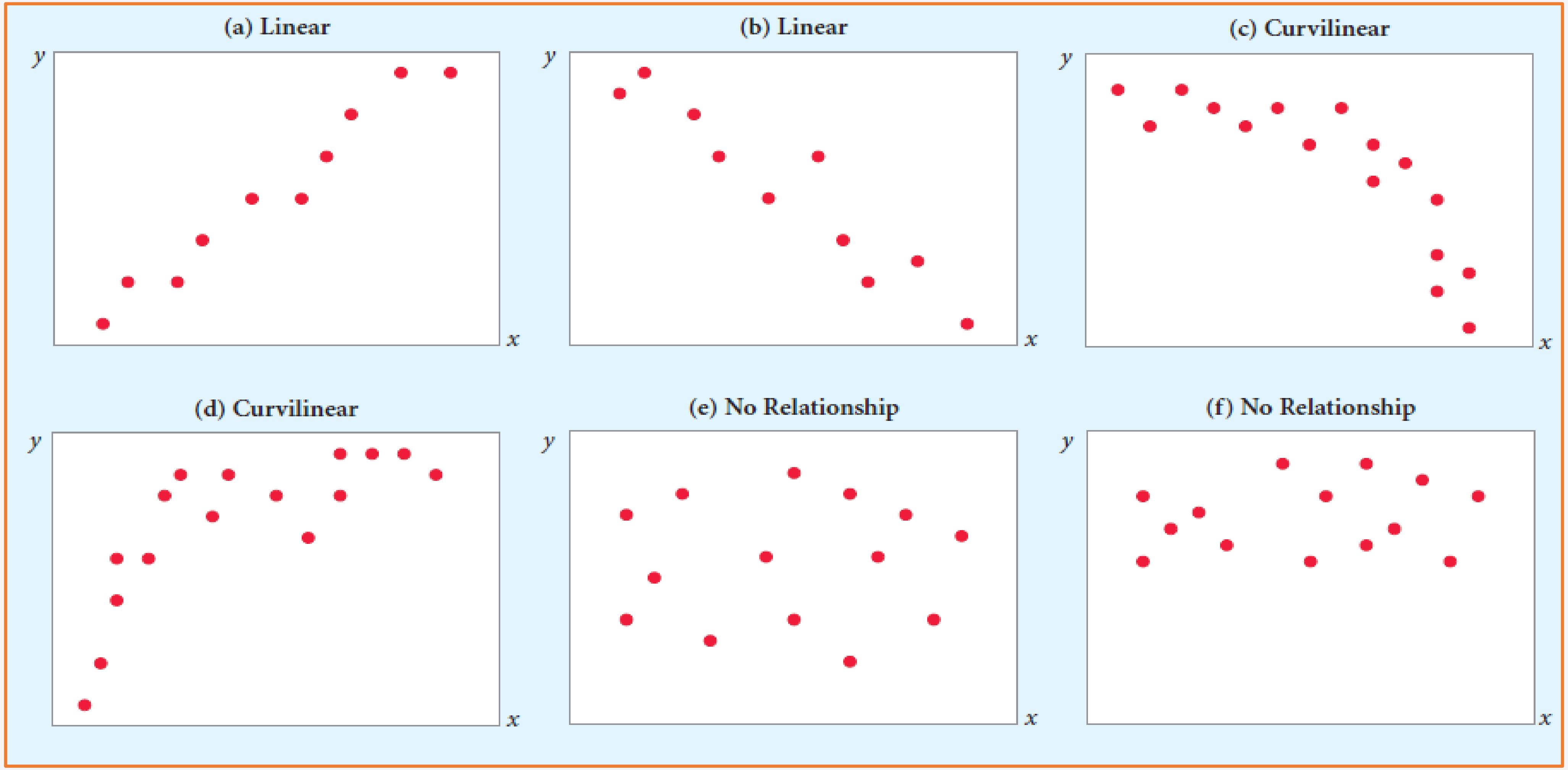
Linear Regression

Read Chapter 7 (Regression) of the Textbook

Regression Models

- To understand the application of regression analysis in data mining
  - Linear/nonlinear
  - Logistic (Logit)
- To understand the key statistical measures of fit

Relationships between variables



When the data shows linear relationship

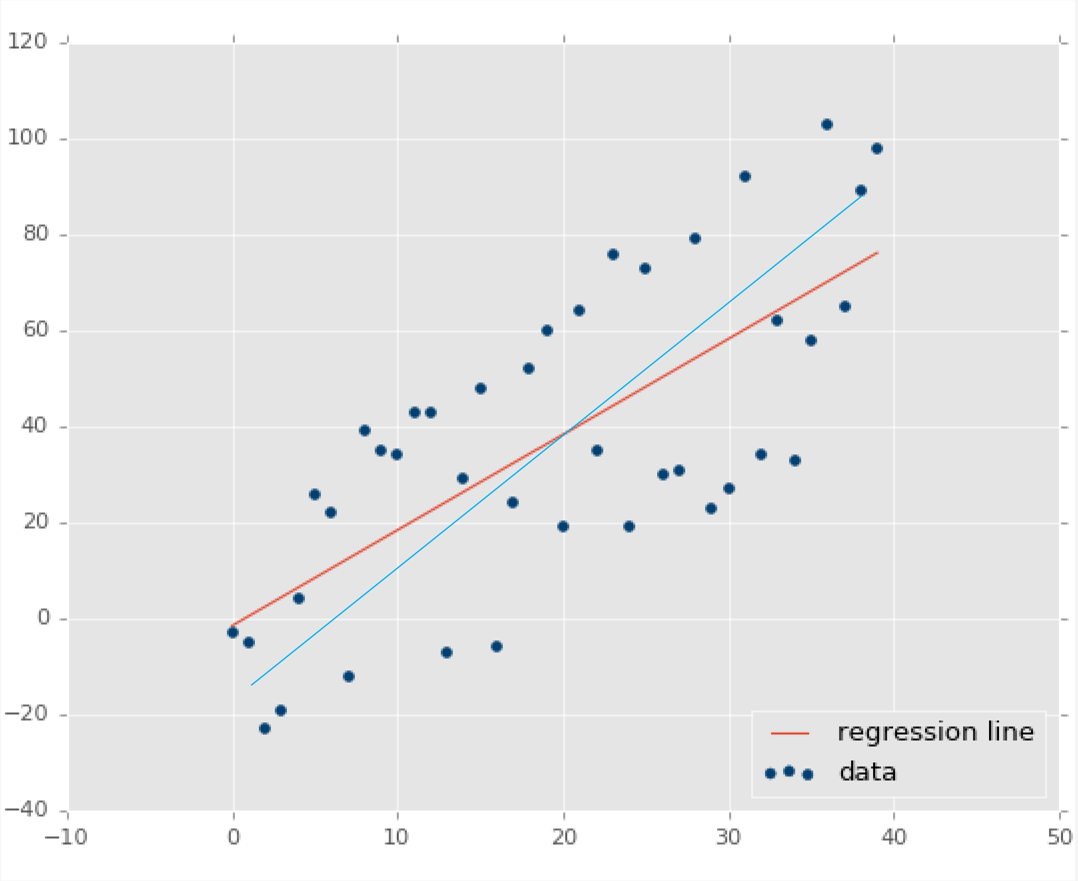
Correlation is high (positive or negative) and Scatter plots display a linear relationship

First model come to mind is

$$Y = m X + b$$

But still, there can be many lines that can "kind of" fit the data as well

Question: How to pick the "best-fit" line?

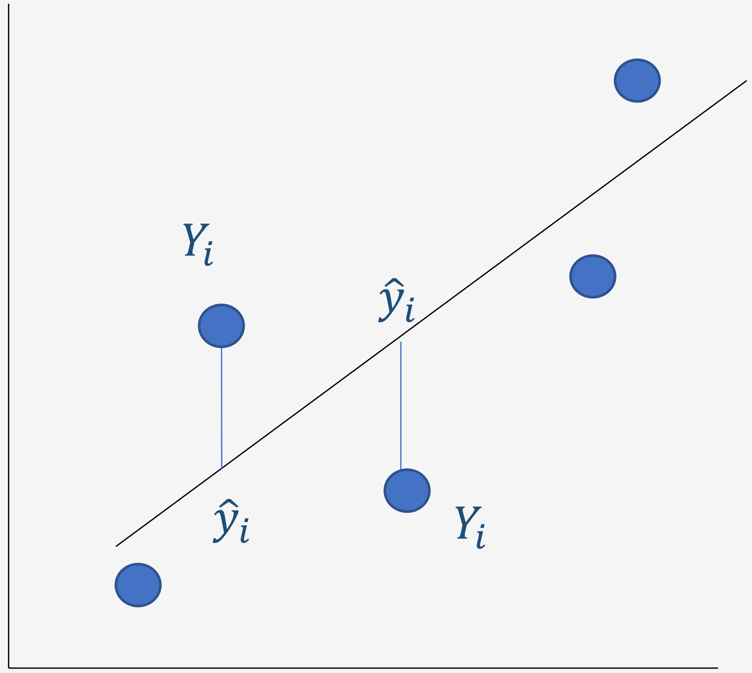


How to find the best fitting line?

Define Mean Squared Error (MSE)
To be the square of the distance between actual and predict Y values

$\hat{y}_i$ = prediction, $Y_i = actual\ value$

$$\text{MSE} = \frac{1}{N}\sum_{i}^{n}(y_i - \hat{y}_i)^2$$



Best fitted line is the line that minimize the MSE =>

Least Square Methods

R-square as metrics for determining "goodness" of the fit

- Determining the relationship between predictor & outcome
- Relationship Among SST, SSR, SSE

$$r^2 = \mathrm{SSR}/\mathrm{SST}$$

$$\mathrm{SST} = \mathrm{SSR} + \mathrm{SSE}$$

$$\sum (y_i - \bar{y})^2 = \sum (\hat{y}_i - \bar{y})^2 + \sum (y_i - \hat{y}_i)^2$$

where:

SST = total sum of squares
SSR = sum of squares due to regression
SSE = sum of squares due to error

Higher R-square => Lower SSE => Better Model

R-square is 0% to 100%, anything > 70% is great

Common Theme, Toolbox and Research workflow in Data Science

Apply different algorithms to solve different problems based on the same <Theme> and <Research Workflow>

Algorithms

- SVM
- KNN
- Naïve Bayes
- Neural Network
- Logistics Regression
- NLP

Theme
Research Workflow
Toolbox

Problems

- Regression
- Classification
- Recommendation System
- Clustering
- Association

Common Theme, Toolbox and Research workflow in Data Science

Will use Linear Regression for many of the general practices in building models, some of them are

- Split the dataset into training set and a testing set
- Use standard metrics to judge model performance
- K-fold cross validation

Linear Regression

Learning by doing

Linear Regression Continued

Challenges Number 1 multi-linear regressions

$$Y = beta\_0 + beta\_1 \; X\_1 + beta\_2 \; X\_2 + \ldots$$

- Collinearity
  - Pick the factors with highest correlation first, but what about the second factors?
    - Second highest correlation coefficients or lowest correlation with the first factor, but with high enough correlation with the dependent variable

  - Solution is: find an Orthogonal independent vectors
    - PCA (Principal Components Analysis)

=> Features Engineering

Common Theme, Toolbox and Research workflow in Data Science

Apply different algorithms to solve different problems based on the same <Theme> and <Research Workflow>

Algorithms

- SVM
- KNN
- Naïve Bayes
- Neural Network
- Logistics Regression
- NLP

Theme
Research Workflow
Toolbox

Problems

- Regression
- Classification
- Recommendation System
- Clustering
- Association

Common Theme in Machine Learning

Linear Regression

Challenge Number 2:

- Relationship is NOT linear
- Solution: may become linear after transformation

$Y = a X^2 + b X + c \Rightarrow Y = b_1 Z_1 + b_2 Z_2 + b_3$

where $Z_1 = X^2$ and $Z_2 = X$

$N = N_0 \exp(-\lambda \cdot t) \Rightarrow \ln(N/N_0) = -\lambda \cdot t + c$

$\Rightarrow Y = m X + b$

where $Y = \ln(N)$

$X = t$

Polynomial Regression

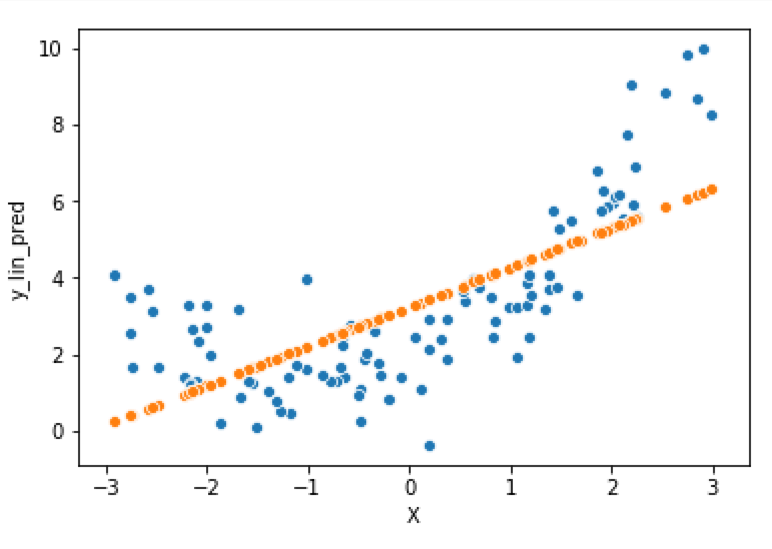
Learning by doing

Simple vs More complicated model

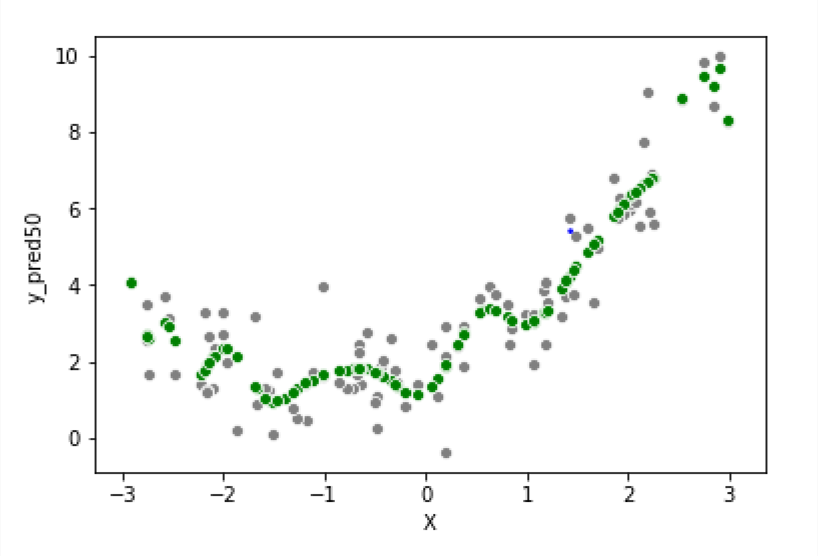
- Using a model with more parameters (more features, more predictors), you are guaranteed to fit your in-sample data (training data) better
  - More parameters => R-squares increases
- BUT it doesn’t mean you have a better model
  - Adjusted R-squares (R-squares adjusted by penalizing models with more parameters)

Lesson Learned from Polynomial Regression

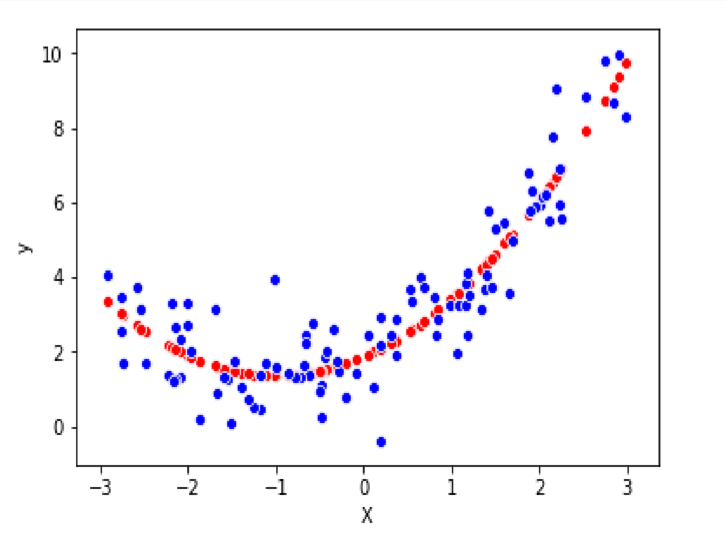
Underfit



Overfit



Good fit



A more sophisticated model tends to have smaller errors in the training set, but can perform worse in testing dataset because it overfit

A too simplistic model will never be able to fit well on both the training set as well as the testing dataset

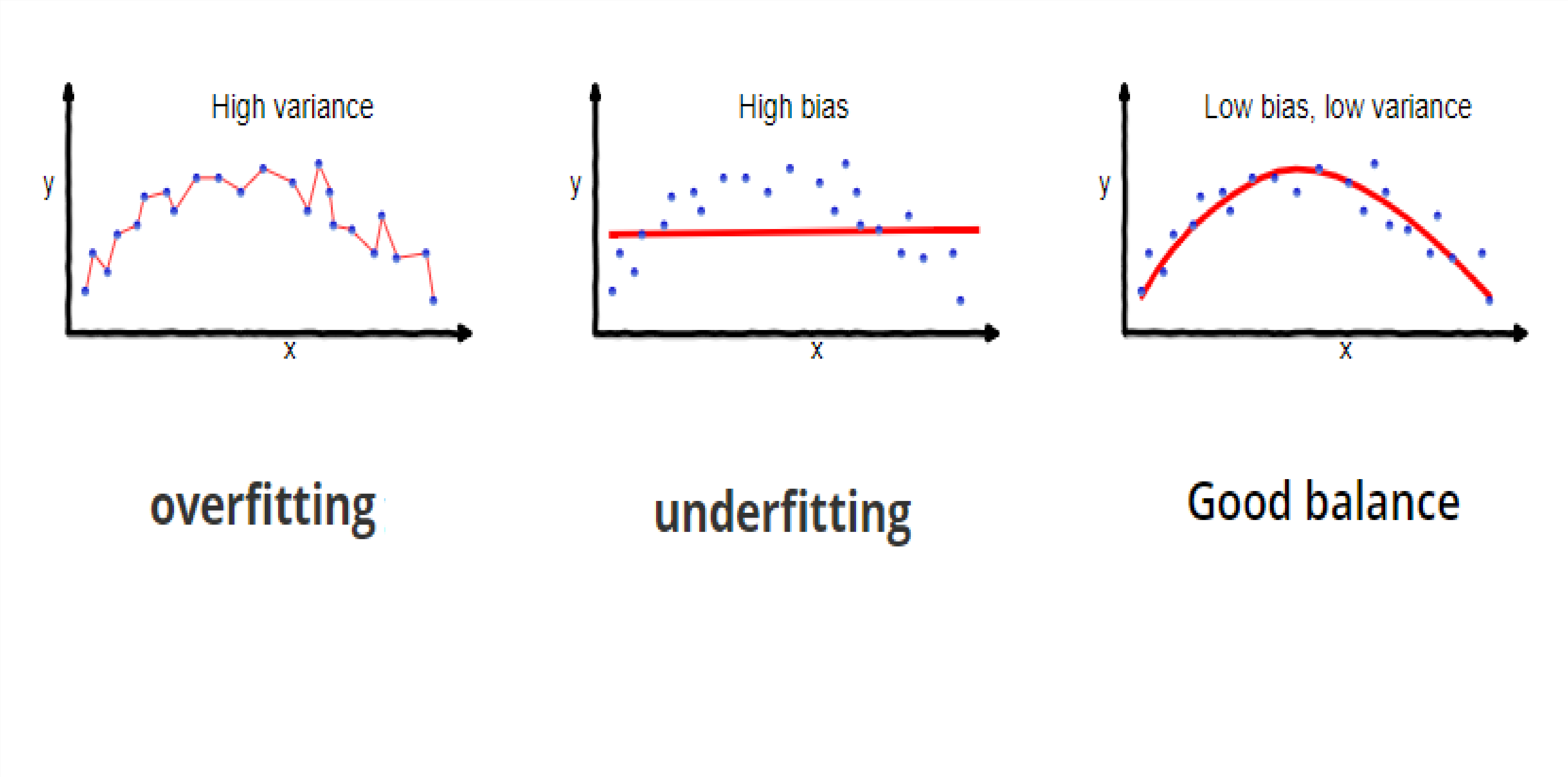
Bias vs Variance

- Bias means your model is intrinsically wrong (off, biased) that you will not fit the data well. If you use a too simplistic model, you will have high bias.

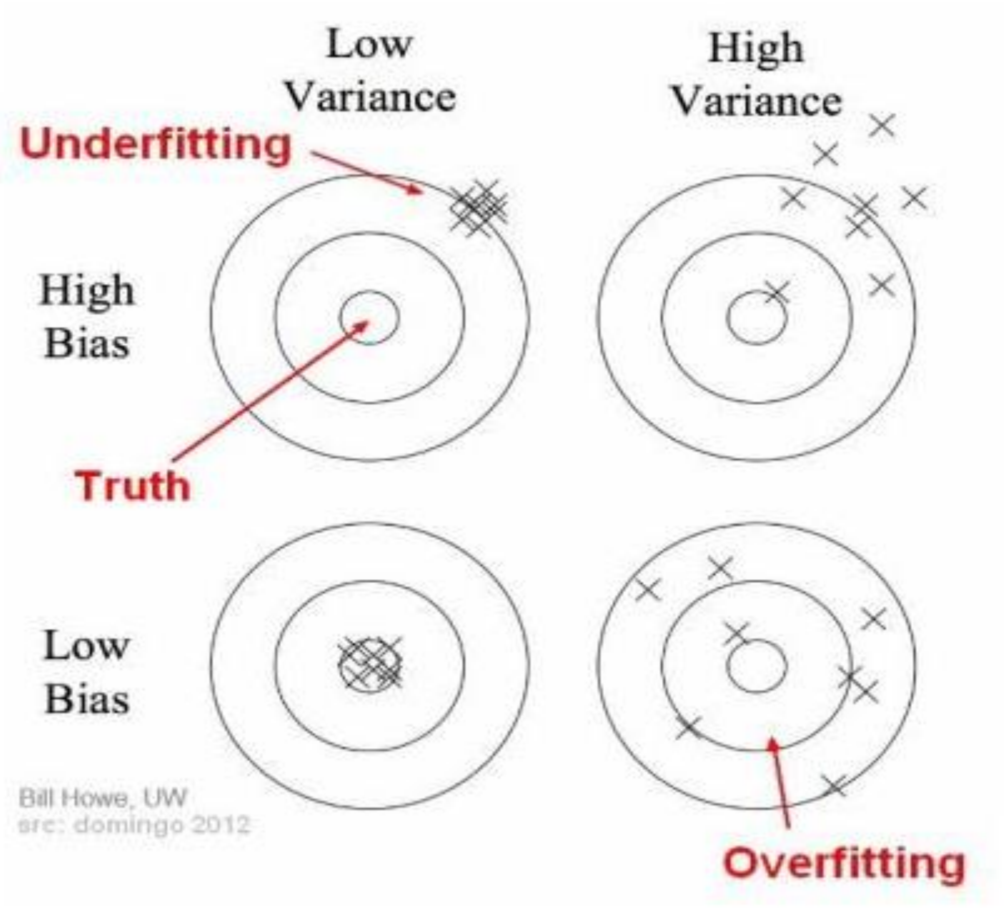
- On the other hand, using a more complicated model, you will have low bias. However, your model will not generalize well to testing dataset (out-of-sample data). The “variance” of your prediction will be high

- We call this the Bias vs Variance trade-off

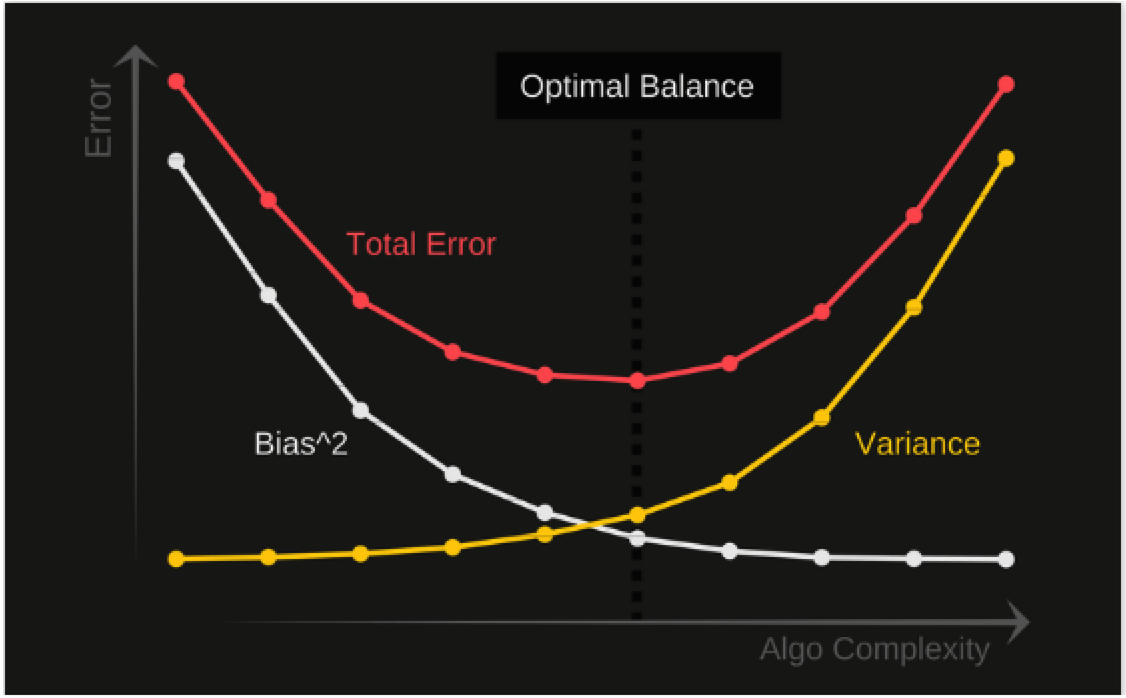
Bias vs Variance Tradeoff



Bias vs Variance Tradeoff



Total Error = Bias^2 + Variance + Irreducible Error



Recall Linear Regression can still apply to non-linear relationship

$Y = a X^2 + b X + c \Rightarrow Y = b1\, Z1 + b2\, Z2 + b3$
where $Z1 = X^2$ and $Z2 = X$

$N = N_0 \exp(-\lambda * t)$
$\Rightarrow \ln(N/N_0) = -\lambda * t + c$
$\Rightarrow Y = m X + b$ where $Y = \ln(N)$ and $X = t$

If $Y = \log(P / (1-P)) = beta_0 + beta_1 * X_1 + beta_2 * X_2 + \ldots. Beta_N + X_N$

where P is the probability of something happens

It is called Logistic Regression, which we will cover next

Classification Problem

Linear Regression: Target variable can take any numeric value

Binary Classification Problem: Target variable is either 1 or 0, Yes or No

Multi-class Classification Problem: Target variable is a list of possible values
(such as classify a picture of animal as a cat, dog, bird, fish picture)

$\Rightarrow$ NEXT TOPICS

=>Classification Problem and Logistics Regression