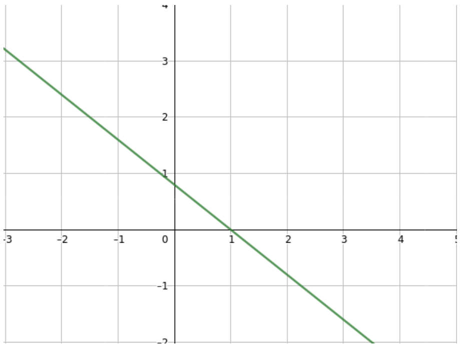
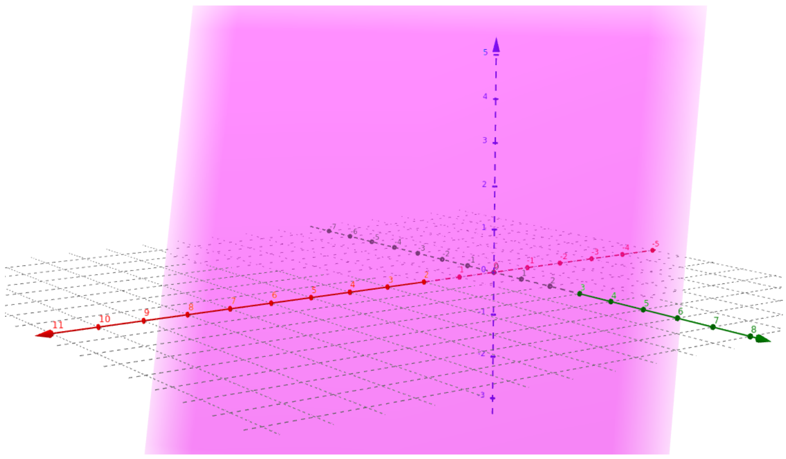
Linear Systems & Augmented Matrices

Geometrical Representation of Solutions of Linear Equations

Given an equation, we can represent its solutions geometrically.
For example, given the formula $4x+5y=4$, we can represent all the solutions as a line.



Similarly, for the general equation $3x+2y+z=6$, we can represent it's solutions as a plane.



Given n real numbers $s_1, s_2, \ldots, s_n$, we say that $x_1=s_1, x_2=s_2, \ldots, x_n=s_n$ is a solution to a linear equation $a_1 x_1=s_1, a_2 x_2=s_2, \ldots, a_n x_n=b$ if the equation is satisfied when we substitute the values into the equation accordingly.

Example: $a_1 s_1+a_2 s_2+\ldots+a_n s_n=b$

General Solutions of Linear Equations

Consider the linear equation $4x-2y=1$ which has infinitely many solutions.

The general solution is
$x=t$
$y=2t-0.5$ where t is an arbitrary parameter.

We can also write the general solution as
$x=0.5s+0.25$
$y=s$ where s is an arbitrary parameter.

Augmented Matrices

A system of linear equations can be represented by an augmented matrix.

$$\begin{array}{l} 1x+2y+3z=11 \\ 4x+5y+6z=16 \\ 7x+8y+9z=63 \end{array} \text{ can be represented as } \left(\begin{array}{ccc|c} 1 & 2 & 3 & 11 \\ 4 & 5 & 6 & 16 \\ 7 & 8 & 9 & 63 \end{array}\right).$$

Homogeneous Linear Systems

A system of linear equations is homogeneous if all the constant terms are zero.

$$\begin{array}{l} 1x+2y+3z=0 \\ 4x+5y+6z=0 \\ 7x+8y+9z=0 \end{array} \text{ or } \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 0 \\ 7 & 8 & 9 & 0 \end{array}\right)$$

A homogeneous linear system always has the origin as the solution. The origin is called the trivial solution. Any other solution is called a non-trivial solution. Since any linear system may only have 0, 1 or inifinte number of solutions, and a homogenous linear system always has the origin as its solution, we can deduce that a homogenous linear system will either have 1 or inifinite number of solutions.