Non-linear models fit w/ OLS

Two best transformations: Squares & Logarithms
we will use logs.

$$\hat{y} = b_0 + b_1 \ln(x)$$

$$X = \begin{bmatrix} 1 & x_{11} \\ 1 & x_{12} \\ \vdots & \vdots \\ 1 & x_{1n} \end{bmatrix} \longrightarrow X_* = \begin{bmatrix} 1 & \ln(x_{11}) \\ 1 & \ln(x_{12}) \\ \vdots & \vdots \\ 1 & \ln(x_{1n}) \end{bmatrix}$$

$$\ln(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + \dots - \dots$$

for x small, $\ln(x+1) \approx x$

$\ln(x) = \ln\big((x+1)-1\big) \approx x - 1$ for x near 1.

$$b_1 \Delta \ln(x) = b_1\big(\ln(x_f) - \ln(x_o)\big)$$

$$= b_1 \ln\left(\frac{x_f}{x_o}\right)$$

$$\approx b_1\left(\frac{x_f}{x_o} - 1\right) \quad \text{if } \frac{x_f}{x_o} \approx 1$$

$$= b_1\left(\frac{x_f - x_o}{x_o}\right)$$

proportional Δ

This means if $x\uparrow$ by 25%, then $\hat{y}\uparrow$ by $0.25 b_1$

- Another transformation (Log-Linear Model)

$$\ln(\hat{y}) = b_0 + b_1 x_1 + b_2 x_2 + \ldots + b_p x_p$$

$$\Rightarrow \hat{y} = \exp(b_0 + b_1 x_1 + b_2 x_2 + \ldots + b_p x_p)$$

$$= \exp(b_0)\exp(b_1 x_1) \cdots \exp(b_p x_p)$$

$$= e^{b_0}(e^{b_1})^{x_1} \cdots (e^{b_p})^{x_p}$$

$$\Rightarrow \hat{y} = m_0 m_1^{x_1} \cdots m_p^{x_p}$$

derived feature

$$- \quad X = \begin{bmatrix} 1 & x_{11} & x_{21} \\ 1 & x_{12} & x_{22} \\ \vdots & \vdots & \vdots \\ 1 & x_{1n} & x_{2n} \end{bmatrix} \longrightarrow X = \begin{bmatrix} 1 & x_{11} & x_{21} & x_{11}x_{21} \\ 1 & x_{12} & x_{22} & x_{12}x_{22} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{1n} & x_{2n} & x_{1n}x_{2n} \end{bmatrix}$$

first order interaction

$$g(\vec{x}) = \hat{y} = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_1 x_2$$

$$= b_0 + (b_1 + b_3 x_2) x_1 + b_2 x_2$$

$$= b_0 + b_1 x_1 + (b_2 + b_3 x_1) x_2$$

$p_{raw} = 2 \Rightarrow p = 3$

$f: \mathbb{R}^2 \rightarrow \mathbb{R} = y$

"non-linear linear model fit w/ OLS"