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2. $\begin{cases} u_{tt}-c^2u_{xx}=0 \\ u(t,0)=0,\ u(t,1)=A\sin\omega t \\ u(0,x)=u_t(0,x)=0 \end{cases}$

$\varphi(x)=u(0,x)=0.\ \psi(x)=u_t(0,x)=0.\ f(t,x)=0.$

$\gamma_1(t)=0 \quad \gamma_2(t)=A\sin\omega t.\ L=1.$

令 $Z(t,x)=\gamma_1(t)+\frac{x}{L}(\gamma_2(t)-\gamma_1(t))=\frac{A}{L}x\sin\omega t=Ax\sin\omega t.$

则 $Z(t,0)=\gamma_1(t).\ Z(t,1)=\gamma_2(t)$

设 $V(t,x)=u(t,x)-Z(t,x).$

则 $\begin{cases} V_{tt}-c^2V_{xx}=A\omega^2x\sin\omega t \\ V(0,x)=0 \quad V_t(0,x)=-A\omega x \\ V(t,0)=0 \quad V(t,1)=0. \end{cases}$

再令 $X(t,x),\ Y(t,x)$ 是以下方程组的解.

① $\begin{cases} X_{tt}-c^2X_{xx}=0 \\ X(0,x)=0 \quad X_t(0,x)=-A\omega x \\ X(t,0)=0 \quad X(t,1)=0. \end{cases}$

② $\begin{cases} Y_{tt}-c^2Y_{xx}=A\omega^2x\sin\omega t \\ Y(0,x)=0.\ Y_t(0,x)=0 \\ Y(t,0)=0.\ Y(t,1)=0. \end{cases}$

则 $X(t,x)=\sum_{k=1}^{\infty}(A_k\cos ck\pi t+B_k\sin ck\pi t)\sin k\pi x.$

其中. $A_k=2\int_0^1 0\cdot\sin k\pi t\,dt=0.$

$B_k=\frac{2}{k\pi c}\int_0^1 -A\omega\eta\sin k\pi\eta\,d\eta=\frac{2A\omega(-1)^k}{c\,k^2\pi^2}$

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$\Rightarrow X(t,x)=\sum_{k=1}^{\infty}\frac{2Aw(-1)^k}{ck^2\pi^2}\sin ck\pi t\sin k\pi x.$

令 $W(s,x)$ 为以下方程组的解.

$$\begin{cases} w_{ss}-c^2w_{ss}=0 \\ w(0,x)=0 \quad w_s(0,x)=f(\tau,x)=Aw^2x\sin w\tau \\ w(t,0)=w(t,1)=0. \end{cases}$$

$\Rightarrow W(s,x:\tau)=\sum_{k=1}^{\infty}B_k(\tau)\sin k\pi c(t-\tau)\sin k\pi x.$

其中. $B_k=\frac{2}{k\pi c}\int_0^1 Aw^2\xi\sin w\tau\cdot\sin k\pi\xi\, d\xi.$

$\Rightarrow B_k=\frac{2(-1)^{k+1}Aw^2\sin w\tau}{k^2\pi^2c}.$

故 $W(s,x:\tau)=\sum_{k=1}^{\infty}2(-1)^{k+1}\frac{Aw^2\sin w\tau}{k^2\pi^2c}\cdot\sin k\pi c(t-\tau)\sin k\pi x.$

$\Rightarrow Y(t,x)=\int_0^t w(t,x:\tau)\,d\tau=\frac{2(-1)^{k+1}Aw^2}{k^2\pi^2c(w^2-k^2\pi^2c^2)}.$

$=\sum_{k=1}^{\infty}\frac{2(-1)^{k+1}Aw^2}{k^2\pi^2c(w^2-k^2\pi^2c^2)}\sin k\pi x\,(w\sin k\pi c\cancel{t}-k\pi c\sin wt).$

$V(t,x)=X(t,x)+Y(t,x)$. 而 $u(t,x)=V(t,x)+Z(t,x)$.

故 $u(t,x)=\sum_{k=1}^{\infty}\frac{2(-1)^{k+1}Aw\sin k\pi x}{w^2-k^2\pi^2c^2}\left(c\sin k\pi ct-w\sin wt\right)$

$+Ax\sin wt.$

3\.

解: $\begin{cases} u_{tt} - c^2 u_{xx} = 0 \\ u(t,0) = u(t,l) = 0 \\ u(0,x) = \sin \frac{3}{2l}\pi x \quad u_t(0,x) = \sin\frac{5}{2l}\pi x. \end{cases}$

$\varphi(x) = \sin\frac{3\pi}{2l}x.\ \psi(x) = \frac{5\pi}{2l}x.$

$$u(t,x) = \sum_{k=1}^{\infty} \left(A_k \cos\frac{k\pi c}{l}t + B_k \sin\frac{k\pi c}{l}\cdot t\right)\sin\frac{k\pi}{l}\cdot x.$$

$$A_k = \frac{2}{l}\int_0^l \varphi(\eta)\sin\frac{k\pi\eta}{l}\,d\eta = \frac{8k(-1)^{k+1}}{\pi(9-4k^2)}.$$

$$B_k = \frac{2}{k\pi c}\int_0^l \psi(\eta)\sin\frac{k\pi\eta}{l}\,d\eta = \frac{8(-1)^k l}{c(15\pi^2 - 4k^2\pi^2)}$$

$$故\ u(t,x) = \sum_{k=1}^{\infty}\left(\frac{8k(-1)^{k+1}}{\pi(9-4k^2)}\cos\frac{k\pi c}{l}t + \frac{8(-1)^k l}{c(15\pi^2-4k^2\pi^2)}\sin\frac{k\pi c}{l}t\right)\sin\frac{k\pi}{l}x.$$

四. 同上: 求得

$$u(t,x) = \sum_{k=1}^{\infty}\frac{2(-1)^k k\pi\sin l}{l^2 - k^2\pi^2}\cdot\cos\frac{k\pi c}{l}t\sin\frac{k\pi}{l}\cdot x.$$