Lab 4

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We now move on to simple linear modeling using the ordinary least squares algorithm.

Let's quickly recreate the sample data set from practice lecture 7:

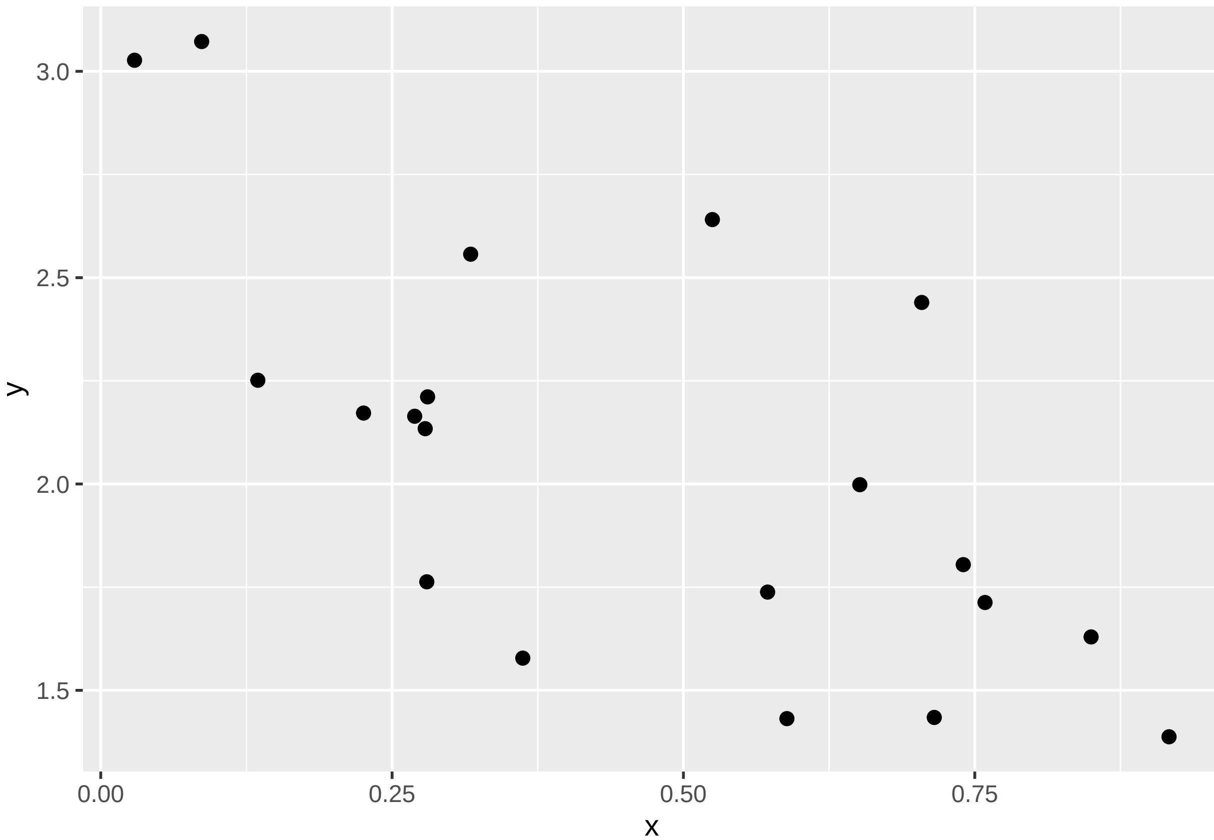
```
n = 20
x = runif(n)
beta_0 = 3
beta_1 = -2
y = beta_0 + beta_1 * x + rnorm(n, mean = 0, sd = 0.33)
```

Rewrite the computation of y so that it is h*(x) + epsilon.

```
h_star_x = beta_0 + beta_1 * x
epsilon = rnorm(n, mean = 0, sd = 0.33)
y = h_star_x + epsilon
```

Graph the data by running the following chunk:

```
pacman::p_load(ggplot2)
simple_df = data.frame(x = x, y = y)
simple_viz_obj = ggplot(simple_df, aes(x, y)) +
  geom_point(size = 2)
simple_viz_obj
```



Does this make sense given the values of $beta_0$ and $beta_1$?

Yes.

Write a function `my_simple_ols` that takes in a vector `x` and vector `y` and returns a list that contains the `b_0` (intercept), `b_1` (slope), `yhat` (the predictions), `e` (the residuals), `SSE`, `SST`, `MSE`, `RMSE` and `Rsq` (for the R-squared metric). Internally, you can only use the functions `sum` and `length` and other basic arithmetic operations. You should throw errors if the inputs are non-numeric and not the same length. You should also name the class of the return value 'my_simple_ols_obj`by using the`class' function as a setter. No need to create ROxygen documentation here.

```
my_simple_ols = function(x,y){
 if (class(x) != "numeric" | class(y) != "numeric") {stop("argument x or y is not numeric")}
 n = length(x)
 if (n != length(y)){stop("x and y must be same length")}

  y_bar = sum(y)/length(y)
  x_bar = sum(x)/length(x)
  s_x_squared = (1/(n-1) * sum((x - x_bar)^2))
  s_xy = (1/(n-1)) * sum((x - x_bar)*(y - y_bar))
  b1= s_xy/s_x_squared
  b0= y_bar - b1*x_bar
  y_hat = b0 + b1*x

  e = y - y_hat
  SSE = sum(e^2)
  SST = sum((y-y_bar)^2)
  Rsq = 1 - SSE/SST # 1 - NA
  MSE = SSE / (n-2)
```

```
  RMSE = sqrt(MSE)

  mod = list(b_0 = b0, b_1 = b1, y_hat = y_hat, e = e, SSE = SSE, SST = SST, Rsq = Rsq, MSE = MSE, RMSE
  class(mod) = "my_simple_ols_obj"
  mod
}
```

Verify your computations are correct for the vectors `x` and `y` from the first chunk using the `lm` function in R:

```
lm_mod = lm(y ~ x)
my_lm_mod = my_simple_ols(x, y)
#run the tests to ensure the function is up to spec
pacman::p_load(testthat)
expect_equal(my_lm_mod$b_0, as.numeric(coef(lm_mod)[1]), tol = 1e-4)
expect_equal(my_lm_mod$b_1, as.numeric(coef(lm_mod)[2]), tol = 1e-4)
expect_equal(my_lm_mod$RMSE, summary(lm_mod)$sigma, tol = 1e-4)
expect_equal(my_lm_mod$Rsq, summary(lm_mod)$r.squared, tol = 1e-4)
```

Verify that the average of the residuals is 0.

```
expect_equal(mean(my_lm_mod$e), 0, tol = 1e-4)
```

Create the X matrix for this data example.

```
X = cbind(1, x)
X
```

```
##                x
##  [1,] 1 0.70447080
##  [2,] 1 0.65136226
##  [3,] 1 0.27842115
##  [4,] 1 0.28047660
##  [5,] 1 0.26938924
##  [6,] 1 0.57225371
##  [7,] 1 0.13480810
##  [8,] 1 0.52485040
##  [9,] 1 0.71524499
## [10,] 1 0.02900179
## [11,] 1 0.75880682
## [12,] 1 0.22553375
## [13,] 1 0.91669713
## [14,] 1 0.31744254
## [15,] 1 0.84983519
## [16,] 1 0.27978700
## [17,] 1 0.74013728
## [18,] 1 0.08660859
## [19,] 1 0.36215297
## [20,] 1 0.58880895
```

Use the `model.matrix` function to compute the matrix `X` and verify it is the same as your manual construction.

```
model.matrix(~ x)
```

```
##    (Intercept)          x
## 1            1 0.70447080
## 2            1 0.65136226
## 3            1 0.27842115
```

```
## 4            1 0.28047660
## 5            1 0.26938924
## 6            1 0.57225371
## 7            1 0.13480810
## 8            1 0.52485040
## 9            1 0.71524499
## 10           1 0.02900179
## 11           1 0.75880682
## 12           1 0.22553375
## 13           1 0.91669713
## 14           1 0.31744254
## 15           1 0.84983519
## 16           1 0.27978700
## 17           1 0.74013728
## 18           1 0.08660859
## 19           1 0.36215297
## 20           1 0.58880895
## attr(,"assign")
## [1] 0 1
```

Using matrix algebra, verify the OLS estimate is the same as you computed from the `my_simple_ols` function.

```
XtXinvX = solve(t(X) %*% X) %*% t(X)
b = XtXinvX %*% y
b
```

```
##        [,1]
##    2.639744
## x -1.254370
```

Find the hat matrix H.

```
H = X %*% XtXinvX
H
```

```
##               [,1]         [,2]         [,3]         [,4]         [,5]
##  [1,]  0.092019911  0.082727951  0.017477539  0.017837164  0.015897301
##  [2,]  0.082727951  0.075490743  0.024669306  0.024949406  0.023438510
##  [3,]  0.017477539  0.024669306  0.075171650  0.074893308  0.076394718
##  [4,]  0.017837164  0.024949406  0.074893308  0.074618045  0.076102852
##  [5,]  0.015897301  0.023438510  0.076394718  0.076102852  0.077677214
##  [6,]  0.068886983  0.064710461  0.035381902  0.035543545  0.034671622
##  [7,] -0.007649244  0.005098867  0.094619211  0.094125824  0.096787220
##  [8,]  0.060593221  0.058250717  0.041801087  0.041891748  0.041402709
##  [9,]  0.093904982  0.084196172  0.016018537  0.016394295  0.014367407
## [10,] -0.026161296 -0.009319573  0.108947121  0.108295300  0.111811310
## [11,]  0.101526633  0.090132429  0.010119551  0.010560538  0.008181795
## [12,]  0.008224270  0.017462234  0.082333471  0.081975936  0.083904525
## [13,]  0.129151391  0.111648461 -0.011261387 -0.010583975 -0.014238024
## [14,]  0.024304788  0.029986829  0.069887513  0.069667603  0.070853830
## [15,]  0.117453112  0.102537049 -0.002207184 -0.001629891 -0.004743885
## [16,]  0.017716511  0.024855434  0.074986691  0.074710394  0.076200772
## [17,]  0.098260179  0.087588294  0.012647713  0.013060745  0.010832798
## [18,] -0.016082313 -0.001469378  0.101146216  0.100580655  0.103631366
## [19,]  0.032127401  0.036079610  0.063832987  0.063680026  0.064505120
## [20,]  0.071783517  0.066966478  0.033140050  0.033326483  0.032320840
##              [,6]         [,7]        [,8]         [,9]        [,10]         [,11]
```

```
##  [1,] 0.06888698 -0.007649244 0.06059322  0.093904982 -0.026161296  0.101526633
##  [2,] 0.06471046  0.005098867 0.05825072  0.084196172 -0.009319573  0.090132429
##  [3,] 0.03538190  0.094619211 0.04180109  0.016018537  0.108947121  0.010119551
##  [4,] 0.03554355  0.094125824 0.04189175  0.016394295  0.108295300  0.010560538
##  [5,] 0.03467162  0.096787220 0.04140271  0.014367407  0.111811310  0.008181795
##  [6,] 0.05848926  0.024087991 0.05476141  0.069734279  0.015767250  0.073160036
##  [7,] 0.02408799  0.129091918 0.03546662 -0.010235468  0.154489539 -0.020691997
##  [8,] 0.05476141  0.035466624 0.05267055  0.061068447  0.030799735  0.062989865
##  [9,] 0.06973428 -0.010235468 0.06106845  0.095874619 -0.029577996  0.103838188
## [10,] 0.01576725  0.154489539 0.03079974 -0.029577996  0.188042725 -0.043392277
## [11,] 0.07316004 -0.020691997 0.06298987  0.103838188 -0.043392277  0.113184187
## [12,] 0.03122277  0.107314240 0.03946834  0.006350153  0.125718717 -0.001227208
## [13,] 0.08557673 -0.058591802 0.06995407  0.132702231 -0.093462287  0.147058860
## [14,] 0.03845059  0.085252563 0.04352224  0.023152065  0.096572699  0.018491419
## [15,] 0.08031862 -0.042542341 0.06700493  0.120479151 -0.072259099  0.132713925
## [16,] 0.03548931  0.094291354 0.04186133  0.016268229  0.108513983  0.010412589
## [17,] 0.07169184 -0.016210583 0.06216639  0.100425196 -0.037471812  0.109178719
## [18,] 0.02029752  0.140661672 0.03334065 -0.019046856  0.169774519 -0.031032991
## [19,] 0.04196668  0.074520323 0.04549432  0.031325612  0.082394173  0.028083846
## [20,] 0.05979119  0.020114089 0.05549163  0.072760756  0.010517267  0.076711892
##              [,12]        [,13]       [,14]        [,15]       [,16]
##  [1,]  0.008224270  0.129151391 0.024304788  0.117453112  0.01771651
##  [2,]  0.017462234  0.111648461 0.029986829  0.102537049  0.02485543
##  [3,]  0.082333471 -0.011261387 0.069887513 -0.002207184  0.07498669
##  [4,]  0.081975936 -0.010583975 0.069667603 -0.001629891  0.07471039
##  [5,]  0.083904525 -0.014238024 0.070853830 -0.004743885  0.07620077
##  [6,]  0.031222770  0.085576729 0.038450593  0.080318622  0.03548931
##  [7,]  0.107314240 -0.058591802 0.085252563 -0.042542341  0.09429135
##  [8,]  0.039468338  0.069954067 0.043522236  0.067004931  0.04186133
##  [9,]  0.006350153  0.132702231 0.023152065  0.120479151  0.01626823
## [10,]  0.125718717 -0.093462287 0.096572699 -0.072259099  0.10851398
## [11,] -0.001227208  0.147058860 0.018491419  0.132713925  0.01041259
## [12,]  0.091532968 -0.028691436 0.075545895 -0.017061137  0.08209589
## [13,] -0.028691436  0.199094620 0.001598855  0.177058995 -0.01081124
## [14,]  0.075545895  0.001598855 0.065712645  0.008752363  0.06974138
## [15,] -0.017061137  0.177058995 0.008752363  0.158280152 -0.00182357
## [16,]  0.082095887 -0.010811244 0.069741382 -0.001823570  0.07480309
## [17,]  0.002020265  0.140905958 0.020488859  0.127470398  0.01292217
## [18,]  0.115698303 -0.074476870 0.090409392 -0.056079655  0.10077040
## [19,]  0.067768740  0.016334031 0.060929110  0.021309735  0.06373134
## [20,]  0.028343068  0.091032826 0.036679360  0.084968328  0.03326394
##              [,17]        [,18]      [,19]      [,20]
##  [1,]  0.098260179 -0.016082313 0.03212740 0.07178352
##  [2,]  0.087588294 -0.001469378 0.03607961 0.06696648
##  [3,]  0.012647713  0.101146216 0.06383299 0.03314005
##  [4,]  0.013060745  0.100580655 0.06368003 0.03332648
##  [5,]  0.010832798  0.103631366 0.06450512 0.03232084
##  [6,]  0.071691840  0.020297521 0.04196668 0.05979119
##  [7,] -0.016210583  0.140661672 0.07452032 0.02011409
##  [8,]  0.062166392  0.033340647 0.04549432 0.05549163
##  [9,]  0.100425196 -0.019046856 0.03132561 0.07276076
## [10,] -0.037471812  0.169774519 0.08239417 0.01051727
## [11,]  0.109178719 -0.031032991 0.02808385 0.07671189
## [12,]  0.002020265  0.115698303 0.06776874 0.02834307
```

```
## [13,]  0.140905958 -0.074476870 0.01633403 0.09103283
## [14,]  0.020488859  0.090409392 0.06092911 0.03667936
## [15,]  0.127470398 -0.056079655 0.02130973 0.08496833
## [16,]  0.012922174  0.100770399 0.06373134 0.03326394
## [17,]  0.105427172 -0.025896025 0.02947319 0.07501853
## [18,] -0.025896025  0.153923877 0.07810721 0.01574231
## [19,]  0.029473188  0.078107215 0.05760187 0.04073468
## [20,]  0.075018531  0.015742307 0.04073468 0.06129278
```

Verify that this specific hat matrix is symmetric.

```
expect_equal(H, t(H))
```

Using the `diag` function, find the trace of the hat matrix.

```
diag(H)
```

```
##  [1] 0.09201991 0.07549074 0.07517165 0.07461804 0.07767721 0.05848926
##  [7] 0.12909192 0.05267055 0.09587462 0.18804273 0.11318419 0.09153297
## [13] 0.19909462 0.06571264 0.15828015 0.07480309 0.10542717 0.15392388
## [19] 0.05760187 0.06129278
```

```
sum(diag(H))
```

```
## [1] 2
```

Create a prediction method `g` that takes in a vector `x_future` and `my_simple_ols_obj`, an object of type `my_simple_ols_obj` and predicts y values for each entry in `x_future`.

```
g = function(x_future, my_simple_ols_obj){
  my_simple_ols_obj$b_0 + my_simple_ols_obj$b_1 * x_future
}
```

Use this function to verify that when predicting for the average x, you get the average y.

```
expect_equal(g(mean(x), my_lm_mod), mean(y))
```

Create a prediction method `g` that takes in a vector `x_future` and the dataset $\mathbb{D}$ i.e. `X` where the first column is the one vector and `y` and returns the OLS predictions.

```
g = function(x_future, X, y){
b = solve(t(X) %*% X) %*% t(X) %*% y
b[1] +b[2]*x_future
}
```

In class we spoke about error due to ignorance, misspecification error and estimation error. Show that as n grows, estimation error shrinks. Let us define an error metric that is the difference between b_0 and b_1 and β_0 and β_1. How about $h = ||b - \beta||^2$ where the quantities are now the vectors of size two. Show as n increases, this shrinks.

```
ns = 10^(1:7)
errors = array(dim=length(ns))
beta = c(beta_0, beta_1)
for (i in 1:length(ns)) {
  n = ns[i]
  x = runif(n)
  h_star_x = beta_0 + beta_1 * x
  epsilon = rnorm(n, mean = 0, sd = 0.33)
  y = h_star_x + epsilon
```

```
  mod = lm(y ~ x)
  b = coef(mod)
  errors[i] = sum((beta - b)^2)
}
errors
```

```
## [1] 4.874365e-03 7.308998e-03 1.986545e-04 9.853330e-05 8.258596e-06
## [6] 8.827203e-07 6.993816e-07
```

We are now going to repeat one of the first linear model building exercises in history — that of Sir Francis Galton in 1886. First load up package `HistData`.

```
pacman::p_load(HistData)
```

In it, there is a dataset called `Galton`. Load it up.

```
data(Galton)
```

You now should have a data frame in your workspace called `Galton`. Summarize this data frame and write a few sentences about what you see. Make sure you report n, p and a bit about what the columns represent and how the data was measured. See the help file `?Galton`. p is 1 and n is 928 the number of observations

```
Galton
```

```
##     parent child
## 1     70.5  61.7
## 2     68.5  61.7
## 3     65.5  61.7
## 4     64.5  61.7
## 5     64.0  61.7
## 6     67.5  62.2
## 7     67.5  62.2
## 8     67.5  62.2
## 9     66.5  62.2
## 10    66.5  62.2
## 11    66.5  62.2
## 12    64.5  62.2
## 13    70.5  63.2
## 14    69.5  63.2
## 15    68.5  63.2
## 16    68.5  63.2
## 17    68.5  63.2
## 18    68.5  63.2
## 19    68.5  63.2
## 20    68.5  63.2
## 21    68.5  63.2
## 22    67.5  63.2
## 23    67.5  63.2
## 24    67.5  63.2
## 25    67.5  63.2
## 26    67.5  63.2
## 27    66.5  63.2
## 28    66.5  63.2
## 29    66.5  63.2
## 30    65.5  63.2
## 31    65.5  63.2
## 32    65.5  63.2
```

```
## 33    65.5  63.2
## 34    65.5  63.2
## 35    65.5  63.2
## 36    65.5  63.2
## 37    65.5  63.2
## 38    65.5  63.2
## 39    64.5  63.2
## 40    64.5  63.2
## 41    64.5  63.2
## 42    64.5  63.2
## 43    64.0  63.2
## 44    64.0  63.2
## 45    69.5  64.2
## 46    69.5  64.2
## 47    69.5  64.2
## 48    69.5  64.2
## 49    69.5  64.2
## 50    69.5  64.2
## 51    69.5  64.2
## 52    69.5  64.2
## 53    69.5  64.2
## 54    69.5  64.2
## 55    69.5  64.2
## 56    69.5  64.2
## 57    69.5  64.2
## 58    69.5  64.2
## 59    69.5  64.2
## 60    69.5  64.2
## 61    68.5  64.2
## 62    68.5  64.2
## 63    68.5  64.2
## 64    68.5  64.2
## 65    68.5  64.2
## 66    68.5  64.2
## 67    68.5  64.2
## 68    68.5  64.2
## 69    68.5  64.2
## 70    68.5  64.2
## 71    68.5  64.2
## 72    67.5  64.2
## 73    67.5  64.2
## 74    67.5  64.2
## 75    67.5  64.2
## 76    67.5  64.2
## 77    67.5  64.2
## 78    67.5  64.2
## 79    67.5  64.2
## 80    67.5  64.2
## 81    67.5  64.2
## 82    67.5  64.2
## 83    67.5  64.2
## 84    67.5  64.2
## 85    67.5  64.2
## 86    66.5  64.2
```

```
## 87    66.5  64.2
## 88    66.5  64.2
## 89    66.5  64.2
## 90    66.5  64.2
## 91    65.5  64.2
## 92    65.5  64.2
## 93    65.5  64.2
## 94    65.5  64.2
## 95    65.5  64.2
## 96    64.5  64.2
## 97    64.5  64.2
## 98    64.5  64.2
## 99    64.5  64.2
## 100   64.0  64.2
## 101   64.0  64.2
## 102   64.0  64.2
## 103   64.0  64.2
## 104   71.5  65.2
## 105   70.5  65.2
## 106   69.5  65.2
## 107   69.5  65.2
## 108   69.5  65.2
## 109   69.5  65.2
## 110   68.5  65.2
## 111   68.5  65.2
## 112   68.5  65.2
## 113   68.5  65.2
## 114   68.5  65.2
## 115   68.5  65.2
## 116   68.5  65.2
## 117   68.5  65.2
## 118   68.5  65.2
## 119   68.5  65.2
## 120   68.5  65.2
## 121   68.5  65.2
## 122   68.5  65.2
## 123   68.5  65.2
## 124   68.5  65.2
## 125   68.5  65.2
## 126   67.5  65.2
## 127   67.5  65.2
## 128   67.5  65.2
## 129   67.5  65.2
## 130   67.5  65.2
## 131   67.5  65.2
## 132   67.5  65.2
## 133   67.5  65.2
## 134   67.5  65.2
## 135   67.5  65.2
## 136   67.5  65.2
## 137   67.5  65.2
## 138   67.5  65.2
## 139   67.5  65.2
## 140   67.5  65.2
```

```
## 141   66.5  65.2
## 142   66.5  65.2
## 143   65.5  65.2
## 144   65.5  65.2
## 145   65.5  65.2
## 146   65.5  65.2
## 147   65.5  65.2
## 148   65.5  65.2
## 149   65.5  65.2
## 150   64.5  65.2
## 151   64.0  65.2
## 152   71.5  66.2
## 153   71.5  66.2
## 154   71.5  66.2
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## 812   69.5  71.2
## 813   69.5  71.2
## 814   69.5  71.2
## 815   69.5  71.2
## 816   69.5  71.2
## 817   69.5  71.2
## 818   69.5  71.2
## 819   69.5  71.2
## 820   69.5  71.2
## 821   69.5  71.2
## 822   69.5  71.2
## 823   69.5  71.2
## 824   69.5  71.2
## 825   69.5  71.2
## 826   68.5  71.2
## 827   68.5  71.2
## 828   68.5  71.2
## 829   68.5  71.2
## 830   68.5  71.2
## 831   68.5  71.2
## 832   68.5  71.2
## 833   68.5  71.2
## 834   68.5  71.2
## 835   68.5  71.2
## 836   68.5  71.2
## 837   68.5  71.2
## 838   68.5  71.2
## 839   68.5  71.2
## 840   68.5  71.2
## 841   68.5  71.2
## 842   68.5  71.2
```

```
## 843   68.5  71.2
## 844   67.5  71.2
## 845   67.5  71.2
## 846   67.5  71.2
## 847   67.5  71.2
## 848   67.5  71.2
## 849   67.5  71.2
## 850   67.5  71.2
## 851   67.5  71.2
## 852   67.5  71.2
## 853   67.5  71.2
## 854   67.5  71.2
## 855   65.5  71.2
## 856   65.5  71.2
## 857   73.0  72.2
## 858   72.5  72.2
## 859   72.5  72.2
## 860   72.5  72.2
## 861   72.5  72.2
## 862   72.5  72.2
## 863   72.5  72.2
## 864   72.5  72.2
## 865   71.5  72.2
## 866   71.5  72.2
## 867   71.5  72.2
## 868   71.5  72.2
## 869   71.5  72.2
## 870   71.5  72.2
## 871   71.5  72.2
## 872   71.5  72.2
## 873   71.5  72.2
## 874   70.5  72.2
## 875   70.5  72.2
## 876   70.5  72.2
## 877   70.5  72.2
## 878   69.5  72.2
## 879   69.5  72.2
## 880   69.5  72.2
## 881   69.5  72.2
## 882   69.5  72.2
## 883   69.5  72.2
## 884   69.5  72.2
## 885   69.5  72.2
## 886   69.5  72.2
## 887   69.5  72.2
## 888   69.5  72.2
## 889   68.5  72.2
## 890   68.5  72.2
## 891   68.5  72.2
## 892   68.5  72.2
## 893   67.5  72.2
## 894   67.5  72.2
## 895   67.5  72.2
## 896   67.5  72.2
```

```
## 897   65.5  72.2
## 898   73.0  73.2
## 899   73.0  73.2
## 900   73.0  73.2
## 901   72.5  73.2
## 902   72.5  73.2
## 903   71.5  73.2
## 904   71.5  73.2
## 905   70.5  73.2
## 906   70.5  73.2
## 907   70.5  73.2
## 908   69.5  73.2
## 909   69.5  73.2
## 910   69.5  73.2
## 911   69.5  73.2
## 912   68.5  73.2
## 913   68.5  73.2
## 914   68.5  73.2
## 915   72.5  73.7
## 916   72.5  73.7
## 917   72.5  73.7
## 918   72.5  73.7
## 919   71.5  73.7
## 920   71.5  73.7
## 921   70.5  73.7
## 922   70.5  73.7
## 923   70.5  73.7
## 924   69.5  73.7
## 925   69.5  73.7
## 926   69.5  73.7
## 927   69.5  73.7
## 928   69.5  73.7
```

TO-DO

Find the average height (include both parents and children in this computation).

```
avg_height = (mean(Galton$parent) + mean(Galton$child)) / 2
```

If you were to use the null model, what would the RMSE be of this model be?

```
rmse_null = sqrt(mean((Galton$child - avg_height) ^ 2))
```

Note that in Math 241 you learned that the sample average is an estimate of the "mean", the population expected value of height. We will call the average the "mean" going forward since it is probably correct to the nearest tenth of an inch with this amount of data.

Run a linear model attempting to explain the childrens' height using the parents' height. Use `lm` and use the R formula notation. Compute and report b_0, b_1, RMSE and R^2. Use the correct units to report these quantities.

```
mod = lm(child ~ parent, Galton)
b_0 = mod$coefficients[1]
b_1 = mod$coefficients[2]
b_0
```

```
## (Intercept)
##    23.94153
```

```r
b_1
```

```
##    parent
## 0.6462906
```

```r
summary(mod)$sigma
```

```
## [1] 2.238547
```

```r
summary(mod)$r.sq
```

```
## [1] 0.2104629
```

Interpret all four quantities: b_0, b_1, RMSE and R^2.

TO-DO

How good is this model? How well does it predict? Discuss.

TO-DO

It is reasonable to assume that parents and their children have the same height? Explain why this is reasonable using basic biology and common sense.

TO-DO

If they were to have the same height and any differences were just random noise with expectation 0, what would the values of β_0 and β_1 be?

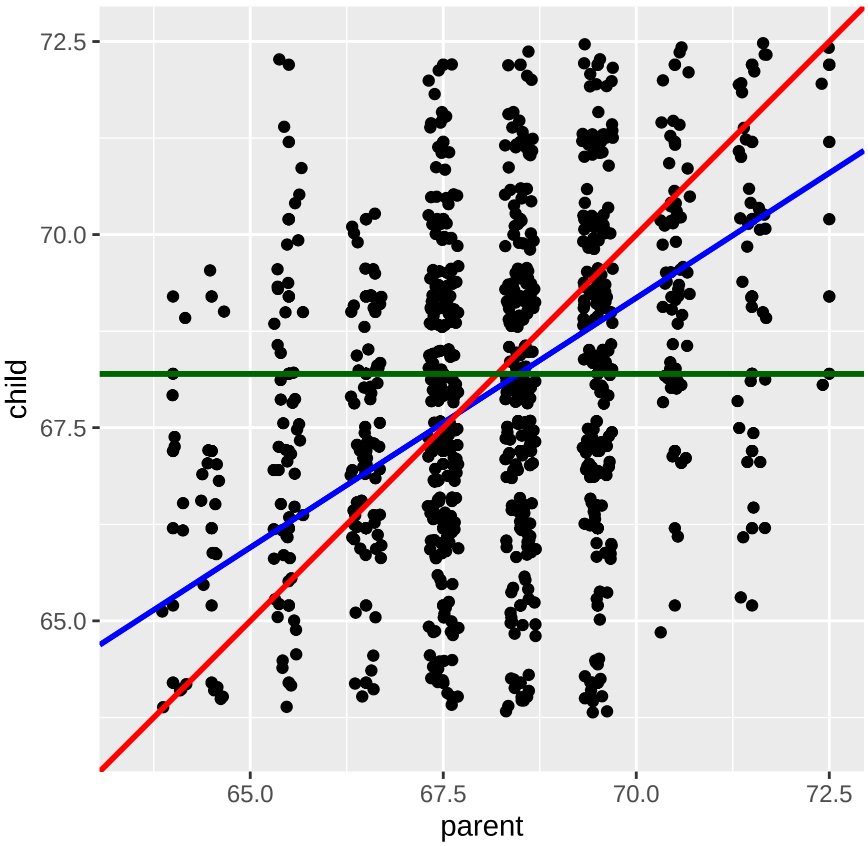
TO-DO

Let's plot (a) the data in $\mathbb{D}$ as black dots, (b) your least squares line defined by b_0 and b_1 in blue, (c) the theoretical line β_0 and β_1 if the parent-child height equality held in red and (d) the mean height in green.

```r
pacman::p_load(ggplot2)
ggplot(Galton, aes(x = parent, y = child)) +
  geom_point() +
  geom_jitter() +
  geom_abline(intercept = b_0, slope = b_1, color = "blue", size = 1) +
  geom_abline(intercept = 0, slope = 1, color = "red", size = 1) +
  geom_abline(intercept = avg_height, slope = 0, color = "darkgreen", size = 1) +
  xlim(63.5, 72.5) +
  ylim(63.5, 72.5) +
  coord_equal(ratio = 1)
```

```
## Warning: Removed 76 rows containing missing values (geom_point).

## Warning: Removed 90 rows containing missing values (geom_point).
```



Fill in the following sentence:

Children of short parents became taller on average and children of tall parents became shorter on average.

Why did Galton call it "Regression towards mediocrity in hereditary stature" which was later shortened to "regression to the mean"?

Because children tend to move toward the mean

Why should this effect be real?

Genes?

You now have unlocked the mystery. Why is it that when modeling with y continuous, everyone calls it "regression"? Write a better, more descriptive and appropriate name for building predictive models with y continuous.

Everyone calls it regression,because of Galton discovery of Parents and Child height moving towards the mean.

Create a dataset $\mathbb{D}$ which we call `Xy` such that the linear model as R^2 about 50% and RMSE approximately 1.

```
x = c(1,2,3,4,5,6,4,1,2,3)
y = c(1,4,3,6,4,7,3,2,3,3)
Xy = data.frame(x = x, y = y)
first_model = lm(y~ x)
summary(first_model)$r.sq
```

```
## [1] 0.6476045
```

```
summary(first_model)$sigma
```

```
## [1] 1.118483
```

Create a dataset $\mathbb{D}$ which we call `Xy` such that the linear model as R^2 about 0% but x, y are clearly associated.

```
x = c(1,1,1,1)
y = c(20,40,20,40)
Xy = data.frame(x = x, y = y)
sec_model = lm(y~ x)
summary(sec_model)$r.sq
```

```
## [1] 0
```

```
summary(sec_model)$sigma
```

```
## [1] 11.54701
```

Extra credit: create a dataset $\mathbb{D}$ and a model (hint: not a lienar model) that can give you R^2 arbitrarily close to 1 but RMSE arbitrarily high.

```
data(iris)
```

Load up the famous iris dataset. We are going to do a different prediction problem. Imagine the only input x is Species and you are trying to predict y which is Petal.Length. What would a reasonable, naive prediction be under all Species? Hint: it's what we did in class.

```
x = iris$Species
y = iris$Petal.Length
petallen_model = lm(y ~ x)
summary(petallen_model)$r.sq
```

```
## [1] 0.9413717
```

```
summary(petallen_model)$sigma
```

```
## [1] 0.4303345
```

Prove that this is the OLS model by fitting an appropriate `lm` and then using the predict function to verify you get the same answers as you wrote previously. Show this by doing a linear regression with and without the intercept.

```
petallen_model = lm(y ~ x)
summary(petallen_model)$r.sq
```

```
## [1] 0.9413717
```

```
summary(petallen_model)$sigma
```

```
## [1] 0.4303345
```

```
predict(petallen_model)
```

```
##     1     2     3     4     5     6     7     8     9    10    11    12    13
## 1.462 1.462 1.462 1.462 1.462 1.462 1.462 1.462 1.462 1.462 1.462 1.462 1.462
##    14    15    16    17    18    19    20    21    22    23    24    25    26
## 1.462 1.462 1.462 1.462 1.462 1.462 1.462 1.462 1.462 1.462 1.462 1.462 1.462
##    27    28    29    30    31    32    33    34    35    36    37    38    39
## 1.462 1.462 1.462 1.462 1.462 1.462 1.462 1.462 1.462 1.462 1.462 1.462 1.462
##    40    41    42    43    44    45    46    47    48    49    50    51    52
## 1.462 1.462 1.462 1.462 1.462 1.462 1.462 1.462 1.462 1.462 1.462 4.260 4.260
##    53    54    55    56    57    58    59    60    61    62    63    64    65
## 4.260 4.260 4.260 4.260 4.260 4.260 4.260 4.260 4.260 4.260 4.260 4.260 4.260
```

```
##    66    67    68    69    70    71    72    73    74    75    76    77    78
## 4.260 4.260 4.260 4.260 4.260 4.260 4.260 4.260 4.260 4.260 4.260 4.260 4.260
##    79    80    81    82    83    84    85    86    87    88    89    90    91
## 4.260 4.260 4.260 4.260 4.260 4.260 4.260 4.260 4.260 4.260 4.260 4.260 4.260
##    92    93    94    95    96    97    98    99   100   101   102   103   104
## 4.260 4.260 4.260 4.260 4.260 4.260 4.260 4.260 4.260 5.552 5.552 5.552 5.552
##   105   106   107   108   109   110   111   112   113   114   115   116   117
## 5.552 5.552 5.552 5.552 5.552 5.552 5.552 5.552 5.552 5.552 5.552 5.552 5.552
##   118   119   120   121   122   123   124   125   126   127   128   129   130
## 5.552 5.552 5.552 5.552 5.552 5.552 5.552 5.552 5.552 5.552 5.552 5.552 5.552
##   131   132   133   134   135   136   137   138   139   140   141   142   143
## 5.552 5.552 5.552 5.552 5.552 5.552 5.552 5.552 5.552 5.552 5.552 5.552 5.552
##   144   145   146   147   148   149   150
## 5.552 5.552 5.552 5.552 5.552 5.552 5.552
```

Use the `model.matrix` function to compute the matrix `X` for the regression with the intercept and without the intercept. What is different?

```r
model.matrix(x ~ 0,iris)
```

```
##
## 1
## 2
## 3
## 4
## 5
## 6
## 7
## 8
## 9
## 10
## 11
## 12
## 13
## 14
## 15
## 16
## 17
## 18
## 19
## 20
## 21
## 22
## 23
## 24
## 25
## 26
## 27
## 28
## 29
## 30
## 31
## 32
## 33
## 34
```

```
## 35
## 36
## 37
## 38
## 39
## 40
## 41
## 42
## 43
## 44
## 45
## 46
## 47
## 48
## 49
## 50
## 51
## 52
## 53
## 54
## 55
## 56
## 57
## 58
## 59
## 60
## 61
## 62
## 63
## 64
## 65
## 66
## 67
## 68
## 69
## 70
## 71
## 72
## 73
## 74
## 75
## 76
## 77
## 78
## 79
## 80
## 81
## 82
## 83
## 84
## 85
## 86
## 87
## 88
```

```
## 89
## 90
## 91
## 92
## 93
## 94
## 95
## 96
## 97
## 98
## 99
## 100
## 101
## 102
## 103
## 104
## 105
## 106
## 107
## 108
## 109
## 110
## 111
## 112
## 113
## 114
## 115
## 116
## 117
## 118
## 119
## 120
## 121
## 122
## 123
## 124
## 125
## 126
## 127
## 128
## 129
## 130
## 131
## 132
## 133
## 134
## 135
## 136
## 137
## 138
## 139
## 140
## 141
## 142
```

```
## 143
## 144
## 145
## 146
## 147
## 148
## 149
## 150
## attr(,"assign")
## integer(0)
```

```r
model.matrix(x ~ ., iris)
```

```
##     (Intercept) Sepal.Length Sepal.Width Petal.Length Petal.Width
## 1             1          5.1         3.5          1.4         0.2
## 2             1          4.9         3.0          1.4         0.2
## 3             1          4.7         3.2          1.3         0.2
## 4             1          4.6         3.1          1.5         0.2
## 5             1          5.0         3.6          1.4         0.2
## 6             1          5.4         3.9          1.7         0.4
## 7             1          4.6         3.4          1.4         0.3
## 8             1          5.0         3.4          1.5         0.2
## 9             1          4.4         2.9          1.4         0.2
## 10            1          4.9         3.1          1.5         0.1
## 11            1          5.4         3.7          1.5         0.2
## 12            1          4.8         3.4          1.6         0.2
## 13            1          4.8         3.0          1.4         0.1
## 14            1          4.3         3.0          1.1         0.1
## 15            1          5.8         4.0          1.2         0.2
## 16            1          5.7         4.4          1.5         0.4
## 17            1          5.4         3.9          1.3         0.4
## 18            1          5.1         3.5          1.4         0.3
## 19            1          5.7         3.8          1.7         0.3
## 20            1          5.1         3.8          1.5         0.3
## 21            1          5.4         3.4          1.7         0.2
## 22            1          5.1         3.7          1.5         0.4
## 23            1          4.6         3.6          1.0         0.2
## 24            1          5.1         3.3          1.7         0.5
## 25            1          4.8         3.4          1.9         0.2
## 26            1          5.0         3.0          1.6         0.2
## 27            1          5.0         3.4          1.6         0.4
## 28            1          5.2         3.5          1.5         0.2
## 29            1          5.2         3.4          1.4         0.2
## 30            1          4.7         3.2          1.6         0.2
## 31            1          4.8         3.1          1.6         0.2
## 32            1          5.4         3.4          1.5         0.4
## 33            1          5.2         4.1          1.5         0.1
## 34            1          5.5         4.2          1.4         0.2
## 35            1          4.9         3.1          1.5         0.2
## 36            1          5.0         3.2          1.2         0.2
## 37            1          5.5         3.5          1.3         0.2
## 38            1          4.9         3.6          1.4         0.1
## 39            1          4.4         3.0          1.3         0.2
## 40            1          5.1         3.4          1.5         0.2
## 41            1          5.0         3.5          1.3         0.3
```

```
## 42              1           4.5          2.3           1.3          0.3
## 43              1           4.4          3.2           1.3          0.2
## 44              1           5.0          3.5           1.6          0.6
## 45              1           5.1          3.8           1.9          0.4
## 46              1           4.8          3.0           1.4          0.3
## 47              1           5.1          3.8           1.6          0.2
## 48              1           4.6          3.2           1.4          0.2
## 49              1           5.3          3.7           1.5          0.2
## 50              1           5.0          3.3           1.4          0.2
## 51              1           7.0          3.2           4.7          1.4
## 52              1           6.4          3.2           4.5          1.5
## 53              1           6.9          3.1           4.9          1.5
## 54              1           5.5          2.3           4.0          1.3
## 55              1           6.5          2.8           4.6          1.5
## 56              1           5.7          2.8           4.5          1.3
## 57              1           6.3          3.3           4.7          1.6
## 58              1           4.9          2.4           3.3          1.0
## 59              1           6.6          2.9           4.6          1.3
## 60              1           5.2          2.7           3.9          1.4
## 61              1           5.0          2.0           3.5          1.0
## 62              1           5.9          3.0           4.2          1.5
## 63              1           6.0          2.2           4.0          1.0
## 64              1           6.1          2.9           4.7          1.4
## 65              1           5.6          2.9           3.6          1.3
## 66              1           6.7          3.1           4.4          1.4
## 67              1           5.6          3.0           4.5          1.5
## 68              1           5.8          2.7           4.1          1.0
## 69              1           6.2          2.2           4.5          1.5
## 70              1           5.6          2.5           3.9          1.1
## 71              1           5.9          3.2           4.8          1.8
## 72              1           6.1          2.8           4.0          1.3
## 73              1           6.3          2.5           4.9          1.5
## 74              1           6.1          2.8           4.7          1.2
## 75              1           6.4          2.9           4.3          1.3
## 76              1           6.6          3.0           4.4          1.4
## 77              1           6.8          2.8           4.8          1.4
## 78              1           6.7          3.0           5.0          1.7
## 79              1           6.0          2.9           4.5          1.5
## 80              1           5.7          2.6           3.5          1.0
## 81              1           5.5          2.4           3.8          1.1
## 82              1           5.5          2.4           3.7          1.0
## 83              1           5.8          2.7           3.9          1.2
## 84              1           6.0          2.7           5.1          1.6
## 85              1           5.4          3.0           4.5          1.5
## 86              1           6.0          3.4           4.5          1.6
## 87              1           6.7          3.1           4.7          1.5
## 88              1           6.3          2.3           4.4          1.3
## 89              1           5.6          3.0           4.1          1.3
## 90              1           5.5          2.5           4.0          1.3
## 91              1           5.5          2.6           4.4          1.2
## 92              1           6.1          3.0           4.6          1.4
## 93              1           5.8          2.6           4.0          1.2
## 94              1           5.0          2.3           3.3          1.0
## 95              1           5.6          2.7           4.2          1.3
```

```
## 96               1           5.7         3.0          4.2          1.2
## 97               1           5.7         2.9          4.2          1.3
## 98               1           6.2         2.9          4.3          1.3
## 99               1           5.1         2.5          3.0          1.1
## 100              1           5.7         2.8          4.1          1.3
## 101              1           6.3         3.3          6.0          2.5
## 102              1           5.8         2.7          5.1          1.9
## 103              1           7.1         3.0          5.9          2.1
## 104              1           6.3         2.9          5.6          1.8
## 105              1           6.5         3.0          5.8          2.2
## 106              1           7.6         3.0          6.6          2.1
## 107              1           4.9         2.5          4.5          1.7
## 108              1           7.3         2.9          6.3          1.8
## 109              1           6.7         2.5          5.8          1.8
## 110              1           7.2         3.6          6.1          2.5
## 111              1           6.5         3.2          5.1          2.0
## 112              1           6.4         2.7          5.3          1.9
## 113              1           6.8         3.0          5.5          2.1
## 114              1           5.7         2.5          5.0          2.0
## 115              1           5.8         2.8          5.1          2.4
## 116              1           6.4         3.2          5.3          2.3
## 117              1           6.5         3.0          5.5          1.8
## 118              1           7.7         3.8          6.7          2.2
## 119              1           7.7         2.6          6.9          2.3
## 120              1           6.0         2.2          5.0          1.5
## 121              1           6.9         3.2          5.7          2.3
## 122              1           5.6         2.8          4.9          2.0
## 123              1           7.7         2.8          6.7          2.0
## 124              1           6.3         2.7          4.9          1.8
## 125              1           6.7         3.3          5.7          2.1
## 126              1           7.2         3.2          6.0          1.8
## 127              1           6.2         2.8          4.8          1.8
## 128              1           6.1         3.0          4.9          1.8
## 129              1           6.4         2.8          5.6          2.1
## 130              1           7.2         3.0          5.8          1.6
## 131              1           7.4         2.8          6.1          1.9
## 132              1           7.9         3.8          6.4          2.0
## 133              1           6.4         2.8          5.6          2.2
## 134              1           6.3         2.8          5.1          1.5
## 135              1           6.1         2.6          5.6          1.4
## 136              1           7.7         3.0          6.1          2.3
## 137              1           6.3         3.4          5.6          2.4
## 138              1           6.4         3.1          5.5          1.8
## 139              1           6.0         3.0          4.8          1.8
## 140              1           6.9         3.1          5.4          2.1
## 141              1           6.7         3.1          5.6          2.4
## 142              1           6.9         3.1          5.1          2.3
## 143              1           5.8         2.7          5.1          1.9
## 144              1           6.8         3.2          5.9          2.3
## 145              1           6.7         3.3          5.7          2.5
## 146              1           6.7         3.0          5.2          2.3
## 147              1           6.3         2.5          5.0          1.9
## 148              1           6.5         3.0          5.2          2.0
## 149              1           6.2         3.4          5.4          2.3
```

```
## 150            1           5.9          3.0          5.1          1.8
##     Speciesversicolor Speciesvirginica
## 1                   0                0
## 2                   0                0
## 3                   0                0
## 4                   0                0
## 5                   0                0
## 6                   0                0
## 7                   0                0
## 8                   0                0
## 9                   0                0
## 10                  0                0
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## 12                  0                0
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## 14                  0                0
## 15                  0                0
## 16                  0                0
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## 47                  0                0
## 48                  0                0
## 49                  0                0
## 50                  0                0
## 51                  1                0
## 52                  1                0
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## 53                   1                 0
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## 104                  0                 1
## 105                  0                 1
## 106                  0                 1
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## 107                0                1
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## 143                0                1
## 144                0                1
## 145                0                1
## 146                0                1
## 147                0                1
## 148                0                1
## 149                0                1
## 150                0                1
## attr(,"assign")
## [1] 0 1 2 3 4 5 5
## attr(,"contrasts")
## attr(,"contrasts")$Species
## [1] "contr.treatment"
```