$y \subseteq R$

$p = 1$

$w_0 + w_1 x_1$

$\mathcal{H} = \{ \vec{w} \cdot \vec{x} : \vec{w} \in \mathbb{R}^{p+1} \}$ all linear model

$\vec{X} = [\vec{1} \ \ \vec{x}]$

ORDINARY least Squares (OLS) regression

$$b_0, b_1 = \underset{\vec{w} \in R^2}{\arg\min} \overbrace{\left\{ \sum_{i=1}^{n} (y_i - \vec{w} \cdot \vec{x}_i)^2 \right\}}^{\text{SSE} \to \text{sum of square error}}$$

(P+1) Here 2

using Calculus

$b_0 = \bar{y} - b_1 \bar{x}$

$b_1 = \dfrac{\sum x_i y_i - n \bar{x} \bar{y}}{\sum x_i^2 - n \bar{x}^2}$

Var (X) is estimated by S_x^2

$$S_x^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$$

$$= \frac{1}{n-1} \sum_{i=1}^{n} x_i^2 - 2\bar{x}x_i + \bar{x}^2)$$

$$= \frac{1}{n-1} \left(\sum x_i^2 - 2n\bar{x}^2 + n\bar{x}^2 \right)$$

$$\Rightarrow \frac{1}{n-1} \left(\sum x_i - n\bar{X}^2 \right)$$

For 2 r.v's : X, Y

Correlation : $\text{Corr}[X,Y] = \frac{\text{Cov}[X,Y]}{\sqrt{\text{Var}[X]\,\text{Var}[Y]}}$

Covariance : $\text{Cov}[X,Y] = E[(X-\mu_x)(Y-\mu_y)]$

$Cov[X, Y]$ estimate by S_{XY}

$$S_{XY} = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})$$

$$= \frac{1}{n-1} \sum_{i=1}^{n} \left(X_i Y_i - \bar{X} Y_i - X_i \bar{Y} + \bar{X}\bar{Y} \right)$$

$$= \frac{1}{n-1} \left(\sum_{i=1}^{n} X_i Y_i - n\bar{X}\bar{Y} - \cancel{n\bar{X}\bar{Y}} + \cancel{n\bar{X}\bar{Y}} \right)$$

$$= \frac{1}{n-1} \left(\sum_{i=1}^{n} X_i Y_i - n\bar{X}\bar{Y} \right)$$

For 2 r.v's X, Y

Corr[X, Y] is estimated by

$$r := \frac{S_{xy}}{\sqrt{S_x^2 S_y^2}} = \frac{S_{xy}}{S_x S_y}$$

Using Calculator

$$b_0 = \bar{Y} - b_1 \bar{X} = \bar{Y} - r \frac{S_y}{S_x} \bar{X}$$

$$b_1 = \frac{\sum x_i y_i - n \bar{x} \bar{y}}{\sum x_i^2 - n \bar{x}^2}$$

$$= \frac{(n-1) S_{xy}}{(n-1) S_x^2} = \frac{S_{xy}}{S_x^2} = \frac{r S_x S_y}{S_x^2}$$

$$= r \frac{S_y}{S_x}$$

2) $g(x) = b_0 + b_1 \bar{X}$ (How to Predict)