HW7

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BB
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1. Manhattan Walk

Let $P(x, y)$ be: "$x \leq 2y$". We prove $P(x, y)$ holds for all pairs $(x, y) \in S$ by structural induction.

Base Cases:
When $x = 3, y = 2$, $3 \leq 2 \cdot 2$. Thus, $P(3, 2)$ holds.

Inductive hypothesis:
Suppose $P(a, b)$ holds for arbitrary $a, b \in S$.

Inductive step:
Case 1: We show $P(a, b + 1)$ holds by inductive steps
By the definition of sets, we know $(a, b) \in S$. By inductive hypothesis, we have $a \leq 2 \cdot b$. By algebra, we have $a \leq 2 \cdot b + 2$ and thus $a \leq 2 \cdot (b + 1)$. This gives us $P(a, b + 1)$.
Case 2: We show $P(a + 3, b + 2)$ holds by inductive steps
By the definition of sets, we know $(a, b) \in S$. By induction hypothesis, we have $a \leq 2 \cdot b$. By algebra, we have $a + 3 \leq 2b + 4$ and thus we have $a + 3 \leq 2 \cdot (b + 2)$. This gives us $P(a + 3, b + 2)$.

Conclusion:
Therefore $P(x, y)$ holds for all pairs $(x, y) \in S$

2. What doesn't kill you makes you stronger

(a)
I spent 10 minutes trying.

(b)
When $a = 1.5, b = 2$, $3a + 2 = 6.5 \not< 3b = 6$

(c)
$f(n) - 1 < f(n)$, so if we could show $g(n) \leq f(n) - 1$, we could also prove $g(n) < f(n)$

(d)
Let $Q(n)$ be $g(n) \leq f(n) - 1$. We prove $Q(n)$ holds for all $n \geq 1$ by strong induction.

Base case: When $n = 1$, $g(1) = 2 \leq f(1) = 3$. Thus, $Q(1)$ holds.

Inductive hypothesis: Suppose $Q(1) \wedge Q(2) \wedge ... \wedge Q(n)$ holds for all $n \geq 1$, which is $g(n) \leq f(n) - 1$

Inductive step: We show $Q(k+1)$ holds which is $g(k+1) \leq f(k+1) - 1$
By inductive hypothesis, we know:

$$\begin{aligned}
g(k) &\leq f(k) - 1 && \text{Inductive hypothesis} \\
3 \cdot g(k) &\leq 3 \cdot (f(k) - 1) && \text{Algebra} \\
3 \cdot g(k) &\leq 3 \cdot f(k) - 3 && \text{Algebra} \\
3 \cdot g(k) + 2 &\leq 3 \cdot f(k) - 3 + 2 && \text{Algebra} \\
3 \cdot g(k) + 2 &\leq 3 \cdot f(k) - 1 && \text{Algebra} \\
g(k+1) &\leq f(k+1) - 1 && \text{Definition of g and f}
\end{aligned}$$

Thus we prove $Q(k+1)$

Conclusion: $Q(n)$ holds for all integers n 1 by the principle of strong induction.

3. Recursion – See: Recursion

(a)

Let S be a set of binary strings defined recursively as:
Basis Step: $1 \in S$ where
Recursive Step: if $x \in S$, then $x \cdot w \in S$ for arbitrary binary string w with $len(w) = 2$.
Justify: The basis step is 1 since 1 has the length of 1 and starts with 1. And concatenating any binary strings with length 2 to elements in the set could new element that satisfies the requirement.

(b)

Let T be a set of binary strings defined recursively as:
Basis Step: $a \in T$ where for arbitrary binary string a where $len(a) = 2$
Recursive Step: if $x \in T$ then $x \cdot b \in T$ for arbitrary binary string b where $len(b) = 3$
Justify: The basis step is a binary string with length 2 since 2 is equivalent to 2 $(mod 3)$. Then, concatenating arbitrary string with length 3 to elements in the set could produce new element that satisfies the requirement.

(c)

Let U be a set of binary strings defined recursively as:
Basis Step: $0 \in U$
Recursive Step: if $x \in U$, then $1x, x1, 0x0 \in U$
Justify: The basis step is a binary string with only one 0 which has odd number of 0s. Then, concatenating one 1 and two 0s in the form written above could produces binary string with odd number of 0s.

6. feedback

(a) 9 hours

(b) question 4

(c) No