COSC 290 Discrete Structures

Lecture 4: Propositional Logic

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Wednesday, Sep. 6, 2017

Colgate University

Plan for today

1. CS Connections: logic-based AI agents
2. Logic and Entailment
3. Propositional Logic

CS Connections: logic-based AI agents

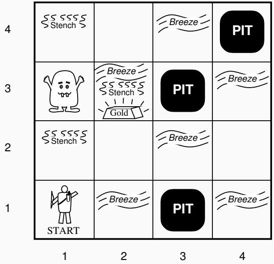
Wumpus World

Performance measure

- gold +1000, death -1000
- -1 per step, -10 for using the arrow

Environment

- Squares adjacent to wumpus are smelly
- Squares adjacent to pit are breezy
- Glitter when gold is in the same square
- Shooting kills wumpus if you are facing it
- Shooting uses up the only arrow
- Grabbing picks up gold if in same square
- Releasing drops the gold in same square



Actuators

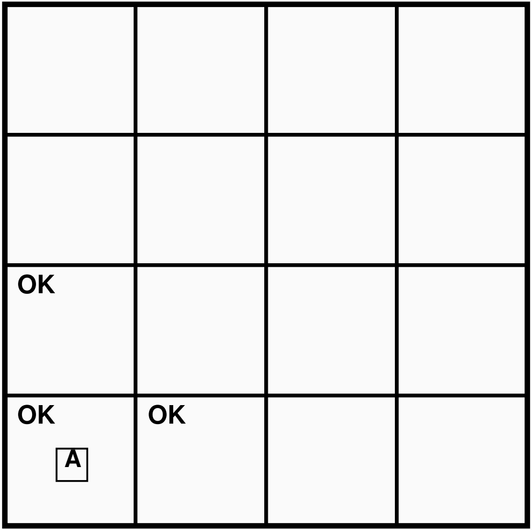
- Left turn, Right turn, Forward, Grab, Release, Shoot

Sensors

- Breeze, Glitter, Smell

Credit: slides adapted from Russell & Norvig, AI: A Modern Approach

Exploring a wumpus world



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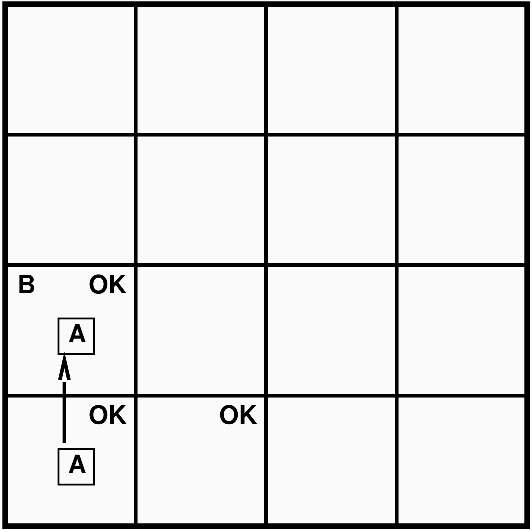

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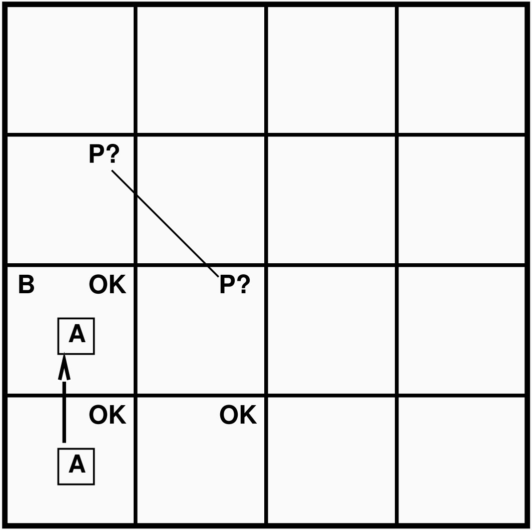

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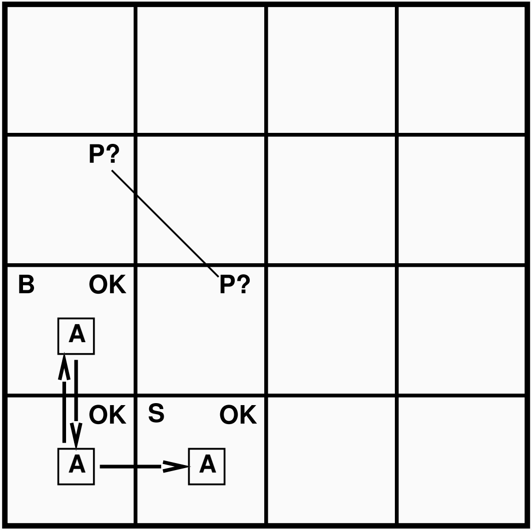

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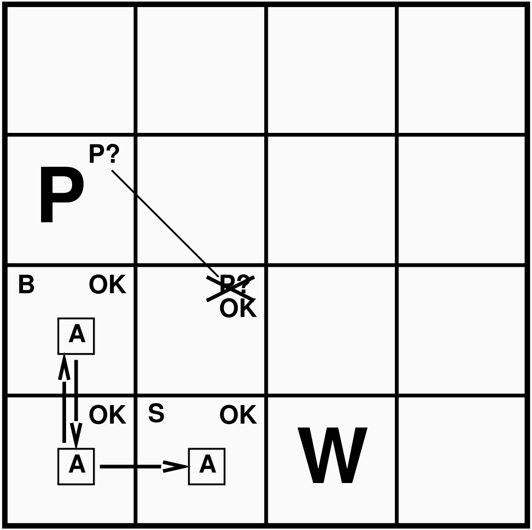

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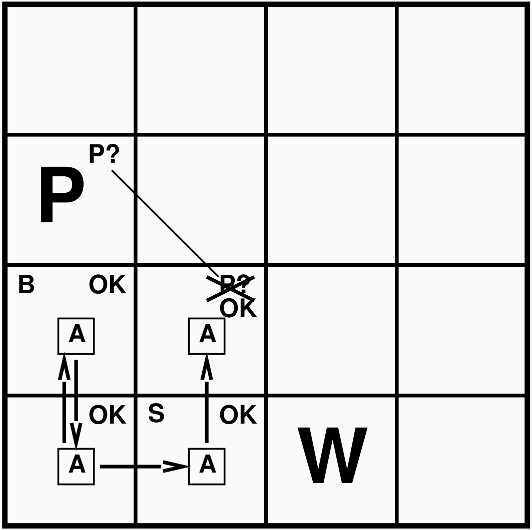
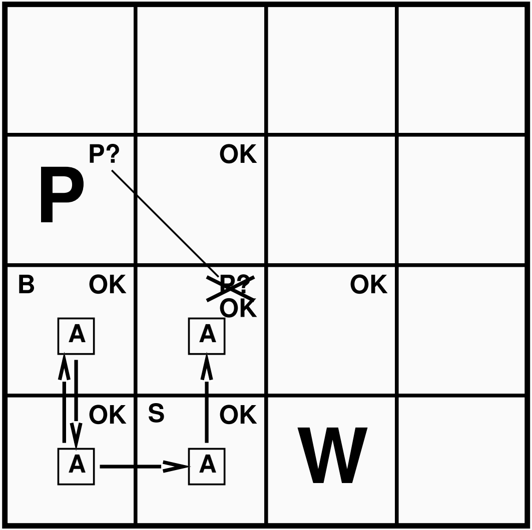


Exploring a wumpus world



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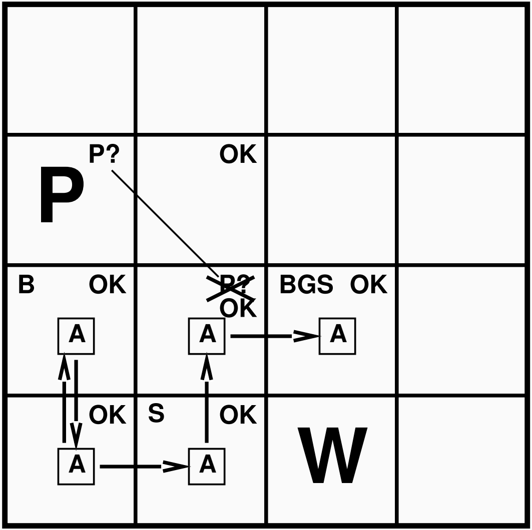

Some tight spots

Breeze in (1, 2) and (2, 1)

Some tight spots

Breeze in (1, 2) and (2, 1) $\implies$ no safe actions

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Smell in (1, 1)

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Breeze in (1, 2) and (2, 1) $\implies$ no safe actions

Smell in (1, 1) $\implies$ cannot move.

However, there's hope:

Some tight spots

Breeze in (1, 2) and (2, 1) $\implies$ no safe actions

Smell in (1, 1) $\implies$ cannot move.

However, there's hope: shoot straight ahead.

- If wumpus was there, it's now dead, so it's safe.
- If wumpus wasn't there, it's safe.

Logic and Entailment

Logic in a general

A formal logic is a language for representing information such that conclusions can be drawn.

Syntax

Syntax defines what sentences are permissible in the language.

Semantics

Semantics defines the "meaning" of sentences.
It defines the rules for determining the *truth* of a sentence with respect to each *possible world*.

Example: arithmetic

- Syntax: $x + y = 4$ is a sentence; $x4 + y =$ is not.
- Semantics: $x + y = 2$ is true in a world x is 2 and y is 2, but false in a world x is 2 and y is 3.

Entailment

Entailment means that one thing *follows from* another:

$$KB \models \alpha$$

Knowledge base *KB* entails sentence α if and only if α is true in all worlds where *KB* is true.

Ex: the KB containing "the Patriots won" entails "Either the Patriots won or the Packers won."

Ex: the KB containing rules of algebra and the fact $x + y = 4$ entails $y = 4 - x$.

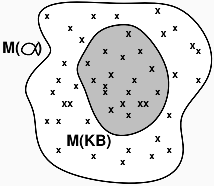
Models

A model is a mathematical abstraction that represents a possible world. A model contains the relevant information to evaluate the truth or falsehood of any sentence.

Model *m* satisfies sentence α if α is true in *m*.

$M(\alpha)$ set of all models that satisfy α.

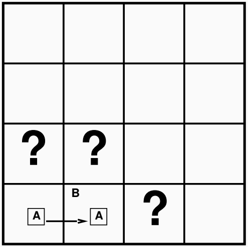
$KB \models \alpha$ if and only if $M(KB) \subseteq M(\alpha)$.



Models in the wumpus world

Situation after...
detecting nothing in [1,1],
moving right, breeze in [2,1]

Consider possible models for ?s
assuming only pits.



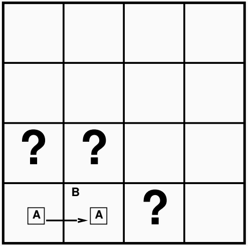
3 Boolean choices $\implies$ 8 possible models

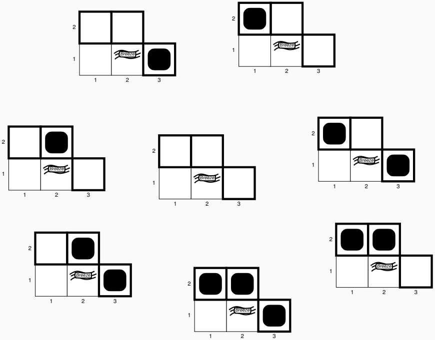
Entailment in the wumpus world

Entailment: Given our *knowledge base* (rules of wumpus world plus info shown in figure), can we determine...

... that [1,2] is safe?

... that [2, 2] is safe?




2
1
Breeze
1 2 3
2
1
Breeze
1 2 3
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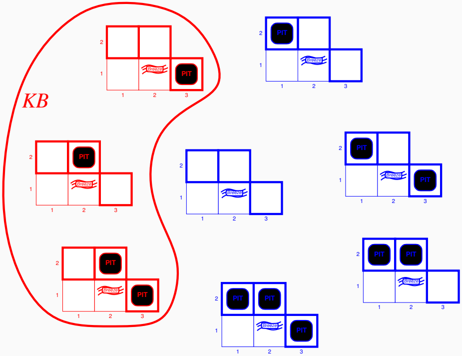
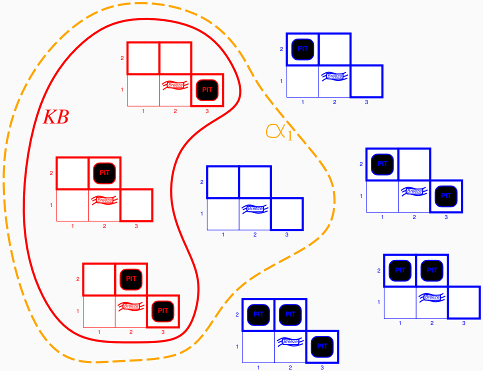
Wumpus models



KB = wumpus-world rules + observations

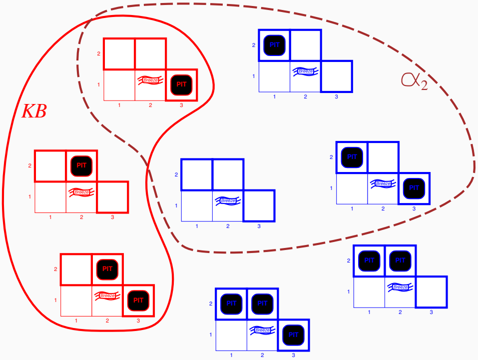
Wumpus models



KB = wumpus-world rules + observations

α_1 = "[1,2] is safe", $KB \models \alpha_1$, proved by model checking.

Wumpus models



KB = wumpus-world rules + observations

α_2 = “[2,2] is safe”, $KB \not\models \alpha_2$.

Poll

Assume you are in a 4 x 4 wumpus world.

You observe a breeze [1,2] and a breeze in [2,1]. Your knowledge base *KB* consists of these facts plus the wumpus-world rules.

Consider the sentence α_3 = "[2,2] has a pit." Does $KB \models \alpha_3$?

A) Yes, the models where α_3 is true, the *KB* is also true.
B) Yes, the models where *KB* is true, the α_3 is also true.
C) No, there are models where α_3 is true but *KB* is not.
D) No, there are models where *KB* is true but α_3 is not.
E) We don't have enough information.

Inference

An inference algorithm is a procedure that takes *KB* and α and attempts to prove that α follows from *KB* or conclude that it does not.

Analogy: consequences of *KB* are a haystack; α is a need.

- Entailment = needle in haystack
- Inference = a procedure for finding it

We just performed inference by model checking. Enumerate all possible models and if α is true in all models where *KB* is true, then $KB \models \alpha$.

We will look at other inference algorithms – in particular, ones that can be applied to *propositional logic*.

Propositional Logic

Poll: what is a proposition?

Propositional logic is based around the concept of a proposition.

Why isn't

"Where is does COSC 290 lab meet?"

considered a proposition?

A) Because only "yes/no" questions can be propositions.
B) Because questions can never be propositions.
C) Because the answer changes over time (last semester it was held in a different room).
D) More than one of the above / None of the above.

Proposition

A proposition is a sentence that is either true or false.

- $3 + 4 = 7$
- My middle name is Herbert or my dog's name is Rufus.
- One of these three propositions is true.

These are *not* propositions:

- Questions: is $3 + 4 = 7$?
- Imperatives: You should major in computer science.
- Opinions: CS majors have more fun.

Syntax of propositional logic

A sentence in propositional logic must conform to the following syntax.

A sentence can consist of a single atomic proposition. Example: "The chair is red." Such propositions are represented using variables p, q, r, etc.

More complex sentences can be constructed from simpler sentences using logical connectives.

- If S is a sentence, $\neg S$ is a sentence (negation)
- If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)
- If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (disjunction)
- If S_1 and S_2 are sentences, $S_1 \implies S_2$ is a sentence (implication)
- If S_1 and S_2 are sentences, $S_1 \iff S_2$ is a sentence (iff)

Semantics of propositional logic

Recall: the semantics defines the rules for determining the *truth* of a sentence.

A simple sentence consisting of a single atomic proposition p is either true or false.

For more complex sentences, the truth can be evaluated using these rules:

$\neg S$	is true iff	S	is false		
$S_1 \wedge S_2$	is true iff	S_1	is true *and*	S_2	is true
$S_1 \vee S_2$	is true iff	S_1	is true *or*	S_2	is true
$S_1 \implies S_2$	is true iff	S_1	is false *or*	S_2	is true
$S_1 \iff S_2$	is true iff	$S_1 \implies S_2$	is true *and*	$S_2 \implies S_1$	is true

In propositional logic, a model is simply an assignment of truth values to the atomic variables.

Truth tables

A truth table can be used to compactly represent semantics of a connective.

Recall from reading: “exlusive or” which is denoted using symbol $\oplus$.

p	q	$p \oplus q$
T	T	?
T	F	?
F	T	?
F	F	?

Example: “I’m either at the office or at home.”

Truthiness of a sentence

Consider the sentence $(p \vee q) \implies r \wedge \neg q$.

How do we evaluate whether this sentence is true?

1. Assign true/false values to atomic propositions p, q, r.
2. Apply recursive algorithm (on the sentence represented as a tree).

Shown on board

Poll: Wumpus world sentences

Let atomic propositions be denoted as follows:

- p_{ij} denotes "there is a **p**it in [i, j]."
- b_{ij} denote "there is a **b**reeze in [i, j]."
- ...

Write this sentence in propositional logic: "There may be a pit in [1,2] or [2,2] and the wumpus is either in [2,1] or [3,1]."

A) $(p_{1,2} \lor p_{2,2}) \land (b_{2,1} \lor b_{3,1})$

B) $(p_{1,2} \lor p_{2,2}) \land (b_{2,1} \oplus b_{3,1})$

C) $(p_{1,2} \land p_{2,2}) \lor (b_{2,1} \land b_{3,1})$

D) $(p_{1,2} \land p_{2,2}) \lor (b_{2,1} \land b_{3,1})$

E) More than one of the above / None of the above

Poll: Wumpus world sentences

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- ...

Suppose we know that "pits cause breezes in adjacent cells." Which of the following sentences could we add to our *KB*?

A) $p_{2,1} \implies (b_{1,1} \wedge b_{2,2} \wedge b_{3,1})$
B) $b_{1,1} \implies (p_{1,2} \vee p_{2,1})$
C) $b_{1,1} \iff (p_{1,2} \vee p_{2,1})$
D) All of the above
E) None of the above unless we also know that $b_{1,1}$ is true or $p_{2,1}$ is true.

Bonus question: would you answer change if you also knew that pits were the sole causes of breezes?