COSC 290 Discrete Structures

Lecture 4: Propositional Logic

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Wednesday, Sep. 6, 2017

Colgate University

Plan for today

1. CS Connections: logic-based AI agents
2. Logic and Entailment

CS Connections: logic-based AI agents

Wumpus World

Performance measure

- gold +1000, death -1000
- -1 per step, -10 for using the arrow

Environment

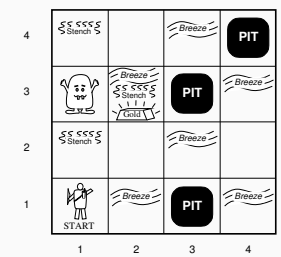
- Squares adjacent to wumpus are smelly
- Squares adjacent to pit are breezy
- Glitter when gold is in the same square
- Shooting kills wumpus if you are facing it
- Shooting uses up the only arrow
- Grabbing picks up gold if in same square
- Releasing drops the gold in same square

Actuators

- Left turn, Right turn, Forward, Grab, Release, Shoot

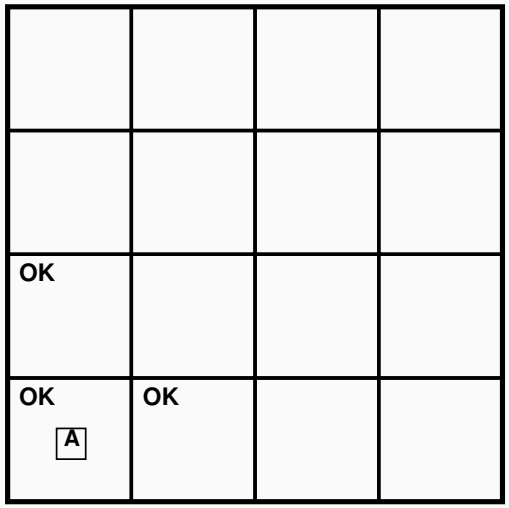
Sensors

- Breeze, Glitter, Smell



Credit: slides adapted from Russell & Norvig, AI: A Modern Approach

Exploring a wumpus world



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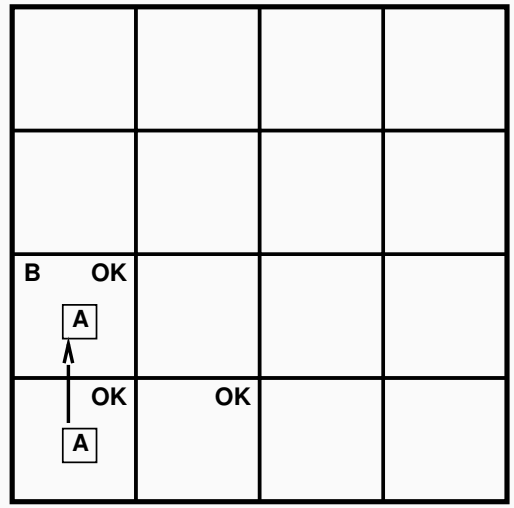

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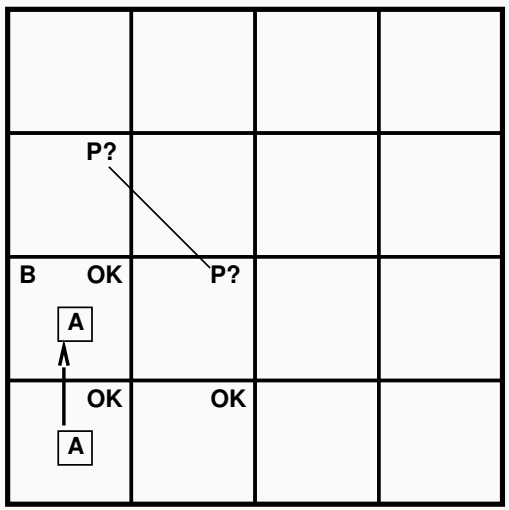

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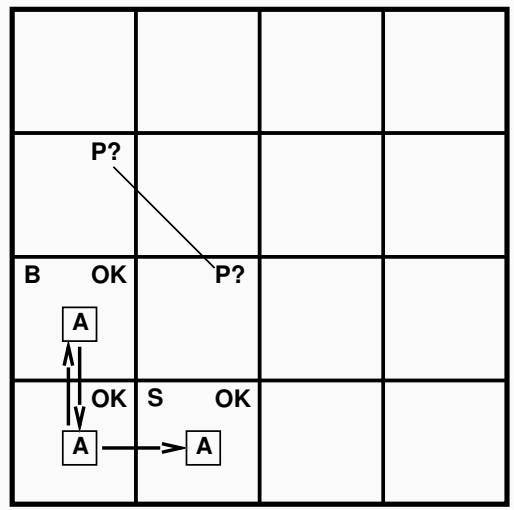

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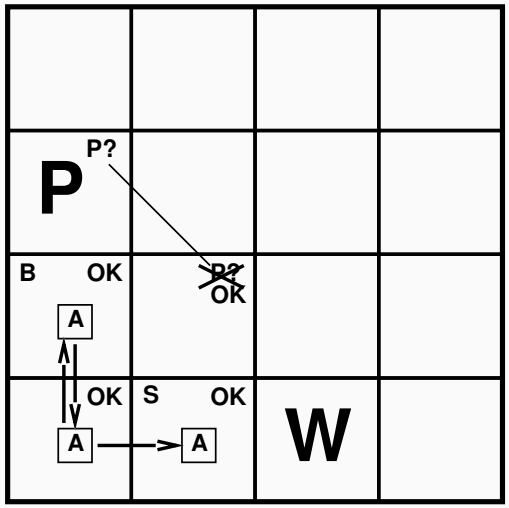

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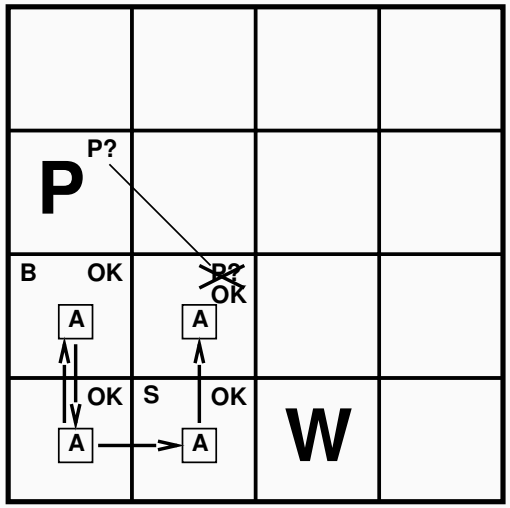

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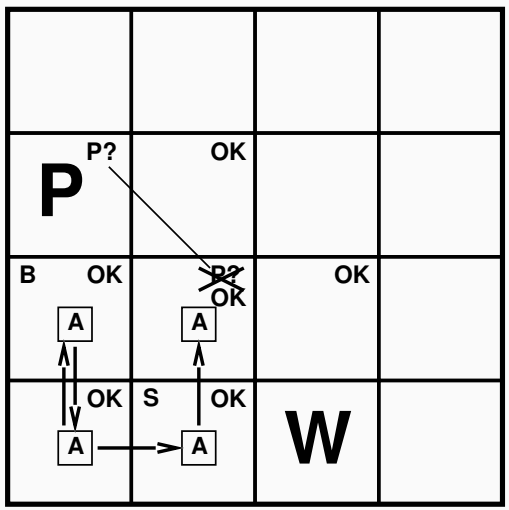

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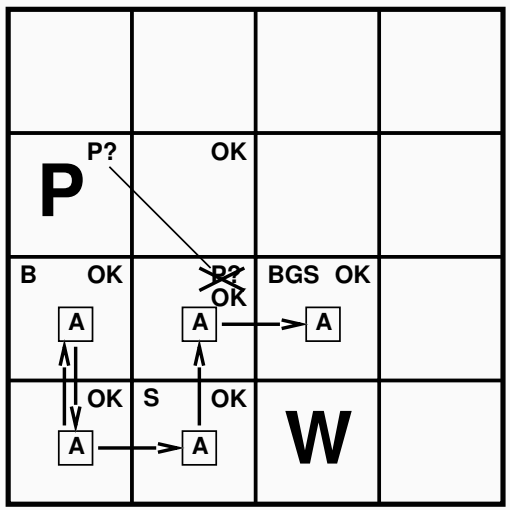


Some tight spots

Breeze in (1, 2) and (2, 1) $\implies$ no safe actions

Smell in (1, 1) $\implies$ cannot move.

However, there's hope: shoot straight ahead.

- If wumpus was there, it's now dead, so it's safe.
- If wumpus wasn't there, it's safe.

Logic and Entailment

Logic in a general

A formal logic is a language for representing information such that conclusions can be drawn.

Syntax

Syntax defines what sentences are permissible in the language.

Semantics

Semantics defines the "meaning" of sentences.
It defines the rules for determining the *truth* of a sentence with respect to each *possible world*.

Example: arithmetic

- Syntax: $x + y = 4$ is a sentence; $x4 + y =$ is not.
- Semantics: $x + y = 4$ is true in a world x is 2 and y is 2, but false in a world x is 2 and y is 3.

Entailment

Entailment means that one thing *follows from* another:

$$KB \models \alpha$$

Knowledge base *KB* entails sentence α if and only if α is true in all worlds where *KB* is true.

Ex: the KB containing "the Patriots won" entails "Either the Patriots won or the Packers won."

Ex: the KB containing rules of algebra and the fact $x + y = 4$ entails $y = 4 - x$.

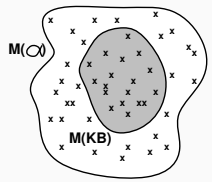
Models

A model is a mathematical abstraction that represents a possible world. A model contains the relevant information to evaluate the truth or falsehood of any sentence.

Model *m* satisfies sentence α if α is true in *m*.

$M(\alpha)$ set of all models that satisfy α.

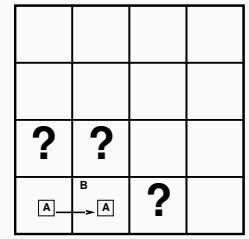
$KB \models \alpha$ if and only if $M(KB) \subseteq M(\alpha)$.



Models in the wumpus world

Situation after...
detecting nothing in [1,1],
moving right, breeze in [2,1]

Consider possible models for ?s
assuming only pits.



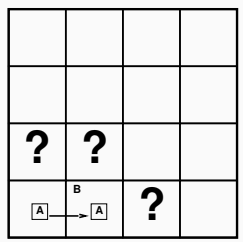
3 Boolean choices $\Longrightarrow$ 8 possible models

Entailment in the wumpus world

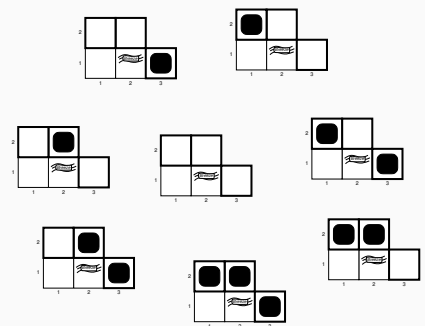
Entailment: Given our *knowledge base* (rules of wumpus world plus info shown in figure), can we determine...

... that [1,2] is safe?

... that [2, 2] is safe?



Wumpus models



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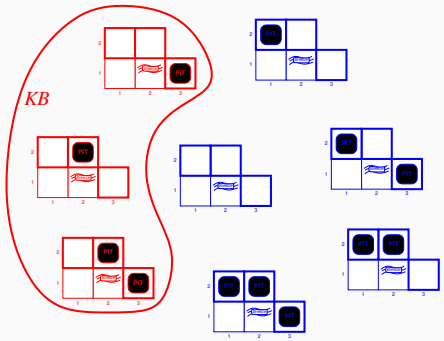
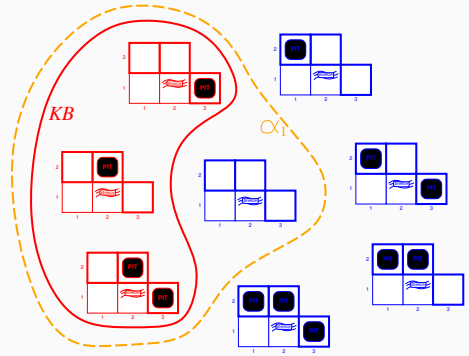


KB = wumpus-world rules + observations

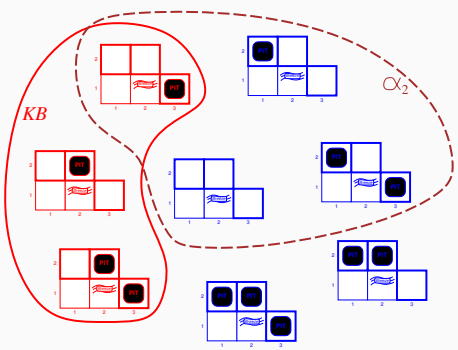
Wumpus models



KB = wumpus-world rules + observations

α_1 = "[1,2] is safe", $KB \models \alpha_1$, proved by model checking.

Wumpus models



KB = wumpus-world rules + observations

α_2 = "[2,2] is safe", $KB \not\models \alpha_2$.

Poll

Assume you are in a 4 x 4 wumpus world.

You observe a breeze [1,2] and a breeze in [2,1]. Your knowledge base *KB* consists of these facts plus the wumpus-world rules.

Consider the sentence α_3 = "[2,2] has a pit." Does $KB \models \alpha_3$?

A) Yes, the models where α_3 is true, the *KB* is also true.
B) Yes, the models where *KB* is true, the α_3 is also true.
C) No, there are models where α_3 is true but *KB* is not.
D) No, there are models where *KB* is true but α_3 is not.
E) We don't have enough information.

Inference

An inference algorithm is a procedure that takes *KB* and α and attempts to prove that α follows from *KB* or conclude that it does not.

Analogy: consequences of *KB* are a haystack; α is a needle.

- Entailment = needle in haystack
- Inference = a procedure for finding it

We just performed inference by model checking. Enumerate all possible models and if α is true in all models where *KB* is true, then $KB \models \alpha$.

We will look at other inference algorithms – in particular, ones that can be applied to *propositional logic*.