8. v的概率分布为 $p(x)$.

故 $Ev=\int_{-\infty}^{\infty} xp(x)dx = \int_0^{+\infty} \frac{2x^2}{a^3\sqrt{\pi}} e^{-x^2/a^2} dx^2 = \frac{2}{a^3\sqrt{\pi}}\int_0^{+\infty} t e^{-t/a^2} dt$

其中 $I=\int_0^{+\infty} t e^{-t/a^2}dt = -a^2 t e^{-t/a^2}\Big|_0^{+\infty} + a^2\int_0^{+\infty} e^{-t/a^2}dt$

$= -a^4 e^{-t/a^2}\Big|_0^{+\infty} = a^4$

故 $Ev = \frac{2a}{\sqrt{\pi}}$ $\qquad \int_0^{+\infty} x^3 e^{-x^2/a^2}dx = a^4/2$

由 $e=\frac12 mv^2 \Rightarrow Ee=\frac12 m Ev^2$

故 $Ee = \frac12 m\int_{-\infty}^{+\infty} x^2 p(x)dx = \frac12 m\int_0^{+\infty}\frac{4x^4}{a^3\sqrt\pi}e^{-x^2/a^2}dx = \frac{2m}{a^3\sqrt\pi}\int_0^{+\infty}x^4 e^{-x^2/a^2}dx.$

$=\frac{2m}{a^3\sqrt\pi}\times 2a^6 = \frac{4}{\sqrt\pi}ma^3$

9. 记 $\xi=\xi_1$, $\eta=\xi_2$. $\xi,\eta \sim N(a,\sigma^2)$.

$\max\{\xi,\eta\} = \frac12|\xi-\eta| + \frac12(\xi+\eta)$ $\quad \xi+\eta \sim N(2a,2\sigma^2)$ $\quad \xi-\eta\sim N(0,2\sigma^2)$

$E\frac12(\xi+\eta) = a$

$E|\xi-\eta| = \int_{-\infty}^{+\infty}|x|\frac{1}{\sqrt{2\pi}\sqrt2\sigma}e^{-\frac{x^2}{4\sigma^2}}dx = \int_{-\infty}^{+\infty}\frac{x}{\sqrt\pi\sigma}e^{-\frac{x^2}{4\sigma^2}}dx = \frac{1}{\sqrt\pi\sigma}\left(-2\sigma^2 e^{-\frac{x^2}{4\sigma^2}}\Big|_0^{+\infty}\right) = \frac{2\sigma}{\sqrt\pi}.$

故 $E\max\{\xi,\eta\} = a + \frac{\sigma}{\sqrt\pi}$.

11. 记 $u(k)$ 为有放回的抽 k 张卡的号码和.

显然 $Eu(1) = \sum_{i=1}^{n}\frac1n\cdot i = \frac1n\cdot\frac{1+n}{2}\cdot n = \frac{1+n}{2}$. $\quad u(k) = k\,u(1)$.

故 $Eu(k) = E\,k\,u(1) = kEu(1) = \frac{1+n}{2}k$.

即 $Eu = \frac{1+n}{2}k$

14. 设白球有 n 个. 则 $p = P(\text{任取一球为白球}) = \frac{n}{N}$. 则 $En = a$

$p = Ep = E\frac{n}{N} = \frac1N En = \frac{a}{N}$

故概率为 $\frac{a}{N}$.