61. $\forall x, y$. $p(x, y) \geq 0$. 故 $p(x, y) = \begin{cases} 4xy, & x>0, 0<y<1 \\ 0 & \text{其它} \end{cases}$

令 $g(u,v)$ 为 (ξ^2, η^2) 的联合概率密度函数. 作变换 $\begin{cases} u = x^2 \\ v = y^2 \end{cases}$ 则 $\begin{cases} x = \sqrt{u} \\ y = \sqrt{v} \end{cases}$

则 $\frac{\partial(u,v)}{\partial(x,y)} = \det\begin{vmatrix} 2x & \\ & 2y \end{vmatrix} = 4xy$. 从而 $|J| = \frac{1}{4xy} = \frac{1}{4\sqrt{uv}}$

故当 $(u,v) \in (0,+\infty) \times (0,1)$ 时.

$g(u,v) = p(x(u,v), y(u,v))|J| = p(\sqrt{u}, \sqrt{v})|J| = 4\sqrt{u}\sqrt{v} \cdot \frac{1}{4\sqrt{u}\sqrt{v}} = 1$

(ξ^2, η^2) 的联合密度. $g(x,y) = \begin{cases} 1 & u>0, 0<v<1 \\ 0 & \text{其它} \end{cases}$

63: $(\xi, \eta) \sim N(0, 0, \sigma_1^2, \sigma_2^2, r)$. 则 $(\xi+\eta, \xi-\eta)$ 服从二维正态分布.

$cov(\xi-\eta, \xi+\eta) = E((\xi+\eta - E(\xi+\eta))(\xi-\eta - E(\xi-\eta))) = E(\xi^2-\eta^2) = \sigma_1^2 - \sigma_2^2 = 0$.

故 $\sigma_1^2 = \sigma_2^2$ 为 $\xi+\eta$ 与 $\xi-\eta$ 独立的充要条件.

1. (1): $E\xi = \sum_{k=1}^{5} \frac{1}{5} \cdot k = \frac{1}{5}\sum_{k=1}^{5} k = 3$

(2): $p(\xi=k) = \frac{a^k}{(1+a)^k}$. $a>0$. $\forall k = 0, 1, 2 \cdots$

$E\xi = \sum_{k=0}^{\infty} k \cdot \frac{a^k}{(1+a)^k}$ 令 $S_n = \sum_{k=0}^{n} k\left(\frac{a}{1+a}\right)^k = \sum_{k=1}^{n} k\left(\frac{a}{1+a}\right)^k$

$\Rightarrow S_n = a - (n+a+1)\left(\frac{a}{a+1}\right)^{n+1}$

$E\xi = \lim_{n\to\infty} S_n = a - \lim_{n\to\infty}(n+a+1)\left(\frac{a}{a+1}\right)^{n+1} = a$.

$\Rightarrow E\xi = a$.

3. (1): $p(\xi=0) = \frac{1}{n}$ $p(\xi=k) = \frac{1}{n-1} \cdot \frac{1}{n-2} \cdots \frac{1}{n-k} = \frac{1}{A_{n-1}^k}$

$E\xi = \sum_{k=1}^{n-1} \frac{k}{A_{n-1}^k} = \sum_{k=1}^{n-1} \frac{k \cdot k!}{(n-1)!} = \frac{1}{(n-1)!}\sum_{k=1}^{n-1} k \cdot k!$

(2): $p(\xi=0) = \frac{1}{n}$. $p(\xi=k) = \left(\frac{n-1}{n}\right)^k \cdot \frac{1}{n}$

$E\xi = \sum_{k=1}^{n-1} \frac{k}{n}\left(\frac{n-1}{n}\right)^k = (n-1) - (2n-1)\left(\frac{n-1}{n}\right)^n$

故 $E\xi = -(2n-1)\left(\frac{n-1}{n}\right)^n + n - 1$.

6. (1): $E\xi = \int_{-\infty}^{+\infty} x p(x) dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{2}{\pi} x \cos^2 x dx = 0$.

6.(1): $E\xi=\int_{-\infty}^{+\infty} xp(x)dx=\int_{-\frac{\pi}{2}} \frac{1}{2} x\cos x dx = 0.$

(2): $E\xi=\int_{-\infty}^{+\infty} xp(x)dx=\int_0^1 x^2dx+\int_1^2(2x-x^2)dx=\frac{1}{3}x^3\Big|_0^1+(x^2-\frac{1}{3}x^3)\Big|_1^2$

$$=\frac{1}{3}+\frac{2}{3}=1$$

(3): $E\xi=\int_{-\infty}^{+\infty} xp(x)dx=\int_{-\infty}^{+\infty}\frac{x}{2\lambda}e^{-\frac{|x-\mu|}{\lambda}}dx=\int_{-\infty}^{\mu}\frac{x}{2\lambda}e^{\frac{x-\mu}{\lambda}}dx+\int_{\mu}^{+\infty}\frac{x}{2\lambda}e^{-\frac{x-\mu}{\lambda}}dx$

$$=\frac{1}{2}(\mu-\lambda)+\frac{1}{2}(\mu+\lambda)=\mu$$

故 $E\xi=\mu$.