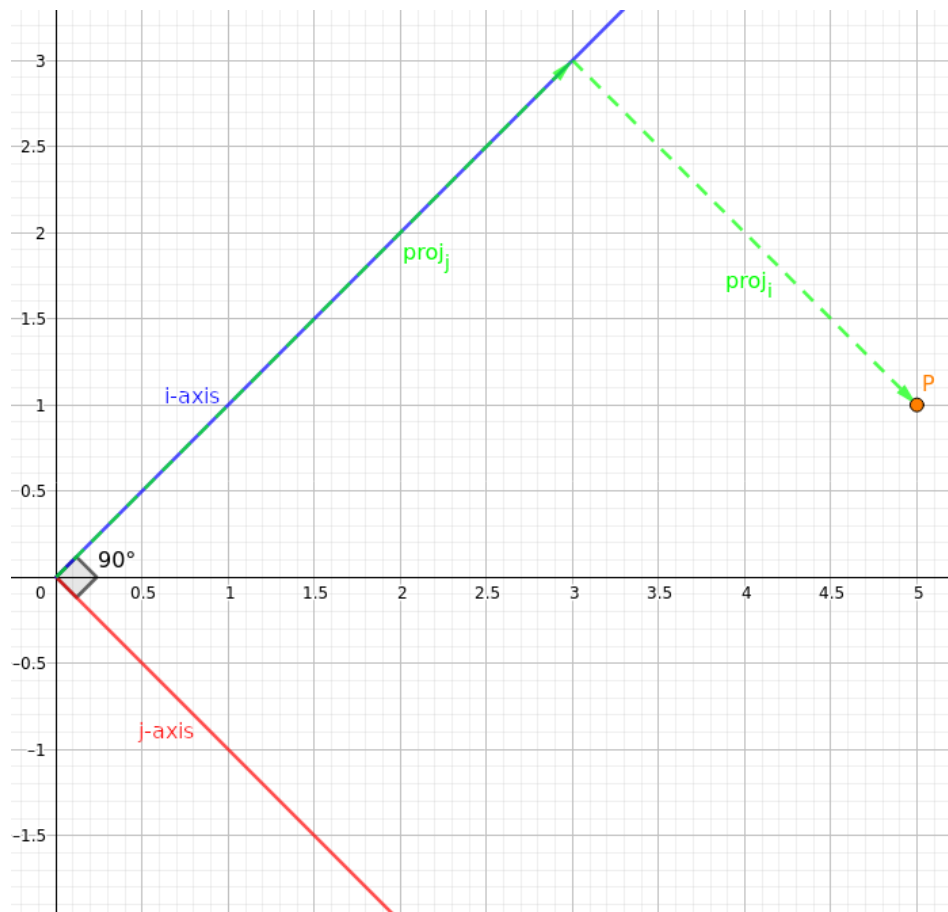
Next, we want to find the direction of the vector. Since \vec{a} is projected onto \vec{b} , we know that $proj_{\vec{b}} \vec{a}$ is in the same direction as \vec{b} . And since we have the length of $proj_{\vec{b}} \vec{a}$, we just need to multiply it with a unit vector in the direction of \vec{b} . To normalise \vec{b} , we simply divide \vec{b} by its length. This gives us $\frac{\vec{b}}{|\vec{b}|}$. And thus, we have found the direction of $proj_{\vec{b}} \vec{a}$.

Finally, multiply the direction and length of $proj_{\vec{b}} \vec{a}$ together, and we have the formula

$$proj_{\vec{b}} \vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \times \frac{\vec{b}}{|\vec{b}|} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \vec{b}.$$

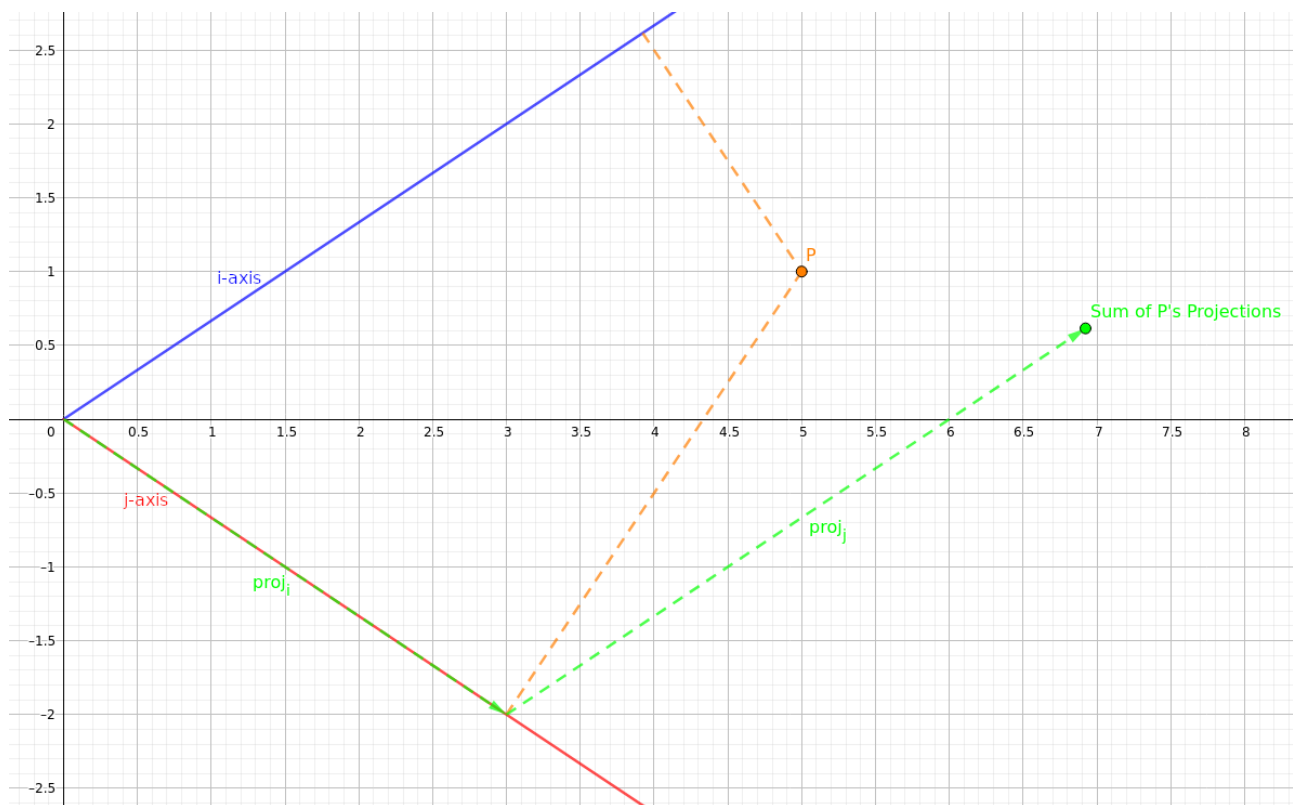
Representing a point as the sum of its projection onto the axes.

For any arbitrary orthogonal axes, a point can be represented as the sum of its projection into the axes.



As can be seen in the figure above, if we take any arbitrary orthogonal axes i and j to be our axes, we can represent point P as the sum of its projection onto axes i and j . Similarly in 3D, as long as all 3 axes are orthogonal, we can represent point P as the sum of its projection onto axes i and j .

However, the above is not true for non-orthogonal axes, where a point cannot be represented as the sum of its projection into the axes.



As can be seen in the figure, point P cannot be represented as the sum of its projection onto axes i and j . Similarly in 3D, as long as 2 of the axes are not orthogonal, point P cannot be represented as the sum of its projection onto axes i and j .