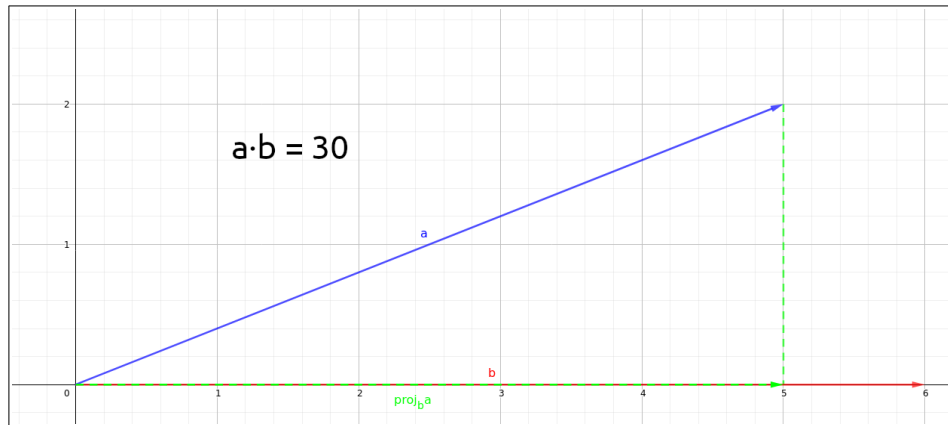


# Vector Projection

The formula for projecting vector  $\vec{a}$  onto vector  $\vec{b}$  is as follows:  $proj_b a = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \vec{b}$ .



But what does it really mean? We can break the formula down into simpler terms by rewriting it as  $proj_b \vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \times \frac{\vec{b}}{|\vec{b}|}$ .

To find the projection of  $\vec{a}$  onto  $\vec{b}$ , we need the length and direction of the projected vector.

Firstly, let's find the length of  $proj_b \vec{a}$ .  $\vec{a} \cdot \vec{b}$  gives us the length of  $proj_b \vec{a}$  multiplied by the length of  $\vec{b}$ . To get the length of  $proj_b \vec{a}$ , we divide  $\vec{a} \cdot \vec{b}$  by the length of  $\vec{b}$  which gives us  $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$ . And there we have it, the length of  $proj_b \vec{a}$  has been found.

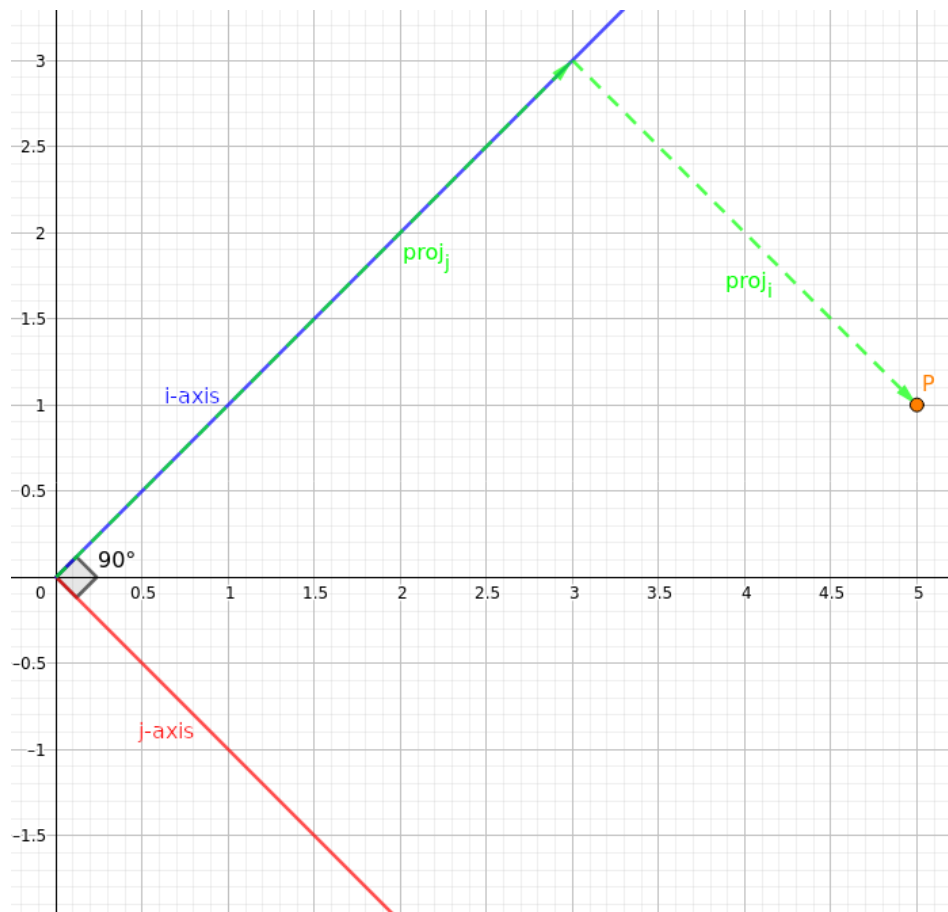
Next, we want to find the direction of the vector. Since  $\vec{a}$  is projected onto  $\vec{b}$ , we know that  $proj_b \vec{a}$  is in the same direction as  $\vec{b}$ . And since we have the length of  $proj_b \vec{a}$ , we just need to multiply it with a unit vector in the direction of  $\vec{b}$ . To normalise  $\vec{b}$ , we simply divide  $\vec{b}$  by its length. This gives us  $\frac{\vec{b}}{|\vec{b}|}$ . And thus, we have found the direction of  $proj_b \vec{a}$ .

Finally, multiply the direction and length of  $proj_b \vec{a}$  together, and we have the formula

$$proj_b \vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \times \frac{\vec{b}}{|\vec{b}|} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \vec{b}.$$

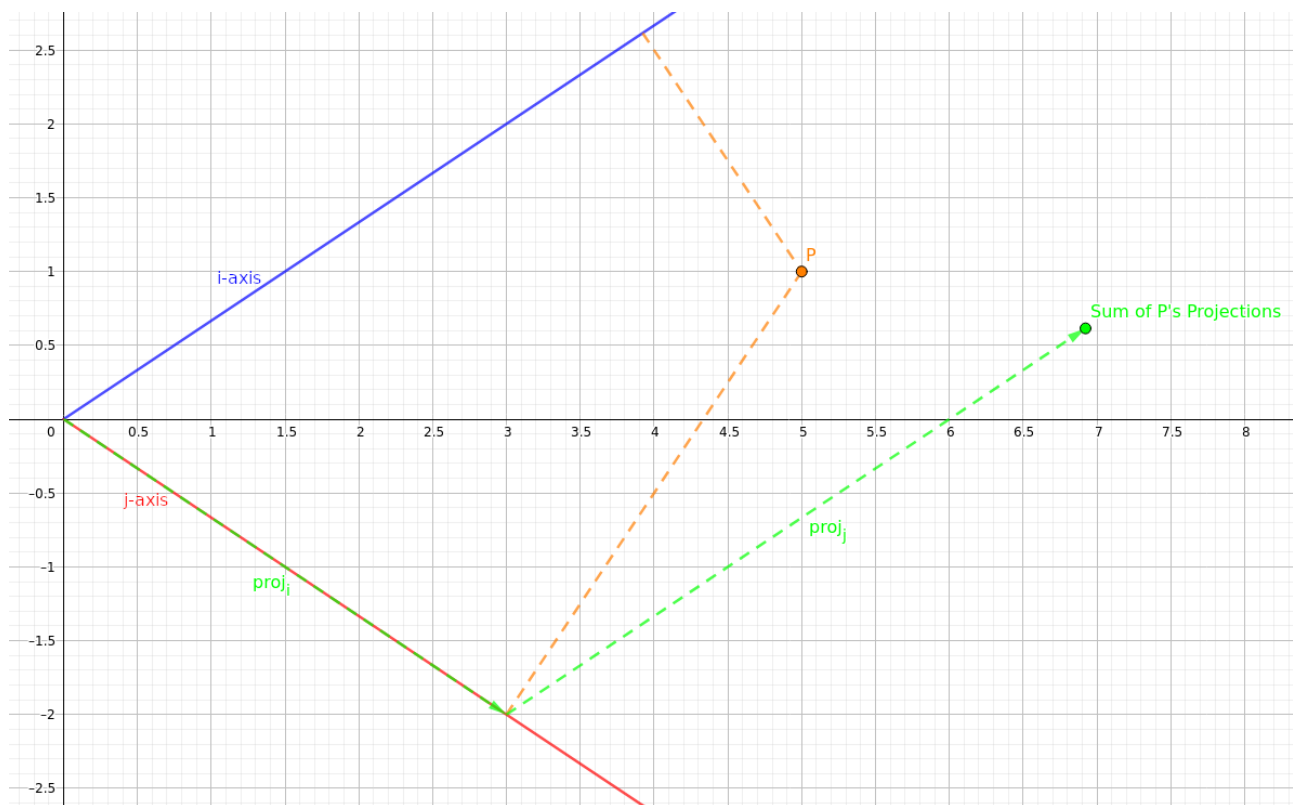
# Representing a point as the sum of its projection onto the axes.

For any arbitrary orthogonal axes, a point can be represented as the sum of its projection into the axes.



As can be seen in the figure above, if we take any arbitrary orthogonal axes  $i$  and  $j$  to be our axes, we can represent point  $P$  as the sum of its projection onto axes  $i$  and  $j$ . Similarly in 3D, as long as all 3 axes are orthogonal, we can represent point  $P$  as the sum of its projection onto axes  $i$  and  $j$ .

However, the above is not true for non-orthogonal axes, where a point cannot be represented as the sum of its projection into the axes.



As can be seen in the figure, point  $P$  cannot be represented as the sum of its projection onto axes  $i$  and  $j$ . Similarly in 3D, as long as 2 of the axes are not orthogonal, point  $P$  cannot be represented as the sum of its projection onto axes  $i$  and  $j$ .