HW4

qw43@cs.washington.edu
BB

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1.1 What's wrong with this picture?

(a)
It is incorrect because it proves from wrong direction. We can not assume the conclusion and start the proof from it. In the proof, it starts from assuming a = c, which is incorrect.

(b) The original statement is false. A counterexample is that when b equals to zero, ab and ac would both be zero but a and c can be different.

1.2

(a)
It is incorrect. When $a^2 \geq 0, b^2 \geq 0$, their square root can be positive or negative, so it is not sufficient to prove $a = b$

(b) The original statement is false. A counterexample is that when $a^2 = 4, b^2 = 4$, a can be 2 and b can be -2. In this case, a doesn't equal to b

2. Formal and English

(a)

Mysterious(x): $\exists y(4y = x - 3)$

(b)

Proof.

1.	Let a be arbitrary	
2.1	$Mysterious(a)$	[Assumption]
2.2	$\exists y(4y = a - 3)$	[Definition of Mysterious]
2.3	$4z = a - 3$	[Eliminate $\exists$: 2.2, z special]
2.4	$a = 4z + 3$	[algebra 2.3]
2.5	$a = 2(2z + 1) + 1$	[algebra 2.4]
2.6	$\exists k(a = 2k + 1)$	[Intro $\forall$ 2.5]
2.7	$Odd(a)$	[Definition of Odd]
3.	$Mysterious(a) \to Odd(a)$	[Direct Proof Rule 2.1 - 2.7]
4.	$\forall x(Mysterious(x) \to Odd(x)$	[Intro $\forall$ 3]

□

(c)

Let a be an arbitrary integer and suppose Mysterious(a) is True
By definition of Mysterious, we can get know there is an integer y such that $4y = a - 3$
Let z be some variable that satisfies the equation mentioned above, we can get $4z = a - 3$
By rearranging and factor out a two, we can get$a = 2(2z + 1) + 1$
Since z is an integer, $2z + 1$ is also an integer, so we can know that a is odd by definition of Odd
Thus, by direct proof rule, we can get the implication if a satisfies Mysterious then it is odd
Finally, since a is an arbitrary integer, we can conclude that for every integer x, if 4 divides $(x - 3)$, then x is odd.

(d)

Let a be an arbitrary integer and suppose Mysterious(a) is True. 1, 2.1
By definition of Mysterious, we can get know there is an integer y such that $4y = a - 3$. 2.2
Let z be some variable that satisfies the equation mentioned above, we can get $4z = a - 3$. 2.3
By rearranging and factor out a two, we can get$a = 2(2z + 1) + 1$. 2.4, 2.5
Since z is an integer, $2k + 1$ is also an integer, so we can know that a is odd by definition of Odd. 2.6, 2.7

Thus, by direct proof rule, we can get the implication if a satisfies Mysterious then it is odd. 3
Finally, since a is an arbitrary integer, we can conclude that for every integer x, if 4 divides $(x - 3)$, then x is odd. 4

3. A Subset Proof

Let x be an arbitrary element and suppose that $x \in (A \cap B) \cup (A \cap C)$. By definition of union, $x \in (A \cap B)$ or $x \in (A \cap C)$.

In the case of $x \in (A \cap B)$, by definition of intersection, we know $x \in A$ and $x \in B$ and all we particular care about is that $x \in A$.

In the case of $x \in (A \cap C)$, by definition of intersection, we know $x \in A$ and $x \in C$and all we particular care about is that $x \in A$.

In either case, we know $x \in A$. Since x is an arbitrary element that is in $x \in (A \cap B) \cup (A \cap C)$ and also in A, we know $x \in (A \cap B) \cup (A \cap C) \subseteq A$.

4. Set Proofs

(a)
It is True

Let A, B, and C be arbitrary sets.
Let x be an arbitrary element of $(B\backslash A) \cap (C\backslash A)$. By definition of intersection, we know $x \in (B\backslash A)$ and $x \in (C\backslash A)$. Then by the definition of difference, we know that $x \in B$ and $x \notin A$, and, $x \in C$ and $x \notin A$. Then, by definition of intersection, we know $x \in (B \cap C)$. We also know $x \notin A$, we can get $x \in (B \cap C)\backslash A$ by the definition of difference. Since x is an arbitrary element of $(B\backslash A) \cap (C\backslash A)$, we have proved that $(B\backslash A) \cap (C\backslash A) \subseteq (B \cap C)\backslash A$.

Let y be an arbitrary element of $(B \cap C)\backslash A$. By definition of difference, we know $y \in (B \cap C)$ and $y \notin A$. Then, by definition of intersection, we know $y \in B$ and $y \in C$. Thus, we can get $y \in (B\backslash A)$ and $y \in (C\backslash A)$ by definition of difference. Then, we can get $y \in (B\backslash A) \cap (C\backslash A)$ by utilizing intersection again. Since y is an arbitrary element of $(B \cap C)\backslash A$, we have proved that $(B \cap C)\backslash A \subseteq (B\backslash A) \cap (C\backslash A)$.

Since the subset relation holds in both directions, we have $(B\backslash A) \cap (C\backslash A) = (B \cap C)\backslash A$

(b)
It is False

Consider $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, $C = \{3, 4, 5\}$. Then $B\backslash C = \{2\}$ and thus $A\backslash(B\backslash C) = \{1, 3\}$. On the other hand, $A\backslash B = \{1\}$ and thus $(A\backslash B)\backslash C = \{1\}$. $3 \in A\backslash(B\backslash C)$ but $3 \notin (A\backslash B)\backslash C$. Thus, the original statement is false

5. I've never seen such raw power[sets]

(a)
An error in proof strategy: An error in proof strategy is that it doesn't show subset relation holds in both directions. In order to prove set equality, we need to prove the subset relation holds in both directions.

A false assertion in the middle of the proof is that if we know the $X \subseteq S \cup T$ we can not claim $X \subseteq S \vee X \subseteq T$. Define $S = \{1, 2, 3\}$ and $T = \{2, 3, 4\}$, there is some subset like $X = \{1, 4\}$ from $P(S \cup T)$ that doesn't satisfy $X \subseteq S \vee X \subseteq T$

(b) It is False.

Consider $S = \{1, 2, 3\}$ and $T = \{2, 3, 4\}$. Then $P(S \cup T) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3,$
$P(S) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$, $P(T) = \{\emptyset, \{2\}, \{3\}, \{4\}, \{2, 4\}, \{3, 4\}, \{2, 3\}, \{2, 3, 4\}\}$, and $P(S \cap T) = \{\emptyset, \{2\}, \{3\}, \{2, 3\}\}$. From the example, we can found that there is some element $\{1, 2, 3, 4\}$ from $P(S \cap T)$ that is not in the set $P(S) \cup P(T) \cup P(S \cap T)$.

6. Cartesian Products

(a)
The reason that $x + y$ is not an arbitrary positive integer is because it depends on x and y. We can not plug in any positive integer value since x and y have been determined.

(b)
Let S, T, and V be arbitrary sets.

Let (x, y) be an arbitrary element of $(S \cup T) \times V$
...
Hence, (x, y) is an element of $(S \times V) \cup (T \times V)$.
Since (x, y) was arbitrary, every element of $(S \cup T) \times V$ is an element of $(S \times V) \cup (T \times V)$, so $(S \cup T) \times V \subseteq (S \times V) \cup (T \times V)$

Let (i, j) be an arbitrary element of $(S \times V) \cup (T \times V)$
...
Hence, (i, j) is an element of $(S \cup T) \times V$.
Since (i, j) was arbitrary, every element of $(S \times V) \cup (T \times V)$ is an element of $(S \cup T) \times V$, so $(S \times V) \cup (T \times V) \subseteq (S \cup T) \times V$

Since the subset relation holds in both directions, we have $(S \cup T) \times V = (S \times V) \cup (T \times V)$

7. Like -3 but better!

(a)
Let n be an arbitrary integer where $n > 3$.
Consider $b = x$ (where $1 \leq x \leq n$).
By the definition of undoes 3, we can get $a + 3 + b \equiv a(mod\ n)$.
Then, by the definition of mod, we can get $n|a - (a + 3 + b)$
Simplifying the formula, we can get $n| - 3 - b$
Thus, we can know that there is some integer z such that $nz = (-3 - b)$ by the definition of divides.
Since $b = x$ where $(1 \leq x \leq n)$ and $n > 3$, we can know that from the there is some integer z where $1 \leq b \leq (n - 3)$ that could satisfy $nz = (-3 - b)$.

Since n was arbitrary, we have that for every integer that is greater than 3, there exists some integer b, where $1 \leq b \leq n$, which undoes 3 for (mod n) addition.

(b)
Predicate logic: $\forall b \forall b'[(Undoes3(b, n) \wedge Undoes3(b', n)) \rightarrow (n|b' - b)]$

English proof:
Let b, b' both be an arbitrary integers that undo 3 for (mod n) addition.

By the definition of mod, we can know that $n| - 3 - b$ and $n| - 3 - b'$. Thus, we can know there is some integer x that satisfies $nx = -3 - b$ and some integer y that satisfies $ny = -3 - b'$. By subtracting second equation from the first equation, we can get $nx - ny = (-3 - b) - (-3 - b')$. Simplifying the equation, we can get $n(x - y) = b' - b$. Since x and y are integers, x-y is integer. Therefore, we can get $b \equiv b'(mod\ n)$.

Since b and b' are arbitrary, we can conclude that for all integers b, b' where both b and b' undo 3 for (mod n) addition, that $b \equiv b'(mod\ n)$

8. feedback

(a) I spent 7 hours working on this assignment. I spend the most time on question 7.