LAB 8: Verification of convolution theorem and inverse using convolution theorem

Program 1: Program to verify the convolution theorem of the Laplace transform of the functions f(t) = t and g(t) = e^t.

```
from sympy import *
t,s,u = symbols("t,s,u")
f=t
g=exp(t)
#finding convolution f*g (using o instead of *)
fog=integrate(f.subs(t,u)*g.subs(t,t-u),(u,0,t))
F=laplace_transform(f,t,s,noconds=True)
G=laplace_transform(g,t,s,noconds=True)
FOG=laplace_transform(fog,t,s,noconds=True)
print("Laplace transfrom of f is")
display(F)
print("Laplace transfrom of g is")
display(G)
print("Laplace transfrom of f*g is")
display(FOG)
if(F*G==FOG):
    print("Convolution theorem is verified")
```

Output:

```
Laplace transfrom of f is
```

$$\frac{1}{s^2}$$

```
Laplace transfrom of g is
```

$$\frac{1}{s-1}$$

```
Laplace transfrom of f*g is
```

$$\frac{1}{s^2\,(s-1)}$$

```
Convolution theorem is verified
```

Program 2: Program to find inverse Laplace Transform of $\frac{s}{(s-1)(s^2+4)}$ using convolution theorem.

```
from sympy import *
s,u = symbols("s,u")
t = Symbol("t",positive=True)
F=1/(s-1)
G=s/(s**2+4)
f=inverse_laplace_transform (F, s, t)
display(f)
g=inverse_laplace_transform (G, s, t)
display(g)
fg=integrate(f.subs(t,u)*g.subs(t,t-u),(u,0,t))
print("Inverse Laplace transfrom of given function is:")
display(fg)
```

Output:

e^t

$\cos(2t)$

```
Inverse Laplace transfrom of given function is:
```

$$\frac{e^t}{5} + \frac{2\sin(2t)}{5} - \frac{\cos(2t)}{5}$$

Exercise: Write a Python program for the following

1. Verify the convolution theorem of the Laplace transform of
   (a) $f(t) = \sin t$ and $g(t) = \dfrac{\sin 3t}{3}$
   (b) $f(t) = \cosh 3t$ and $g(t) = t^5$
2. Find the inverse Laplace Transform of the following using the convolution theorem
   (a) $\dfrac{s^2}{(s^2+1)(s^2+4)}$
   (b) $\dfrac{s}{(s^2+4)(s^2+9)}$
   (c) $\dfrac{1}{s^2(s^2+4)}$