Problem 1

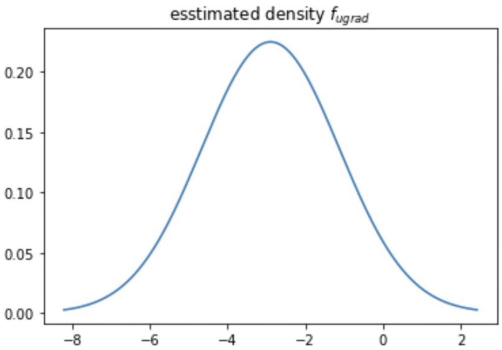
In [39]:
```
import numpy as np
import math
import matplotlib.pyplot as plt
import scipy.stats as stats
```

In [41]:
```
ugrad = np.loadtxt('hw4-ugrad.dat', unpack = True)
chair = np.loadtxt('hw4-chair.dat', unpack = True)
coke = np.loadtxt("hw4-coke.dat", unpack = True)
unknown = np.loadtxt("hw4-unknown.dat", unpack = True)
n_ugrad = len(ugrad)
n_chair = len(chair)
n_coke = len(coke)
n_unknown = len(unknown)
```

(a)

In [47]:
```
# 1 (a)
# mean and standard deviation of ugrad
mean_ugrad = sum(ugrad)/len(ugrad)
var_ugrad = 0
for i in ugrad:
    var_ugrad = var_ugrad + (i-mean_ugrad)**2
var_ugrad = var_ugrad/len(ugrad)
sd_ugrad = math.sqrt(var_ugrad)
print(mean_ugrad, var_ugrad)

# plot
plot_xa = np.linspace(mean_ugrad - sd_ugrad * 3, mean_ugrad + sd_ugrad * 3, 1000)
plt.plot(plot_xa, stats.norm.pdf(plot_xa, mean_ugrad, sd_ugrad))
plt.title("esstimated density $f_{ugrad}$")
plt.savefig("f_ugrad")
```

```
-2.893530685999996 3.142711789810604
```



mean ≈ -2.894

variance ≈ 3.143

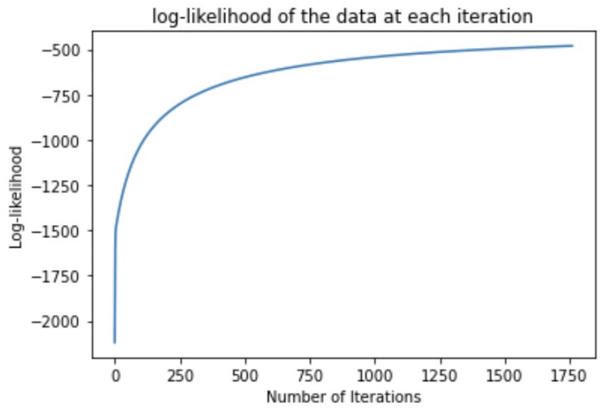
(b)

In [48]:
```
# 1 (b)
a = 1
b = 0
tolerance = 0.0001
step_size = 0.0001
dl_da = 100
dl_db = 100
log_likelihood = []

e = 3

while np.abs(e) > tolerance:
    a_temp = 0
    b_temp = 0
    for xi in chair:
        a_temp += xi * ((1 - np.exp(-a*xi-b))/(1 + np.exp(-a*xi-b)))
        b_temp += (1 - np.exp(-a*xi-b))/(1 + np.exp(-a*xi-b))
    dl_da = n_chair/a - a_temp
    dl_db = - b_temp
    a = a + step_size*dl_da
    b = b + step_size*dl_db
    la = 0
    lb = 0
    for xi in chair:
        la += xi
        lb += np.log(1 + np.exp(-a*xi-b))
    l = n_chair*np.log(a) - a*la - n_chair*b - 2*lb
    log_likelihood.append(l)
    if len(log_likelihood) >= 2:
        e = log_likelihood[len(log_likelihood)-1]/log_likelihood[len(log_likelihood)-2] - 1
print(a, b)
```

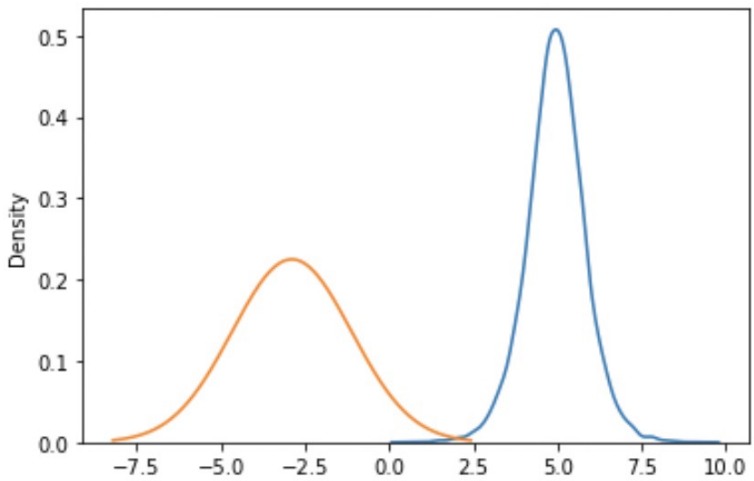
```
2.08380759580565 -10.337514280116958
```

$a \approx 2.084$ $b \approx -10.338$

```
In [46]: # 1 (b)
         # The plot of the log-likelihood of the data
         plot_xb1 = np.linspace(0, len(log_likelihood), len(log_likelihood))
         plt.plot(plot_xb1, log_likelihood)
         plt.title("log-likelihood of the data at each iteration")
         plt.xlabel("Number of Iterations")
         plt.ylabel("Log-likelihood")
         plt.show()
```




```
In [45]: # 1 (b)
         import seaborn as sb
         sb.kdeplot(np.random.logistic(-b/a, 1/a, 10000))
         plt.plot(plot_xa, stats.norm.pdf(plot_xa, mean_ugrad, sd_ugrad))
```



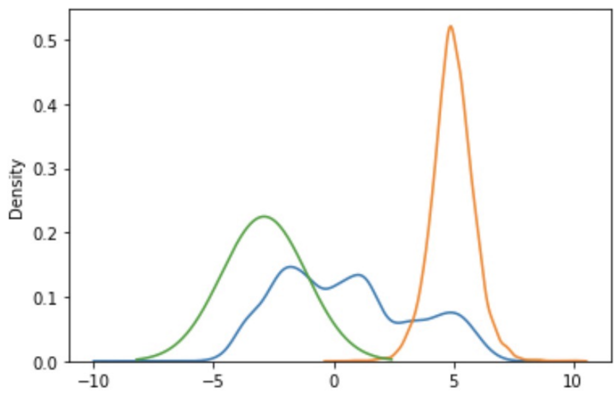
— logistic density

— normal density

c)

```
In [7]: # 1 (c)
        plot_xc = np.linspace(-10, 10, 1000)
        plot_yc = []
        h = 0.5

        for x in plot_xc:
            temp = 0
            for xi in coke:
                temp = temp + (1/math.sqrt(2*np.pi))*np.exp(-((x-xi)/h)**2/2)
            y = (1/(len(coke)*h))*temp
            plot_yc.append(y)
```

```
In [152]: plt.plot(plot_xc, plot_yc)
          sb.kdeplot(np.random.logistic(-b/a, 1/a, 10000))
          plt.plot(plot_xa, stats.norm.pdf(plot_xa, mean_ugrad, sd_ugrad))
          # sb.kdeplot()
```



— Normal density

— logistic density

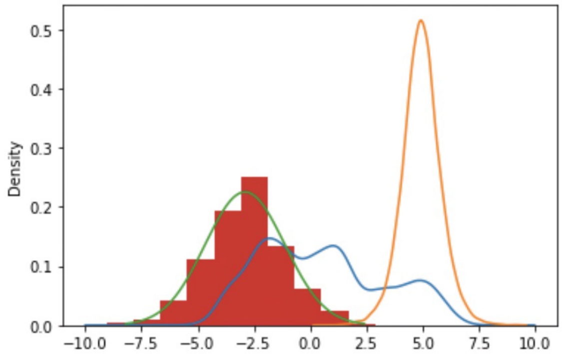
— kernel density estimator.

d)

In [53]:
```
# 1 (d)
log_norm = 0
log_logistics = 0
log_lo1 = 0
log_lo2 = 0
log_kde = 0
log_temp = 0
for xi in unknown:
    log_norm = log_norm + (-1/2)*np.log(sd_ugrad**2) + (-1/2)*np.log(2*np.pi) - (xi - mean_ugrad)**2/(2*sd_ugrad**2)
    log_lo1 = log_lo1 + xi
    log_lo2 = log_lo2 + np.log(1 + np.exp(-a*xi-b))
    kde_temp = 0
    for i in coke:
#           kde_temp = kde_temp + (1/math.sqrt(2*np.pi))*np.exp((-((xi-i)/h)**2)/2)
        kde_temp = kde_temp + (1/math.sqrt(2*np.pi))*np.exp((-((xi-i)/h)**2)/2)
    log_temp = (1.0/(n_coke*h)) * kde_temp
    log_kde = log_kde + np.log(log_temp)
log_logistics = n_unknown * np.log(a) - a*log_lo1 - n_unknown*b - 2*log_lo2
print(log_logistics)
print(log_norm)
print(log_kde)
```

```
-5151.290450105144
-665.2023398471927
-1151.797558584534
```

In [54]:
```
plt.plot(plot_xc, plot_yc)
sb.kdeplot(np.random.logistic(-b/a, 1/a, 10000))
plt.plot(plot_xa, stats.norm.pdf(plot_xa, mean_ugrad, sd_ugrad))
plot_gridx = np.linspace(min(unknown), max(unknown), 10)
plot_gridy = np.zeros(len(plot_gridx))
plt.hist(unknown, density = True)
plt.show()
```



— Normal density

— logistic density

— kernel density estimator.

— data Dunknown

Problem 2:

(a) $f(x_i) = r \cdot e^{-r \cdot x_i}$

When $x_i > 1$, $y_i = f(x_i) = r \cdot e^{-r} \cdot e^{x_i} = 1$

$f_r(0) = F(1) = \int_0^1 r \cdot e^{-rx} \, dx = 1 - e^{-r}$

$f_r(1) = F(\infty) = 1 - (1 - e^{-r}) = e^{-r}$

$P(y_i = 0) = 1 - e^{-r}$ $\quad P(y_i = 1) = e^{-r}$

(b)

Let n_0 be the number of censored data where $y_i = 0$

n_1 be the number of censored data where $y_i = 1$

$L(r) = \prod_{i=1}^{n_0} (1 - e^{-r}) \cdot \prod_{i=1}^{n_1} (e^{-r})$

$l(r) = \ln[(1 - e^{-r})^{n_0} \cdot (e^{-r})^{n_1}]$

$= n_0 (1 - e^{-r}) - n_1 r$

(c) $l'(r) = n_0 e^{-r} - n_1 = 0$

$n_0 e^{-r} = n_1$

$e^{-r} = \frac{n_1}{n_0}$

$\ln(e^{-r}) = \ln(\frac{n_1}{n_0})$

$-r = \ln(\frac{n_1}{n_0})$

$r = -\ln(\frac{n_1}{n_0})$

d) Yes, I think there are 2 sufficient statistics.

The first sufficient statistics is n_0

The second one is n_1