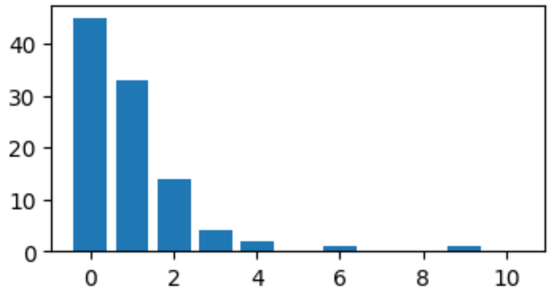
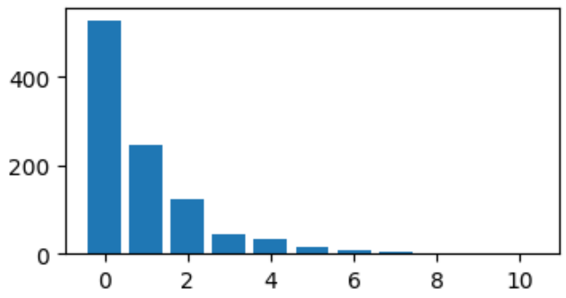
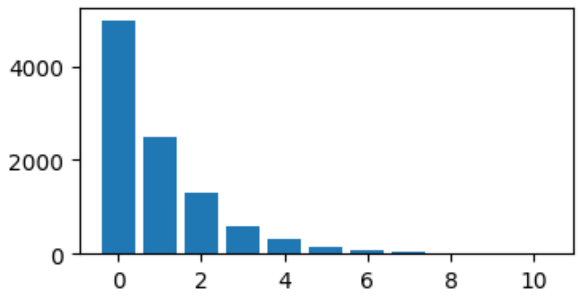
1.圣彼得堡问题

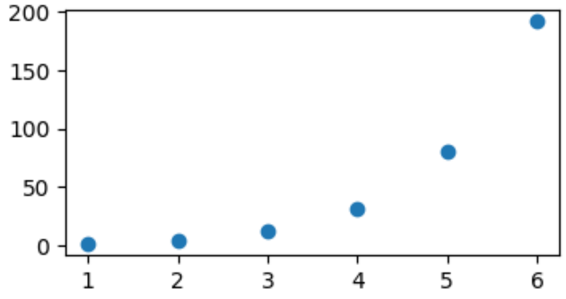
```python
import numpy as np
import math
import random as rd
import matplotlib.pyplot as plt

# 硬币
def isFront():
    return rd.choice([True, False])

# St. Petersburg question
def saint():
    k = 0
    while isFront() and k < 10:
        k += 1
    return k

# k次实验
def kSaint(k):
    result = [0 for i in range(11)]
    for i in range(k):
        positive = saint()
        result[positive] += 1
    return result

# 作图
def kSaintPlot(k):
    result = kSaint(k)
    plt.figure(figsize=(4, 2))
    plt.bar([i for i in range(len(result))],result)
    plt.show()

kSaintPlot(100)
kSaintPlot(1000)
kSaintPlot(10000)
```







```
def earn():
    earn = 0
    k = 1
    earn_values = []
    while k < 7:
        earn = k * 2 ** (k - 1)
        earn_values.append(earn)
        k += 1

    plt.figure(figsize=(4, 2))
    x_values = list(range(1, len(earn_values) + 1))
    plt.scatter(x_values, earn_values)
    plt.show()

earn()
```



3.

$\int_{-\infty}^{\infty} g(x)dx = 0.1c + 0.6*2c + 0.3*c/3 = 1 \Rightarrow c = \frac{5}{7}$

当y取值为$c=\frac{5}{7}$时，在[0,0.1)之间随机取一个数即得到对应的样本，同理，当y为2c,c/3时，在区间[0.1,0.7),[0,7,1)随机取一个数即为样本

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$\int_{-\infty}^{\infty} cg(t)dt = c(\int_{-2}^{2} \frac{8}{7} + \frac{118}{63}x^2 - \frac{74}{63}x^4 + \frac{10}{63}x^6)dx = c\frac{2}{63}(36x + \frac{59}{3}x^3 - \frac{37}{5}x^5 + \frac{5}{7}x^7)|_{-2}^{2} = c\frac{35264}{6615} = 1$

$\Rightarrow c = \frac{6615}{35264}$

显然偶函数$f(x)$在[-2,0]递减，[0,2]递增，其在$[0,\infty]$的反函数显然存在，记为$f^{-1}(x) = \begin{cases} 0 & x < -2 \\ g_1^{-1}(x) & x \in [-2,\ 0] \\ g_2^{-1}(x) & x \in (0,2] \\ 0 & x > 2 \end{cases}$,

那么取$h(x) = \begin{cases} 0 & x < -2 \\ -g_1^{-1}(x) & x \in [-2,\ 0] \\ g_2^{-1}(x) & x \in (0,2] \\ 0 & x > 2 \end{cases}$,再取一$(-\infty,+\infty)$的均匀分布$X$，那么$h(X)$即为所需抽样

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不妨记$A=(a_{ij})_{2\times 2}, \mu=(\mu_1,\mu_2)^T$那么$Y_1$$N(\mu_1,a_{11}^2+a_{12}^2)$,$Y_2$$N(\mu_2,a_{21}^2+a_{22}^2)$

其概率密度函数为$p(y_1,y_2) = \frac{1}{2\pi\sqrt{(a_{11}^2+a_{12}^2)(a_{21}^2+a_{22}^2)}}\exp\{-\frac{1}{2}[\frac{(x_1-\mu_1)^2}{a_{11}^2+a_{12}^2} + \frac{(x_2-\mu_2)^2}{a_{21}^2+a_{22}^2}]\}$

Gibbs采样模拟圆周率随机投点

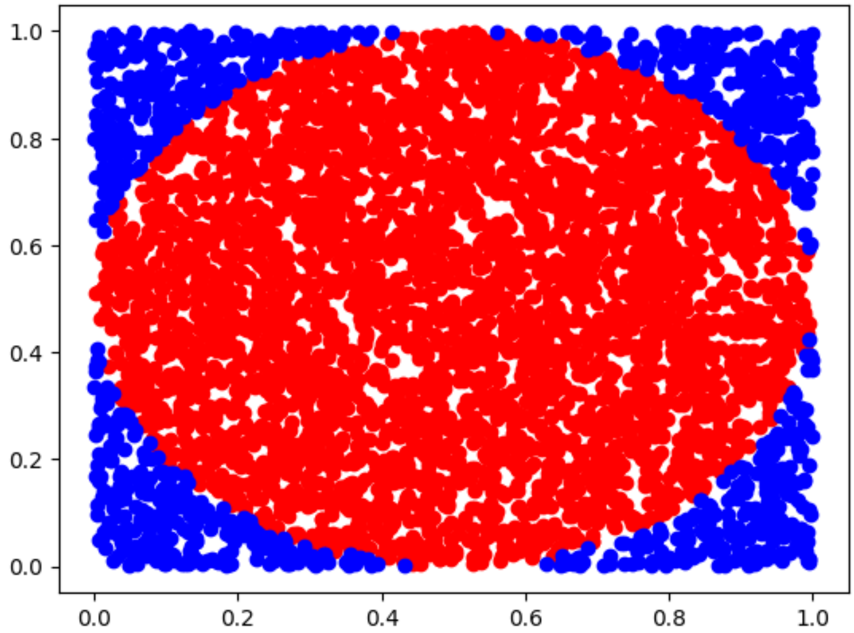
```python
def Gibbs_pi(num):
    def In_cir(x,y):
        result = math.sqrt((x-0.5)**2 + (y-0.5)**2)
        return result < 0.5

    n = 0
    fig, ax = plt.subplots()
    inLstX, inlstY, outLstX, outLstY = [], [], [], []
    for _ in range(num):
        x, y = rd.uniform(0,1), rd.uniform(0,1)
        if (In_cir(x,y)):
            n += 1
            inLstX.append(x)
            inlstY.append(y)
        else:
            outLstX.append(x)
            outLstY.append(y)
    plt.scatter(inLstX,inlstY,color = 'red')
    plt.scatter(outLstX,outLstY,color = 'blue')
```

```python
    plt.show()
    return 4*n/num

print(Gibbs_pi(5000))


```




```
3.1744
```

蓄水池算法

1.先准备一个大小为m的返回结果ans，即蓄水池。

2.对于先来到的前m个数，先来先得，直接丢入蓄水池中，占好位置。

3.对于后面来到的第cnt（cnt>m）个数，有m/cnt的概率保留，如果保留，则用其替换出蓄水池中的任意一个数。

对于样本中的每个数来说，保留在蓄水池中的概率都相同

```python
sample = [rd.randint(1,100) for _ in range(100)]
def reserSamp(lst, m):
    if m >= len(lst):
        return sample
    else:
        cnt = len(lst) - m
        mindex = list(range(m))
        def isSave(m):
            return rd.uniform(0,1) < m/cnt
        def swap(lst, n):
            del lst[rd.choice(mindex)]
            lst.append(n)
            return 0
```

```python

        ans = [lst[i] for i in range(m)]
        for i in range(m,len(lst)):
            if isSave(cnt):
                swap(ans, lst[i])
        return ans

print(reserSamp(sample,5))

```

```
[29, 82, 68, 22, 73]
```