

Introduction to CNN

Convolutional Neural Network



Review: Convolution

- Convolution is a mathematical way of combining two signals to form a third signal
- As we saw in our previous lectures, it is one of the most important techniques in signal processing
- In case of **2D** data (grayscale images), the convolution operation between a filter $W^{k \times k}$ and an image $X^{N_1 \times N_2}$ can be expressed as:

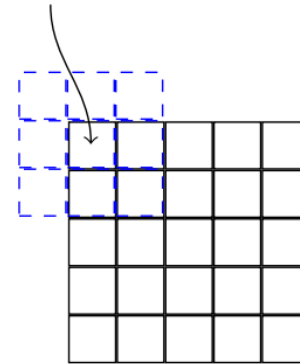
$$Y(i, j) = \sum_{u=-k}^k \sum_{v=-k}^k W(u, v) X(i - u, j - v)$$

Review: Convolution

- More generally, given a $m_1 \times m_2$ filter K , we can write it as:
- This allows kernel to be centered on pixel of interest

$$S_{ij} = (I * K)_{ij} = \sum_{a=\lfloor -\frac{m}{2} \rfloor}^{\lfloor \frac{m}{2} \rfloor} \sum_{b=\lfloor -\frac{n}{2} \rfloor}^{\lfloor \frac{n}{2} \rfloor} I_{i-a, j-b} K_{\frac{m}{2}+a, \frac{n}{2}+b}$$

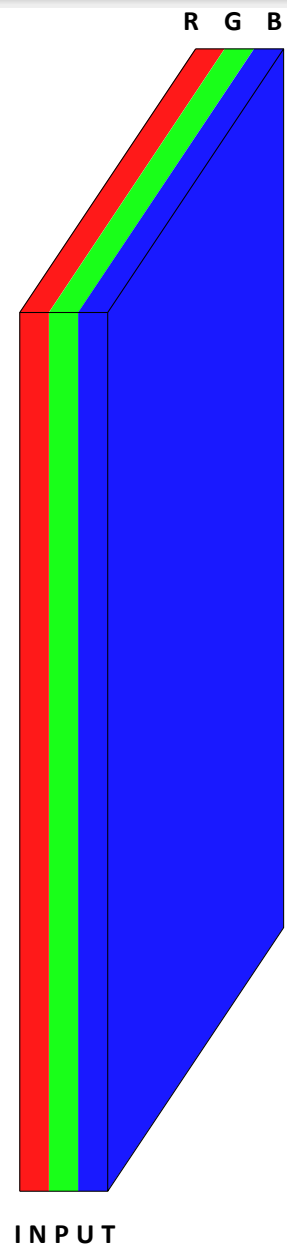
pixel of interest



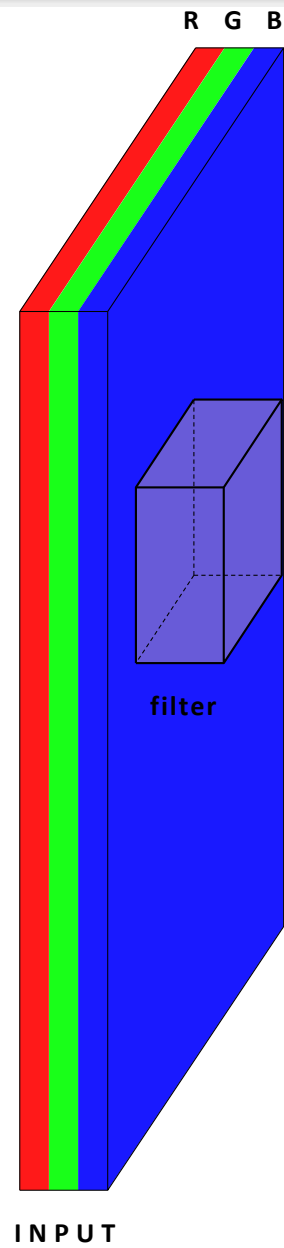
Pause and ponder

- In the 1D case, we slide a one dimensional filter over a one dimensional input
- In the 2D case, we slide a two dimensional filter over a two dimensional output
- What would happen in the 3D case, where images have RGB channels?

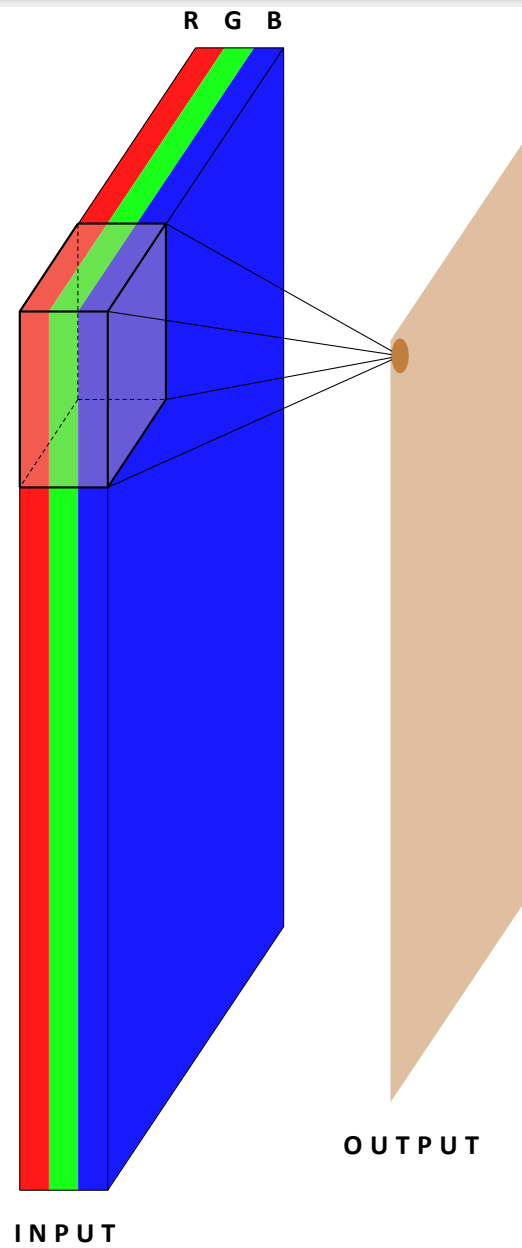




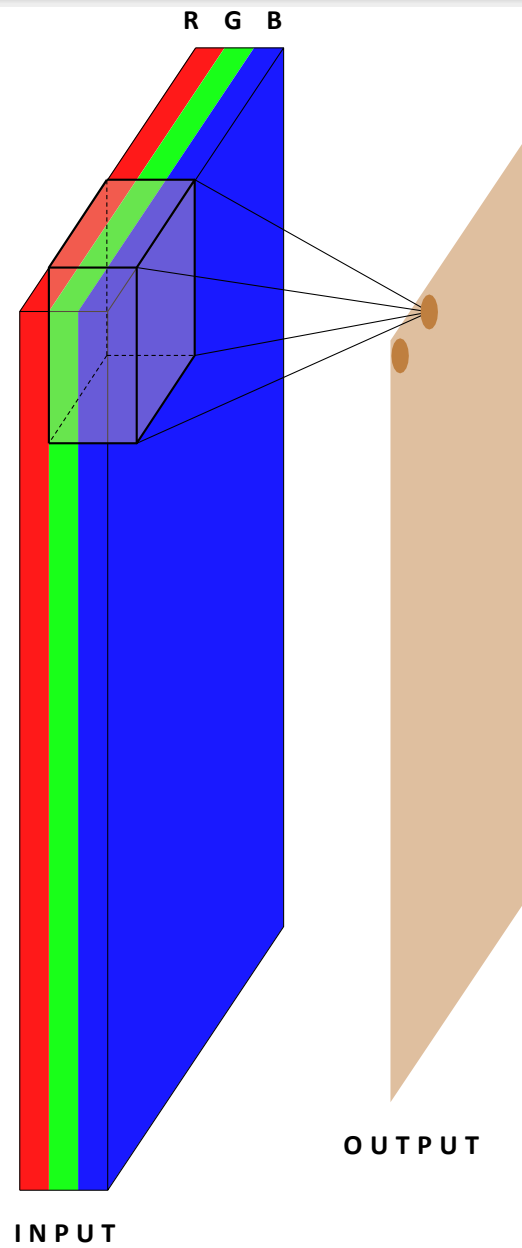
- What would a 3D filter look like?



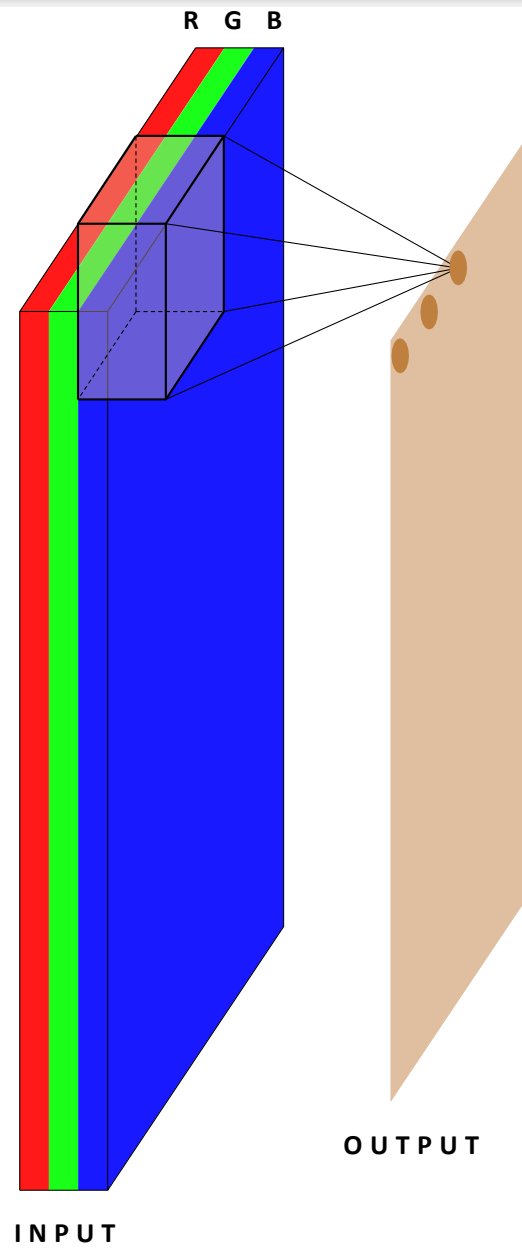
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- It will be 3D and we will refer to it as a volume



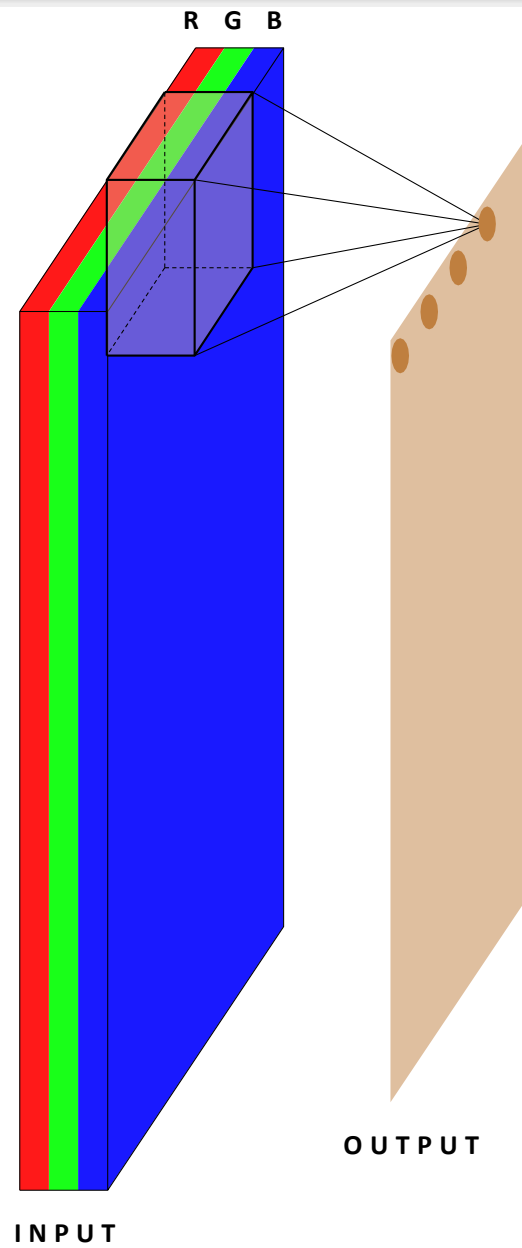
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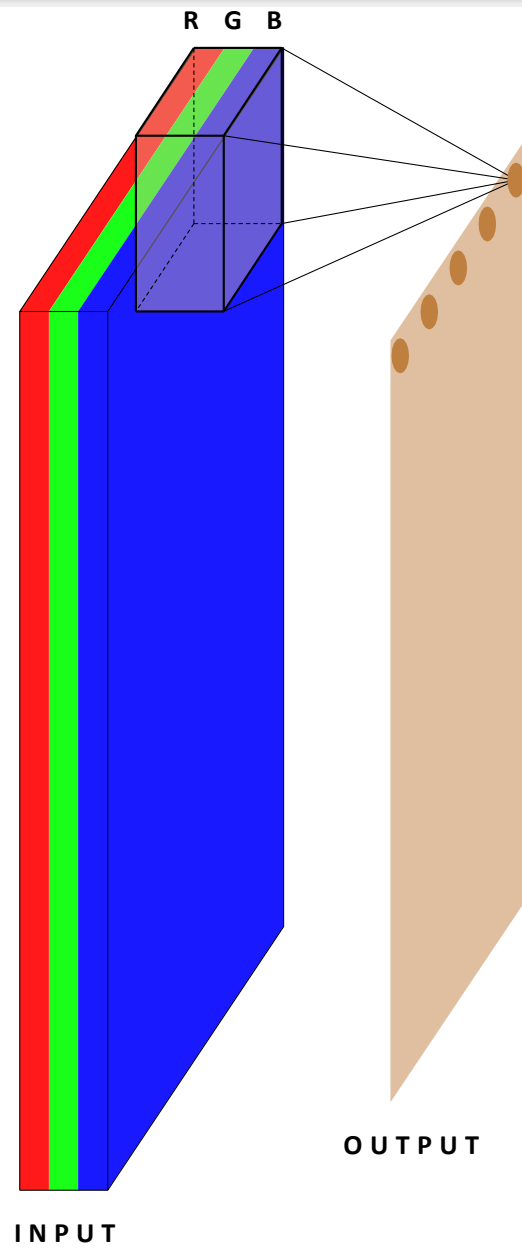
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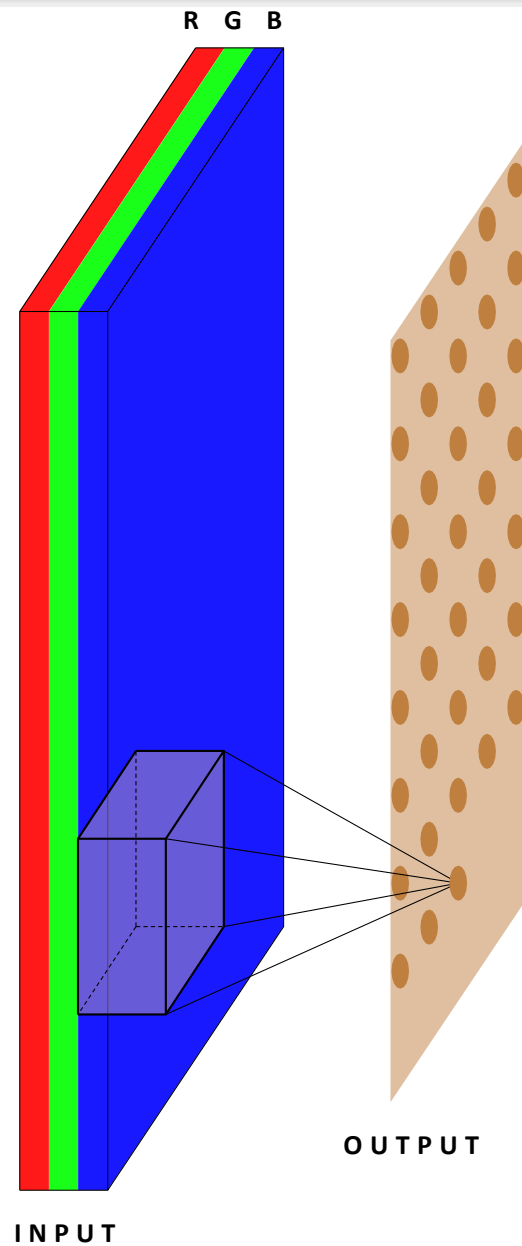
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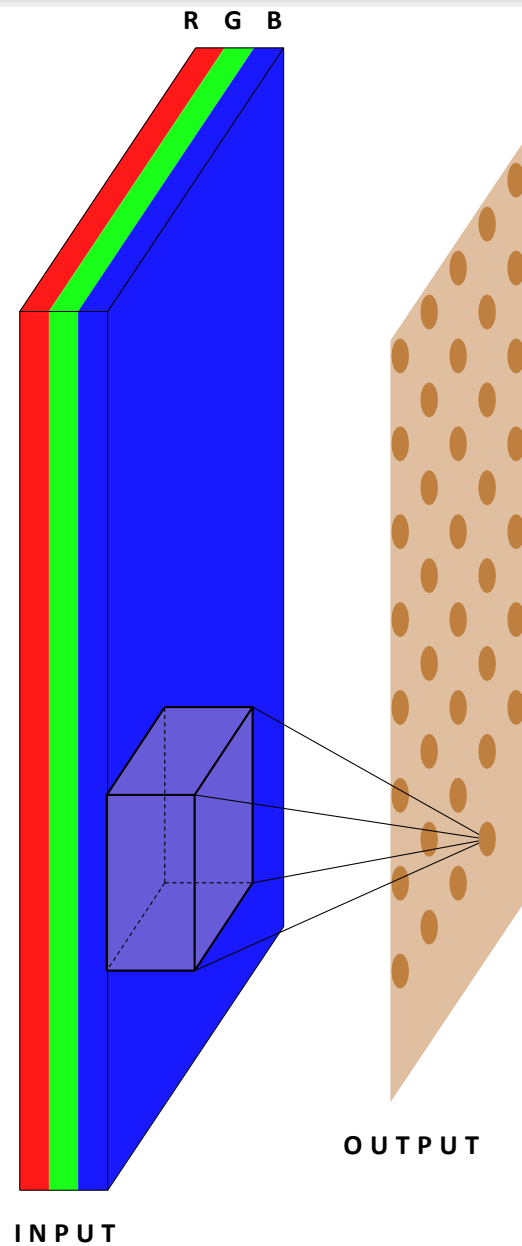
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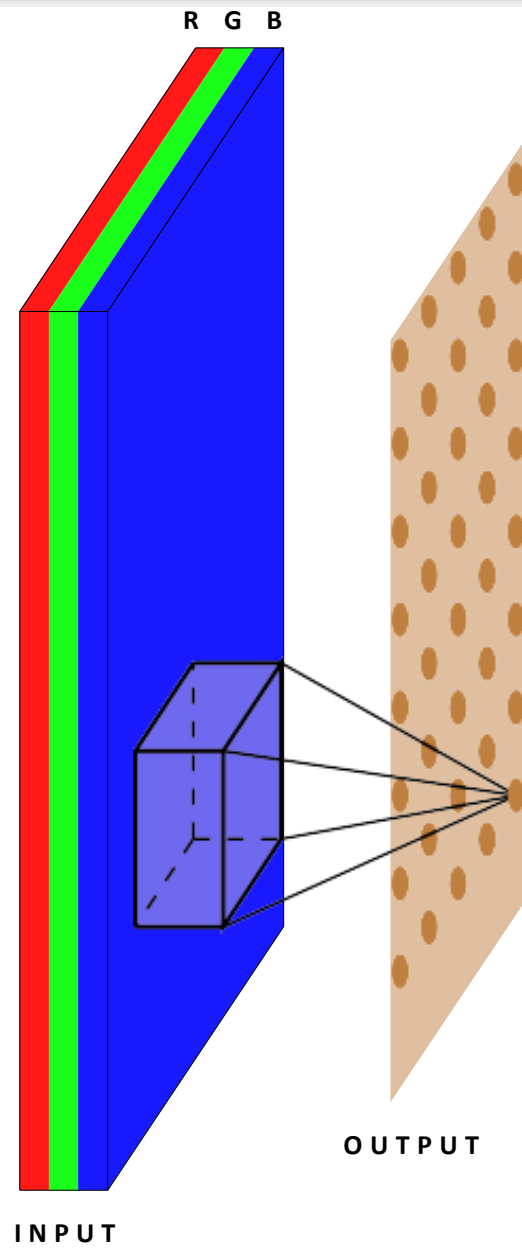
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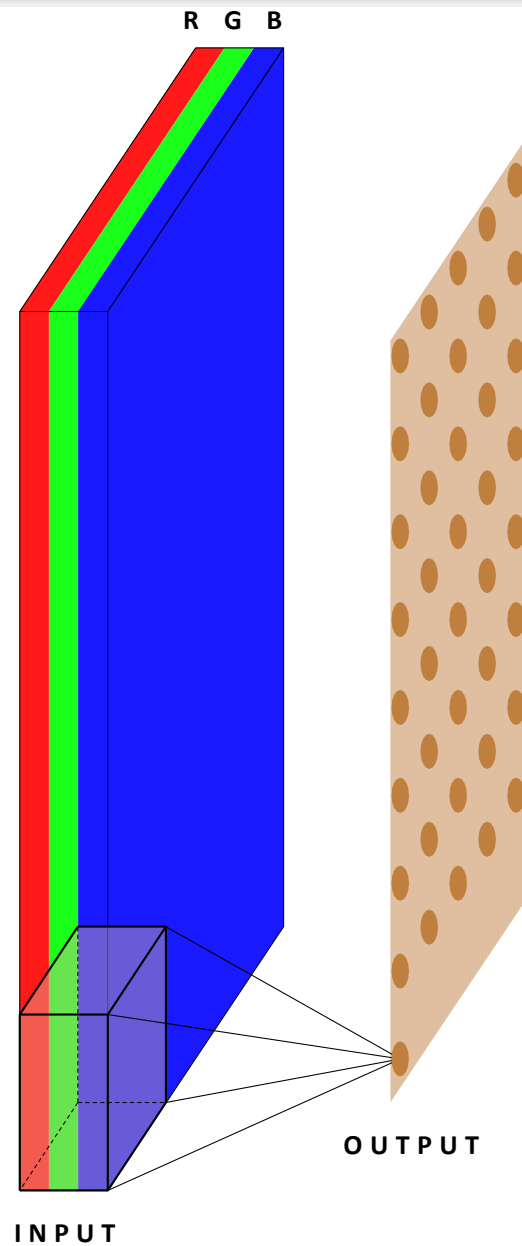
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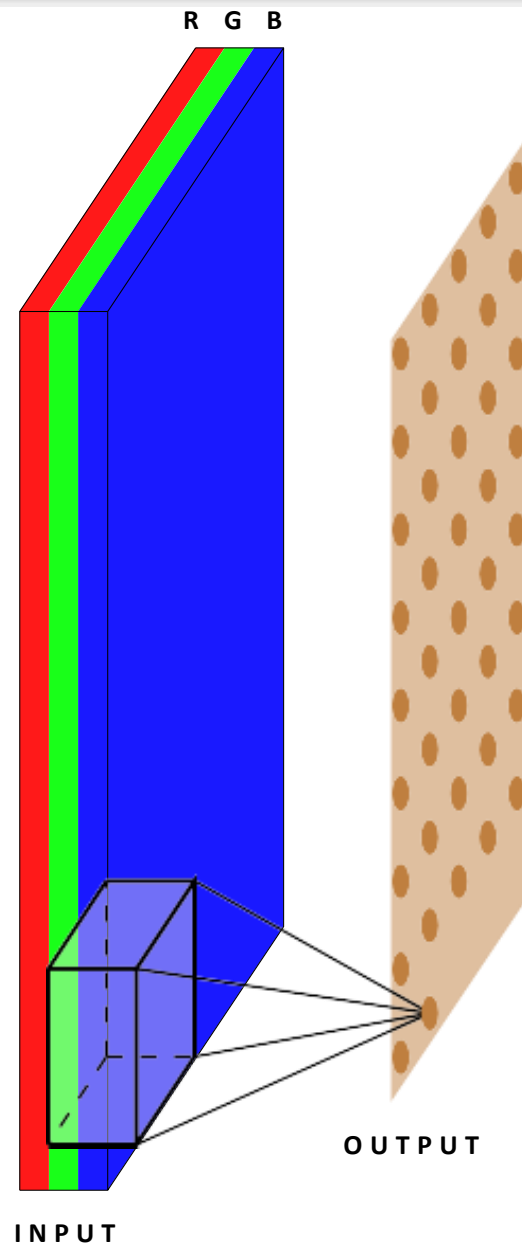
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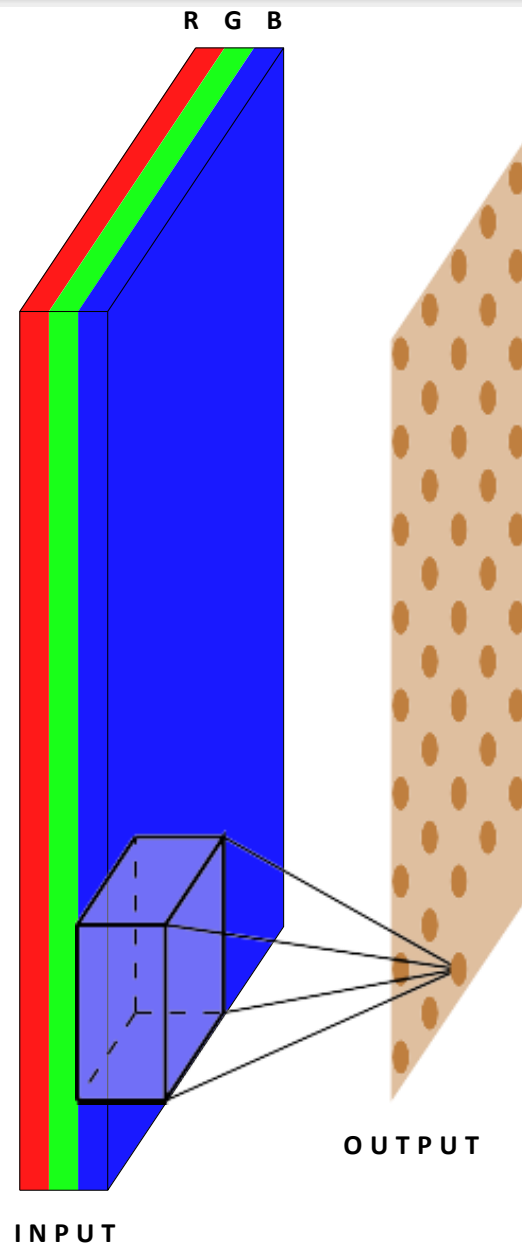
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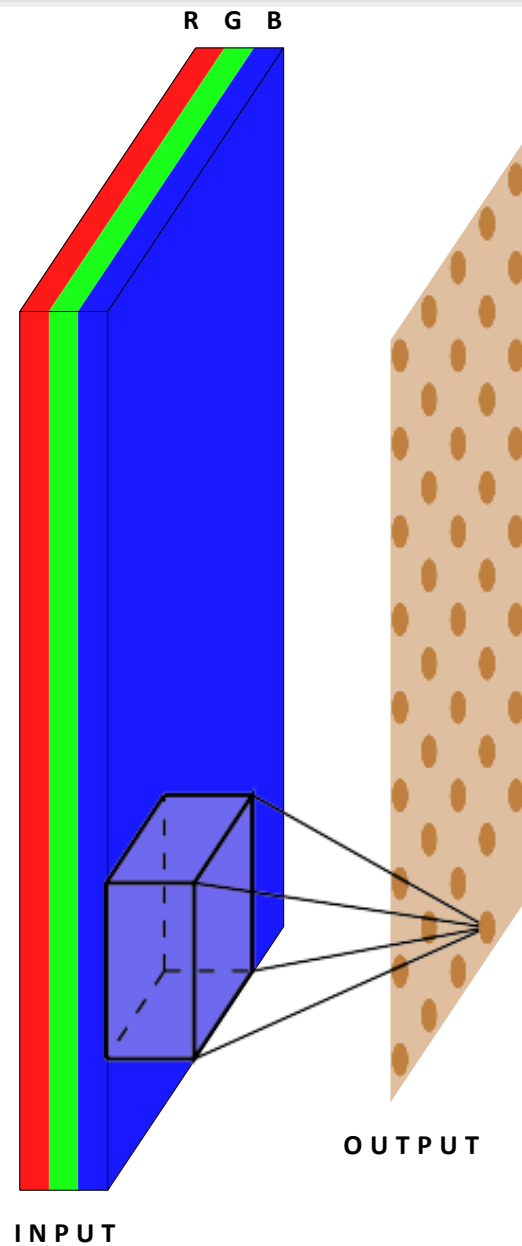
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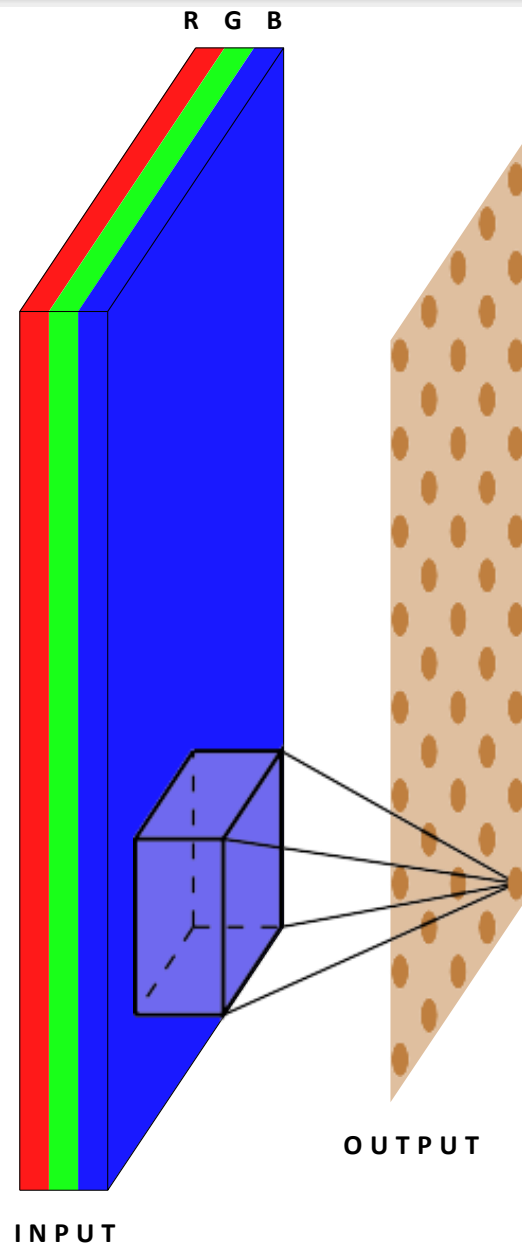
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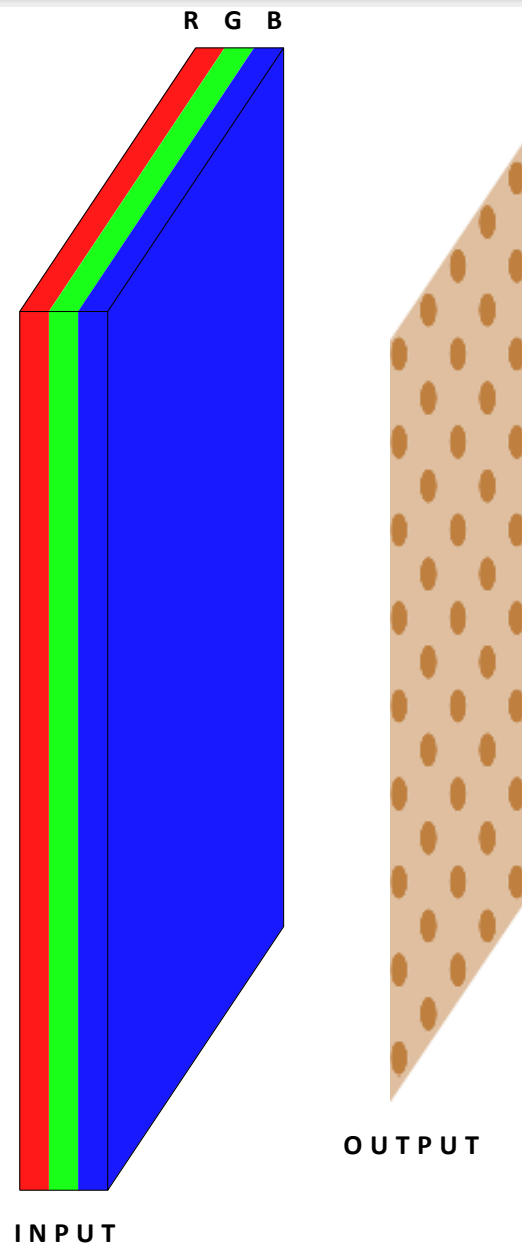
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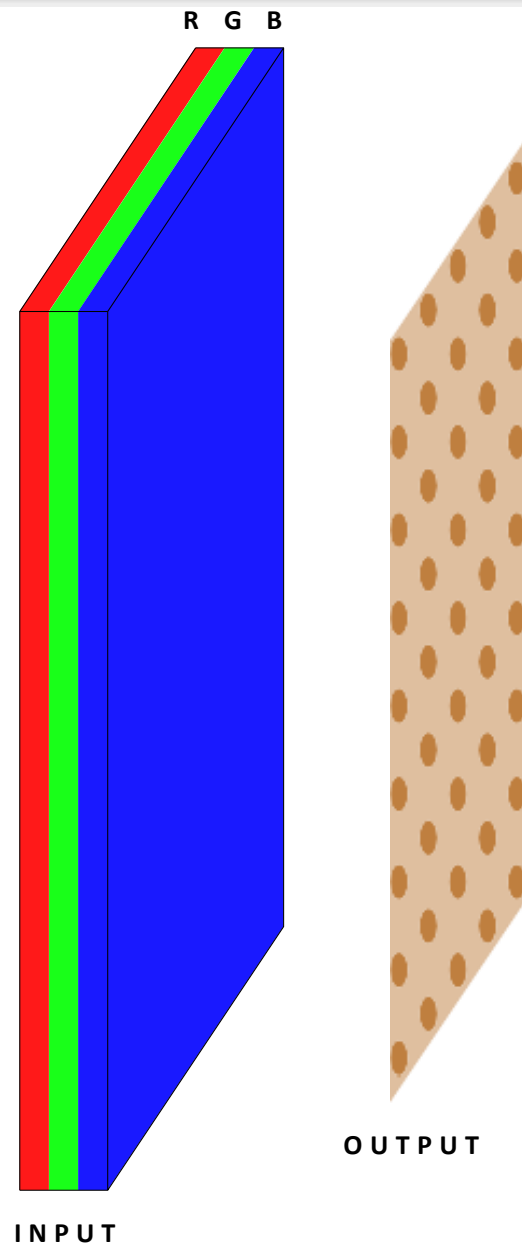
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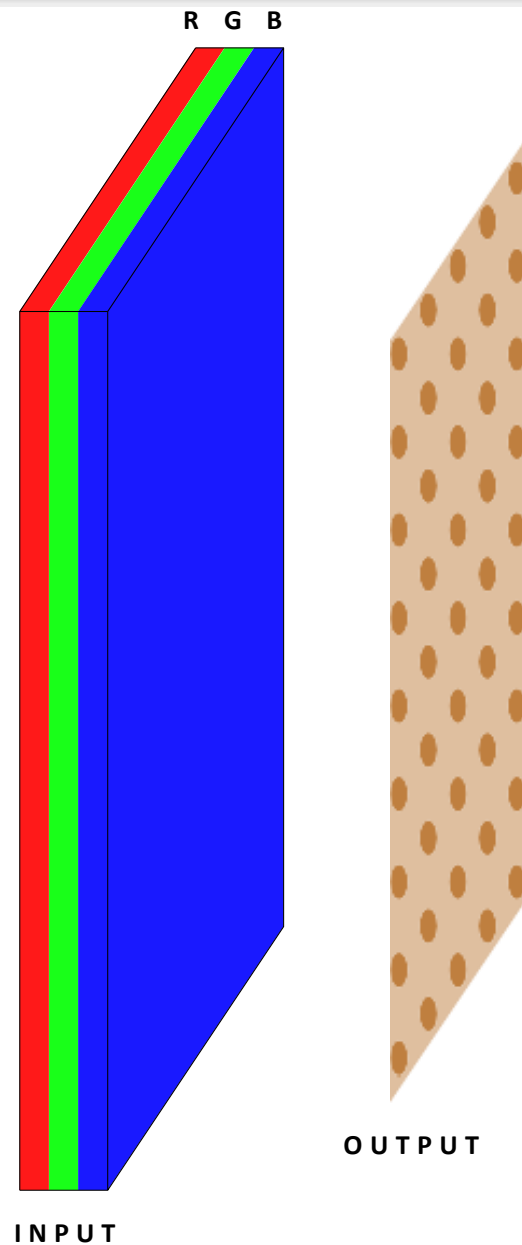
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- Once again we can apply multiple filters to get multiple feature maps

Relation between input size, output size and filter size



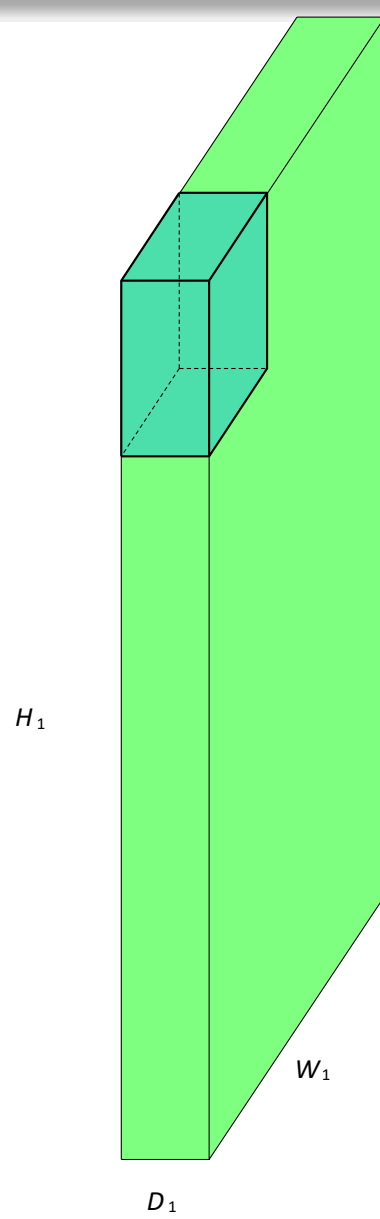


H_1

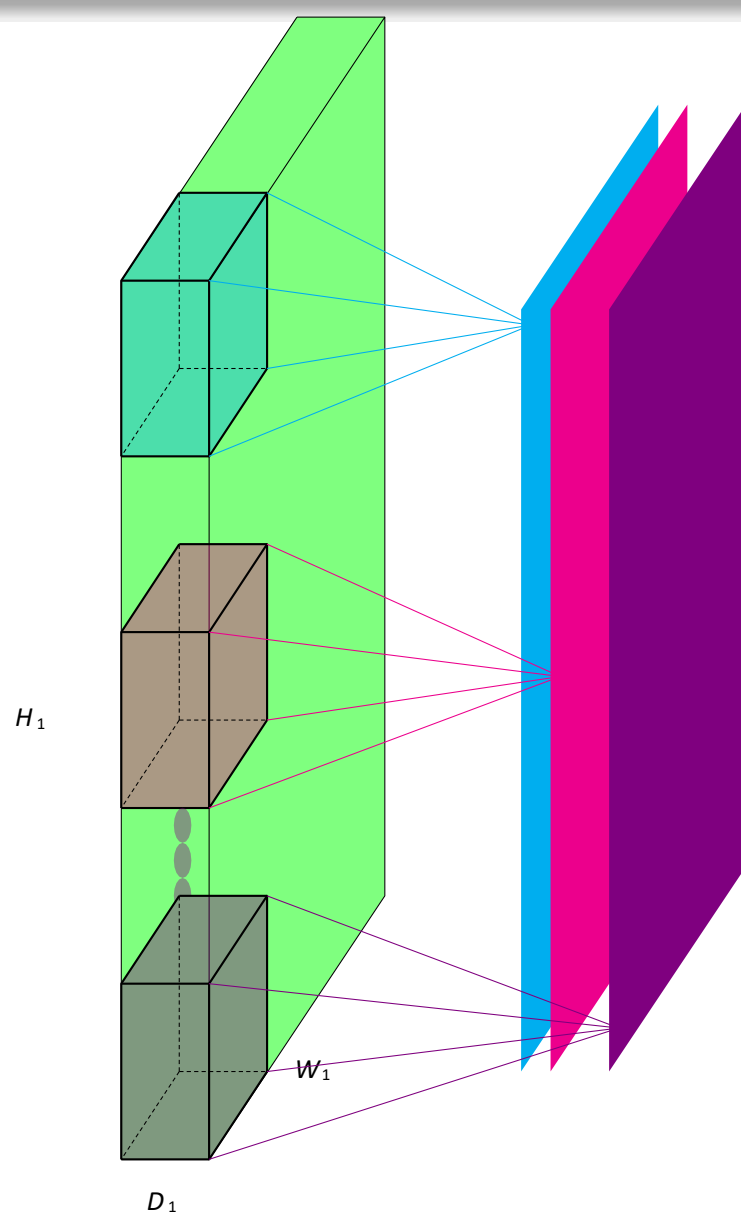
W_1

D_1

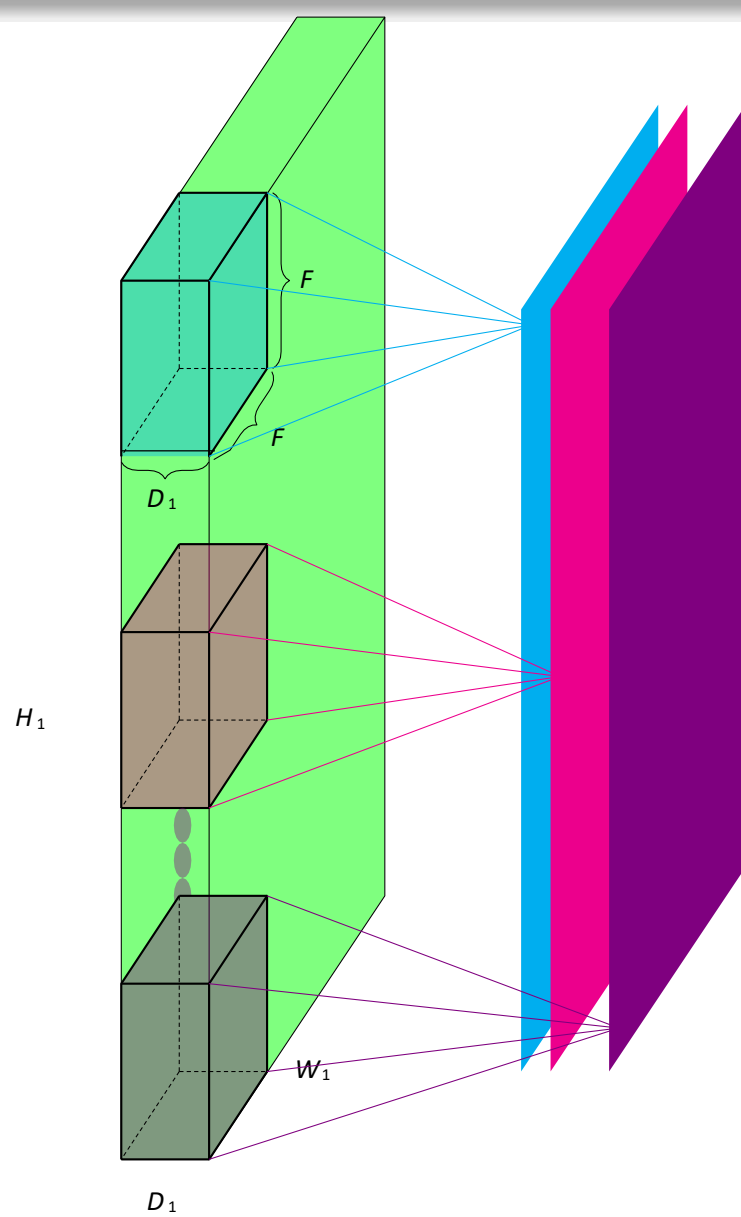
- Width (W_1), Height (H_1) and Depth (D_1) of the original input
- The Stride S (We will come back to this later)



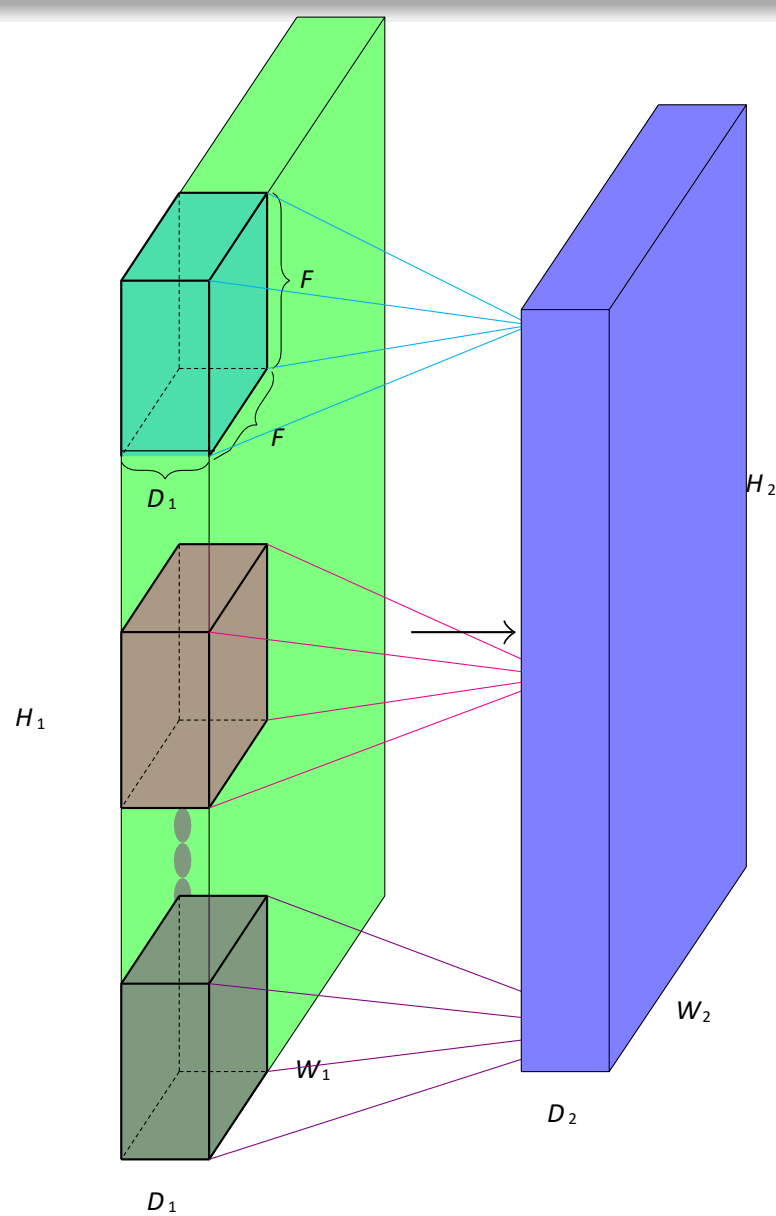
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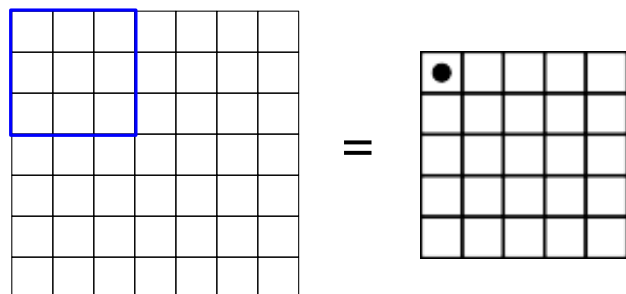


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- The spatial extent (F) of each filter (the depth of each filter is same as the depth of each input)

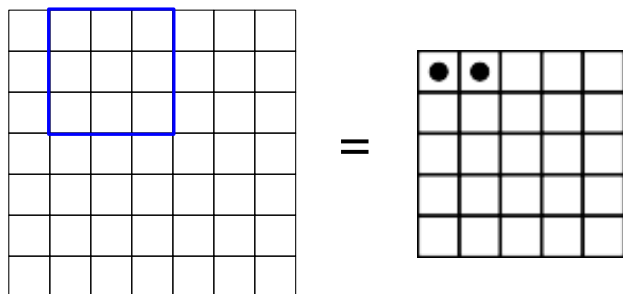


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- The Stride S (We will come back to this later)
- The number of filters K
- The spatial extent (F) of each filter (the depth of each filter is same as the depth of each input)
- The output is $W_2 \times H_2 \times D_2$ (we will soon see a formula for computing W_2 , H_2 and D_2)

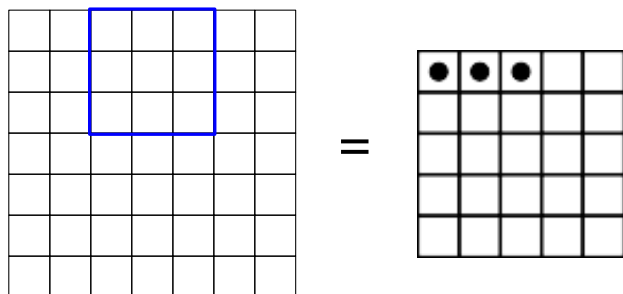
- Let us compute the dimension (W_2, H_2) of the output



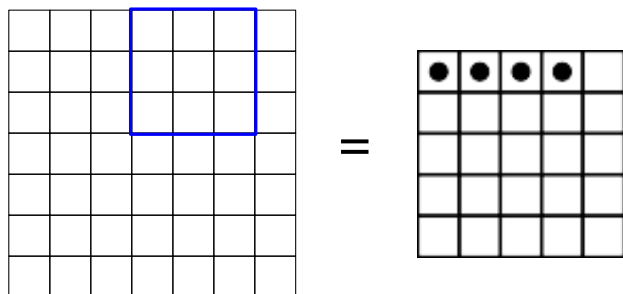
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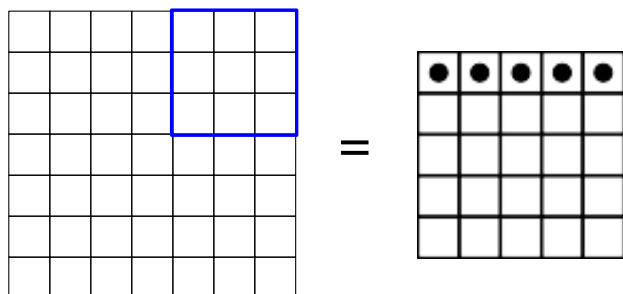
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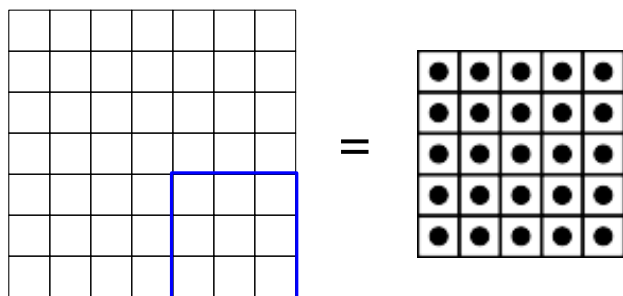
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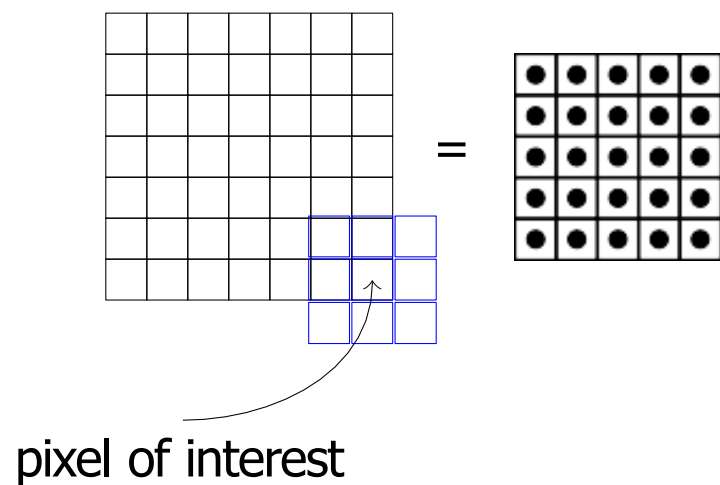
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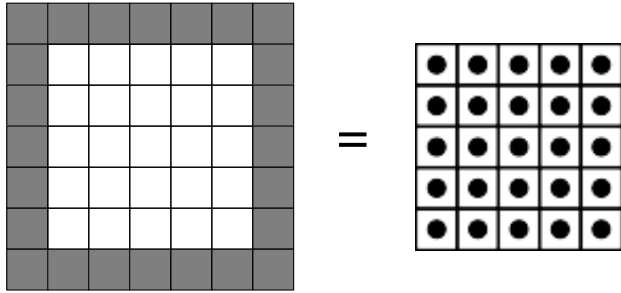
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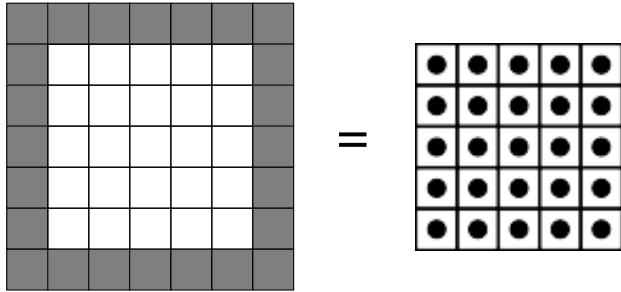
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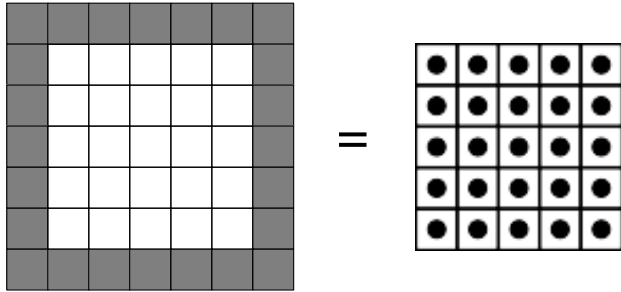
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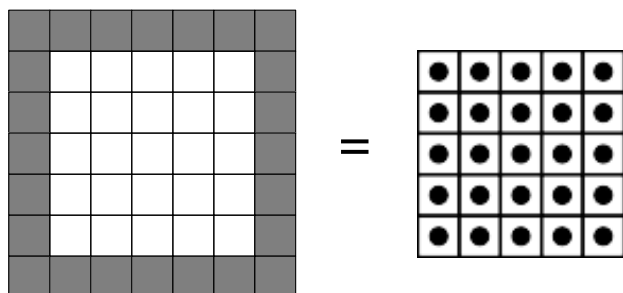
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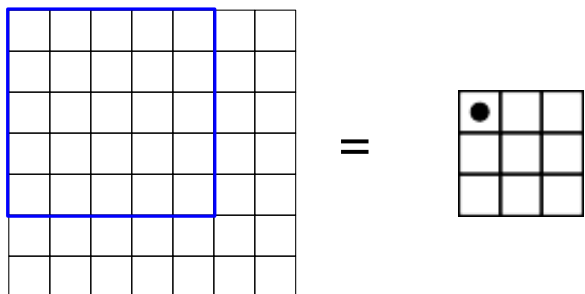
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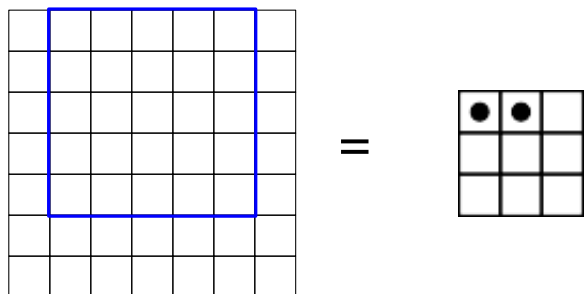
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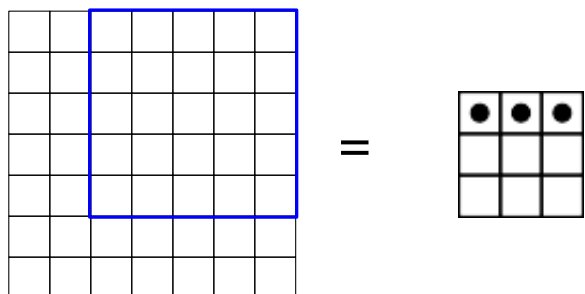
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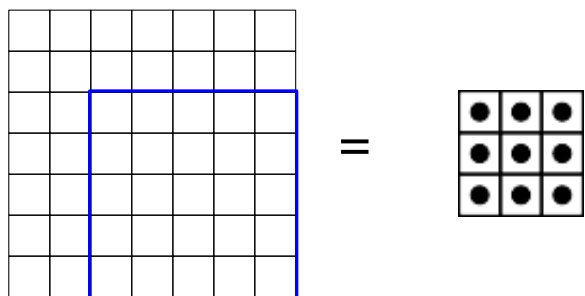
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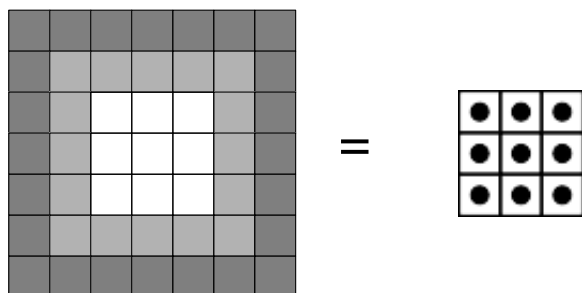
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In general, $W_2 = W_1 - F + 1$

$$H_2 = H_1 - F + 1$$

We will refine this formula further

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0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
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0	0	0	0	0	0	0	0	0

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0	0	0	0	0	0	0	0	0
0								0
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0	0	0	0	0	0	0	0	0
0								0
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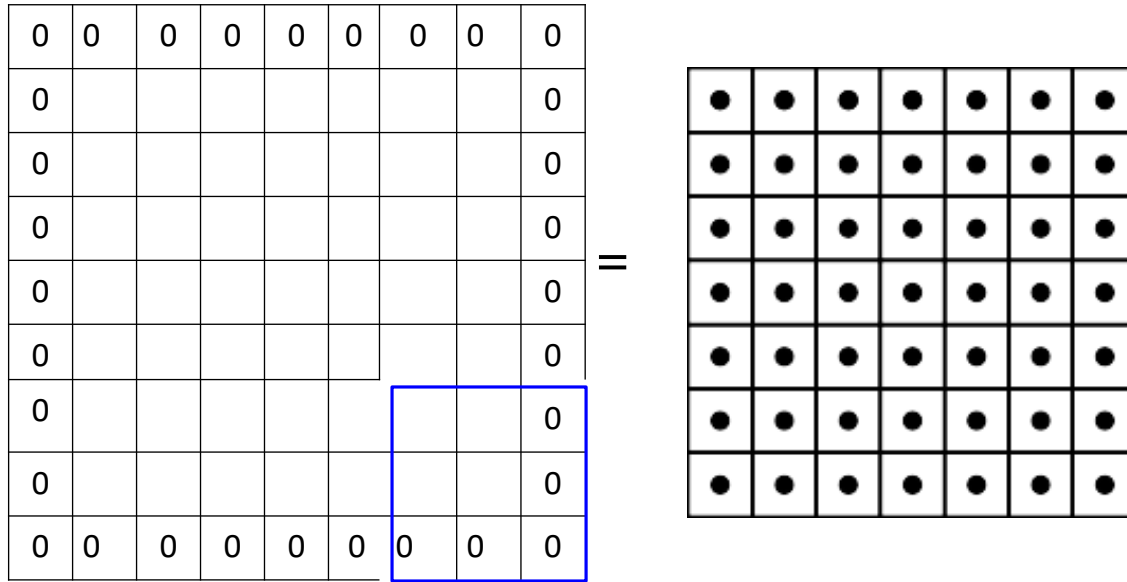
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We now have,

$$W_2 = W_1 - F + 2P + 1$$

$$H_2 = H_1 - F + 2P + 1$$

We will refine this formula further

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0	0	0	0	0	0	0	0	0	0
0									0
0									0
0									0
0									0
0									0
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0	0	0	0	0	0	0	0	0	0

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- Here, we are essentially skipping every 2nd pixel which will again result in an output which is of smaller dimensions

0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
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0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

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•	•		

- What does the stride S do?
- It defines the intervals at which the filter is applied (here $S = 2$)
- Here, we are essentially skipping every 2nd pixel which will again result in an output which is of smaller dimensions

0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
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0	0	0	0	0	0	0	0	0

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0	0	0	0	0	0	0	0	0
0								0
0								0
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- What does the stride S do?
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0	0	0	0	0	0	0	0	0
0								0
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0								0
0								0
0								0
0								0
0								0
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- What does the stride S do?
- It defines the intervals at which the filter is applied (here $S = 2$)
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0	0	0	0	0	0	0	0	0
0								0
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0	0	0	0	0	0	0	0	0

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- It defines the intervals at which the filter is applied (here $S = 2$)
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0	0	0	0	0	0	0	0	0
0								0
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0								0
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0								0
0								0
0	0	0	0	0	0	0	0	0

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•	•	•	

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- It defines the intervals at which the filter is applied (here $S = 2$)
- Here, we are essentially skipping every 2nd pixel which will again result in an output which is of smaller dimensions

0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

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•	•	•	•
•	•	•	•
•	•	•	•

- What does the stride S do?
- It defines the intervals at which the filter is applied (here $S = 2$)
- Here, we are essentially skipping every 2nd pixel which will again result in an output which is of smaller dimensions

0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

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•	•	•	•
•	•	•	•
•	•	•	•
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- What does the stride S do?
- It defines the intervals at which the filter is applied (here $S = 2$)
- Here, we are essentially skipping every 2nd pixel which will again result in an output which is of smaller dimensions

0	0	0	0	0	0	0	0	0
0								0
0								0
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0								0
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0	0	0	0	0	0	0	0	0

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•	•	•	•
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•	•	•	•
•	•		

- What does the stride S do?
- It defines the intervals at which the filter is applied (here $S = 2$)
- Here, we are essentially skipping every 2nd pixel which will again result in an output which is of smaller dimensions

0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

=

•	•	•	•
•	•	•	•
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- What does the stride S do?
- It defines the intervals at which the filter is applied (here $S = 2$)
- Here, we are essentially skipping every 2nd pixel which will again result in an output which is of smaller dimensions

So what should our final formula look like,

0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

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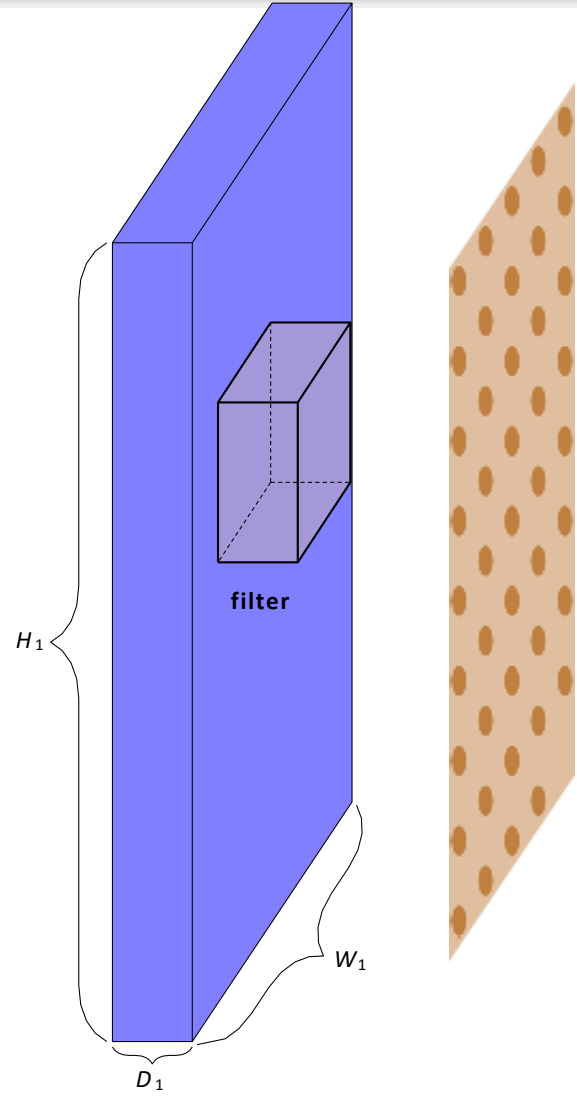
•	•	•	•
•	•	•	•
•	•	•	•
•	•	•	•

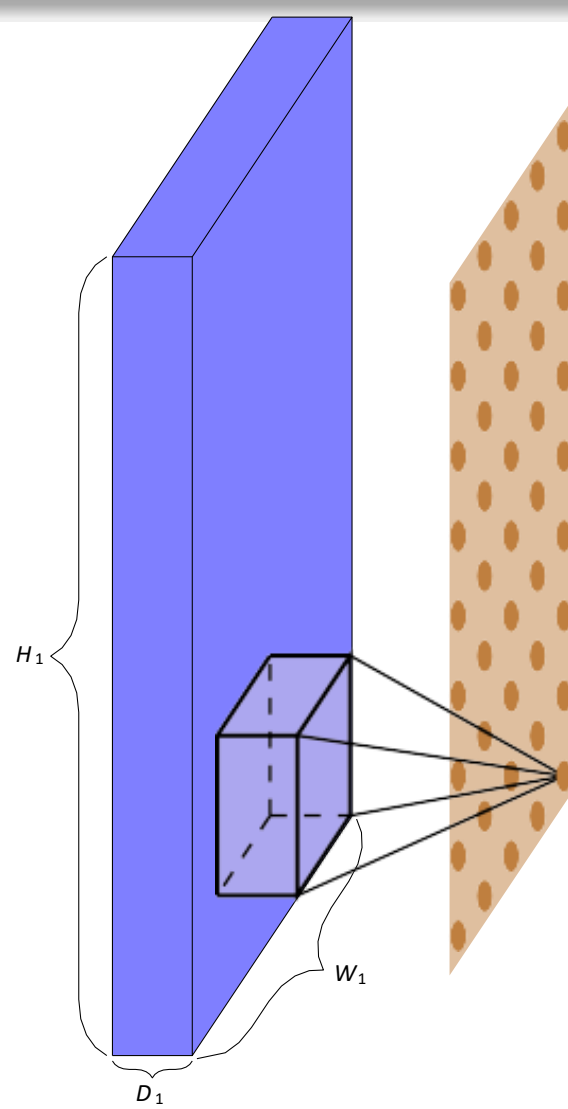
- What does the stride S do?
- It defines the intervals at which the filter is applied (here $S = 2$)
- Here, we are essentially skipping every 2nd pixel which will again result in an output which is of smaller dimensions

So what should our final formula look like,

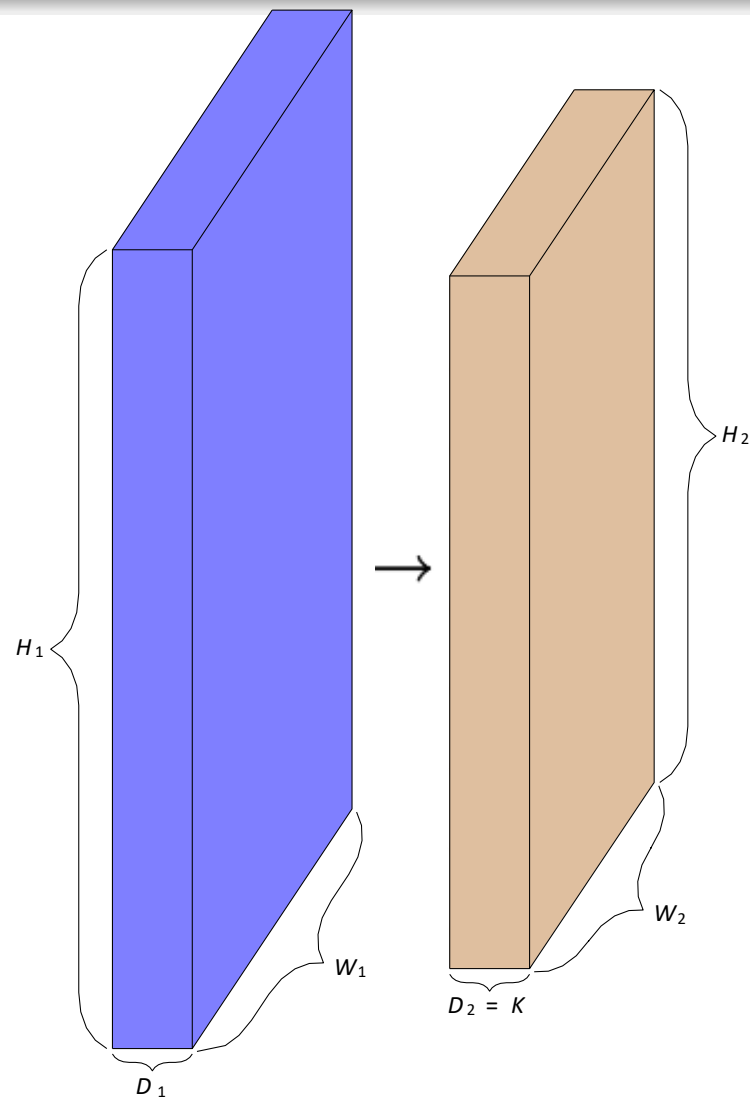
$$W_2 = \frac{W_1 - F + 2P}{S} + 1$$

$$H_2 = \frac{H_1 - F + 2P}{S} + 1$$





- Each filter gives us one 2D output.

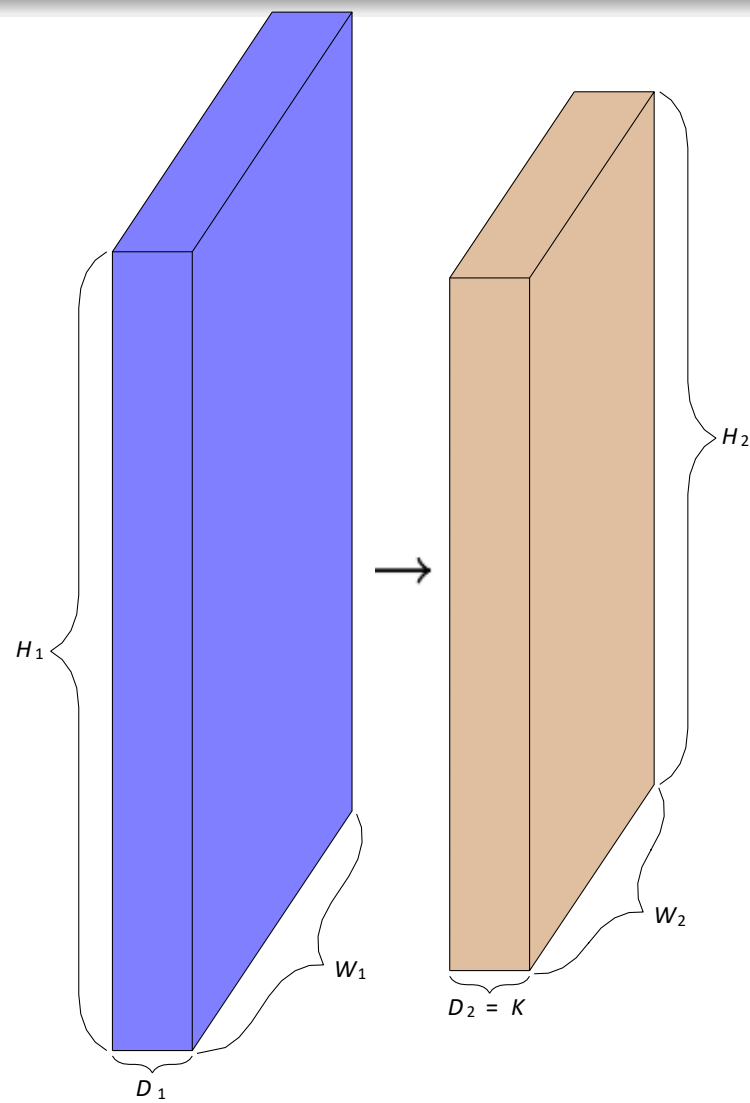


$$W_2 = \frac{W_1 - F + 2P}{S} + 1$$

$$H_2 = \frac{H_1 - F + 2P}{S} + 1$$

$$D_2 = K$$

- Each filter gives us one 2D output.
- K filters will give us K such 2D outputs

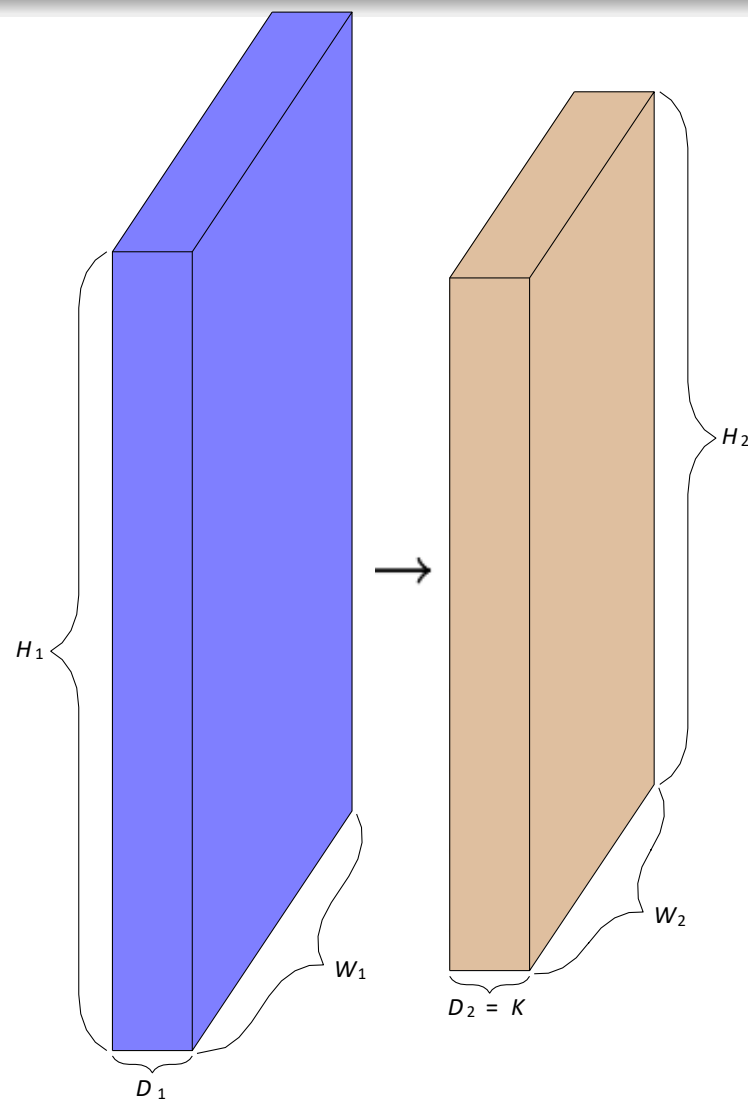


$$W_2 = \frac{W_1 - F + 2P}{S} + 1$$

$$H_2 = \frac{H_1 - F + 2P}{S} + 1$$

$$D_2 = K$$

- Each filter gives us one 2D output.
- K filters will give us K such 2D outputs
- We can think of the resulting output as $K \times W_2 \times H_2$ volume



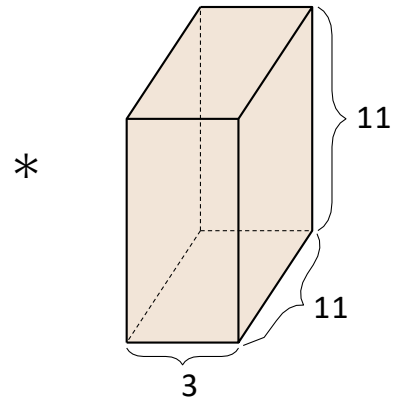
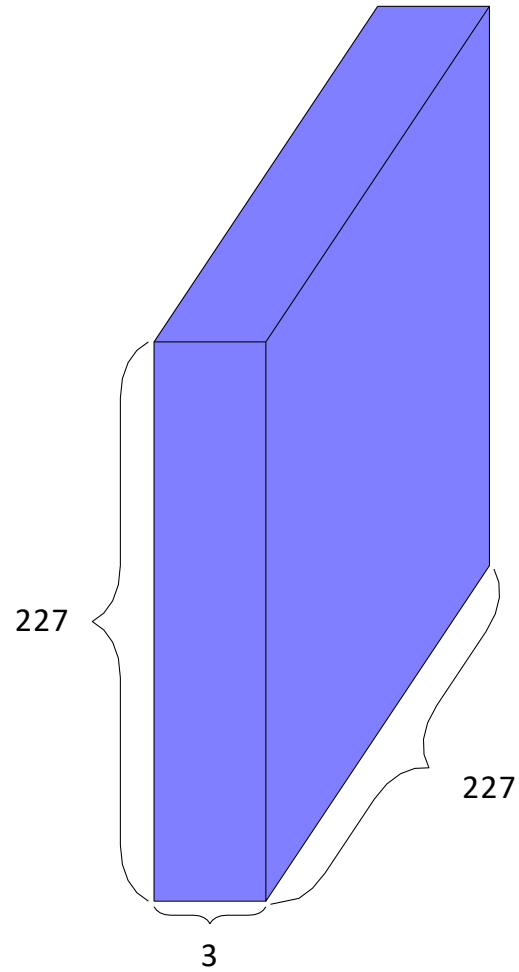
$$W_2 = \frac{W_1 - F + 2P}{S} + 1$$

$$H_2 = \frac{H_1 - F + 2P}{S} + 1$$

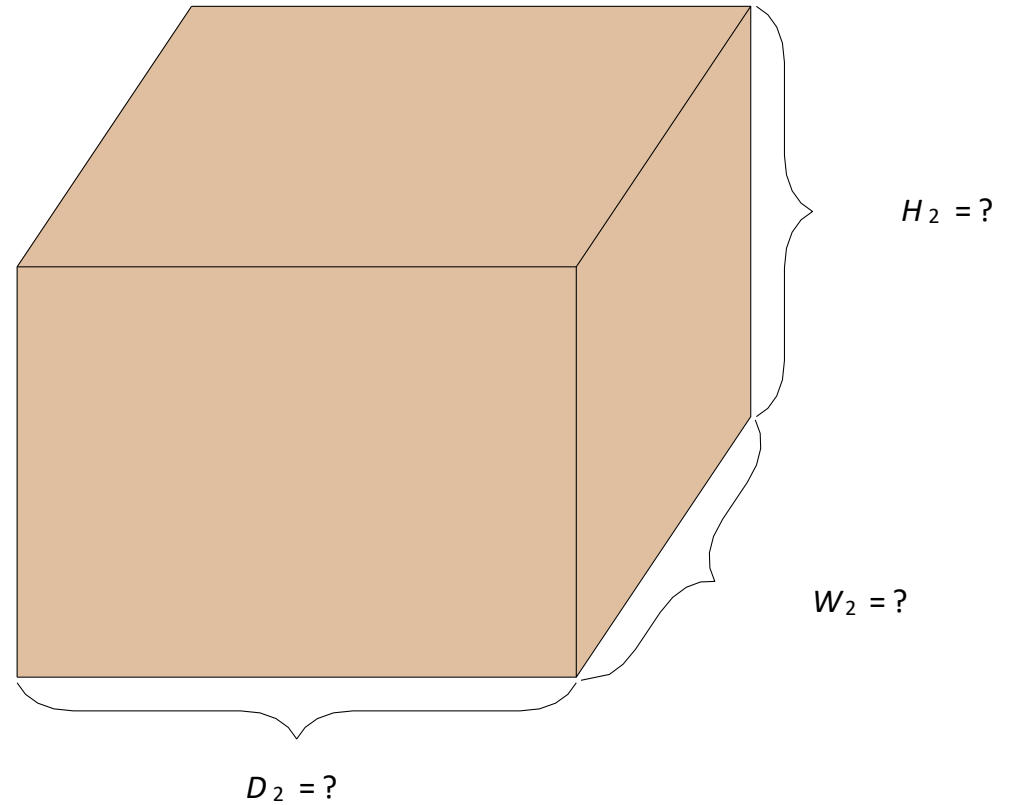
$$D_2 = K$$

- Each filter gives us one 2D output.
- K filters will give us K such 2D outputs
- We can think of the resulting output as $K \times W_2 \times H_2$ volume
- Thus $D_2 = K$

Let us do a few exercises



=



96 filters

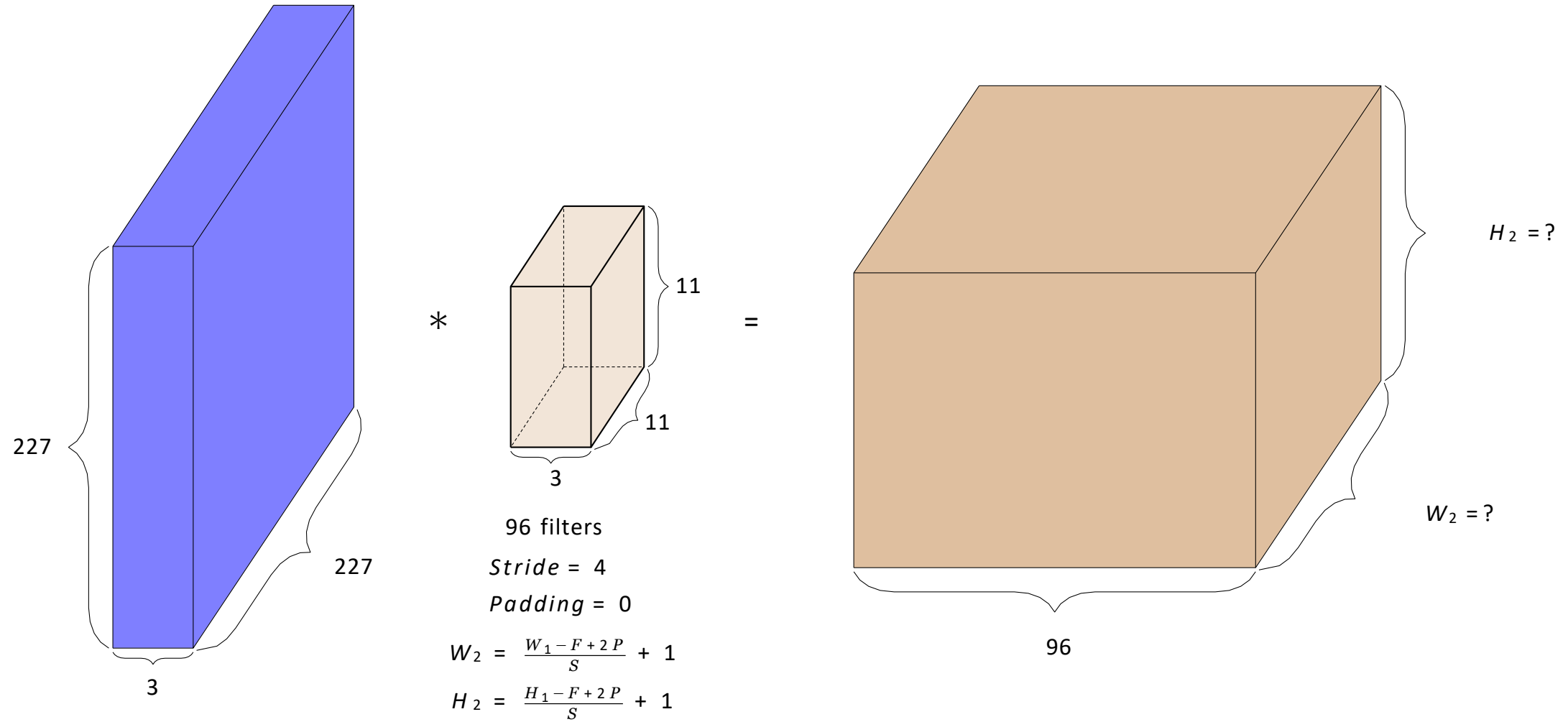
Stride = 4

Padding = 0

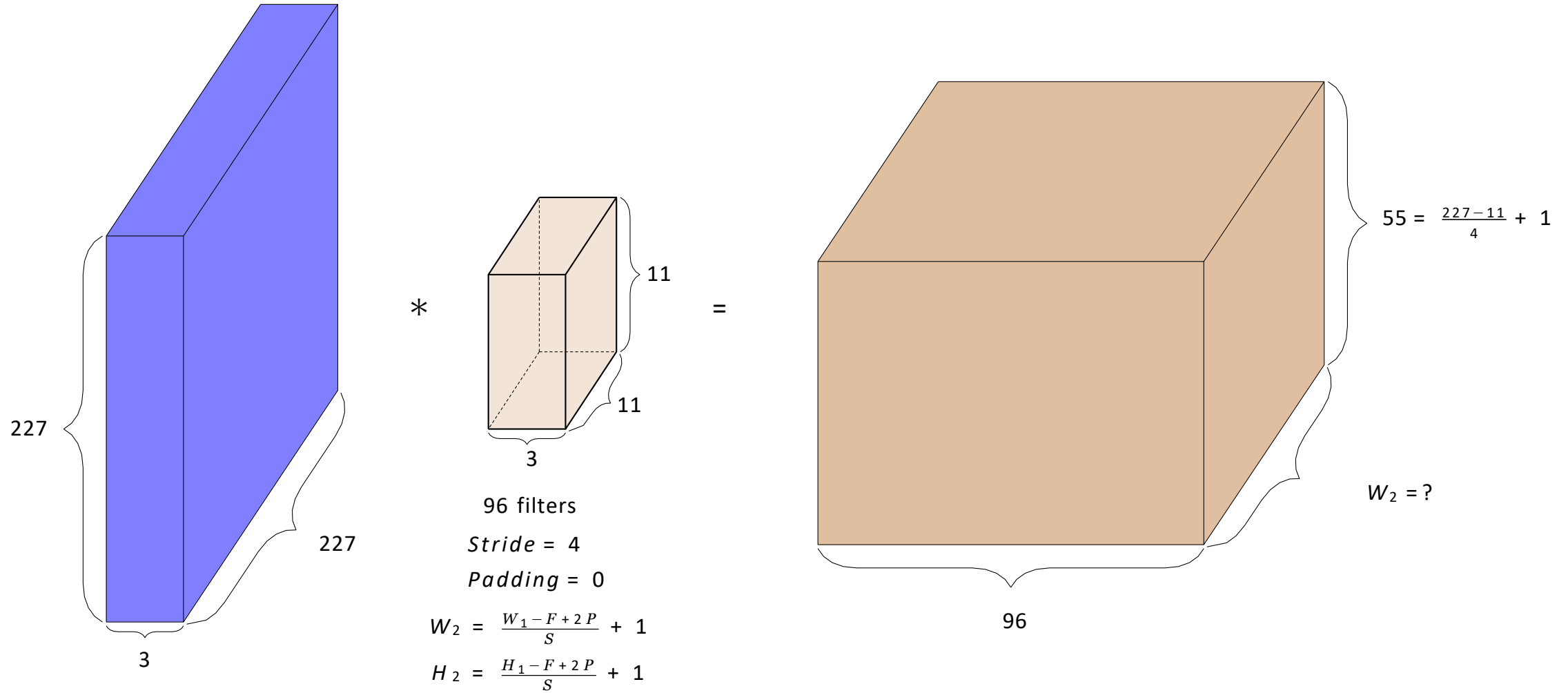
$$W_2 = \frac{W_1 - F + 2P}{S} + 1$$

$$H_2 = \frac{H_1 - F + 2P}{S} + 1$$

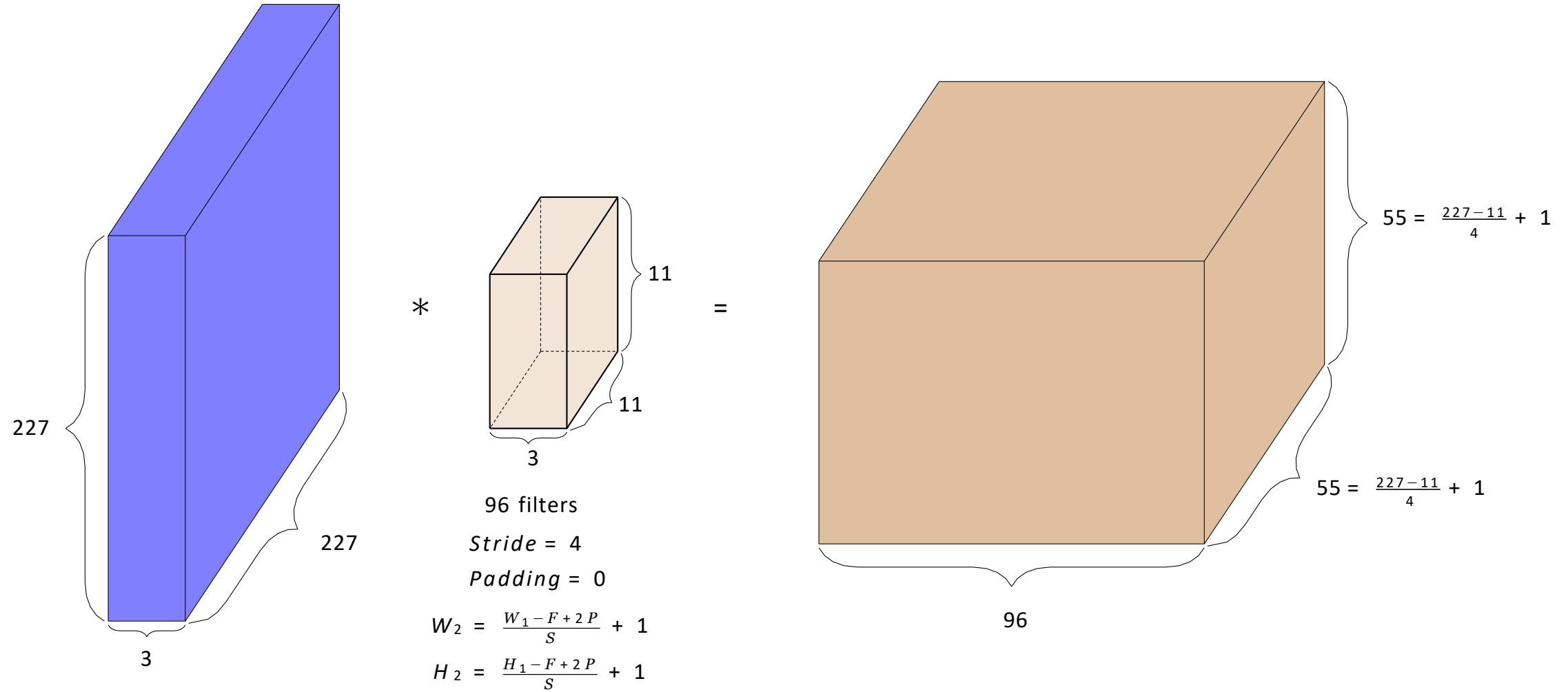
Let us do a few exercises



Let us do a few exercises



Let us do a few exercises



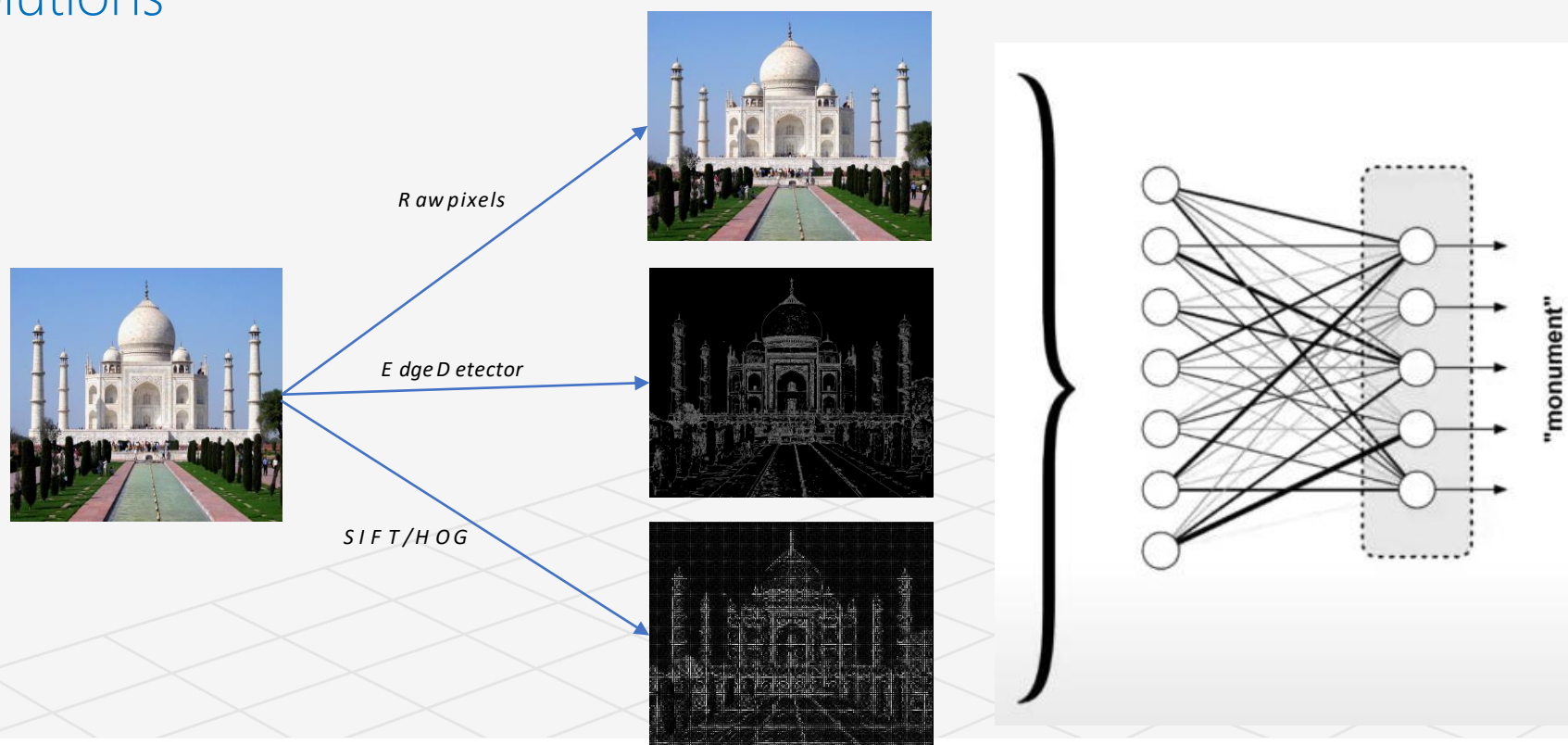
Putting things into perspective

- What is the connection between this operation (convolution) and neural networks?
- We will try to understand this by considering the task of “image classification”



Traditional ML based Vision

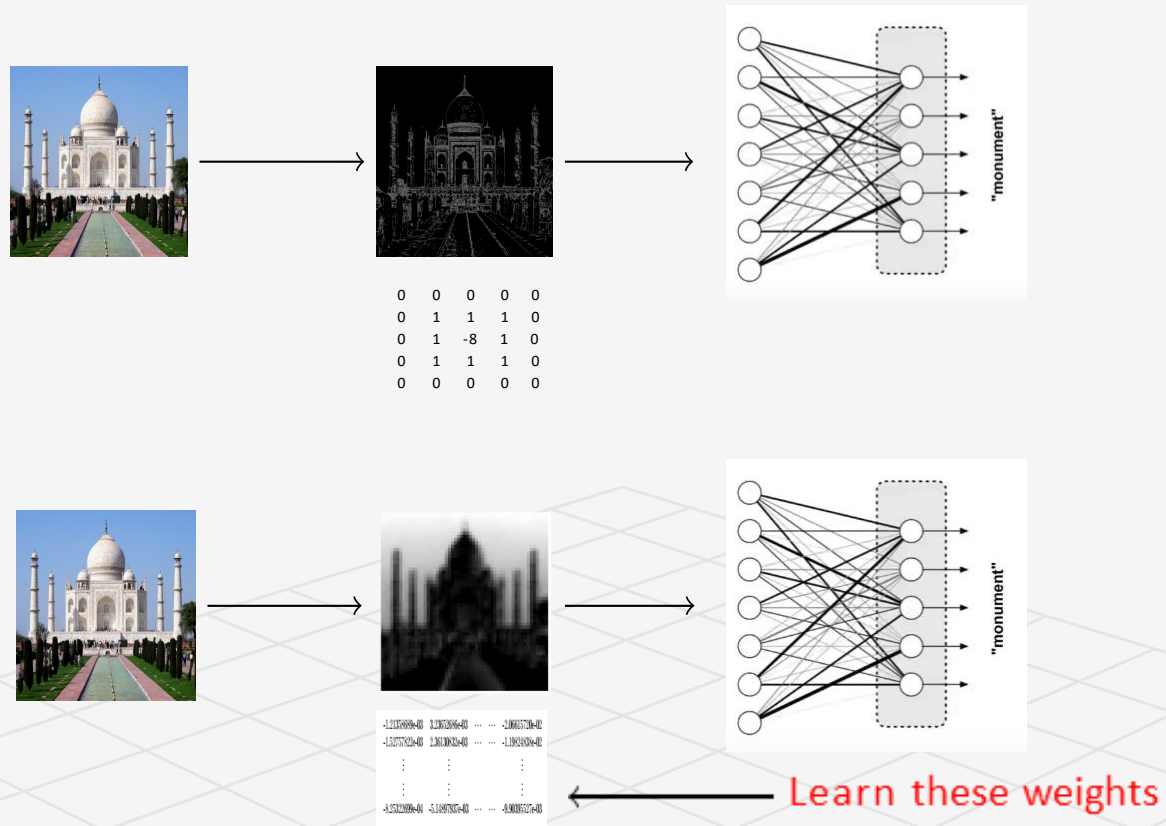
- Traditional ML-based computer vision solutions involve static feature engineering from images (e.g. recall SIFT, LBP, HoG, etc)
- Though effective, static feature engineering was a bottleneck of pre-DL vision solutions



static feature extraction (no learning) learning weights of classifier

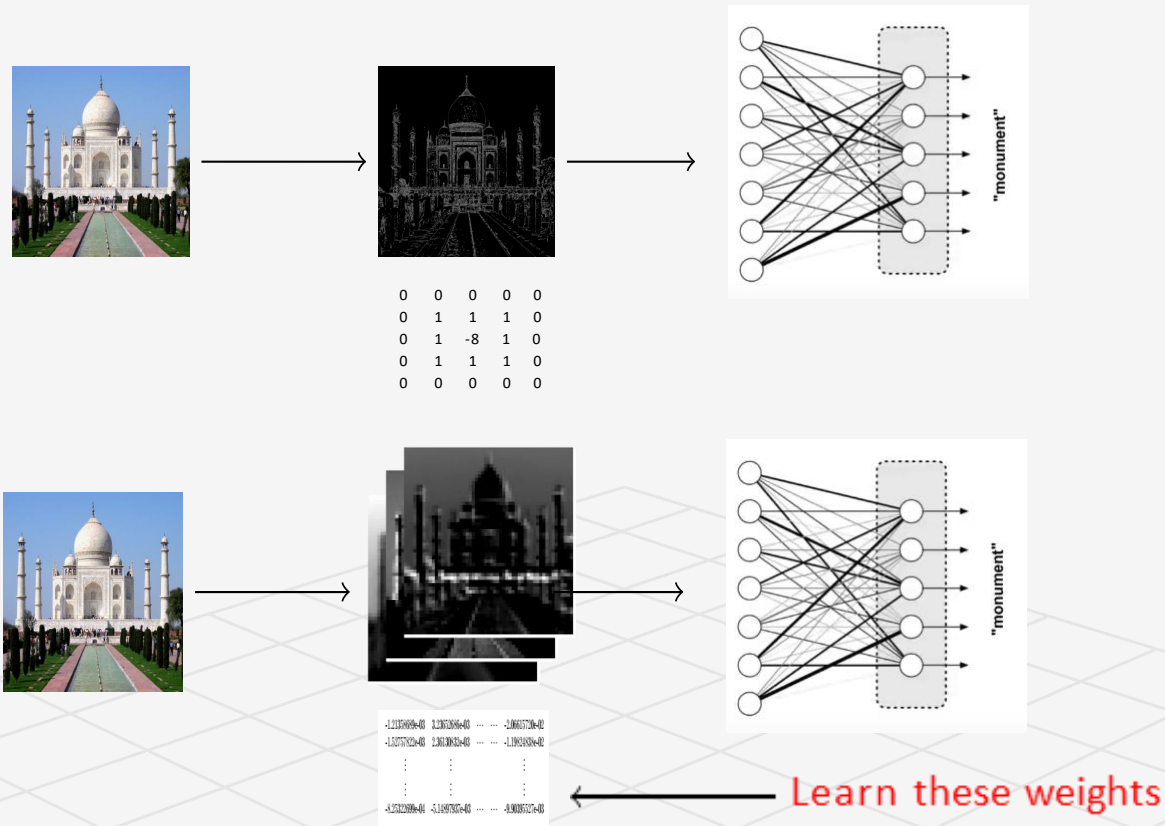
Beyond static feature engineering

- Instead of using handcrafted kernels such as edge detectors can we learn meaningful kernels/filters in addition to learning the weights of the classifier?



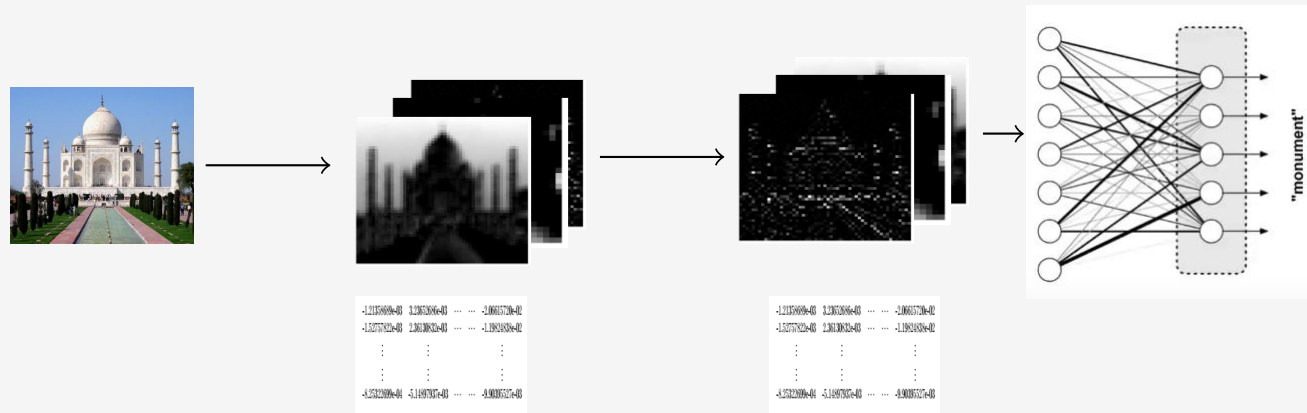
Beyond static feature engineering

- Even better: Instead of using handcrafted kernels (such as edge detectors), can we **learn multiple meaningful kernels/filters** in addition to learning the weights of the classifier?



Beyond static feature engineering

- Can we **learn multiple layers of meaningful kernels/filters** in addition to learning the weights of the classifier?

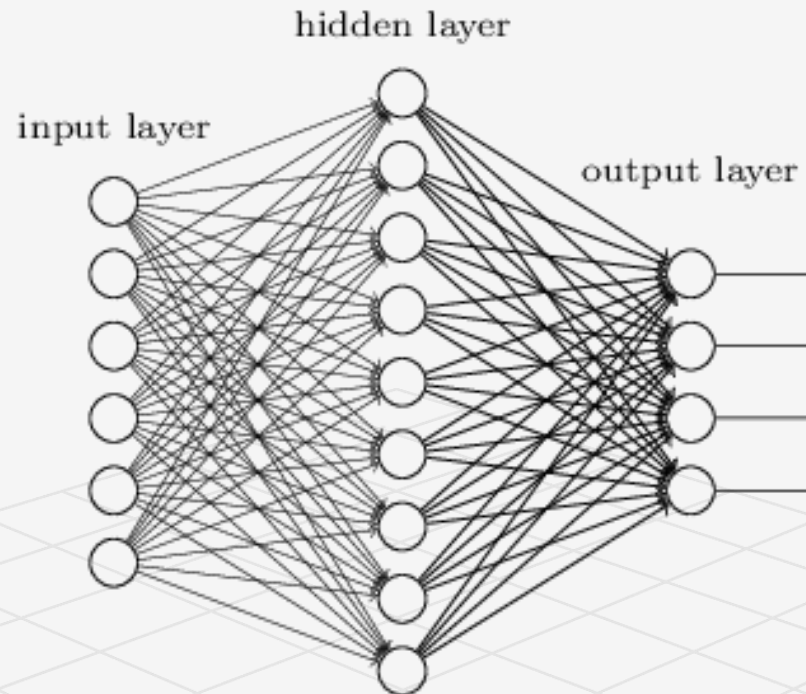


Yes, we can !

Simply by treating these kernels as parameters and learning them in addition to the weights of the classifier (using back propagation)

Pause and ponder

- Okay, I get it that the idea is to learn the kernel/filters by just treating them as parameters of the classification model
- But why not directly use flattened images with fully connected neural network or FNN instead?



Challenges of applying FNNs to images



MNIST Dataset

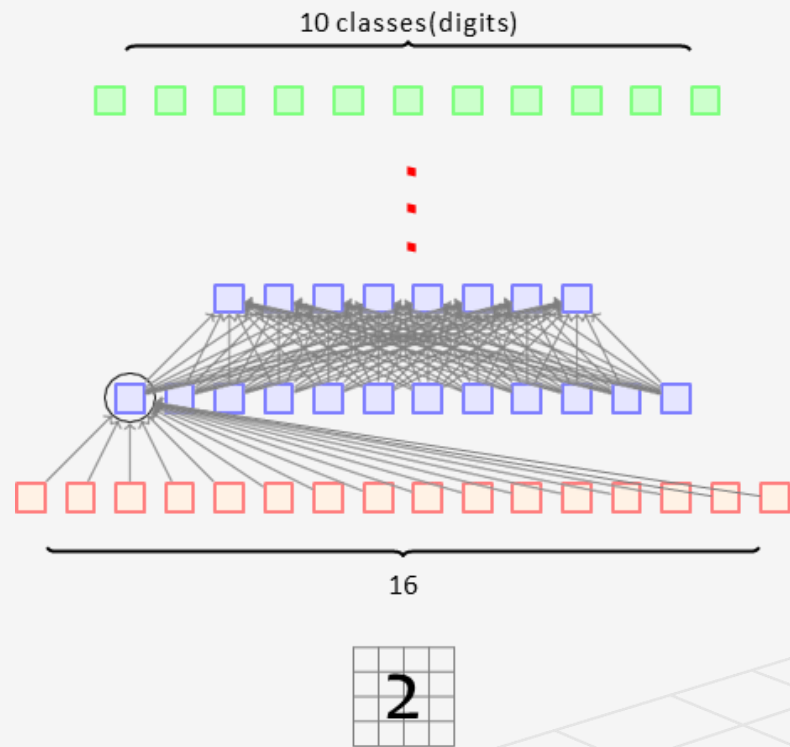
- On a reasonably simple dataset like MNIST, we can get about **2%** error (or even better) using FNNs, but
 - Ignores spatial (2-D) structure of input images - unroll each 28×28 image into a 784-D vector
 - Pixels that are spatially separate are treated the same way as pixels that are adjacent
- No obvious way for networks to learn same features (e.g. edges) at different places in the input image
- Can get computationally expensive for large images
 - For a **1MP** color image with 20 neurons in the first hidden layer, how many weights in the first layer?

60 million

How CNN solves these limitation

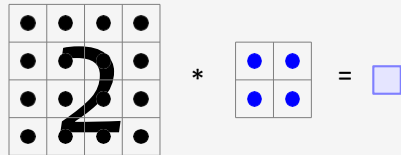
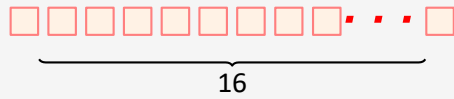
- Local receptive fields, in which hidden units are connected to local patches of the layer below, serve two purposes:
 - Capture local spatial relationships in pixels (which would not be captured by FNNs)
 - Greatly reduces number of parameters in the model
 - For a **1MP** color image a filter size of $K_1 \times K_2$ in the first hidden layer, how many weights in a convolutional layer? $K_1 \times K_2$, compare with 60 million for FNNs on the previous slide!
- Weight sharing, which also serves two purposes:
 - Enables translation-invariance of neural network to objects in images
 - Reduces number of parameters in the modelPooling which condenses information from previous layer, serves two purposes:
- Aggregates information, especially minor variations
 - Reduces size of output of a previous layer, which reduces number of computations in later layers

Local receptive field



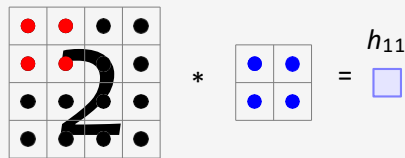
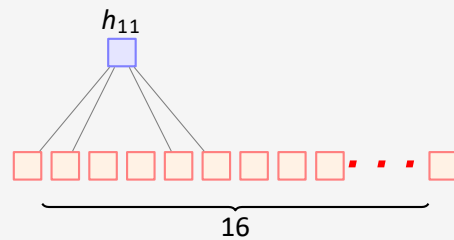
- This is what a regular feed-forward neural network
- There are many dense connections here
- For example all the 16 input neurons are contributing to the computation of h_{11}
- Contrast this to what happens in the case of convolution

Local receptive field

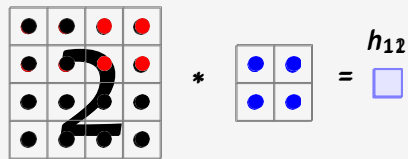
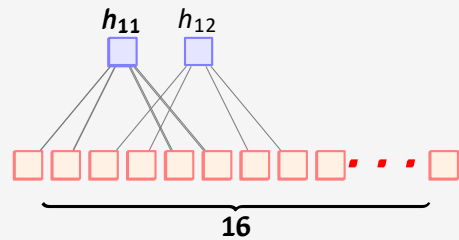


Local receptive field

- Only a few local neurons participate in the computation of h_{11}
- For example, only pixels 1, 2, 5, 6 contribute to h_{11}



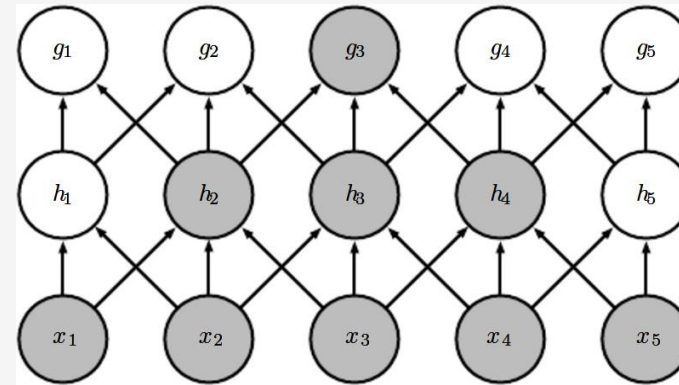
Local receptive field



- Only a few local neurons participate in the computation of h_{11}
- For example, only pixels 1, 2, 5, 6 contribute to h_{11}
- The connections are much sparser
- We are taking advantage of the structure of the image (interactions between neighboring pixels are more interesting)
- This sparse connectivity reduces the number of parameters in the model
- But is sparse connectivity really good thing?

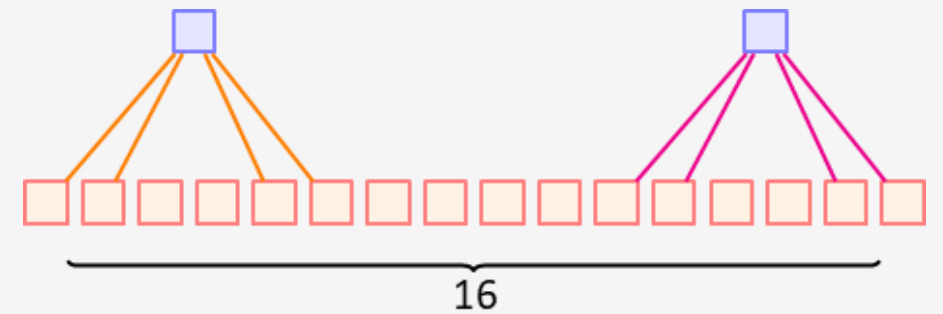
Local receptive field

- Aren't we losing information (by losing interactions between some input pixels)
- Well, not really
- The two highlighted neurons (x_1 & x_5) do not interact in layer 1
- But they indirectly contribute to the computation of g_3 and hence interact indirectly



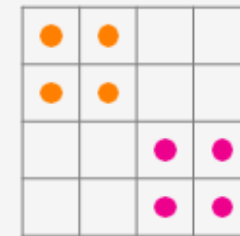
Weight sharing

- Consider the following network
- Do we want the kernel weights to be different for different portions of the image?
 - We would want the filter to respond to an object/artifact same way irrespective of its position
- Imagine that we are trying to learn a kernel that detects edges
- Shouldn't we be applying the same kernel at all the portions of the image?



● Kernel 1

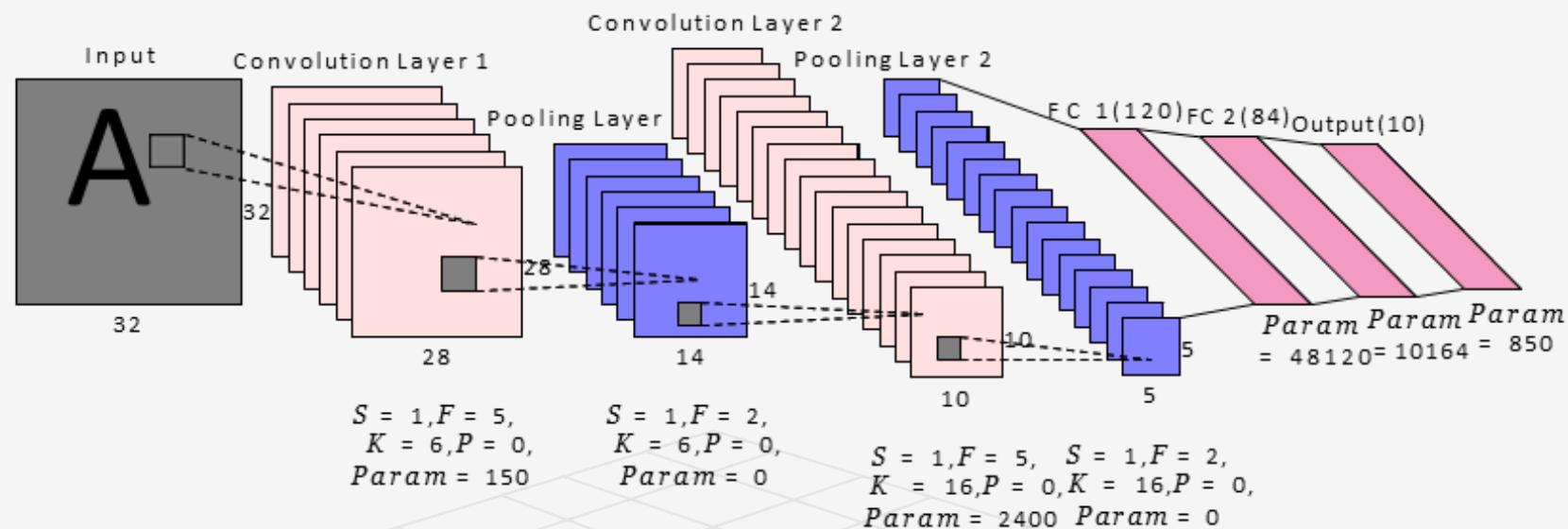
● Kernel 2



4x4 Image

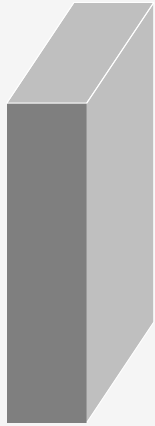
Convolutional neural network

- It has alternate convolution and pooling layers What does a pooling layer do?
- Let us see



Pooling Layer

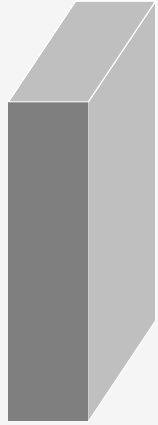
- Parameter free down sampling operation



Input



Pooling Layer



Input

*



1 filter



Pooling Layer



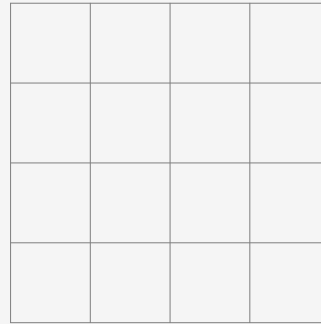
Input

*

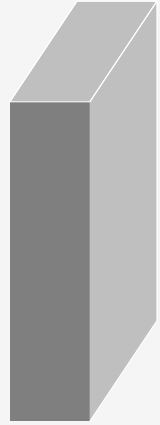


1 filter

=



Pooling Layer



Input

*

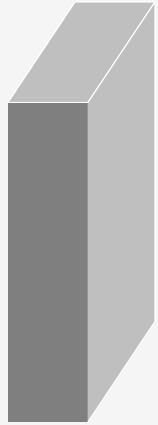


1 filter

=

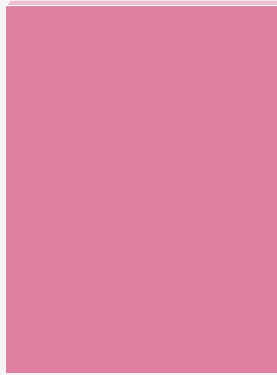
1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

Pooling Layer



Input

*



1 filter

=

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

$\xrightarrow{\text{maxpool}}$
2x2 filters (stride 2)



Pooling Layer



Input

*

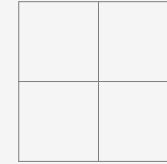


1 filter

=

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 2)

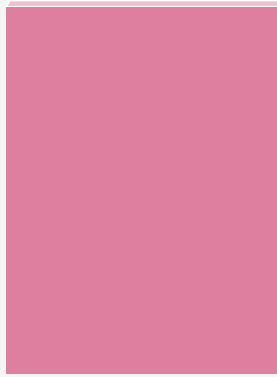


Pooling Layer



Input

*



1 filter

=

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 2)

8	

Pooling Layer



Input

*



1 filter

=

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 2)

8	4

Pooling Layer



Input

*



1 filter

=

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 2)

8	4
7	

Pooling Layer



Input

*



1 filter

=

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 2)

8	4
7	5

Pooling Layer



Input

*



1 filter

=

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 2)

8	4
7	5

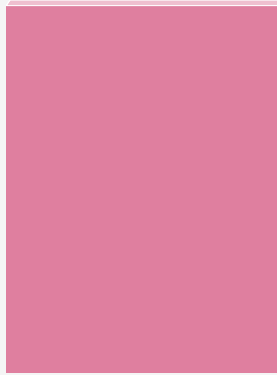
1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

Pooling Layer



Input

*



1 filter

=

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 2)

8	4
7	5

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 1)

Pooling Layer



Input

*



1 filter

=

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 2)

8	4
7	5

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

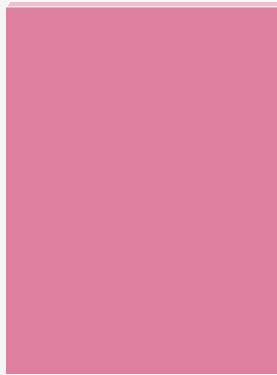
maxpool
2x2 filters (stride 1)

Pooling Layer



Input

*



1 filter

=

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 2)

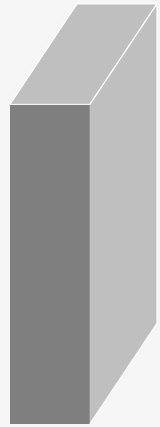
8	4
7	5

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 1)

8		

Pooling Layer



Input

*



1 filter

=

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 2)

8	4
7	5

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 1)

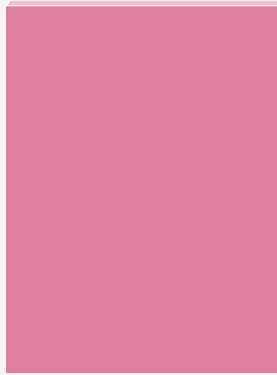
8	8	

Pooling Layer



Input

*



1 filter

=

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 2)

8	4
7	5

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 1)

8	8	4
8	8	5
7	6	

Pooling Layer



Input

*



1 filter

=

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 2)

8	4
7	5

Instead of max pooling we can also do average pooling

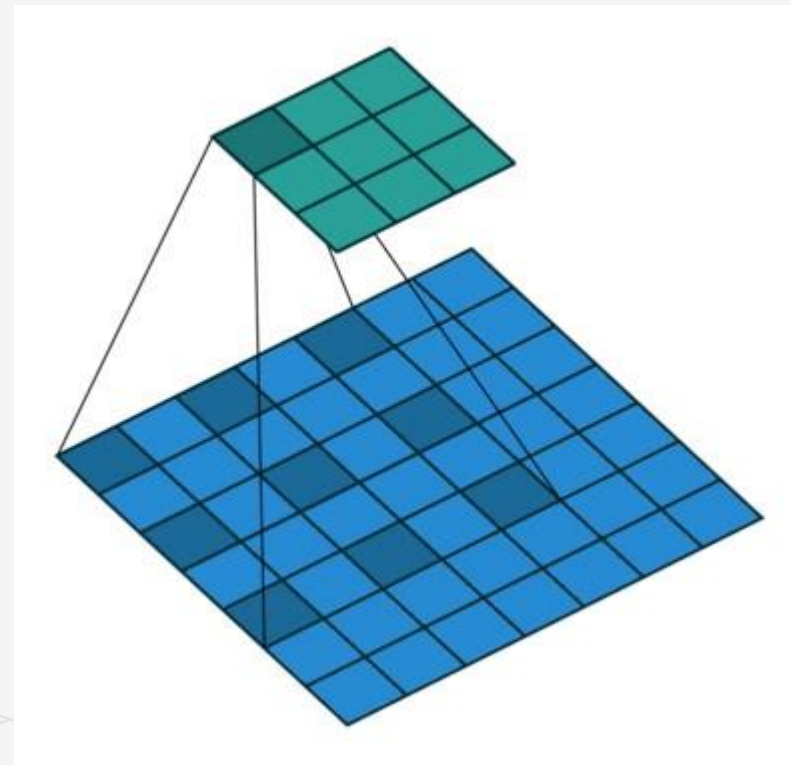
1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 1)

8	8	4
8	8	5
7	6	5

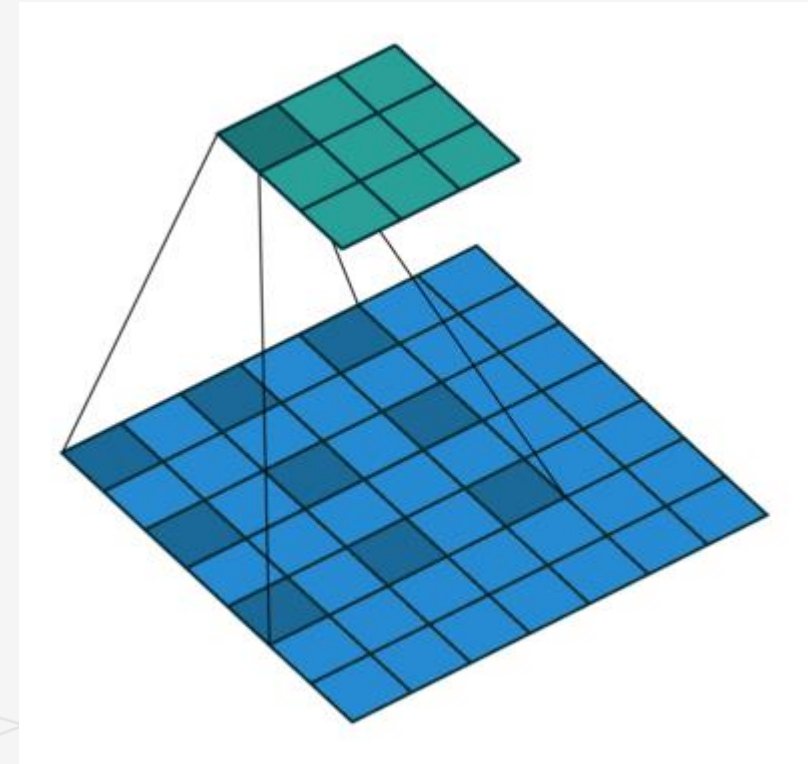
Other variants of convolution: Dilated Convolution

- Introduces another parameter to convolutional layer called dilation rate
- Controls spacing between values in a kernel
- Figure shows 3×3 kernel with dilation rate 2



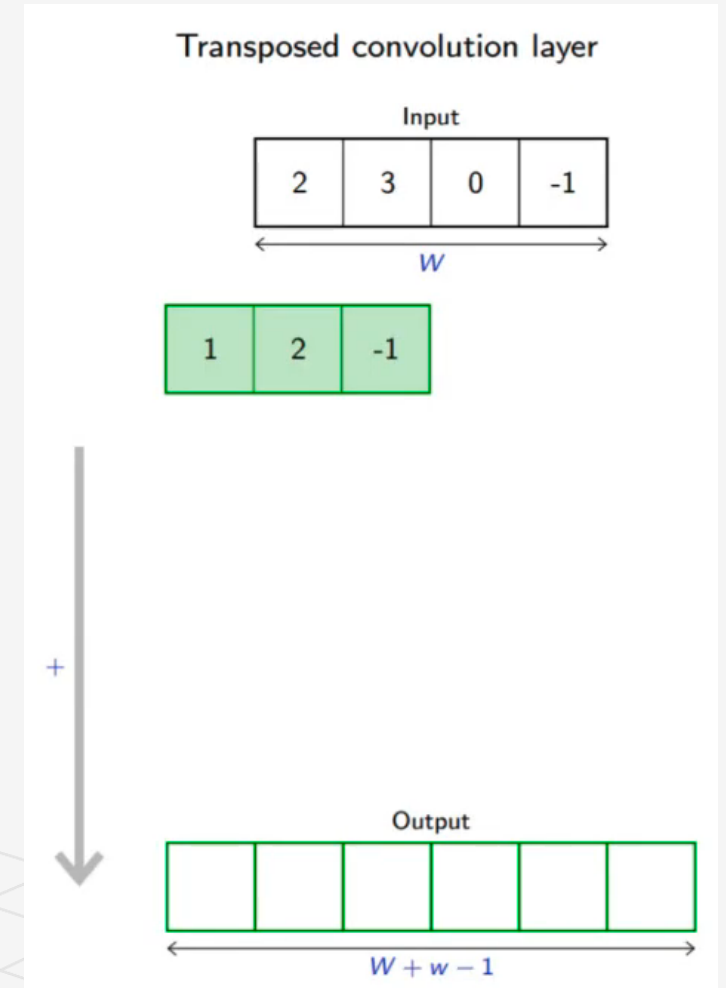
Other variants of convolution: Dilated Convolution

- Introduces another parameter to convolutional layer called dilation rate
- Controls spacing between values in a kernel
- Figure shows 3×3 kernel with dilation rate 2
- Notice that dilated rate 1 is standard convolution
- A subtle difference between dilated convolution and standard convolution with stride > 1 , what is it?



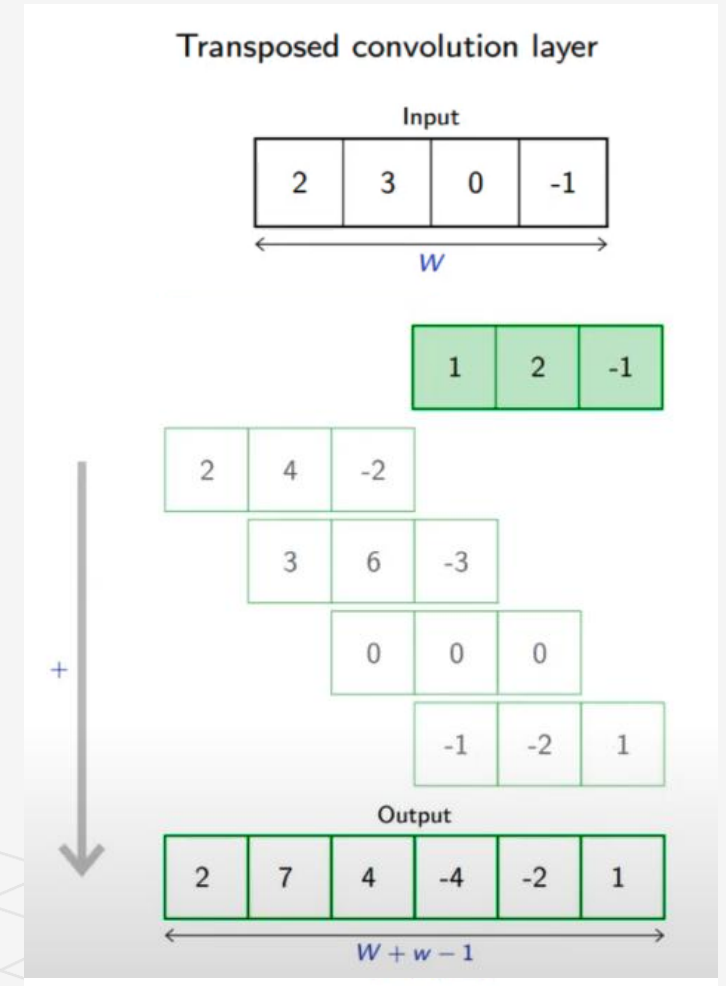
Other variants of convolution: Transpose Convolution

- Allows for learnable upsampling
- Also known as Deconvolution (bad) or Upconvolution
- Traditionally, we could achieve upsampling through interpolation or similar rules
- Why not allow the network to learn the rules by itself?
- Let us see a 1D example



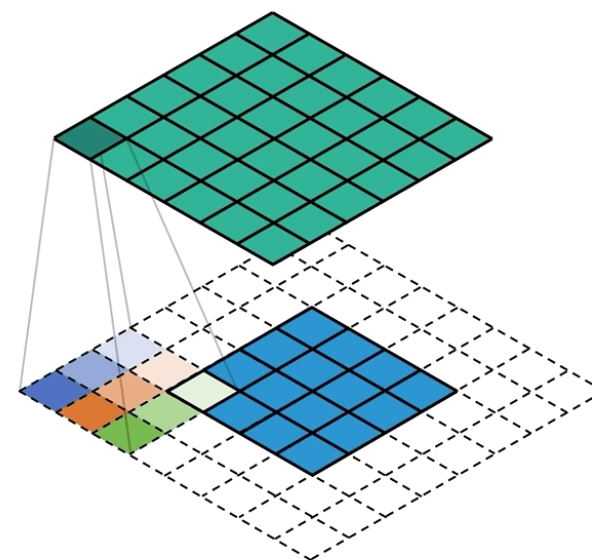
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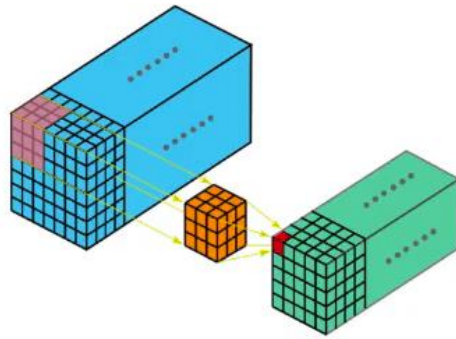


Other variants of convolution: Transpose Convolution

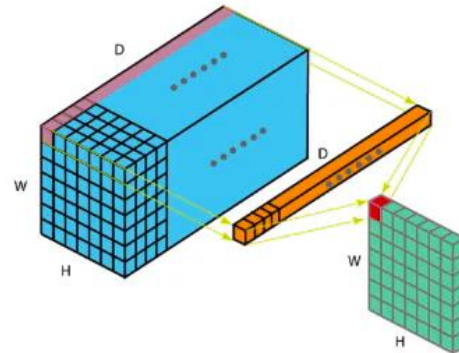
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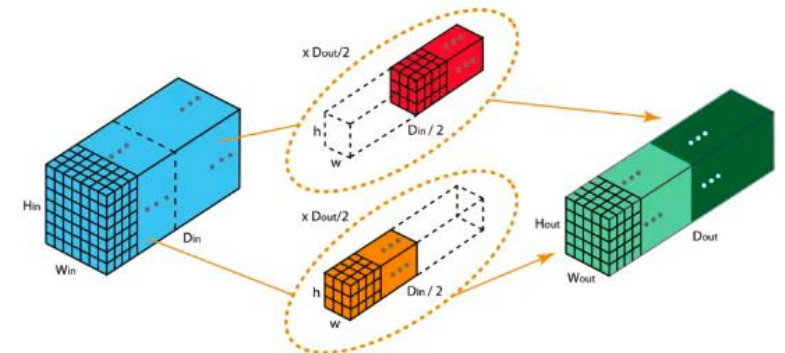
Other variants of convolution



3D Convolution

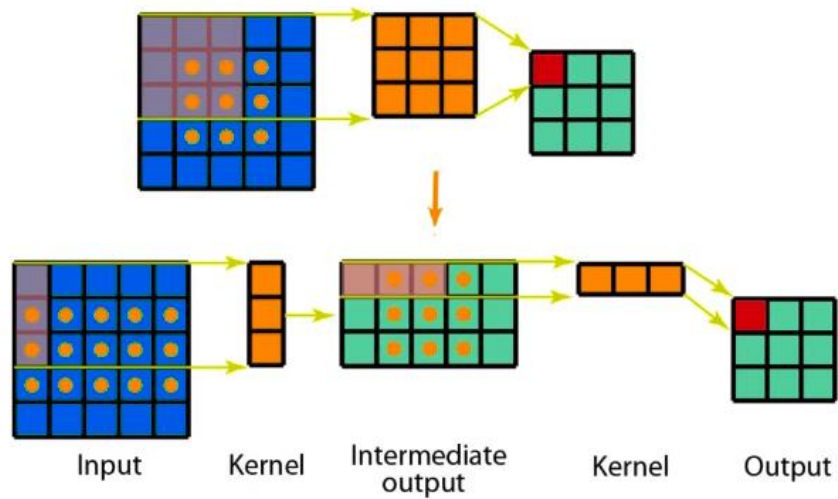


1×1 Convolution
Pointwise Convolution

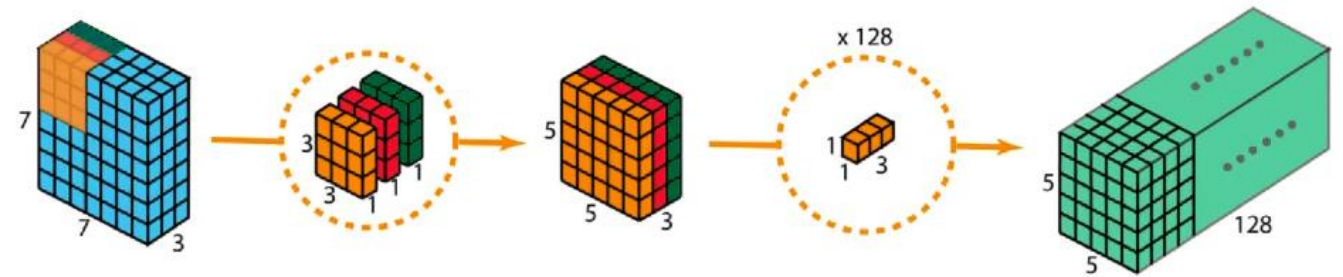


Grouped Convolution

Other variants of convolution

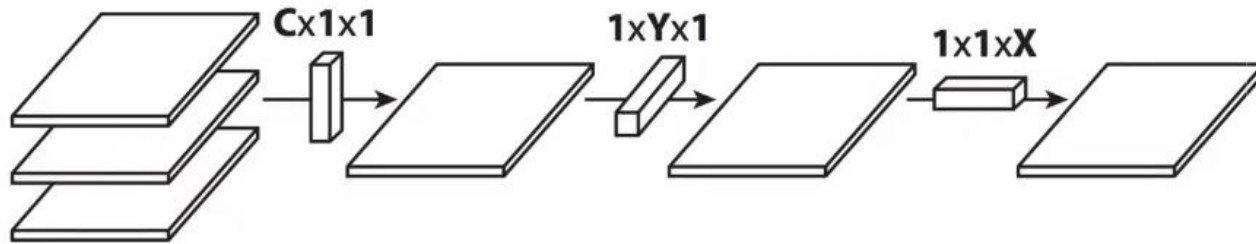


Spatial Separable Convolution

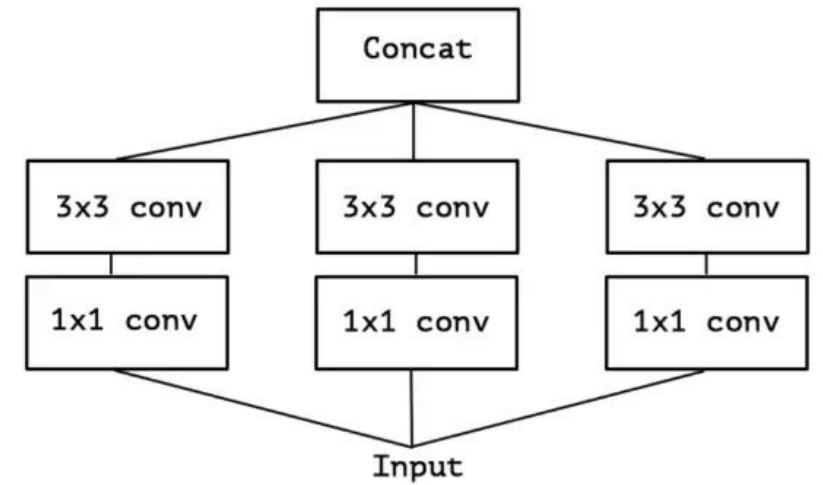


Depthwise Separable Convolution

Other variants of convolution



Flattened Convolutions



Spatial and Cross-Channel Convolutions

Resources and homework

- For an interactive illustration of the convolution operation, visit <https://setosa.io/ev/image-kernels/>
- Deep Learning Book: Chapter 9 - Convolutional Networks
- Stanford CS231n Notes
- Questions
 - Given a $32 \times 32 \times 3$ image and 6 filters of size $5 \times 5 \times 3$, what will be the dimension of the output volume when a stride of 1 and a padding of 0 is considered?
 - Is the max-pooling layer differentiable? How to backpropagate across it?