Exercise 1

Since the times between arrivals of cars independent with mean 5 minutes and finite variance, so the arrival rate of car is $\frac{1}{5}$, and the process rate $\frac{1}{2}$.

Thus $\rho = \frac{\text{arrival rate}}{\text{process rate}} = \frac{1/5}{1/2} = \frac{2}{5}$

And, the probability of having no vehicles in the toll booth:

$P_{no} = 1 - \rho = \frac{3}{5}$

Thus, the probability having n customers in is

$P_n = (\rho)^n \cdot P_{no} = \left(\frac{2}{5}\right)^n \times \frac{3}{5}$

$$\boxed{\text{When } n \to \infty, \quad \lim_{n\to\infty} P_n = \lim_{n\to\infty} \left(\frac{2}{5}\right)^n \times \frac{3}{5} \to 0}$$

Exercise 2:

Let Y denote the number of flips required to get the first tail.

$Y \sim Geo(1-p)$

$$E[X] = \sum_{i=1}^{\infty} E[X \mid Y=i] P(Y=i)$$

$$= \sum_{i=1}^{r} E[X \mid Y=i] P(Y=i) + \sum_{i=r+1}^{\infty} E[X \mid Y=i] P(Y=i)$$

Since $E[X \mid Y=i] \begin{cases} i + E[X] & \text{if } i \le r \\ r & \text{if } i > r \end{cases}$

$$\boxed{\begin{aligned} \text{Thus, } E[X] &= \sum_{i=1}^{r} (i+E[X]) P(Y=i) + \sum_{i=r+1}^{\infty} r\, P(Y=i) \\ &= \sum_{i=1}^{r} (i+E[X]) p^{i-1}(1-p) + \sum_{i=r+1}^{\infty} r p^{i-1} (1-p) \\ &= (1-p) \sum_{i=1}^{r} p^{i-1} (i+E[X]) + (1-p) \sum_{i=r+1}^{\infty} p^{i-1} r \end{aligned}}$$