LAB - 6 : Finding gradient, divergence, curl and Green's Theorem

Gradient of a Scalar Function

$$\phi = \phi(x, y, z)$$

$$\text{grad}\,\phi = \frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}$$

```
#Finding the Gradient of a scalar function
from sympy . physics . vector import *
from sympy import *
x,y,z=symbols('x,y,z')
v= ReferenceFrame ('v')
φ=log(x**2+y**2+z**2)
print ("\n Gradient of φ is")
display(Derivative(φ,x)*v.x+Derivative(φ,y)*v.y+Derivative(φ,z)*v.z)
gradφ=diff(φ,x)*v.x+diff(φ,y)*v.y+diff(φ,z)*v.z
display(gradφ)
```

OUTPUT

```
 Gradient of φ is
```

$$\frac{\partial}{\partial x}\log\left(x^2 + y^2 + z^2\right)\hat{\mathbf{v}}_{\mathbf{x}} + \frac{\partial}{\partial y}\log\left(x^2 + y^2 + z^2\right)\hat{\mathbf{v}}_{\mathbf{y}} + \frac{\partial}{\partial z}\log\left(x^2 + y^2 + z^2\right)\hat{\mathbf{v}}_{\mathbf{z}}$$

$$\frac{2x}{x^2 + y^2 + z^2}\hat{\mathbf{v}}_{\mathbf{x}} + \frac{2y}{x^2 + y^2 + z^2}\hat{\mathbf{v}}_{\mathbf{y}} + \frac{2z}{x^2 + y^2 + z^2}\hat{\mathbf{v}}_{\mathbf{z}}$$

```
#Finding the Normal vector to the surface
from sympy . physics . vector import *
from sympy import *
x,y,z=symbols('x,y,z')
v= ReferenceFrame ('v')
φ=x**3+y**3+3*x*y*z-3
print ("\n Gradient of φ is")
display(Derivative(φ,x)*v.x+Derivative(φ,y)*v.y+Derivative(φ,z)*v.z)
gradφ=diff(φ,x)*v.x+diff(φ,y)*v.y+diff(φ,z)*v.z
display(gradφ)
NV=gradφ.subs({x:1,y:2,z:-1})
print('Normal vector to the suface is ')
display(NV)
```

OUTPUT

Gradient of φ is

$$\frac{\partial}{\partial x}\left(x^3 + 3xyz + y^3 - 3\right)\hat{\mathbf{v}}_{\mathbf{x}} + \frac{\partial}{\partial y}\left(x^3 + 3xyz + y^3 - 3\right)\hat{\mathbf{v}}_{\mathbf{y}} + \frac{\partial}{\partial z}\left(x^3 + 3xyz + y^3 -\right.$$

$$(3x^2 + 3yz)\hat{\mathbf{v}}_{\mathbf{x}} + (3xz + 3y^2)\hat{\mathbf{v}}_{\mathbf{y}} + 3xy\hat{\mathbf{v}}_{\mathbf{z}}$$

Normal vector to the suface is

$$-3\hat{\mathbf{v}}_{\mathbf{x}} + 9\hat{\mathbf{v}}_{\mathbf{y}} + 6\hat{\mathbf{v}}_{\mathbf{z}}$$

Divergence of a Vector point Function

$$f = f_1(x,y,z)\hat{i} + f_2(x,y,z)\hat{j} + f_3(x,y,z)\hat{k}$$

$$divf = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

```
#Finding the divergence of a vector field
from sympy . physics . vector import *
from sympy import *
x,y,z=symbols('x,y,z')
v= ReferenceFrame ('v')
f1=x*y**2
f2=2*x**2*y*z
f3=-3*y*z**2
f=f1*v.x+f2*v.y+f3*v.z
print ("\n Divergence of the vector field f is")
display(Derivative(f1,x)+Derivative(f2,y)+Derivative(f3,z))
divf=diff(f1,x)+diff(f2,y)+diff(f3,z)
display(divf)
```

OUTPUT

```
 Divergence of the vector field f
```

$$\frac{\partial}{\partial x} x y^2 + \frac{\partial}{\partial z}\left(-3 y z^2\right) + \frac{\partial}{\partial y} 2 x^2 y z$$

$$2x^2 z + y^2 - 6yz$$

```python
# To verify the vector is a solenoidal
from sympy . physics . vector import *
from sympy import *
x,y,z=symbols('x,y,z')
v= ReferenceFrame ('v')
f1=3*y**4*z**2
f2=4*x**3*z**2
f3=3*x**2*y**2
f=f1*v.x+f2*v.y+f3*v.z
print ("\n Divergence of the vector f is ")
display(Derivative(f1,x)+Derivative(f2,y)+Derivative(f3,z))
divf=diff(f1,x)+diff(f2,y)+diff(f3,z)
display(divf)
if divf==0:
    print('The vector f is solenoidal')
else:
    print('The vector f is not solenoidal')
```

OUTPUT

```
 Divergence of the vector f
```

$$\frac{\partial}{\partial z} 3x^2 y^2 + \frac{\partial}{\partial y} 4x^3 z^2 + \frac{\partial}{\partial x} 3y^4 z^2$$

$$0$$

```
The vector f is solenoidal
```

Curl of a Vector Point Function

$$f = f_1(x,y,z)\hat{i} + f_2(x,y,z)\hat{j} + f_3(x,y,z)\hat{k}$$

$$curlf = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix} = \left(\frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z}\right)\hat{i} - \left(\frac{\partial f_3}{\partial x} - \frac{\partial f_1}{\partial z}\right)\hat{j} + \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right)\hat{k}$$

```
#Program to find curl of F=xy^2i+2x^2yzj -3yz^2k
from sympy . physics . vector import *
from sympy import *
x,y,z=symbols('x,y,z')
v= ReferenceFrame ('v')
f1=x*y**2
f2=2*x**2*y*z
f3=-3*y*z**2
f=f1*v.x+f2*v.y+f3*v.z
print (" curl of f is ")
display((Derivative(f3,y)-Derivative(f2,z))*v.x-(Derivative(f3,x)-Derivativ
curlf=(diff(f3,y)-diff(f2,z))*v.x-(diff(f3,x)-diff(f1,z))*v.y+(diff(f2,x)-d
display (curlf)
```

OUTPUT

```
 curl of f is
```

$$\left(\frac{\partial}{\partial y}(-3yz^2) - \frac{\partial}{\partial z}2x^2yz\right)\hat{\mathbf{v}}_{\mathbf{x}} + \left(\frac{\partial}{\partial z}xy^2 - \frac{\partial}{\partial x}(-3yz^2)\right)\hat{\mathbf{v}}_{\mathbf{y}} + \left(-\frac{\partial}{\partial y}xy^2 + \frac{\partial}{\partial x}2x^2yz\right)\hat{\mathbf{v}}_{\mathbf{z}}$$

$$(-2x^2y - 3z^2)\hat{\mathbf{v}}_{\mathbf{x}} + (4xyz - 2xy)\hat{\mathbf{v}}_{\mathbf{z}}$$

Green's Theorem

If $M(x,y), N(x,y), M_y$ and N_x be continuous in a region 'R' of the xy-plane bounded by a closed curve 'C', then

$$\int_C M dx + N dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\, dy$$

```
#Using Green's theorem, evaluate Integral of [(x + 2y)dx + (x - 2y)dy] over
#where c is the region bounded by coordinate axes and the line x = 1 and y
from sympy import *
x,y=symbols('x,y')
p=x+2*y
q=x-2*y
f= diff (q,x)- diff (p,y)
soln = integrate (f,[x,0,1],[y,0,1])
display(Eq(Integral (f,[x,0,1],[y,0,1]),soln))
```

OUTPUT

$$\int_0^1 \int_0^1 (-1)\, dx\, dy = -1$$

Exercise : Write the python program for the following

1. If $\phi = x^2 y + y^2 z + z^2 x$ find $\nabla\phi$ at $(1,1,1)$
2. Find $\nabla\phi$ if $\phi = \log(x^2 + y^2 + z^2)$
3. Find the unit normal vector to the surface $x^2 y + 2xz = 4$ at $(2,-2,3)$
4. Find the unit normal vector to the surface $xy^3 z^2 = 4$ at $(-1,-1,2)$
5. Find $\nabla.F$ *and* $\nabla\times F$ if $F = x^2 yz\hat{i} + xy^2 z\hat{j} + xyz^2\hat{k}$
6. Show that the vector $(-x^2 + yz)\hat{i} + (-z^2 x + 4y)\hat{j} + (2xz - 4z)\hat{k}$ is solenoidal.
7. Using Green's theorem, evaluate $\int_C (y^2 + xy)dx + x^2\, dy$ where 'C' is the region bounded by the curves $y = x$ and $y = x^2$