$$\begin{cases} 0.001x_1 + 2x_2 + 3x_3 = 1 \\ -x_1 + 3.712x_2 + 4.623x_3 = 2 \\ -2x_1 + 1.072x_2 + 5.643x_3 = 3 \end{cases}$$

顺序消元:
$$\left(\begin{array}{ccc|c} 0.001 & 2 & 3 & 1 \\ -1 & 3.712 & 4.623 & 2 \\ -2 & 1.072 & 5.643 & 3 \end{array}\right) \to \left(\begin{array}{ccc|c} 0.001 & 2 & 3 & 1 \\ & 2003.712 & 3004.623 & 1002 \\ & 4001.072 & 6005.643 & 2003 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} 0.001 & 2 & 3 & 1 \\ & 2003.712 & 3004.623 & 1002 \\ & & 5.922 & 2.176 \end{array}\right) \to \left(\begin{array}{ccc|c} 0.001 & & & -0.002 \\ & 2003.712 & & -100.927 \\ & & 5.922 & 2.176 \end{array}\right)$$

$$\Rightarrow \bar{x}_1 = (-2.000, -0.05037, 0.3674)^T$$

列主元消元:
$$\left(\begin{array}{ccc|c} 0.001 & 2 & 3 & 1 \\ -1 & 3.712 & 4.623 & 2 \\ -2 & 1.072 & 5.643 & 3 \end{array}\right) \to \left(\begin{array}{ccc|c} -2 & 1.072 & 5.643 & 3 \\ -1 & 3.712 & 4.623 & 2 \\ 0.001 & 2 & 3 & 1 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} -2 & 1.072 & 5.643 & 3 \\ & 3.176 & 1.802 & 0.5 \\ & 2.001 & 3.001 & 1.002 \end{array}\right) \to \left(\begin{array}{ccc|c} -2 & 1.072 & 5.643 & 3 \\ & 3.176 & 1.802 & 0.5 \\ & & 1.866 & 0.687 \end{array}\right)$$

$$\to \cdots \left(\begin{array}{ccc|c} -2 & & & 0.977 \\ & 3.176 & & -0.163 \\ & & 1.866 & 0.687 \end{array}\right)$$

$$\Rightarrow \bar{x}_2 = (-0.4885, -0.05132, 0.3682)$$

$\bar{x}_1 = (-2.000, -0.05037, 0.3674)^T$ 准确值 $\bar{x} = (-0.4904, -0.05104, 0.3675)^T$

$\bar{x}_2 = (-0.4885, -0.05132, 0.3682)^T$

显然,与直接进行顺序消元相比,列主元消元法要准确的多.

平方根法.

$$\begin{cases} 4x_1 + 2x_2 - 2x_3 = 10 \\ 2x_1 + 2x_2 - 3x_3 = 5 \end{cases} \quad \text{设} A = \begin{pmatrix} 4 & 2 & -2 \\ 2 & 2 & -3 \end{pmatrix} \quad L = \begin{pmatrix} l_{11} & & \\ l_{21} & l_{22} & \end{pmatrix}$$

$-2x_1 - 3x_2 + 14x_3 = 4.$ $\quad \begin{pmatrix} \cdots \\ -2 & -3 & 14 \end{pmatrix}.$ $\quad L = \begin{pmatrix} \cdots \\ l_{31} & l_{32} & l_{33} \end{pmatrix}.$

则 $l_{11} = 2 \quad l_{21} = a_{21}/l_{11} = 1 \quad l_{22} = 1$

$l_{31} = -1 \quad l_{32} = -2 \quad l_{33} = 3$ $\qquad$ 故 $L = \begin{pmatrix} 2 & & \\ 1 & 1 & \\ -1 & -2 & 3 \end{pmatrix}$

$LL^T X = B$. 令 $L^T X = Y$. 则 $LY = B$

$\Rightarrow Y = (5, 0, 3)^T \Rightarrow \begin{pmatrix} 2 & 1 & -1 \\ & 1 & -2 \\ & & 3 \end{pmatrix} X = \begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix}$

$\Rightarrow X = (2, 2, 1)^T$

三角分解法: $LU = A$. $\quad L = \begin{pmatrix} 1 & & \\ l_{21} & 1 & \\ l_{31} & l_{32} & 1 \end{pmatrix} \quad U = \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ & u_{22} & u_{23} \\ & & u_{33} \end{pmatrix}$

依次解出 l_{ij}. u_{ij}. $\Rightarrow L = \begin{pmatrix} 1 & & \\ 0.5 & 1 & \\ -0.5 & -2 & 1 \end{pmatrix} \quad U = \begin{pmatrix} 4 & 2 & -2 \\ & 1 & -2 \\ & & 9 \end{pmatrix}$

$LUX = B$ 令 $UX = Y$. $\quad LY = B \Rightarrow Y = (10, 0, 9)$

$UX = Y \Rightarrow X = (2, 2, 1)$