Lecture 10

02.27.2020

Review for Lab

$A = V \Lambda V^{-1}$ $\begin{bmatrix} \lambda & & 0 \\ & \ddots & \\ 0 & & \lambda \end{bmatrix}$ diagnolization

$V = \begin{bmatrix} \vec{v}_1 \cdots \vec{v}_d \end{bmatrix}$

$g_0(\vec{x}) = \bar{y} = b_0 \quad \forall \vec{x}$ Is this a linear model? Yes

$AV = \Lambda V^{-1}$

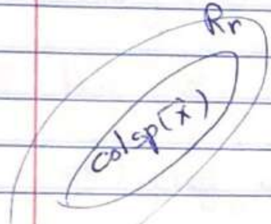
$A\vec{v}_1 = \lambda \vec{v}_1$

$\vec{\hat{y}} = \bar{y}\vec{1}_n = H\vec{y}$

Projection Matrix.

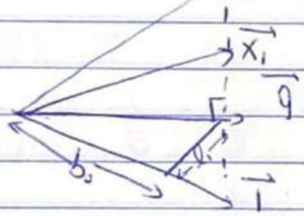
$$H = X(X^T X)^{-1} X^T = \vec{1}(\vec{1}^T \vec{1})\vec{1}^T = \vec{1} n^{-1} \vec{1}^T = \frac{1}{n}\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} = \begin{bmatrix} 1/n & 1/n & \cdots & 1/n \\ \vdots & & & \\ 1/n & \cdots & \cdots & 1/n \end{bmatrix}_{n}$$

Rank = 1



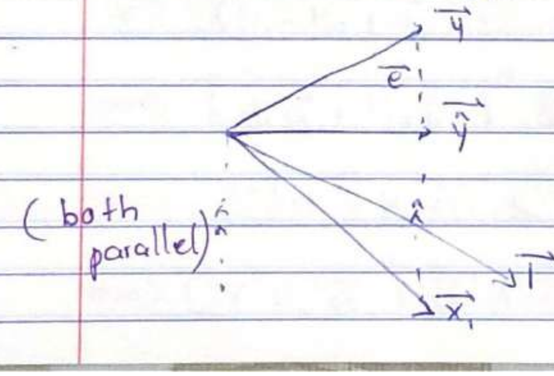
$H\vec{x_i} = \vec{x_j}$

$R^n = \text{colsp}(X) \oplus \text{colsp}(X_0)$



$\text{colsp}[[\vec{1} : \vec{x_1}]]$
$\hat{y} \in$

Imagine $\vec{x_1}'$ s.t. $\text{colsp}[[\vec{1} : \vec{x_1}]] = \text{colsp}[[\vec{1} \mid \vec{x_1}']]$



{column space ⟺ Rank}

$V = [\vec{v}_1 \mid \vec{v}_2]$

$V = \mathrm{Span}[\{\vec{v}_1, \vec{v}_2\}] = \mathrm{colsp}[V]$

$$\mathrm{proj}_V(\vec{a}) \overset{?}{=} \mathrm{proj}_{\vec{v}_1}(\vec{a}) + \mathrm{proj}_{\vec{v}_2}(\vec{a})$$

$$\| \qquad\qquad\qquad \|$$

$$H\vec{a} \qquad\qquad H_1\vec{a} + H_2\vec{a}$$

$$\|$$

$$c_1\vec{v}_1 + c_2\vec{v}_2 \qquad \text{When } \vec{v}_1 \perp \vec{v}_2 \quad \text{OH NO!}$$