LAB - 6 : Programme to compute area & volume

Evaluation of double integral

$$\int_1^4 \int_0^{\sqrt{4-x}} x\, y\, dy\, dx$$

$$\text{Integral}(\text{integrand}, \text{reference variable}) \rightarrow \int$$

$$\text{Integrate}(\text{integrand}, \text{reference variable}) \rightarrow \text{used to do the integration}$$

Program 1: Program for evaluation of double integral.

```
#Evaluation of Double Integral
from sympy import *
x,y=symbols('x,y')
f=x*y
I=Integral(f,(y,0,sqrt(4-x)),(x,1,4))
display(Eq(I,integrate(f,(y,0,sqrt(4-x)),(x,1,4))))
```

OUTPUT:

$$\int_1^4 \int_0^{\sqrt{4-x}} xy\, dy\, dx = \frac{9}{2}$$

Evaluation of triple integral

$$\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\,dx\,dy\,dz$$

Program 2: Program for evaluation of tripe integral.

```
#Evaluation of Triple Integral
from sympy import *
x,y,z,a,b,c=symbols('x,y,z,a,b,c')
f=(x**2)+(y**2)+(z**2)
I=Integral(f,(x,-a,a),(y,-b,b),(z,-c,c))
I1=simplify(integrate(f,(x,-a,a),(y,-b,b),(z,-c,c)))
display(Eq(I,I1))
```

OUTPUT:

$$\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}\left(x^2+y^2+z^2\right)\,dx\,dy\,dz=\frac{8abc\left(a^2+b^2+c^2\right)}{3}$$

Program 3: Program for verification of changing the order of the integration will not affect the integral.

```
#Verification of Changing the order of the integration will not affect the
from sympy import *
x,y=symbols('x,y')
f=(x**2)+(y**2)
I1=simplify(integrate(f,x,y))
display(Eq(Integral(f,x,y),I1))
I2=simplify(integrate(f,y,x))
display(Eq(Integral(f,y,x),I2))
if I1==I2:
    print('Changing the order of the integration will not affect the integr
else:
    print('Changing the order of the integration will affect the integral')
```

OUTPUT:

$$\iint \left(x^2 + y^2\right)\, dx\, dy = \frac{xy\left(x^2 + y^2\right)}{3}$$

$$\iint \left(x^2 + y^2\right)\, dy\, dx = \frac{xy\left(x^2 + y^2\right)}{3}$$

```
Changing the order of the integration will not affect the integral
```

<u>Program 4:</u> Program to find the area in cartesian form.

```
# Finding the Area in cartesian form
from sympy import *
x,y=symbols('x,y')
a=2
b=1
A=4*Integral(1,(y,0,(b/a)*sqrt(a**2-x**2)),(x,0,a))
print('Area of the ellipse ')
A1=(4*(integrate(1,(y,0,(b/a)*sqrt(a**2-x**2)),(x,0,a))))
display(Eq(A,A1))
```

OUTPUT:

```
Area of the ellipse
```

$$4\int_0^2 \int_0^{0.5\sqrt{4-x^2}} 1\, dy\, dx = 2.0\pi$$

Program 5: Program to find the area in polar form.

```
# Finding the Area in polar form
from sympy import *
r,θ,a=symbols('r,θ,a')
f=r
A=2*Integral(f,(r,0,a*(1+cos(θ))),(θ,0,pi))
A1=2*(integrate(r,(r,0,a*(1+cos(θ))),(θ,0,pi)))
print('Area of the cardiod  ')
display(Eq(A,A1))
```

OUTPUT:

```
Area of the cardiod
```

$$2\int\limits_{0}^{\pi}\int\limits_{0}^{a(\cos(\theta)+1)} r\,dr\,d\theta = \frac{3\pi a^2}{2}$$

Program 5: Program to find the volume of the solid using triple integral.

```
#Volume of the solid using triple integral
from sympy import *
x,y,z,a,b,c=symbols('x,y,z,a,b,c')
A=Integral(1,(z,0,c*(1-(x/a)-(y/b))),(y,0,b*(1-(x/a))),(x,0,a))
A1=integrate(1,(z,0,c*(1-(x/a)-(y/b))),(y,0,b*(1-(x/a))),(x,0,a))
print('volume of the tetrahedron  ')
display(Eq(A,A1))
```

OUTPUT:

```
volume of the tetrahedron
```

$$\int\limits_{0}^{a}\int\limits_{0}^{b\left(1-\frac{x}{a}\right)}\int\limits_{0}^{c\left(1-\frac{y}{b}-\frac{x}{a}\right)} 1\,dz\,dy\,dx = \frac{abc}{6}$$

Exercise: Write python program for the following

1. Evaluate:

a) $\int_0^a \int_0^b \left(x^2+y^3\right)dxdy$ b) $\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{dydx}{\sqrt{1+x^2+y^2}}$ c) $\int_0^1 \int_0^{\sqrt{1-y^2}} x^3 ydxdy$

d) $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} \left(x^2+y^2+z^2\right)dzdydx$ e) $\int_0^{\log 2} \int_0^{x} \int_0^{x+\log y} e^{x+y+z}dzdydx$

f) $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{\sqrt{1-x^2-y^2-z^2}}$

2. Evaluate:

a) $\iint_R xy^2dxdy$ over the region R bounded by $y=x^2, y=x$ and $x=1$.

b) $\iint_R x^2y^2dxdy$ over the region R bounded by the y-axis, x-axis and $x^2+y^2=1$.

3. Using double integral,

a) find the area of the circle $x^2+y^2=16$ in the first quadrant.

b) find the area bounded by the circle $x^2+y^2=a^2$ and the line $x+y=a$ in the first quadrant.

c) find the area bounded between the parabolas $y^2=4ax$ and $x^2=4ay$.

d) find the area bounded by the curves $y=x^2$ and the line $x+y=1$.

e) find the area enclosed by the curve $r=a\left(1+\cos\theta\right)$ between $\theta=0$ and $\theta=\pi$.

4. a) Find the volume of the tetrahedron bounded by the planes $x=0, y=0, z=0$ and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

b) Find the volume of the solid bounded by the planes $x=0, y=0, z=0$ and $x+y+z=1$.

c) Find the volume generated by the revolution of the cardioid $r=a(1+\cos\theta)$ about the initial line.