HW8

qw43@cs.washington.edu
BB

June 1, 2022

---

1. Like Divides, but more complicated

(a)

Let a, b, c be arbitrary integers.
Define the following relation R on the positive integers. $(x, y) \in R$ if and only if x/y is an even integer.
We prove R is a transitive relation which means S is transitive if and only if $\forall a \forall b \forall c[(a, b) \in R \wedge (b, c) \in R \rightarrow (a, c) \in R]$
Suppose $(a, b) \in R \wedge (b, c) \in R$.

By definition of R, we could know that there is an even integer z equals to a/b which means $a = b \cdot z$ where $z = 2 \cdot x$ for some integer x. By definition of R again. we could know there is an even integer w equals to b/c which means $b = c \cdot w$ where $w = 2 \cdot y$ for some integer y. By substituting b in $a = b \cdot z$ with $b = c \cdot w$, we have $a = c \cdot w \cdot z$ which is $a = c \cdot 2 \cdot y \cdot 2 \cdot x$. By algebra, we have $a = c \cdot 4 \cdot y \cdot x$. This means there is an even integer $4 \cdot y \cdot x$ equals to a/c by algebra. Since x, y are integers, $4 \cdot y \cdot x$ is also an even integer. Thus, we know $(a, c) \in R$.

Since a, b, c are arbitrary, we can conclude that if $(a, b) \in R \wedge (b, c) \in R$, then $(b, c) \in R$. Therefore, we prove R is a transitive relation.

(b)

We will prove R is not a symmetric relation by counter example.
R is symmetric if and only if $\forall a \forall b[(a, b] \in R \rightarrow (b, a) \in R$.
Let $(4, 2) \in R$. We know that $(4, 2) \in R$ because $4/2 = 2 = 2 \cdot 1$ by the definition of even and R. But $(2, 4) \notin R$ since $2/4 = 1/2$ which is not an even integer. Thus, not all element satisfies $\forall a \forall b[(a, b] \in R \rightarrow (b, a) \in R$.
Thus, R is not a symmetric relation.

2. Com-pair-isons

(a)
Let $X = \{x_1, x_2\}, Y = \{y_1, y_2\}$ with $x_1 < x_2$ and $y_1 < y_2$ be arbitrary elements that satisfy $X \in A \wedge Y \in A$. Suppose $X \preceq Y$ and $Y \preceq X$ satisfy the relation $\preceq$ defined in the problem specification. We prove the relation $\preceq$ is antisymmetry by showing for all $a, b \in A, [(a,b) \in R \wedge (b,a) \in R \rightarrow a = b]$.

Because of $X \preceq Y$, we know that $x_1 \leq y_1$ and $x_2 \leq y_2$ by the definition of $\preceq$. Because of $Y \preceq X$, we could know $x_1 \leq y_1$ and $x_2 \leq y_2$ by the definition of $\preceq$. Thus, since we know $x_1 \leq y_1$ and $x_2 \leq y_2$ and $x_1 \leq y_1$ and $x_2 \leq y_2$, we have $x_1 = y_1$ and $x_2 = y_2$ by algebra. Therefore, we get $X = Y$.

Since X, Y are arbitrary elements in A, we have for all $a, b \in A, [(a,b) \in R \wedge (b,a) \in R \rightarrow a = b]$. Thus, we prove the relation $\preceq$ is antisymmetry.

(b)
No.
We prove it by counter example.
Consider $X = \{1,5\} \in A, Y = \{2,3\} \in A$. We know that $\{1,5\} \not\preceq \{2,3\}$ because $5 > 3$ and $\{2,3\} \not\preceq \{1,5\}$ because $2 > 1$.
Thus, $\preceq$ is not a total order on A.

(c)
We prove it by counter example.
Consider $X = \{3,9\} \in A, Y = \{4,7\} \in A, Z = \{2,8\} \in A$. $X \preceq Y$ because $3 \leq 4$ and $Y \preceq Z$ because $7 \leq 8$. However, $X \not\preceq Z$ since $3 \not\leq 2$ and $9 \not\leq 8$. Thus, we know not all elements in A satisfy $\forall X \forall Y \forall Z[X \preceq Y \wedge Y \preceq Z] \rightarrow X \preceq Z$.

5. Extra-Arden-ary Translations

(a) $\forall A \forall B[\neg Matches(\varepsilon, B) \rightarrow \exists C((C = A \cdot C \vee B) \wedge \forall D((D = A \cdot D \vee B) \rightarrow (C = D))))]$

(b)

$\neg\forall A \forall B[\neg Matches(\varepsilon, B) \rightarrow \exists C((C = A \cdot C \cup B) \wedge \forall D((D = A \cdot D \cup B) \rightarrow (C = D)))]$	Law of Implication
$\neg\forall A \forall B[Matches(\varepsilon, B) \vee \exists C((C = A \cdot C \cup B) \wedge \forall D(\neg(D = A \cdot D \cup B) \vee (C = D)))]$	Law of Implication
$\exists A \neg\forall B[Matches(\varepsilon, B) \vee \exists C((C = A \cdot C \cup B) \wedge \forall D(\neg(D = A \cdot D \cup B) \vee (C = D)))]$	Definition of negation
$\exists A \exists B \neg[Matches(\varepsilon, B) \vee \exists C((C = A \cdot C \cup B) \wedge \forall D(\neg(D = A \cdot D \cup B) \vee (C = D)))]$	Definition of negation
$\exists A \exists B[\neg Matches(\varepsilon, B) \wedge \neg\exists C((C = A \cdot C \cup B) \wedge \forall D(\neg(D = A \cdot D \cup B) \vee (C = D)))]$	DeMorgan's Laws
$\exists A \exists B[\neg Matches(\varepsilon, B) \wedge \forall C \neg((C = A \cdot C \cup B) \wedge \forall D(\neg(D = A \cdot D \cup B) \vee (C = D)))]$	Definition of negation
$\exists A \exists B[\neg Matches(\varepsilon, B) \wedge \forall C(\neg(C = A \cdot C \cup B) \vee \neg\forall D(\neg(D = A \cdot D \cup B) \vee (C = D)))]$	DeMorgan's Laws
$\exists A \exists B[\neg Matches(\varepsilon, B) \wedge \forall C((C \neq A \cdot C \cup B) \vee \exists D((D = A \cdot D \cup B) \wedge (C \neq D)))]$	Definition of negation

(c) There exists some regular expressions A and B such that if the empty string does not match B, then all regular expression C that does not satisfy $C = A \cdot C \cup B$ or there exists some regular expression D that satisfies $D = A \cdot D \cup B$ and are different from C.

6. Proof By Contradiction

(a) $\forall x \forall y((Rational(x) \land \neg Rational(y)) \rightarrow \neg Rational(x+y))$

(b) $\exists x \exists y(Rational(x) \land \neg Rational(y) \land Rational(x+y))$

(c)
I prove this claim by contradiction.
Denote r to be a rational number and i to be an irrational number

Suppose, for the sake of contradiction, the sum of a rational number and an irrational number, s, is rational which is $r + i = s$. By the definition of rational number, r and s can be made by dividing two integers, so $r = \frac{a}{b}$ and $s = \frac{m}{n}$ for some integers a, b, m, and n. Then, subtracting both both sides of equation by r. we have $i = s - r$ which equals to $i = \frac{m}{n} - \frac{a}{b}$. By algebra, we have $i = \frac{mb-na}{nb}$. Since a, b, m, and n are integers, $mb - na$ and nb are also integers. So, we conclude i is a rational number which is a contradiction. Thus, our assumption that s is a rational number must be false. Therefore, I prove the claim.

7. Buggy

(a) The biggest bug is step 2. The negation is not correct, while when we do proof by contradiction, we need to assume the negation and start from that.

8. feedback

(a) 9 hours

(b) question 4

(c) No