A stock-flow consistent model of the Portuguese economy in continuous time

Contents

1 - Introduction

This paper describes a stock-flow consistent model of the Portuguese Economy in continuous time based on National Accounts data. The model uses the same sectoral structure of National Accounts, and its balance sheets and transactions are defined from these as well. The model being presented here tries to depict economic reality in terms of its monetary flows, circumscribing itself to transactions that actually take place and in the form in which they take place. To achieve this, in addition to the expected simplifications done to National Accounts data, some specific modifications are introduced, such as the removal of imputed rents, the consideration of interest payments that actually take place (before FISIM allocation) and the decomposition of government consumption

Section 2 deals with the model's balance sheets and how these are constructed from the National Accounts. As an example, balance sheets for the starting point (end of 2012) of the simulations that will be carried out later are derived.

Section 3 describes and justifies the transactions included in the model and how these compare with National Accounts data. It begins by looking into how GDP's expenditure and income approach components are included in the model, and then it compares each sector's full sequence of accounts with the simplified version of the model.

In section 4 the behavioural equations of the model are presented. First the exogenous variables of the model are presented, with a special focus on an important one, the Portugal Spread, a variable that tries to capture general economic sentiment and credit market conditions. After that we describe how interest rates – floating and fixed - and its dynamics are defined in the model. Finally, for each sector, we present the full sequence of accounts and explain all behavioural equations of the model.

2 – Model Sectors and Balance Sheets

Adopting the structure used in the European System of Accounts (ESA), the model contains five sectors: Households (which includes Non Profit Institutions Serving Households - NPISH), the Government, the Non-financial Sector, the Financial Sector and the Rest of the World.

Regarding the sectors' balance sheets, an important simplification of the model is that the Financial Sector is the counterpart of every financial instrument of the remaining sectors, with the exception of Government Debt owed to the Rest of the World. This can be seen in the balance sheet matrix in table 1 which shows the assets (with a plus sign and on the left) and liabilities (with a minus sign and on the right) of each sector.

	Households	NFS	Government	FS	RoW	Σ
Currency and Transferable Deposits	+CurrTD			-CurrTD		0
Other Assets	+OtherAssets			-OtherAssets		0
Housing Loans	-LoansH			+LoansH		0
Consumption and Other Loans	-LoansCO			+LoansCO		0
Current Account		+CurrAct		-CurrAct		0
Capital Account		+CapAct		-CapAct		0
NFS Loans		-LoansNFS		+LoansNFS		0
Government Deposits			+GovDep	-GovDep		0
Government Debt			-GovDebt$_{Dom}$	+GovDebt$_{Dom}$		0
Government Debt			-GovDebt$_{RoW}$		+GovDebt$_{RoW}$	0
Private External Debt				-PrvtExtDebt	+PrvtExtDebt	0
External Assets				+ExtAssets	-ExtAssets	0
Reserves				+Reserves	-Reserves	0
Net Worth	-HH$_{NW}$	-NFS$_{NW}$	-Gov$_{NW}$	-FS$_{NW}$	-RoW$_{NW}$	0
Σ	0	0	0	0	0	

Table 1 - Balance sheet matrix

We can see that Households have two assets, Currency and Transferable Deposits, on which it receives no interest, and Other Assets on which it does. On the liability side it has two types of Loans, Housing Loans, and Consumption and Other Loans.

The Non-financial Sector has two assets, a Current Account and a Capital Account. This is necessary so that investment (gross capital formation), like every other flow, has an origin and a destination, a defining characteristic of stock-flow consistent models. The Current Account is where all final demand components are paid into, and where these are distributed into taxes (less subsidies) on production, imports and income as will be explained in more detail in section 3.1. The Current Account is always equal to zero, with gross operating surplus of the Non-financial Sector – residually defined after final demand being distributed into other income components – being immediately transferred into the Capital Account. The Non-financial Sector has only one liability, Loans.

Government has one asset, Government Deposits, which is always equal to zero since the difference between Government expenses and receipts is immediately reflected in the levels of Government Debt. Government Debt is split into two in the model, the first being government debt domestically owned and the second owed to Rest of the World.

The Rest of the World sector has two assets, Government Debt and Private External Debt, and two liabilities External Assets and Reserves. Reserves are instrumental in defining external debt levels. All transactions with the Rest of the World are settled with Reserves, and there is a target level for Reserves which is achieved via an increase or decrease in Private External Debt.

The Financial Sector's balance sheet is defined residually as a counterpart of every financial instrument of the remaining sectors, except for Government Debt owed to the Rest of the World. So this means it has six different assets, and six different liabilities, although two of these liabilities are always equal to zero as mentioned before.

2.1 - From the National Accounts' Balance Sheet to the model's Balance Sheet

In the ESA, the balance sheet of a given sector is altered when a transaction takes place or when the value of a financial instrument changes. A significant limitation of the model at this stage is the absence of revaluations of balance sheet items, so all changes of each sector's assets and liabilities are determined by transactions, which can happen between different sectors or between different financial instruments of the same sector. This means that the trajectory of each sector's net worth, in the model, is given by its' net lending/borrowing position which can be very different of the evolution observed in the financial accounts and so it's impossible for the model to capture the actual development of each sector's net worth. In defining the model's balance sheets we use the National Accounts' balance sheets to determine the model's initial position, but after that they will inevitably diverge.

According to the ESA, Financial Assets and Liabilities are divided into 8 types of instruments:

- F1 - Monetary Gold and SDR
- **F2 - Currency and Deposits**
    - **F21 – Currency**
    - **F22 – Transferable Deposits**
    - **F29 – Other Deposits**
- **F3 - Debt Securities**
- **F4 - Loans**
- F5 - Equity and investment fund shares or units
- **F6 - Insurance, Pensions and Standardized Guarantee Schemes**
- F7 - Financial Derivatives and Employee Stock Options
- F8 - Other Accounts Receivable/Payable

In defining each sector's initial balance sheet in the model we consider F2 - Currency and Deposits, F3-Debt Securities, F4-Loans for all sectors and F6-Insurance, Pensions and SGS for the Households sector as well.

Regarding the financial instruments not considered in the model, F1-Monetary Gold and SDR and F7 – Financial Derivatives and Employee Stock Options are almost insignificant in the case of Portugal and so can be safely ignored. F8 – Other Accounts Receivable/Payable are financial assets and liabilities created as counterparts to transactions where there is a timing difference between these transactions and the

corresponding payments which happens due to the accrual recording used in the ESA as opposed to cash recording. This makes little sense to include in the model and, despite being significant in gross terms, it's much smaller when we consider the net position of each sector which means that for every sector the amount owed and due practically cancel each other out. Finally, F5 – Equity and investment fund shares or units is the most problematic financial instrument to exclude because of the significant weight it has on some sector's balance sheet. The main reason for not including this financial instrument is the fact that changes in holdings of equity are mostly due to revaluations which are not a feature of the model at the moment. This is one of the many aspects of the model to look into in the future.

We'll now take a look at how National Accounts balance sheets compare with the model's initial balance sheet, using as an example the starting point of the simulations to be carried out in the second part of this paper, which begin in the end of 2012.

2.1.1 – Households

Assets –> 352 454	Liabilities –> 171 232
F21 – Currency -> 3 063	F4 – Loans -> 152 244
F22 – Transferable Deposits -> 35 728	F7 – Derivatives -> -4
F29 – Other Deposits -> 113 844	F8 – Other -> 18 992
F3 – Debt Securities -> 24 284	
F4 – Loans -> 18 831	
F5 – Equity -> 78 650	
F6 – Ins. Pensions and SGS -> 64 666	
F7 – Derivatives -> 5	
F8 – Other -> 13 384	Net Worth ->181 222

Assets -> 260 415	Liabilities – 152 244
Currency and TD (F21+F22) -> 38 791	Housing Loans (F4)->118 867
Other Assets (F29+F3+F4+F6) -> 221 624	Consumption and Other Loans (F4) -> 33 377
	Net Worth -> 108 171

Table 2 - National Accounts balance sheet (top) and model's balance sheet (bottom)

In the National Accounts, at the end of 2012, Households' assets amounted to € 352 454 million and its liabilities to € 171 232 million which meant a net worth of € 181 222 million. On the asset side of the model we have Currency and Transferable Deposits which, as the name implies, is given by F21 – Currency and F22 – Transferable Deposits, and we have Other Assets which results in the aggregation of F3 – Debt Securities, F4 – Loans and F6 – Insurance, Pensions and SGS. In the case of liabilities, F4 – Loans are split into Housing Loans and Consumption and Other Loans, based on data from the Bank of Portugal. The result of this simplification is assets totalling € 260 415 million and liabilities € 152 244 million, which means a net worth of € 108 171 million.

2.1.2 – Non-financial Sector

Assets –> 166 609	Liabilities –> 403 096
F2 – Currency and Deposits -> 30 130	**F3 – Debt Securities -> 35 086**
F3 – Debt Securities -> 2 380	**F4 – Loans -> 166 892**
F4 – Loans -> 28 996	F5 – Equity -> 141 259
F5 – Equity -> 37 222	F6 – Ins. Pensions and SGS -> 3 916
F6 – Ins. Pensions and SGS -> 2 754	F7 – Derivatives -> 2 374
F7 – Derivatives -> 18	F8 – Other -> 53 568
F8 – Other -> 65 109	Net Worth -> -236 486

Assets -> 30 130	Liabilities – 170 603
Current Account -> 0	Loans (F3n+F4n)-> 170 603
Capital Account (F2) -> 30 130	Net Worth -> -140 473

Table 3- National Accounts balance sheet (top) and model's balance sheet (bottom)

Non-financial Sector's balance sheet, at the end of 2012, had € 166 609 million in assets and € 403 096 in liabilities, meaning a net worth of € -236 486 million. In the model, the Capital Account corresponds to F2 – Currency and Deposits, while Loans include F3 – Debt Securities and F4 – Loans, on a net basis. The model's balance sheet is much smaller when compared to the financial accounts, with assets totalling € 30 130 million and liabilities € 170 603 million for a net worth of € -140 473 million.

2.1.3 – Government

Assets –> 81 482	Liabilities –> 234 045
F2 – Currency and Deposits -> 20 145	**F2 – Currency and Deposits -> 13 688**
F3 – Debt Securities -> 9 385	**F3 – Debt Securities -> 111 491**
F4 – Loans -> 6 569	**F4 – Loans -> 87 311**
F5 – Equity -> 33 071	F5 – Equity -> 1 964
F6 – Ins. Pensions and SGS -> 19	F7 – Derivatives -> 1 200
F7 – Derivatives -> 397	F8 – Other -> 18 391
F8 – Other -> 11 895	Net Worth -> -152 563

Assets -> 0	Liabilities – 176 391
Government Deposits -> 0	Government Debt (F2n+F3n+F4n)-> 176 391
	- Owed to RoW -> 141 167
	- Owed to FS -> 35 224
	Net Worth -> -176 391

Table 4 - National Accounts balance sheet (top) and model's balance sheet (bottom)

By the end of 2012, Government assets totalled € 81 482 million and liabilities € 234 045 million, giving a net worth of € -152 563 million. In the model, Government Debt is obtained by aggregating F2 – Currency and Deposits, F3 – Debt Securities and F4 – Loans on a net basis, which amounts to € 176 391 million, with the division between domestic and foreign ownership being based on Bank of Portugal Data.

2.1.4 – Rest of the World

Assets –> 514 604	Liabilities –> 302 861
F1 – Gold and SDR -> 940	F1 – Gold and SDR -> 925
F2 – Currency and Deposits -> 171 868	**F2 – Currency and Deposits -> 92 499**
F3 – Debt Securities -> 105 458	**F3 – Debt Securities -> 88 066**
F4 – Loans -> 111 345	**F4 – Loans -> 31 751**
F5 – Equity -> 113 681	F5 – Equity -> 77 986
F6 – Ins. Pensions and SGS -> 203	F6 – Ins. Pensions and SGS -> 635
F7 – Derivatives -> -3	F7 – Derivatives -> -3 645
F8 – Other -> 11 112	F8 – Other -> 14 643
	Net Worth -> 211 744

Assets -> 388 671	Liabilities – 212 316
Government Debt -> 141 167	External Assets(F2+F3+F4-Reserves)->209 616
Private External Debt (F2+F3+F4-GovDebt) -> 247 504	Reserves -> 2 700
	Net Worth -> 176 355

Table 5 - National Accounts balance sheet (top) and model's balance sheet (bottom)

The Rest of the World's balance sheet, at the end of 2012, amounted to € 514 604 million on the asset side and to € 302 861 million on the liability side, for a net worth of € 211 744 million. In the model's assets we have the previously defined Government Debt and Private External Debt which is simply the aggregation of F2 – Currency and Deposits, F3 – Debt Securities and F4 – Loans minus Government Debt owed to non-residents. The liabilities of the Rest of the World sector in the model are Reserves (explained when describing the Financial Sector's balance sheet) and External Assets, which are given by F2 – Currency and Deposits, F3 – Debt Securities and F4 – Loans minus Reserves. This amounts to € 388 671 million in assets and € 212 316 million in liabilities for a net worth of € 176 355 million.

2.1.5 – Financial Sector

Assets –> 620 853	Liabilities –> 609 260
F1 – Gold and SDR -> 16 434	F1 – Gold and SDR -> 940
F2 – Currency and Deposits -> 83 044	**F2 – Currency and Deposits -> 351 635**
F3 – Debt Securities -> 134 574	**F3 – Debt Securities -> 41 436**
F4 – Loans -> 291 914	**F4 – Loans -> 19 456**
F5 – Equity -> 60 709	F5 – Equity -> 102 124
F6 – Ins. Pensions and SGS -> 518	**F6 – Ins. Pensions and SGS -> 63 610**
F7 – Derivatives -> -479	F7 – Derivatives -> 13
F8 – Other -> 34 139	F8 – Other -> 30 046
	Net Worth -> 11 593

Assets -> 570 386	Liabilities – 538 049
Loans to HH -> 152 244	Currency and TD -> 38 791
Loans to NFS -> 170 603	Other Assets -> 221 624
Government Debt -> 35 224	Capital Account -> 30 130
External Assets -> 209 616	Private External Debt -> 247 504
Reserves -> 2 700	Net Worth -> 32 337

Table 6 - National Accounts balance sheet (top) and model's balance sheet (bottom)

The National Accounts balance sheets show us that by the end of 2012 the Financial Sector had € 620 853 million in assets and € 609 260 in liabilities, which means a net worth of € 11 593 million. Has stated before, in the model, the Financial Sector's balance sheet is defined residually, as the counterpart of other sectors assets and liabilities which results in assets amounting to € 570 386 million and liabilities totalling € 538 049 million for a net worth of € 32 337 million. Reserves are simply defined as a percentage (0.005%) of total liabilities of the Financial Sector. The chosen percentage comes from the ECB actual minimum reserve requirements, which are 1% for liabilities with a maturity of up to 2 years and 0% for higher maturity liabilities, with liabilities vis-à-vis other credit institutions subject to the Eurosystem's minimum reserve requirements, the ECB and euro area national central banks being excluded from the reserve base. In the model, the level of Reserves adjusts (rapidly) to the minimum required level, which means that it doesn't accommodate the possibility of excess reserves.

3 – Model transactions and how they compare with National Accounts

The amount of flows featured in the National Accounts is enormous, and obviously impossible to include them all in a model. For a first look of the simplifications involved, in table 7 we can see the transactions included in the model - except for the purely financial transactions which we will look into when describing the model's behavioural equations.

	Households	Government	Non-Financial Sector		Financial Sector	RoW
			Current Account	Capital Account		
1 - Households Consumption	-Cons		+Cons			
2 - Government Consumption		-OthConsGov	+OthConsGov			
3 - Households GCF	-GCF$_{HH}$		+GCF$_{HH}$			
4 - Government GCF		-GCF$_{Gov}$	+GCF$_{Gov}$			
5 - NFS GCF			+GCF$_{NFS}$	-GCF$_{NFS}$		
6 - Exports			+Exports			-Exports
7 - Output=1+2+3+4+5=8+9+10+11			Output			
8 - Imports			-Imports			+Imports
9 - Taxes on Production		+TaxProd	-TaxProd			
10 - Private Wages and Mixed Income	+WagesMl		-WagesMl			
11 - Gross operating surplus of NFS			-GOS$_{NFS}$	+GOS$_{NFS}$		
12 - Government Wages	+WagesGov	-WagesGov				
13 - Taxes on Income (and Social Contributions for HH)	-TaxIncSC	+TaxInc_SC +TaxInc$_{NFS}$ +TaxInc$_{FS}$		-TaxInc$_{NFS}$	-TaxInc$_{FS}$	
14 - Dividends	+Div$_{HH}$			-Div$_{HH}$ -Div$_{RoW}$		+Div$_{RoW}$
15 - Interest	+IntDep$_{HH}$ -IntLoansCO$_{HH}$ -IntLoansH$_{HH}$	-IntGovDebt$_{Dom}$ -IntGovDebt$_{RoW}$		-IntLoans$_{NFS}$	+IntLoans$_{NFS}$ -IntDep$_{HH}$ +IntLoansCO$_{HH}$ +IntLoansH$_{HH}$ +IntGovDebt$_{Dom}$ -IntPrvtExtDebt +IntExtAss	+IntGovDebt$_{RoW}$ +IntPrvtExtDebt -IntExtAss
16 - Social Transfers	+SocTrsf	-SocTrsf				
17 - Current Transfers	+CurTrsf					-CurTrsf
18 - Acquisitions less disposals of NPNFA	+Acq-DofNPNFA	-Acq-DofNPNFA				
19 - Income atributted to insurance policy holders	+InsIncome				-InsIncome	

Table 7- Transactions flow matrix

The Current Account of the Non-financial Sector is where all output production takes place, where final demand components are paid into (lines 1 to 6) and from where these are distributed to every sector (lines 8 to 11). In trying to depict in the model what actually happens in the economy on a cash flow basis, these GDP components (on the expenditure and income approach) are slightly different than the ones found in the National Accounts.

We begin by looking at these differences in GDP components. After that we look at each sector full sequence of accounts, identifying which flows are included in the model and how this is done – they can be included without any change, they can be aggregated with other flows, or modified in some way(for instance interest payments in the model are before FISIM) - and which are endogenously and exogenously defined.

3.1 – GDP components

In Figure 1 we can see the average composition of the expenditure (top part) and income (bottom part) approaches to GDP for the 1995-2017 period. During this period, consumption expenditures amounted, on average, to 83.7% of GDP (64.5% of Households and NPISH and 19.2% of Government), gross capital formation to 22.2% (11.6% of the Non-financial Sector, 6.2% of Households, 3.8% of Government and 0.6% of the Financial Sector) and exports to 31.4% which adds up to a final demand of 137.3% of GDP, meaning that imports averaged 37.3%. On the income side, between 1995 and 2017, taxes less subsidies on production represented 12.2% of GDP, compensation of employees 46.6%, and gross operating surplus and mixed income 51.2% (18.1% going to Households and NPISH, 17.4% to NFS, 3.0% to the Financial Sector and 2.8% to the Government).

After the simplifications made to the model are explained we will compare this figure to an analogous one for the aggregates used in the model.

| Government Consumption 19.2% | Consumption HH+NPISH 64.5% | GCF HH 6.2% | GCF FS 0.6% | GCF Gov 3.8% | GCF NFS 11.6% |

| Consumption 83.7% | Gross Capital Formation 22.2% | Exports 31.4% |

Final Demand - 137.3%

| Compensation of employees 46.6% | GOS and Mixed Income 51.2% | Taxes less subsidies on production 12.2% | Imports 37.3% |

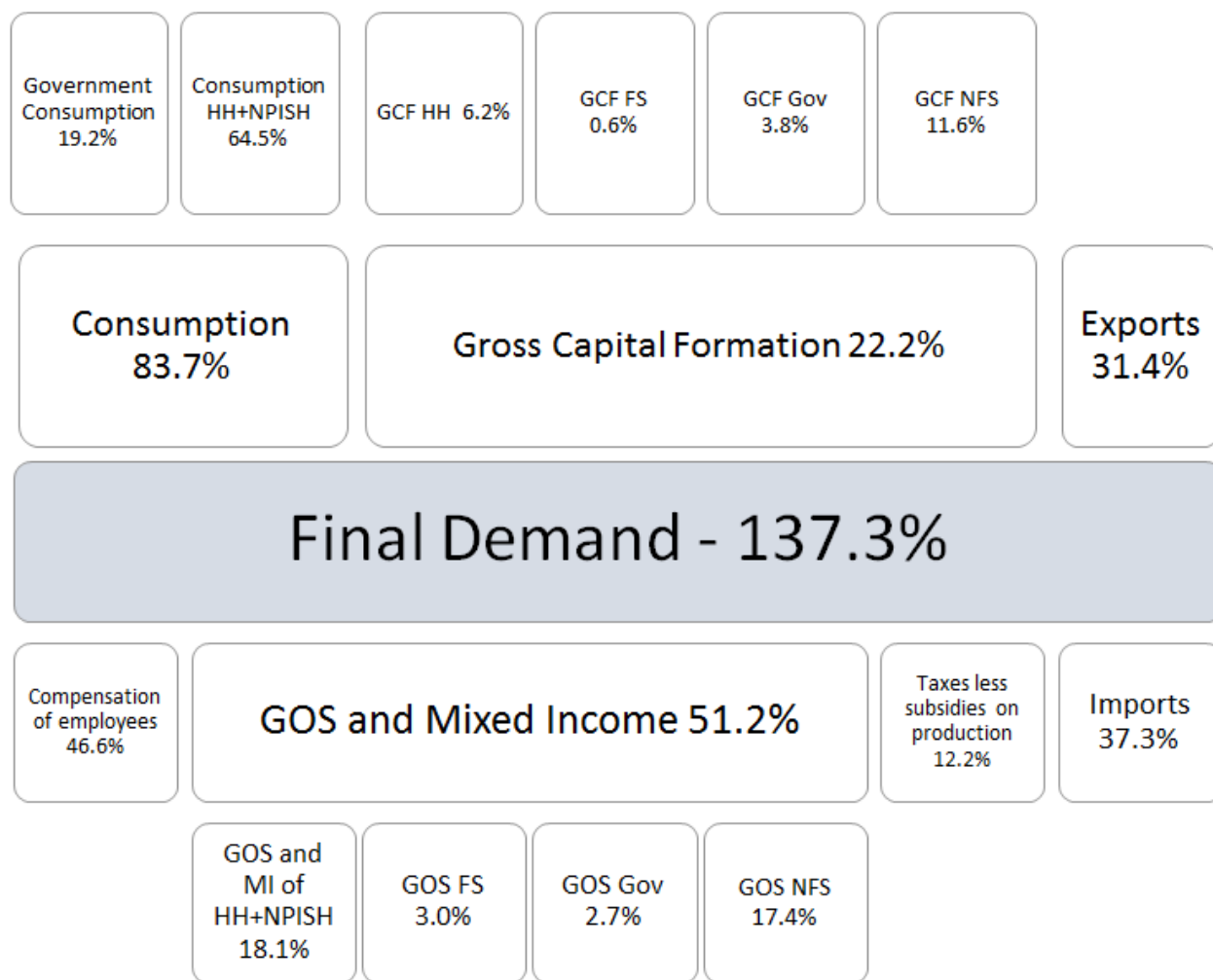| GOS and MI of HH+NPISH 18.1% | GOS FS 3.0% | GOS Gov 2.7% | GOS NFS 17.4% |

Figure 1- From final demand to income (average values 1995-2017 as % of GDP)

As stated before, we try to approximate the model to what really happens in the economy on a cash-flow basis. To achieve this we basically did three modifications regarding final demand components (and consequently the correspondent income that goes into each sector): deconstruct government consumption, remove imputed rents and consider the interest payments that actually take place (before FISIM). We will

now look in more detail into the components of GDP affected by these changes, government consumption and households' consumption.

3.1.1 – Government Consumption

Government consumption is mostly - because it also includes the expenditures on products supplied to households via market producers - an accounting convention created to counterbalance the non-market output of Government which is provided for free (P.132 - other non-market output in the ESA) to other sectors, with no monetary transactions involved. This Government output provided for free is the parcel of Government output without a counterpart on the uses side of any other sector (including the Government). The way non-market output is measured in the ESA, is by making it equal to the sum of its costs. In the case of Government this is done by considering the costs of producing all of its output and then deducting the parts that already have a counterpart on the uses side of some sector.

D.1 Compensation of employees 13.0%	P.2 Intermediate consumption 5.1%	D.29-D39 Other taxes less other subsidies on production -0.2%	P.51c Consumption of fixed capital 2.6%	B.2n Net operating surplus 0.2%

P.1 - Output 20.6%

P.11 - Market output 2.1%	P.12 - Output for own final use 0.7%	P.13 - Non-market output 17.7%

P.131 - Payments for non-market output 0.5%	P.132 - Other non-market output 17.2%	D.632 - Social transfers in kind via market producers 1.9%

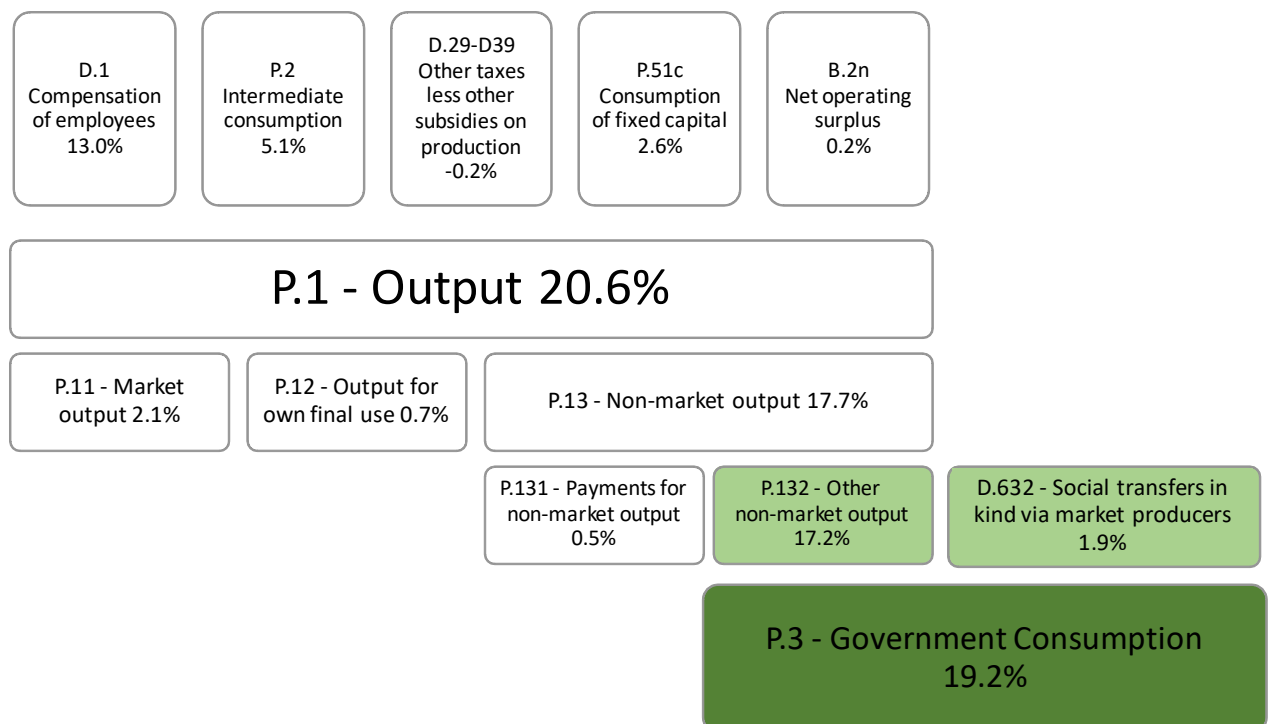**P.3 - Government Consumption 19.2%**

Figure 2 - Government Consumption in ESA

In figure 2 we can see how Government's output (P1) is derived as the sum of its production costs (top part), namely compensation of employees (D.1), intermediate consumption (P.2), other taxes less other subsidies on production (D.29-D.39), consumption of fixed capital (P.51c) and net operating surplus (B.2n). Output can then be divided into market output (P.11), output for own final use (P.12) – which in this case is capital formation and has a counterpart on capital formation of Government - and non-market output (P.13). Deducting from this last component the payments made on non-market output (P.131) we obtain other non-market output (P.132) and adding to this social transfers in kind via market producers (D.632) we finally get to Government consumption (P.3).

$$P.3 = P.132 + D.632$$

$$P.132 = P.1 - (P.11 + P.12 + P.131)$$

$$P.1 = D.1 + P.2 + D.29 - D.39 + P.51c + B.2n$$

We can see that a very small part of Government consumption actually involves an acquisition of goods and services from the productive sector. A very significant part of it consists in the payment of wages while another non negligible parcel refers to the consumption of fixed capital.
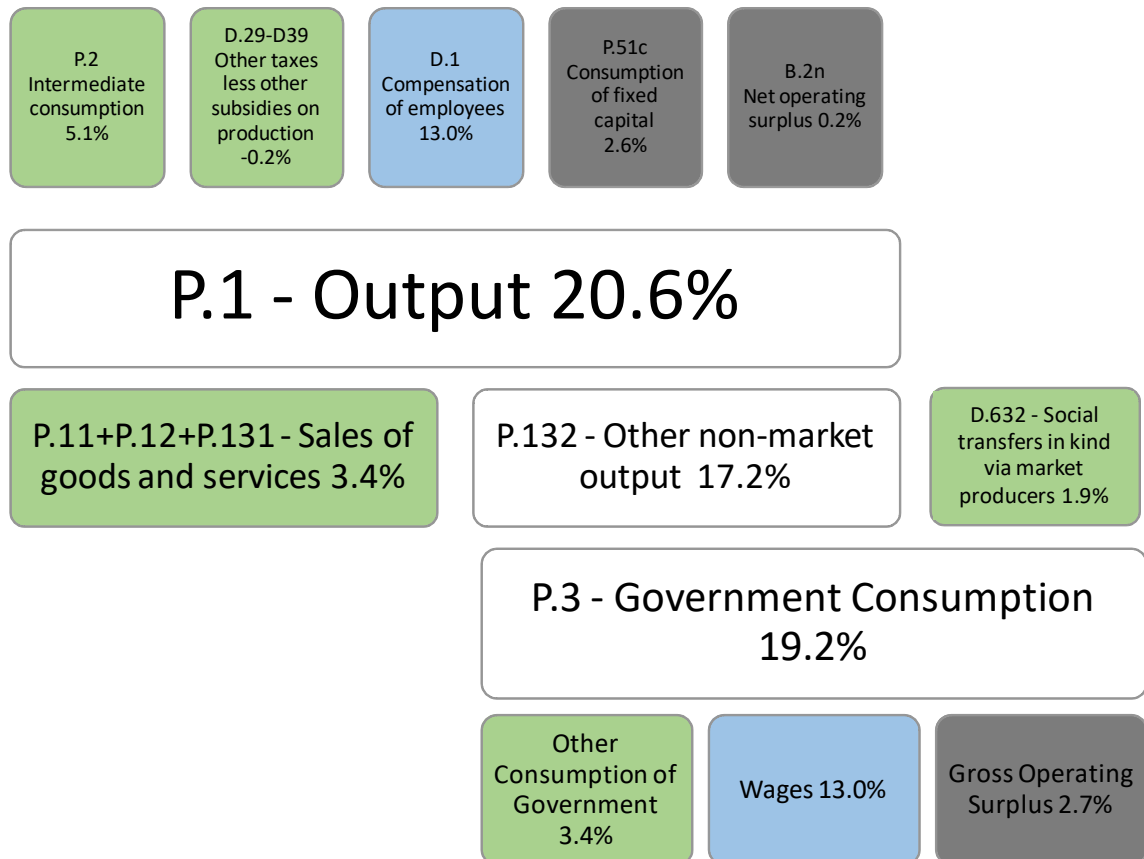

| P.2 Intermediate consumption 5.1% | D.29-D39 Other taxes less other subsidies on production -0.2% | D.1 Compensation of employees 13.0% | P.51c Consumption of fixed capital 2.6% | B.2n Net operating surplus 0.2% |

P.1 - Output 20.6%

| P.11+P.12+P.131 - Sales of goods and services 3.4% | P.132 - Other non-market output 17.2% | D.632 - Social transfers in kind via market producers 1.9% |

P.3 - Government Consumption 19.2%

| Other Consumption of Government 3.4% | Wages 13.0% | Gross Operating Surplus 2.7% |

Figure 3 - Government Consumption - from ESA to the model

To better replicate what happens in reality, in the model we decompose Government consumption into three components: wages, gross operating surplus and other consumption of government as shown in figure 3, where each of these three components is coloured with the same colour of the components that define them. Wages is simply equal to D.1 – compensation of employees and it's paid directly to Households. Gross operating surplus is given by the sum of P.51c - consumption of fixed capital and the B.2n - net operating surplus of Government and it's not included in the model. Consumption of fixed capital accounts for the loss in value of capital used by the Government, so while being relevant when considering the costs of production it makes no sense to include this in a model based on monetary flows. Net operating surplus, which refers to the profits of governmental units involved in market activities, it's a negligible amount and in principle it should be equal to zero (UNA 2014). Since this is in fact a residual revenue for the government sector it's logical to leave it out when transitioning from a production cost framework to a monetary flow perspective. Finally, other consumption of government is basically the net spending of Government on goods and services, and it's equal to D.632 – social transfers in kind via market producers plus P.2 – intermediate consumption plus D.29 other taxes on production paid by the government minus D.39 other subsidies on production received by the government minus P.11+P.12+P.131 – sales of goods and services. In the model

other consumption of government is an expense of the Government and a receipt of the Non-financial Sector.

$$Government\ Consumption = Wages + GOS + OthConsGov$$

$$Wages = D.1$$

$$GOS = P.51c + B.2n$$

$$OthConsGov = P.2 + (D.29 - D.39) - (P.11 + P.12 + P.131) + D.632$$

It should be noted that intermediate consumption incudes FISIM and in the spirit of the model this should not be included in other consumption of government. This option was considered, but not taken, because this is such a small amount (0.2% of GDP) and since the government projections we intend to analyze with this model don't have this discriminated from intermediate consumption we would have to come up with some estimate for this before using other consumption of government as an input of the model. In short, it would overcomplicate things for a very marginal gain.

3.1.2 – Consumption of households

Consumption of Households has two significant imputed components: imputed rents of homeowner-occupiers and FISIM which stands for Financial Intermediation Services Indirectly Measured.

Owner-occupiers rents is an imputed item that tries to capture the value a household derives from living in their own home. The rationale behind this imputation is to avoid changes in GDP due to a person decision to whether buy or lease a house and to avoid distortions in cross-country comparisons. It is included in Households' final consumption and in Households' output. Another consequence of this is the reclassification of expenditures made on the decoration, maintenance or repair of the house of the owner-occupier from consumption to intermediate consumption. The difference between imputed rents and this intermediate consumption (ignoring other taxes less subsidies on production) is gross operating surplus which in the case of Households refers exclusively to this activity. A simplified example of this (ignoring other taxes less subsidies on production) can be seen in table 8.

Uses	Resources
Final Consumption of Households -> 100 - of which maintenance of houses of owner-occupiers -> 5	

Uses	Resources
Final Consumption of Households -> 115 - of which imputed rents of owner-occupiers -> 20 Intermediate Consumption -> 5	Output from imputed rents -> 20

Uses	Resources
Final Consumption of Households -> 115 - of which imputed rents of owner-occupiers -> 20	Gross Operating Surplus -> 15

Table 8 - Imputed rents in ESA – an example in three steps

This is a purely imputed item with no transaction taking place, so it makes sense to exclude it from a model based on monetary flows.

FISIM is an attempt by National Accountants to measure the intermediation services provided by Financial Institutions. It's heavily based on the loanable funds theory and on the idea of the existence of an interest rate that would make lenders and borrowers happy in reaching a deal. This reference interest rate is used to calculate interest payments on loans and deposits in the ESA, which is denominated as 'ESA interest'. The difference between 'ESA interest' and interest payments actually made – 'bank interest' – is FISIM. So for deposits with banks, FISIM is equal to ESA interest less bank interest and for loans FISIM is equal to bank interest less ESA interest. FISIM is then allocated to intermediate consumption or to final consumption depending on the counterpart of the interest payment being made and on the type of financial instrument it pertains to. For instance, FISIM associated with interest payments Households receive on their deposits is allocated to their consumption while FISIM related to housing loans is considered intermediate consumption of owner-occupiers. The difference between ESA interest and bank interest for Households can be seen in figure 4.
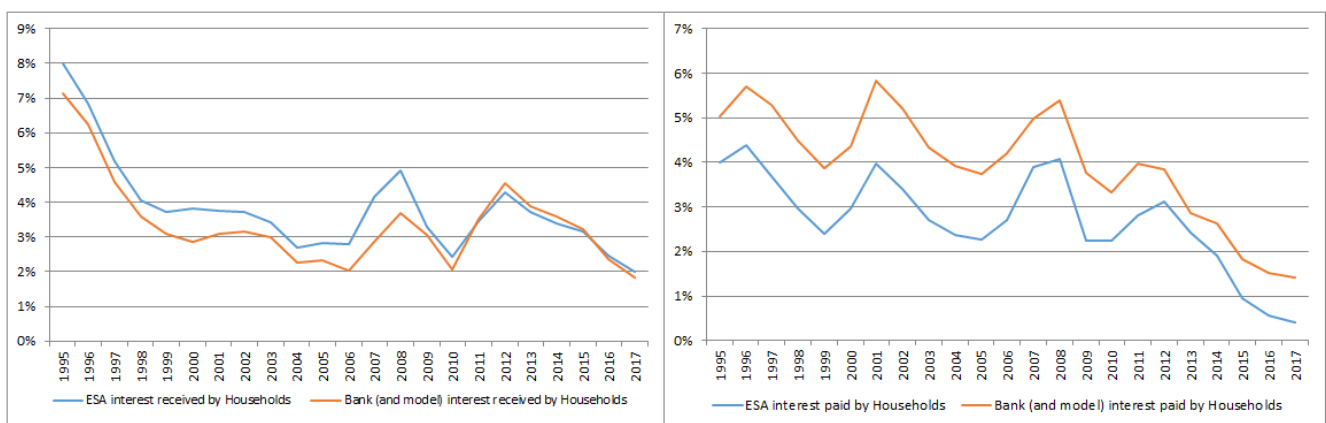

Figure 4 - Interest before and after FISIM as % of GDP

Due to data availability and the fact that FISIM is both a part of Households' consumption and of owner-occupier's intermediate consumption we deal with these two adjustments simultaneously. There are two issues regarding data availability: first, data on imputed rentals is available only when definitive annual accounts are published which happens a year and three quarters after the year is finished; second, from publicly available data we can only obtain total FISIM and not its partition into intermediate consumption and final consumption. Simply put, consumption of Households in the model is given by National Accounts consumption minus gross operating surplus of Households and minus total FISIM of Households.

As shown in figure 5, imputed rents represented, from 1995 to 2016, an average of 5.5% of GDP (in 2016 this reached 7.5%), while Households' consumption except imputed rents averaged 57.2%.
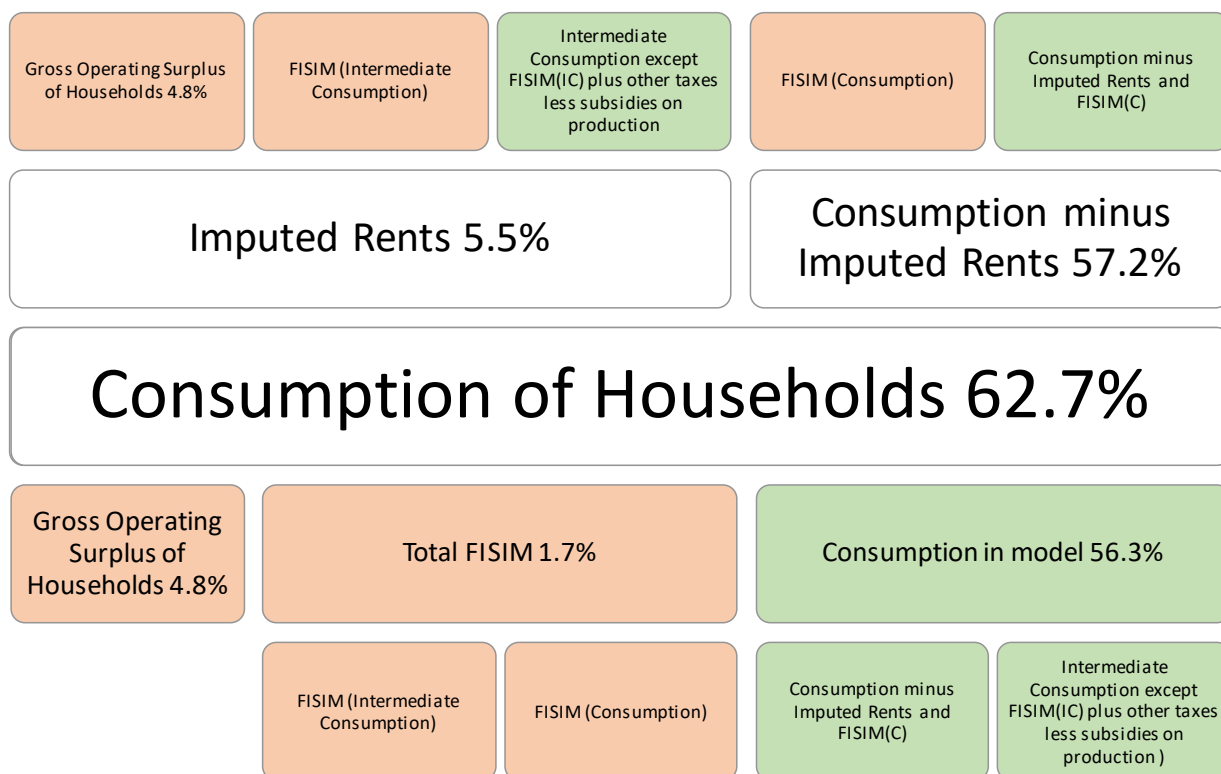
Gross Operating Surplus of Households 4.8%	FISIM (Intermediate Consumption)	Intermediate Consumption except FISIM(IC) plus other taxes less subsidies on production	FISIM (Consumption)	Consumption minus Imputed Rents and FISIM(C)
Imputed Rents 5.5%			Consumption minus Imputed Rents 57.2%	
Consumption of Households 62.7%				
Gross Operating Surplus of Households 4.8%	Total FISIM 1.7%		Consumption in model 56.3%	
	FISIM (Intermediate Consumption)	FISIM (Consumption)	Consumption minus Imputed Rents and FISIM(C)	Intermediate Consumption except FISIM(IC) plus other taxes less subsidies on production)

Figure 5 - Consumption of Households – components included (green) and excluded (red)

Imputed rentals can be partitioned into three components: 1 - gross operating surplus of households, 2 - FISIM allocated to households' intermediate consumption and 3 - intermediate consumption (except FISIM) of owner-occupiers plus other taxes less subsidies on production. The remaining part of consumption can be split into FISIM allocated to consumption and consumption except imputed rents and FISIM.

As stated before, by considering imputed rentals as part of consumption, expenditures made on the decoration, maintenance or repair of the house of the owner-occupier, that would otherwise be considered consumption, are now classified as intermediate consumption. These expenditures actually take place, and if we simply removed imputed rents from consumption, we would be eliminating these as well. So, looking at the five parts decomposition of households' consumption on the top part of figure 4, we can identify one component that doesn't involve an actual transaction, which is the excess of imputed rents over intermediate consumption – gross operating surplus – and two components (both allocations of FISIM) which are already being accounted for by considering in the model the interest payments that actually happen. By removing these parcels (GOS and FISIM) of National Accounts final consumption of households we get households' consumption in the model - which includes expenditures made on the decoration, maintenance or repair of the house of the owner-occupier. This obviously means that on the income side we will also eliminate gross operating surplus of households.

In fact, to be precise, what we eliminate from consumption (of households and NPISH) and from the income side is gross operating surplus of households and NPISH. In short, gross operating surplus of NPISH (a small amount representing on average 0.2% of GDP) is exactly equal to the consumption of fixed capital of NPISH, which is a production cost considered in the calculation of NPISH output (analogously to what happened for government output). This output of NPISH is counter-balanced on the expenditure side by allocating it to final consumption of households. Summing up, in the model, consumption of households and NPISH is given by National Accounts Consumption of households and NPISH minus total FISIM and minus gross operating surplus of households and NPISH.

3.1.3 – GDP components in the model

Figure 6 shows how GDP components look like in the model with the percentages still referring to the original GDP.
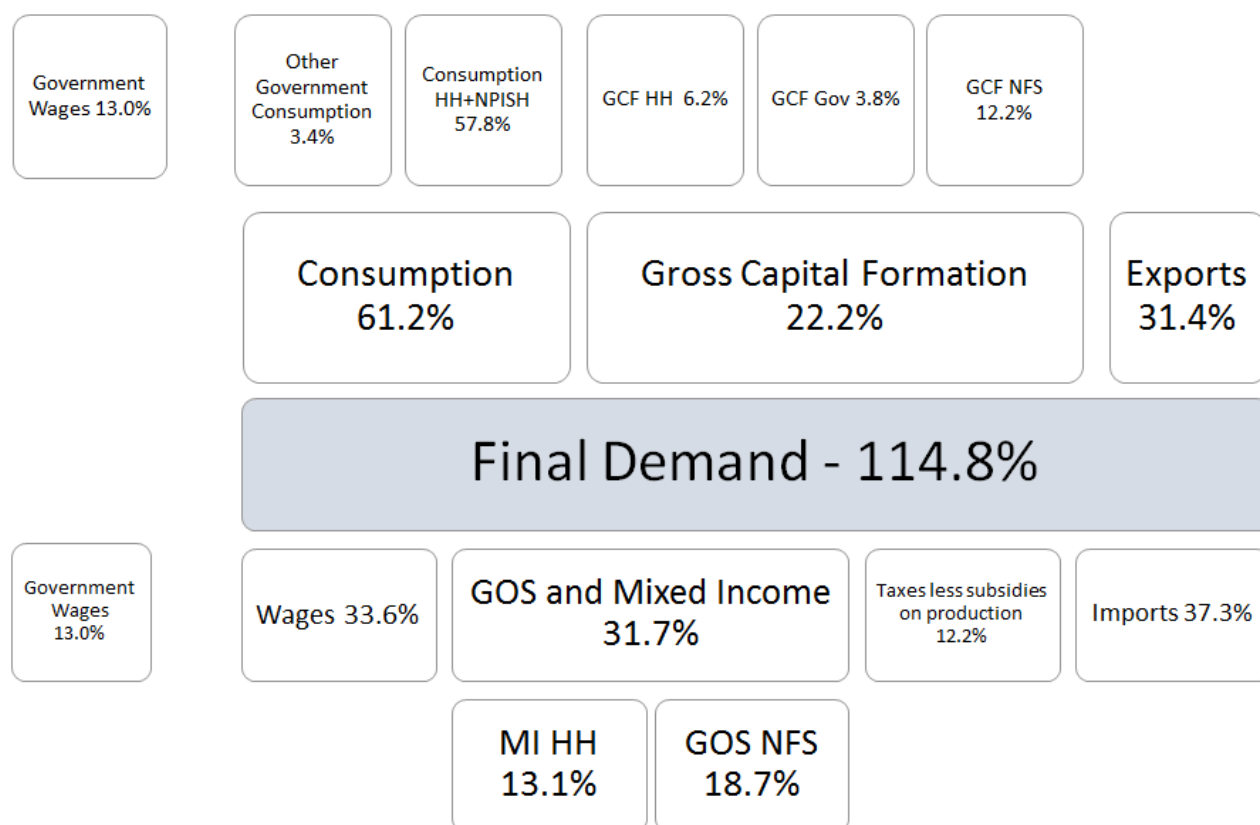
Figure 6 - From final demand to income in the model

The first thing to notice is that government wages is separated from the remaining final demand components. This happens because in the model government wages are not paid into the non-financial sector's current account, like the other components, where the process of decomposition of final demand into imports, taxes on production and income takes place.

The changes made to consumption of households and government, result in a lower level of final demand - 114.8% of GDP instead of 137.3%. Total gross capital formation is unchanged, but in the model the financial sector doesn't invest in real assets. This represents a small amount (0.6% of GDP) and it's included in the gross capital formation of the non-financial sector.

Regarding income distribution, the amount of wages to be paid out of final demand is reduced by the amount being paid by the government directly to households. Gross operating surplus of government and households has been removed, as explained before, and gross operating surplus of the non-financial sector now includes part of the gross operating surplus of the financial sector. To be precise, gross operating surplus of the non-financial sector, in the model, is equal to gross operating surplus of the non-financial and financial sectors minus FISIM paid by households. The higher level of gross operating surplus going to the non-financial sector (1.3% of GDP more), is partly compensated by the inclusion of gross capital formation of the financial sector in the non-financial sector's gross capital formation mentioned before. In spite of these

simplifications hurting the financial sector's net lending/borrowing position, in the model this sector comes out benefitted when compared to the National Accounts, maybe too much even. The overall effect of these simplifications on each sector's net lending/borrowing position can be seen in more detail in the following section.

3.2 – Full sequence of accounts for each sector

In this section we will compare the remainder of the sequence of accounts of each sector with what happens in the model. This begins where the previous section ended, with the balancing item of the generation of income account – Gross Operating Surplus and Mixed Income. For each sector there is a table comparing the National Accounts with the model, where each item is expressed as the average values as a percentage of GDP from 1995 to 2017. In each of these tables, items included in the model are coloured with the same colour of the National Accounts item they are based on.

3.2.1 – Households

We can see in table 9 how the variables included in the model compare with the National Accounts.

Households + NPISH					
National Accounts			Model		
Primary Income Account					
Uses		Resources		Uses	Resources
		B.2g - Operating surplus - 5.0%			
		B.3g Mixed income - 13.1%			Wages + Mixed income - 59.7%
		D.1 - Compensation of employees - 46.6%			
D.4.1 - Interest - 2.7%		D.4.1-Interest - 3.8%		Interest before FISIM - 4.0%	Interest before FISIM - 3.4%
		D.4.2 - Distributed income of corporations - 2.8%			Dividends - 2.8%
		D.4.4 - Other investment income-1.5%			Insurance income - 1.5%
D.4.5 - Rents - 0.0%		D.4.3 + D.4.5 - Reinvested Earnings on FDI and Rents - 0.1%			
B.5g - Balance of primary income - 70.3%				Balance of primary income - 63.5%	
Secondary Income Account					
Uses		Resources		Uses	Resources
		B.5g - Balance of primary income - 70.3%			Balance of primary income - 63.5%
D.5 - Current Taxes on Income and Wealth - 5.9%					
D.6.1 - Social Contributions - 13.9%		D.6.1 - Social Contributions - 0.2%		Income Taxes and Social Contributions - 17.1%	Social Transfers - 14.2%
D.6.2 - Social Benefits other than Social Transfers in kind - 0.2%		D.6.2 - Social Benefits other than Social Transfers in kind - 16.6%			
D.7.1 - Net non-Life Insurance Premiums - 0.6%		D.7.2 - Non-Life Insurance Claims - 0.7%			
D.7.5 - Miscellaneous Current Transfers - 1.9%		D.7.5 - Miscellaneous Current Transfers - 5.2%			Current Transfers - 3.4%
B.6g - Disposable Income - 70.5%				Disposable Income - 63.9%	
Use of Income Account					
Uses		Resources		Uses	Resources
		B.6g - Disposable Income - 70.5%			Disposable Income - 63.9%
P.3 - Final Consumption Expenditure - 64.5%				Consumption - 57.8%	
		D.8 - Adjustment for the change in net equity of households in pension funds - 0.3%			
B.8g - Gross Saving - 6.3%				Gross Saving - 6.2%	
Capital Account					
Uses		Resources		Uses	Resources
		B.8g - Gross Saving - 6.3%			Gross Saving - 6.2%
P.5 - Gross Capital Formation - 6.2%		D.9.2 - Investment Grants - 0.5%		Gross Capital Formation - 6.2%	
D.9 - Capital Transfers - 0.1%		D.9.9 - Other Capital Transfers - 0.6%			
NP - Acquisitions less disposals of non-produced non-financial assets - (1.0%)				NP - Acquisitions less disposals of non-produced non-financial assets - (1.0%)	
B.9g - Net Lending/Borrowing - 2.1%				Net Lending/Borrowing - 1.0%	

Table 9 - Households full sequence of accounts - National Accounts vs Model

In the primary income account, as we saw before, B.2g - gross operating surplus is not included in the model as well as D.43 – reinvested earnings on foreign direct investment and D.45 – rents with these last two items amounting to less than 0.1% of GDP. We can see the difference between interest in the national accounts and interest before FISIM allocation, with interest paid being 1.3% of GDP higher and interest on deposits 0.4% lower. This amount of 1.7% of GDP as well as the 5.0% of GDP of gross operating surplus are

compensated by removing these from consumption as explained in section 3.1.2. D.44 – Other investment income consists mostly of D.441 – property income attributed to insurance policy holders which refers to income earned by the investment of insurance technical reserves which is attributed to the policy holders. This income is not available for households to freely use but it's still a significant amount. For the households' net lending/borrowing position in the model not to differ too much from the National Accounts, this variable – which is fairly regular - is included in the model and it's exogenously defined. However it's not part of Households' current income, being paid directly into Other Assets.

In the secondary income account D.71 – net non-life insurance claims on the uses side and D.72 – non-life insurance claims on the resources side are absent from the model since these are not significant (0.6% and 0.7% of GDP) and practically cancel each other out. The difference between income taxes and social contributions in the model and the sum of D.5 – Current Taxes on Income and Wealth, D.61 – Social Contributions and D.62 – Social benefits other than social transfers in kind comes from the fact that in the model the government is the only sector paying social benefits and receiving social contributions while in reality this is done by all sectors in the form of private pension funds. This can be seen in table 10 with values expressed as average GDP percentages from 1995 to 2017. It's relevant to notice that households' net (of benefits received) contributions to private pension funds are considered as an act of saving in ESA so this is reversed by an adjustment in the use of income account (D.8 – adjustment for the change in net equity of households in pension funds). This way, the excess of contributions over benefits has an effect in B6g – gross disposable income, but it's neutral in terms of B.5 – gross saving. In the model only the transactions between households and the government sector were included, which means lower levels of social contributions and benefits for households than what actually happens in reality. The overall effect of this simplification on households' disposable income is slightly positive (0.3% of GDP) while on gross saving it is neutral since the adjustment made in the use of income account mentioned before is not included in the model.

D.61 - Social contributions (Net Position)						D.61 in the Model				
HH	Gov	NFS	FS	Row		HH	Gov	NFS	FS	Row
-13,7%	11,2%	1,0%	1,5%	0,0%		-11,2%	11,2%	-	-	-

D.62 - Social benefits other than social transfers in kind(Net Position)						D.62 in the Model				
HH	Gov	NFS	FS	Row		HH	Gov	NFS	FS	Row
16,4%	-14,2%	-1,0%	-1,2%	0,0%		14,2%	-14,2%	-	-	-

D.61 + D.62						D.61 + D.62 in the Model				
HH	Gov	NFS	FS	Row		HH	Gov	NFS	FS	Row
2,7%	-3,0%	0,0%	0,3%	0,1%		3,0%	-3,0%	-	-	-

Table 10 - Social benefits and contributions - National Accounts vs Model

Something similar was done regarding D.75 – Miscellaneous current transfers as shown in table 11. Since this is a relevant and very regular part of households' current income - consisting mostly of migrants' remittances - it's included in the model and it's exogenously defined with the counterpart of it being obviously the rest of the world sector.

D.75 - Miscellaneous current transfers (Net Position)				
HH	Gov	NFS	FS	Row
3,4%	-0,6%	-0,7%	0,0%	-2,0%
D.75 - in the Model				
HH	Gov	NFS	FS	Row
3,4%	-	-	-	-3,4%

Table 11- Miscellaneous current transfers - National Accounts vs Model

In the use of income account the differences have already been mentioned. Regarding consumption it comes from the removal of B.2g - gross operating surplus and of FISIM as explained in section 3.1.2. These amount to 6.7% of GDP which is the difference between consumption in the model and in National Accounts. D.8 - adjustment for the change in net equity of households in pension funds is the adjustment made to take into account that net (of benefits received) contributions of households' to private pension funds are an act of saving, adding to a fund which is ultimately owned by households. Since we are only considering social contributions and benefits occurring between households and the government - ignoring private pension funds - it makes no sense to include this adjustment.

In the capital account, D.9 – capital transfers on the uses side are insignificant and are not included in the model. On the resources side, D.92 – investment grants, although being quiet regular since 2003, these have also been almost insignificant during that period (0.3% of GDP) with significant amounts being only observed before 2003. D.99 – other capital transfers is a very irregular flow, varying between 0.2% and 1.6% of GDP, which makes it hard to include in the model in a satisfying way. NP – acquisitions less disposals of non-produced non-financial assets is a fairly regular and non-negligible income flowing to households, mostly from the non-financial sector as can be seen in table 12. Like for D.441, the decision to include this flow in the model was to approximate households' net lending/borrowing in the model with the National Accounts. Similarly to what was done for D.441, this flow is not considered current income and it's deposited into households other assets being exogenously defined.

NP - Acquisitions less disposals of non-produced non-financial assets				
HH	Gov	NFS	FS	Row
-1,0%	0,0%	0,8%	0,2%	0,0%
NP - in the Model				
HH	Gov	NFS	FS	Row
-1,0%	-	1,0%	-	-

Table 12 - Acquisitions less disposals of NPNFA - National Accounts vs Model

Overall, with the variables included in the model, households' net lending/borrowing position is inferior then in the National Accounts with the differences being mostly due to capital transfers and less significant in the more recent years. This can be seen in figure 7 where net lending/borrowing of the model follows very closely the net lending/borrowing of National Accounts after excluding capital transfers.
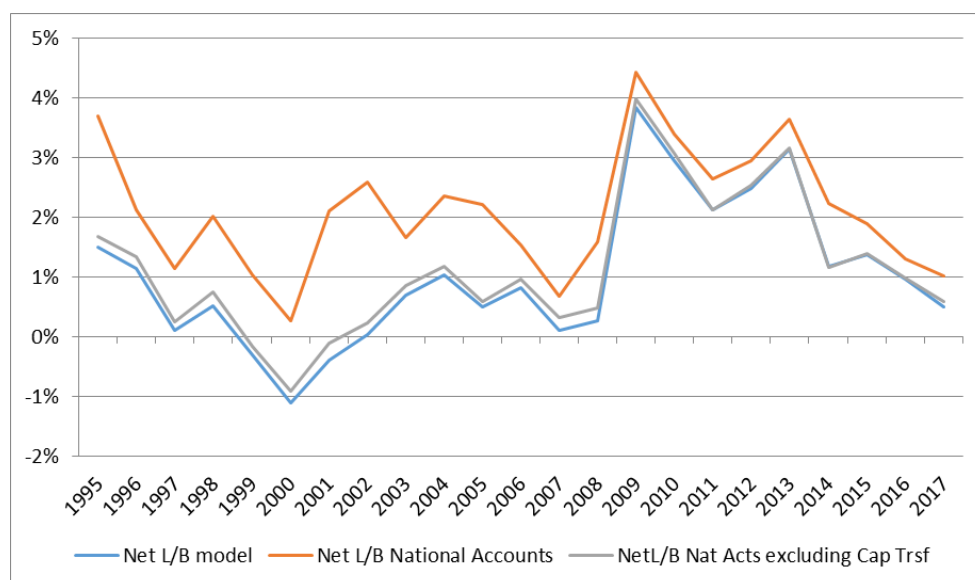

Figure 7 - Households net lending/borrowing - National Accounts vs Model

3.2.2 – Government

Table 13 shows how the variables included in the model relate with the National Accounts.

Government					
National Accounts				Model	
Primary Income Account					
Uses	Resources			Uses	Resources
	B.2g - Operating surplus - 2.7%				
D.3.1 - Subsidies on Products - 0.1%	D.2.1 - Taxes on Products - 12.9%				Taxes on Production - 12.2%
D.3.9 - Other Subsidies on Production - 0.8%	D.2.9 - Other Taxes on Production - 1.0%				
D.4.1 - Interest - 3.6%	D.4.1-Interest - 0.4%			Interest before FISIM - 3.4%	
	D.4.2 - Distributed income of corporations - 0.4%				
	D.4.4 - Other Investment Income - 0.0%				
D.4.5 - Rents - 0.0%	D.4.5 - Rents - 0.1%				
B.5g - Balance of primary income - 13.1%				Balance of primary income - 8.8%	
Secondary Income Account					
Uses	Resources			Uses	Resources
	B.5g - Balance of primary income - 13.1%				Balance of primary income - 8.8%
D.5 - Current Taxes on Income and Wealth - 0.0%	D.5 - Taxes on Income and Wealth - 9.2%				Income Taxes and SC of HH - 17.1%
D.6.2 - Social Benefits other than Social Transfers in kind - 14.2%	D.6.1 - Social Contributions - 11.2%			Social Transfers - 14.2%	Income Taxes of NFS - 2.5%
D.7.1 - Net non-Life Insurance Premiums - 0.0%	D.7.2 - Non-life insurance Claims - 0.0%				Income Taxes of FS - 0.7%
D.7.4 - Current International Cooperation - 0.2%	D.7.4 - Current International Cooperation - 0.3%				
D.7.5 - Miscellaneous Current Transfers - 1.4%	D.7.5 - Miscellaneous Current Transfers - 0.8%				
D.7.6 - VAT- and GNI-based EU own resources - 0.8%					
B.6g - Disposable Income - 17.9%				Disposable Income - 14.9%	
Use of Income Account					
Uses	Resources			Uses	Resources
	B.6g - Disposable Income - 17.9%				Disposable Income - 14.9%
P.3 - Final Consumption Expenditure - 19.2%				Other Consumption of Gov - 3.4%	
				Wages - 13.0%	
B.8g - Gross Saving - (1.3%)				Gross Saving - (1.6%)	
Capital Account					
Uses	Resources			Uses	Resources
	B.8g - Gross Saving - (1.3%)				Gross Saving - (1.6%)
P.5 - Gross Capital Formation - 3.8%	D.9.1 - Capital Taxes - 0.0%			Gross Capital Formation - 3.8%	
D.92 - Investment Grants - 0.5%	D.9.2 - Investment Grants - 0.9%				
D.99 - Other Capital Transfers - 0.7%	D.9.9 - Other Capital Transfers - 0.2%				
NP - Acquisitions less disposals of non-produced non-financial assets - 0.0%					
B.9g - Net Lending/Borrowing - (5.0%)				Net Lending/Borrowing - (5.4%)	

Table 13 - Government full sequence of accounts - National Accounts vs Model

In the primary income account, as explained in section 3.1.1, governments' gross operating surplus is not included in the model. Taxes (less subsidies) on production, in the model, differ from D.2 – taxes on production minus D.3 – subsidies on production of National Accounts because in reality taxes on production are paid to the government and the rest of the world sector and subsidies on production are paid by these two sectors as well, while in the model the only sector collecting taxes and paying subsidies is the government. In order to have GDP equal to income plus taxes less subsidies on production, we have that taxes (less subsidies) on production collected by the government in the model is in fact given by the total of taxes (less subsidies) on production. This can be seen in table 14. The simplification made regarding taxes on production "harms" the government position in 0.8% of GDP, but this is later compensated by not including D.76 – VAT-and GNI- based EU own resources in the secondary income account, which is a flow to the rest of the world sector with the same magnitude. In the model, government doesn't receive any interest so the interest paid is the net interest before FISIM allocation. While D.44 – other investment income and D.45 – rents are insignificant items, D.42 – distributed income of corporations is a bit less insignificant and it's a fairly regular flow. There's a case for including it in the model, but, for now, we don't think it's relevant enough.

D.2 - Taxes on Production		
to Gov	to RoW	Total
13,9%	0,1%	14,0%

D.3 - Subsidies on Production		
from Gov	from RoW	Total
0,9%	0,9%	1,8%

Taxes less subsidies on production		
Gov	RoW	Total
13,0%	-0,8%	12,2%

Taxes on production in the Model		
Gov	RoW	Total
12,2%	-	12,2%

Table 14 - Taxes and subsidies on production - National Accounts vs Model

In the secondary income account, D.61 – Social contributions and D.62 – Social benefits other than social transfers in kind have already been discussed in the households' sector. D.5 – taxes on income and wealth are given by what each of the remaining domestic sectors pay, which is very close (but not exactly equal since the rest of the world sector also pays and receives income taxes) to the amount collected by the government in the National Accounts. As mentioned before the non-inclusion of D.76 – VAT – and GNI – based EU own resources compensates for the simplification made regarding taxes less subsidies on production. The remaining items in the secondary income account almost cancel each other out and in net terms are not significant enough to include in the model.

In the use of income account we only have government's consumption which was already analysed in section 3.1.1.

In the capital account, only P.5 – gross capital formation is included in the model, and the items not included in the model (D9 – capital transfers and NP - acquisitions less disposals of non-produced non-financial assets) practically cancel each other out.

In figure 8 we can see how net lending/borrowing of the government, considering the variables included in the model, compares with the data from National Accounts. The bigger differences occur when there are significant capital transfers like in 2014 and 2017 when huge bank bailouts occurred.
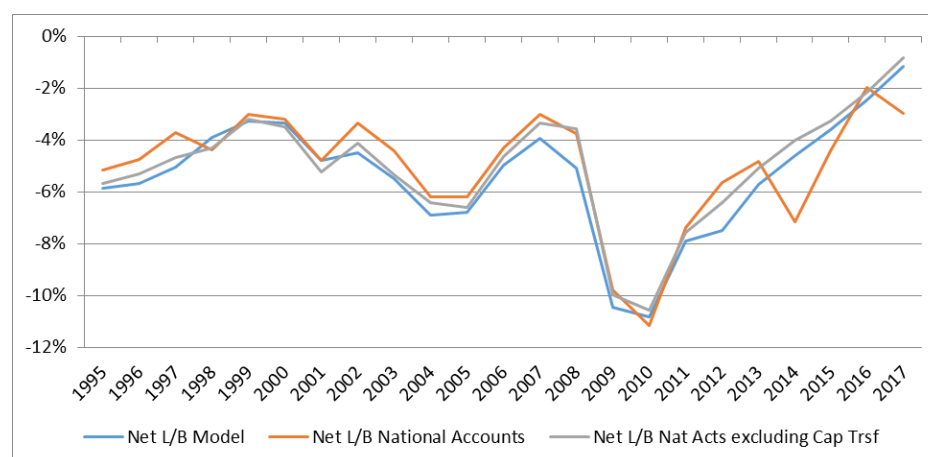

Figure 8 - Government net lending/borrowing - National Accounts vs Model

3.2.3 – Non-financial sector

Table 15 shows how the variables included in the model compare with the National Accounts.

Non Financial Sector				
National Accounts			Model	
Primary Income Account				
Uses	Resources		Uses	Resources
	B.2g - Operating surplus - 17.4%			B.2g - Operating surplus -18.7%
D.4.1-Interest - 3.2%	D.4.1-Interest - 1.8%		Interest before FISIM - 2.9%	
D.4.2 - Distributed income of corporations - 5.8%	D.4.2 - Distributed income of corporations - 1.5%		Dividends - 4.0%	
D.4.3 - Reinvested Earnings on FDI - 0.2%	D.4.3 - Reinvested Earnings on FDI - 0.1%			
	D.4.4 - Other investment income - 0.1%			
D.4.5 - Rents - 0.2%	D.4.5 - Rents - 0.0%			
B.5g - Balance of primary income - 11.5%			Balance of primary income - 11.8%	
Secondary Income Account				
Uses	Resources		Uses	Resources
	B.5g - Balance of primary income - 11.5%			Balance of primary income - 11.8%
D.5 - Current Taxes on Income and Wealth - 2.5%			Taxes on Income - 2.5%	
D.6.2 - Social Benefits other than Social Transfers in kind 1.0%	D.6.1 - Social Contributions - 1.0%			
D.7.1 - Net non-life insurance premiums - 0.4%	D.7.2 - Non-Life Insurance Claims - 0.3%			
D.7.5 - Miscellaneous Current Transfers - 0.8%	D.7.5 - Miscellaneous Current Transfers - 0.0%			
B.6g and B.8g - Disposable Income and Gross Saving - 8.1			Disposable Income and Gross Saving - 9.3%	
Capital Account				
Uses	Resources		Uses	Resources
	B.8g - Gross Saving - 8.1%			Gross Saving - 9.3%
P.5 - Gross Capital Formation - 11.6%	D.9.2 - Investment Grants - 0.6%		Gross Capital Formation - 12.2%	
D.99 - Other Capital Transfers - 0.2%	D.9.9 - Other Capital Transfers - 0.2%			
NP - Acquisitions less disposals of non-produced non financial assets - 0.8%			NP - Acquisitions less disposals of non-produced non financial assets - 1.0%	
B.9g - Net Lending/Borrowing - (3.7%)			Net Lending/Borrowing - (3.9%)	

Table 15 - Non-financial sector full sequence of accounts - National Accounts vs Model

In the primary income account, gross operating surplus differs from National Accounts because it includes part of gross operating surplus of the financial sector as explained in section 3.1.3. In the model, the non-financial sector doesn't receive interest, so interest paid is given by net interest before FISIM allocation. Regarding D.42 – distributed income of corporations, in the model, the non-financial sector is the only sector paying dividends and households and the rest of the world sector the only ones receiving, while in reality we have three sectors paying dividends and every sector receiving them. This can be seen in table 16 where additivity differences are due to rounding. The values used in the model are defined by the net position of the sector receiving dividends. The remaining flows in the primary income account are almost insignificant and practically cancel each other out. The only worth mentioning is D-43 - reinvested earnings on FDI, which although being negligible on average, it's a pretty irregular flow reaching in some years close to 1.0% of GDP.

D.42 - Distributed income of corporations (Resources-Uses)				
HH	Gov	NFS	FS	Row
2,8%	0,4%	1.5%-5.8%=-4.4%	1.9%-1.9%=0.0%	1.9%-0.7%=1.1%
Dividends in the Model				
HH	Gov	NFS	FS	Row
2,8%	-	-4,0%	-	1,1%

Table 16 - Distributed income of corporations - National Accounts vs Model

In the secondary income account, the only flow included in the model is D.5 – taxes on income and wealth. D.61 – social contributions and D.72 – non-life insurance claims on the resources side are practically compensated by D.62 – social benefits other than social transfers in kind and D.71 – net non-life insurance premiums on the uses side. The simplifications made regarding D.75 – miscellaneous current transfers were described when dealing with the household sector, and are not included in the model in the case of the non-financial sector.

In the capital account, P.5 – gross capital formation is obviously included in the model, but it also includes non-financial sector's gross capital formation as mentioned in section 3.1.3. NP - acquisitions less disposals of non-produced non-financial assets is included as explained in section 3.2.1 and it's exogenously defined. The remaining components of the capital account are not included in the model.

Overall, net lending/borrowing given by the variables included in the model follows very closely actual net lending/borrowing as shown in figure 9.
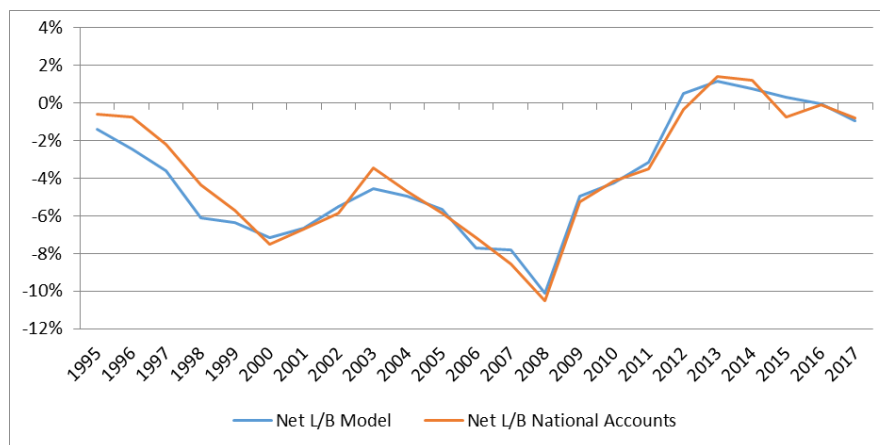

Figure 9 - Non-financial sector net lending/borrowing - National Accounts vs Model

3.2.4 – Financial Sector

The variables included in the model and how these compare with the National Accounts can be seen in table 17.

Financial Sector						
National Accounts				**Model**		
Primary Income Account						
Uses		Resources			Uses	Resources
		B.2g - Operating surplus - 3.0%				
D.4.1-Interest - 8.1%		D.4.1-Interest - 9.9%			Interest paid to HH - 3.4%	Interest from HH - 4.0%
D.4.2 - Distributed income of corporations - 1.9%		D.4.2 - Distributed income of corporations - 1.9%			Interest paid to RoW - 2.3%	Interest from NFS - 2.9%
D.4.3 - Reinvested Earnings on FDI - 0.1%		D.4.3 - Reinvested Earnings on FDI - 0.2%			Insurance Income paid to HH - 1.5%	Interest from Gov - 1.2%
D.4.4 -Other investment income - 1.7%		D.4.4 - Other investment income - 0.2%				Interest from RoW - 2.8%
D.4.5 - Rents - 0.0%		D.4.5 - Rents - 0.0%				
B.5g - Balance of primary income - 3.4%					Balance of primary income - 3.6%	
Secondary Income Account						
Uses		Resources			Uses	Resources
		B.5g - Balance of primary income - 3.4%				Balance of primary income -3.6%
D.5 - Current Taxes on Income and Wealth - 0.7%					Income Taxes - 0.7%	
D.6.2 - Social Benefits other than Social Transfers in kind - 1.2%		D.6.1 - Social Contributions - 1.5%				
D.7.1 - Net non-Life Insurance Premiums - 0.3%		D.7.1 - Net non-Life Insurance Premiums - 1.3%				
D.7.2 - Non-Life Insurance Claims - 1.3%		D.7.2 - Non-Life Insurance Claims - 0.2%				
D.7.5 - Miscellaneous Current Transfers - 0.3%		D.7.5 - Miscellaneous Current Transfers - 0.3%				
B.6g - Disposable Income - 2.9%					Disposable Income - 3.0%	
Use of Income Account						
Uses		Resources			Uses	Resources
		B.6g - Disposable Income - 2.9%				Disposable Income - 3.0%
D.8 - Adjustment for the change in net equity of households in pension funds - 0.3%						
B.8g - Gross Saving - 2.6%					Gross Saving - 3.0%	
Capital Account						
Uses		Resources			Uses	Resources
		B.8g - Gross Saving - 2.6%				Gross Saving - 3.0%
P.5 - Gross Capital Formation - 0.6%		D.9.2 - Investment Grants - 0.0%				
D.99 - Other Capital Transfers - 0.8%		D.9.9 - Other Capital Transfers - 0.8%				
K.2 - Acquisitions less disposals of non-produced non financial assets - 0.2%						
B.9g - Net Lending/Borrowing - 1.8%					Net Lending/Borrowing - 3.0%	

Table 17 - Financial sector full sequence of accounts - National Accounts vs Model

In the primary income account, gross operating surplus is not included in the model, as mentioned before, but this is more than compensated by considering the interest payments that actually happen before FISIM allocation. The effect of this can be better seen in figure 10 where the balance of interest payments considered in the model (before FISIM) is compared with this same balance in the National Accounts (after FISIM) plus gross operating surplus of the financial sector. Before 2004, this simplification benefits significantly the financial sector, but since then it's relatively neutral regarding the sectors' net lending/borrowing position.
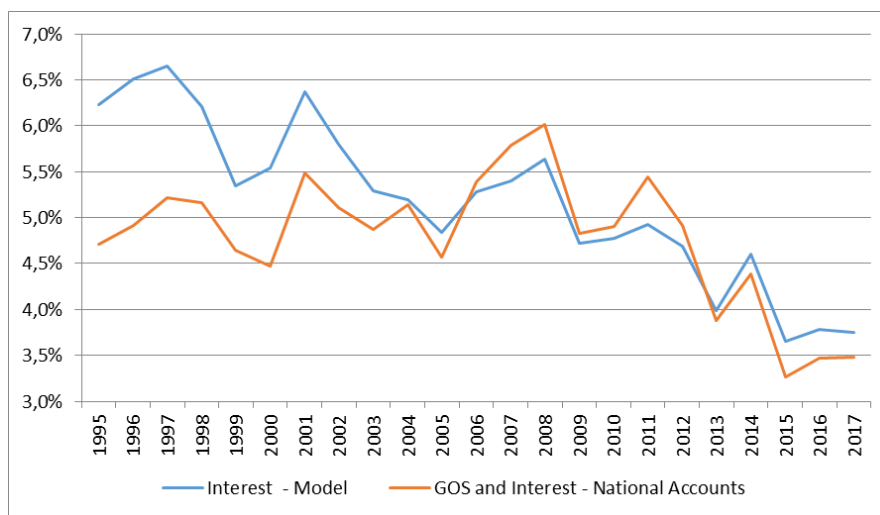
Figure 10 - Interest in the model vs interest and GOS in National Accounts

Regarding interest payments only, figure 11 shows us the difference between interest payments featured in the National Accounts and the ones considered in the model in terms of the financial sector's net position as a percentage of GDP.
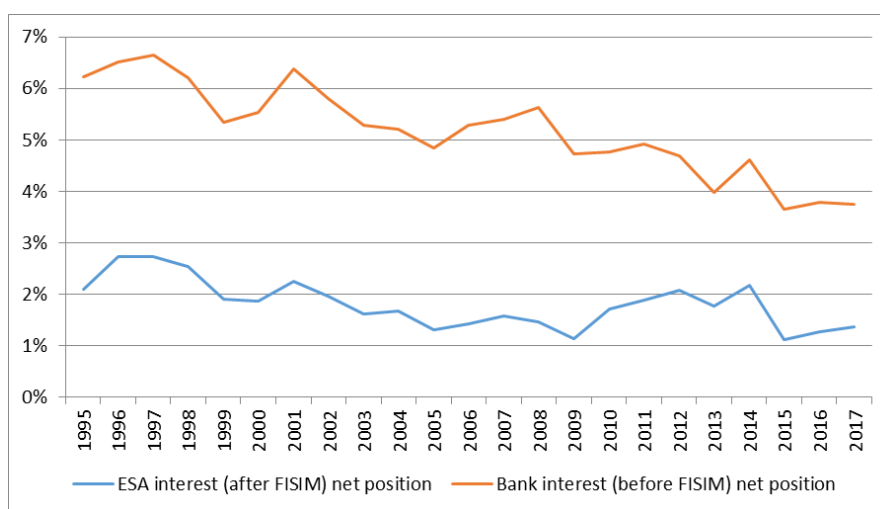

Figure 11 – Interest before and after FISIM as % of GDP

In gross terms, table 18 displays interest payments (before FISIM) paid and received for each sector and how these were included in the model. The values in green were the ones used as reference for the model, so for households and the rest of the world sector, gross interest was considered while for the government and for the non-financial sector net payments were included.

D.41 - Interest paid (before FISIM)						Interest paid in the Model				
HH	Gov	NFS	FS	Row		HH	Gov	NFS	FS	Row
4,0%	3,8%	4,5%	7,4%	2,8%		4,0%	3,4%	2,9%	5,7%	2,8%
D.41 - Interest received (before FISIM)						Interest received in the Model				
HH	Gov	NFS	FS	Row		HH	Gov	NFS	FS	Row
3,4%	0,4%	1,6%	12,6%	4,4%		3,4%	-	-	10,9%	4,4%
D.41 - Interest (Net Position)						Net interest in the Model				
HH	Gov	NFS	FS	Row		HH	Gov	NFS	FS	Row
-0,6%	-3,4%	-2,9%	5,1%	1,7%		-0,6%	-3,4%	-2,9%	5,2%	1,7%

Table 18 - Interest payments - National Accounts vs Model

D.44 – other investment income is included in the model as a payment to households - referring to income earned by the investment of insurance technical reserves which is attributed to the policy holders - since it's a non-negligible part of households' income as explained before. D.42 – distributed income of corporations paid and received by the Financial Sector cancel each other out when looking at average values between 1995 and 2017, but the net position is in fact pretty irregular during this period. The same happens with D.43 – reinvested earnings of foreign direct investment, which despite being pretty insignificant, even in gross terms, when looking at average values, it can reach significant amounts for any given year. This can be seen in figure 12. This irregular behaviour of these flows makes it hard to include in the model in a satisfying way, so they are left out.
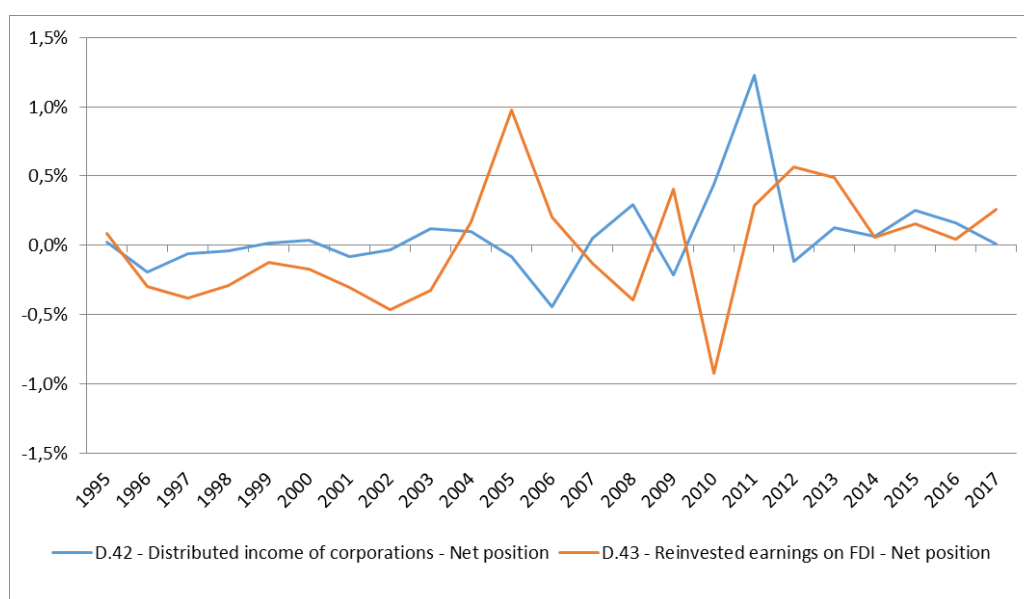
Figure 12 - D.42 and D.43 net positions as % of GDP

In the secondary income account and in the use of income account, the only flow included in the model is D.5 – taxes on income and wealth. D.62 – social benefits other than social transfers in kind, D.61 – social contributions and D.8 – adjustment for the change in households' pension funds (from use of income account) considered together are exactly neutral in terms of the financial sector's net lending/borrowing position as explained in section 3.2.1. The remaining flows in the secondary income account, D.7 – other current transfers - which are miscellaneous transfers and insurance claims and premiums - are practically equivalent through time, on the uses and resources side, and are not included in the model.

None of the flows of the capital account are included in the model. P.5 - gross capital formation is included in the non-financial sector's gross capital formation. Although being relatively small on average, D.99 – other capital transfers is a very significant flow (on the uses and resources side) in many instances as shown in figure 13. These are too irregular to include in the model.
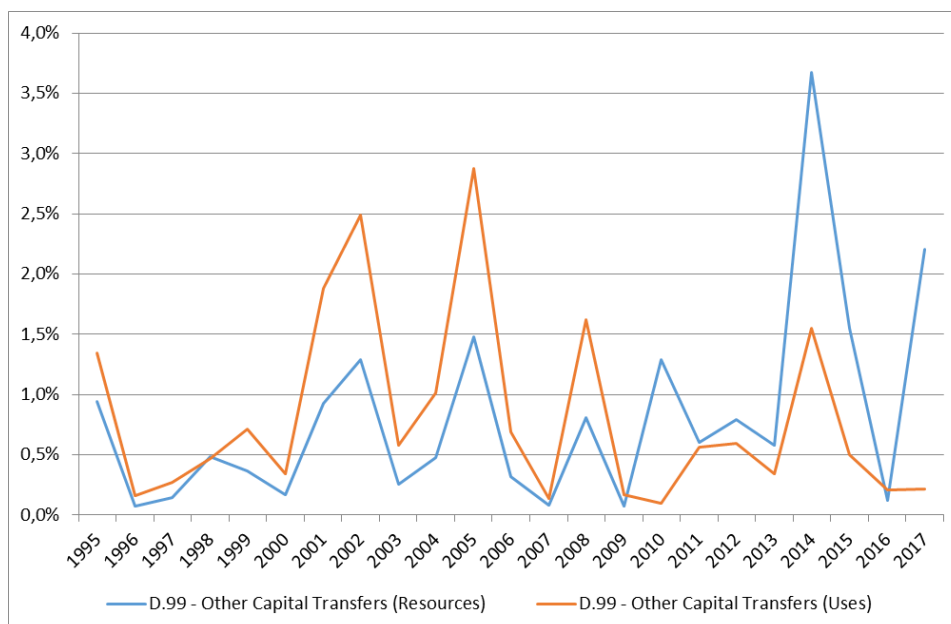
Figure 13 - Other Capital Transfers - Financial Sector

In figure 14 we can see how the model's net lending/borrowing compares with real net lending/borrowing for the financial sector.
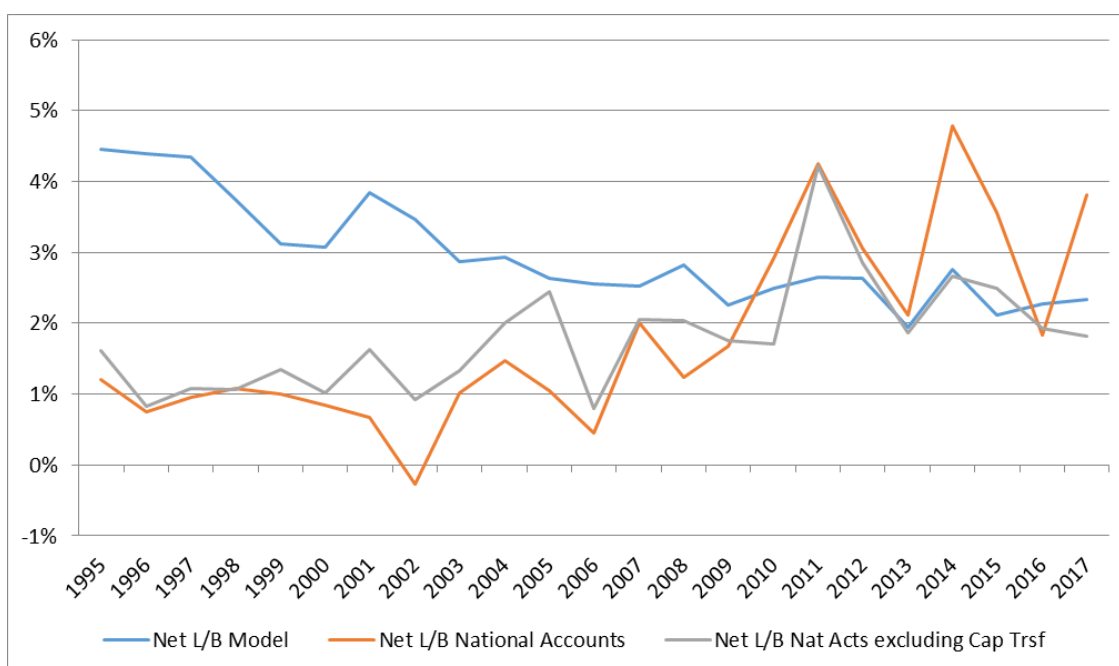

Figure 14 - Financial sector net lending/borrowing - National Accounts vs Model

3.2.5 – Rest of the world sector

In table 19 we can see how the model variables compare with the National Accounts for the rest of the world sector.

Rest of the World			
National Accounts		**Model**	
Primary and Secondary Income Account			
Uses	Resources	Uses	Resources
	B.11 - External Balance of Goods and Services - 6.2%		External Balance of Goods and Services - 6.2%
D.1-Compensation of Employees - 0.2%	D.1-Compensation of Employees - 0.2%		
D.3.1 - Subsidies on Products - 0.3%	D.2.1 - Taxes on Products - 0.1%		
D.3.9 - Other Subsidies on Production - 0.6%	D.2.9 - Other Taxes on Production - 0.0%		Interest from Gov - 2.2%
D.4.1-Interest - 2.8%	D.4.1-Interest - 4.4%	Interest - 2.8%	Interest from FS - 2.3%
D.4.2 - Distributed income of corporations - 0.7%	D.4.2 - Distributed income of corporations - 1.9%		Dividends - 1.2%
D.4.3 - Reinvested Earnings on FDI - 0.3%	D.4.3 - Reinvested Earnings on FDI - 0.3%		
D.4.4 - Other investment income - 0.2%	D.4.4 - Other investment income - 0.0%		
D.5 - Current Taxes on Income and Wealth - 0.2%	D.5 - Current Taxes on Income and Wealth - 0.1%		
D.6 - Social Contributions and Benefits- 0.1%	D.6 - Social Contributions and Benefits- 0.1%		
D.7.1 - Net non-Life Insurance Premiums - 0.1%	D.7.1 - Net non-Life Insurance Premiums - 0.1%		
D.7.2 - Non-Life Insurance Claims - 0.1%	D.7.2 - Non-Life Insurance Claims - 0.1%		
D.7.4 - Current International Cooperation - 0.3%	D.7.4 - Current International Cooperation - 0.2%		
D.7.5 - Miscellaneous Current Transfers - 2.8%	D.7.5 - Miscellaneous Current Transfers - 0.7%	Current Transfers - 3.4%	
	D.7.6 - VAT- and GNI-based EU own resources - 0.8%		
B.12 - Current External Balance - 6.5%		Current External Balance - 5.3%	
Capital Account			
Uses	Resources	Uses	Resources
	B.12 - Current External Balance - 6.5%		Current External Balance - 5.3%
D.92 - Investment Grants - 1.6%	D.9.2 - Investment Grants - 0.0%		
D.99 - Other Capital Transfers - 0.1%	D.9.9 - Other Capital Transfers - 0.1%		
K.2 - Acquisitions less disposals of non-produced non financial assets - 0.0%			
B.9g - Net Lending/Borrowing - 4.9%		Net Lending/Borrowing - 5.3%	

Table 19 - Rest of the world sector full sequence of accounts - National Accounts vs Model

The variables included besides the obvious B.11 – external balance of goods and services are D.41 – interest, D.42 – distributed income of corporations and D.75 – miscellaneous current transfers which were explained in previous sections. Of the missing flows the most significant is D.92 – investment grants which is paid to households, the non-financial sector and the government in almost equal parts. This is not included, because if we did, the rest of the world net lending/borrowing position in the model would deviate too much from National Accounts and because it would have to be paid to three sectors which would add an unnecessary complexity to the model. In fact, the omitted flows practically cancel each other out, as shown in figure 15. When this doesn't happen, like between 2011 and 2014, it's due to irregular flows as D.42 – distributed income of corporations and D.43 – reinvested earnings on FDI, which would be difficult to include in the model.
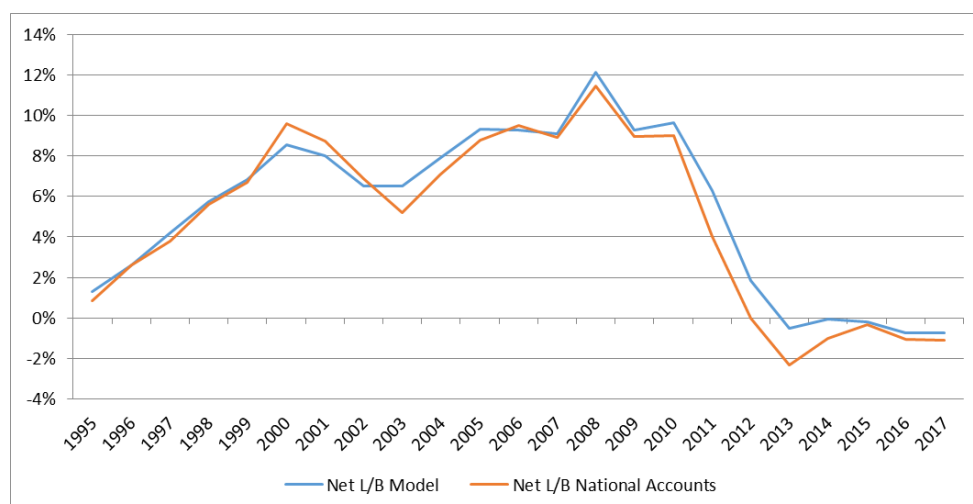

Figure 15 - Rest of the world sector net lending/borrowing - National Accounts vs Model

4 – Behavioural equations

In this section we describe the model's behavioural equations. We begin by presenting the exogenous variables in the model. Then we focus on an important exogenous variable of the model the Portugal Spread and how it is used along with a benchmark interest rate to define the interest rates of the model. After that we describe each sector's behavioural equations, presenting the full transaction matrix for each sector. Throughout this section, the same Greek letters are repeatedly used for different parameters in distinct equations.

4.1 – Exogenous variables

The exogenous variables of the model can be seen in table 20.

Variable Description			Variable in Model
Government related	Tax Rates	on households' income	$SC+IncTaxHH_{Rate}$
		on non-financial sector's income	$IncTaxNFS_{Rate}$
		on financial sector's income	$IncTaxFS_{Rate}$
		on production	$TaxProd_{Rate}$
	Expenditures	Government Wages	$WagesGov$
		Other Government Consumption	$OthConsGov$
		Gross Capital Formation	GCF_{Gov}
		Social Transfers	$SocTrsf$
	Debt Redenomination		$GovDebtReden$
Interest Rates related	Portugal Spread		$PortugalSpread$
	1 month Euribor		$Euribor$
	Interest rate on private external debt		$IntRatePrvtExtDebt$
	Interest rate on external assets		$IntRateFA$
Households related	Current transfers from abroad		$CurrTrsf$
	Acquisitions less disposals of NPNFA		$Aq-DofNPNFA$
	Insurance income attributed to policy holders		$InsIncome$
	Early repayments of housing loans		$EarlyRepHLoans$
Non-financial sector related	Weight of domestic dividends on total dividends		$WgtDomDiv$
	Wage share (including mixed income)		$WageMIShare$
Rest of the world related	Exports		$Exports$
	Brent price in Euros		$Brent$

<p align="center">Table 20 - Exogenous variables</p>

A significant part of the model's exogenous variables are related with government decisions. On the income side we have tax rates while on the expenditure side we have all spending variables. Another government related exogenous variable is government debt redenomination, which was necessary to include to deal with significant early repayments of foreign debt domestically financed that have happened in recent times.

The interested rates related variables are the Portugal Spread (which we will describe in detail in the next section) the one month Euribor (the benchmark interest rate of the model) and the interest rates on private external debt and on external assets. These two interest rates are exogenously defined for now, because we haven't rea

Related with households, we have three variables that were included in the model to bring households' net lending/borrowing closer to what's published in the National Accounts and because they are fairly predictable flows: Current transfers; Acquisitions less disposals of non-produced non-financial assets; and

Insurance income attributed to policy holders. The other exogenous variable is the early repayment of housing loans. The low interest rates of recent times has resulted in an expressive level of early repayments of housing loans. Instead of overcomplicating the repayment function of housing loans we decided to include this exogenous variable to address this issue.

Regarding the non-financial sector, we have two exogenous variables. The first simply determines the division between dividends paid to households and paid to the rest of the world sector. The second one is the wage (and mixed income) share, which defines how income from production, after taxes on production and imports being paid, is divided into households and the non-financial sector.

The remaining two exogenous variables of the model are Exports and the price of Brent in Euros. This last variable appears only in the imports function.

4.2 – Portugal Spread

The Portugal Spread is an exogenous variable that represents general economic sentiment as well as credit market conditions.

Beau et al (2014) tells us that "The cost of (bank) funding can be decomposed into a risk-free component, a combination of credit risk and liquidity risk premia, and other costs. The risk premia are influenced by a combination of general, 'macro' factors (such as the broad economic outlook, or an increase in the riskiness of the banking sector) and factors that are idiosyncratic to any given bank (…) Taken together, the risk-free rate and the risk premia generally account for the bulk of overall funding costs." Extending this idea to the whole financial sector we can say that its average funding cost is given by a risk free rate plus a risk premia (or spread) that is influenced by general macro factors such as the ones mentioned by Beau et all (2014). This risk premia is the Portugal Spread in the model, the excess of the financial sectors' funding costs over some risk-free interest rate, which in this case is the 1-month Euribor.

The question is then how to estimate this spread. According to Cadamagnani et all (2015), banks determine their interest rates on loans and deposits through an internal pricing mechanism, where there is a bank's treasury function that acts as a central risk management hub overseeing different business lines. This treasury function works like a bank within a bank, borrowing deposits from the bank's deposit taking business lines and lending them to the loans conceding business lines. The price the treasury function pays for the deposits and charges on the loans – the transfer price - is largely based on the bank's funding costs and the price appearing (to the bank's costumers) on the end of each business line is set to make them profitable. This means that the interest rate charged on loans conceded by the bank has a positive spread on top of the transfer price while the interest rate offered on deposits has a negative spread. A stylized example of this can be seen in figure 16 taken from Cadamagnani et all (2015).
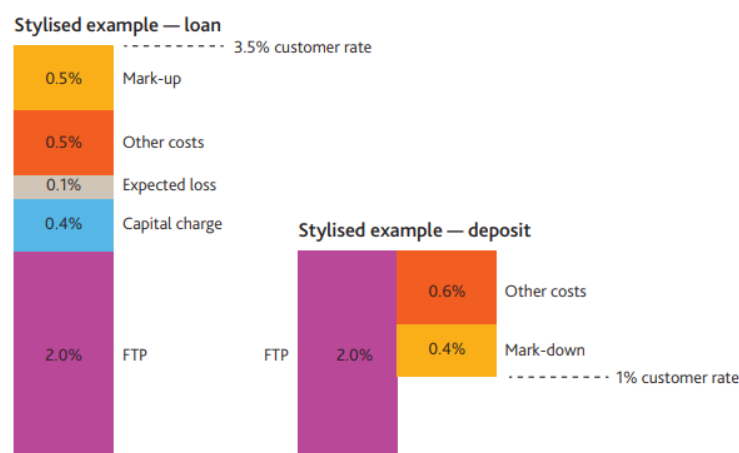
Figure 16 - Stylised example of interest rate pricing from Cadamagnani et all (2015)

Using this idea, we can use available data on interest rates on loans and deposits to obtain an estimate of the financial sector's funding costs. Figure 17 has the weighted average interest rates on new loans and on new deposits offered by the Portuguese monetary financial institutions to non-financial corporations and private individuals. The estimated financing rate is simply given by a weighted average between these two, with a weight of 0.75 for the deposits rate and 0.25 to the loans rate.
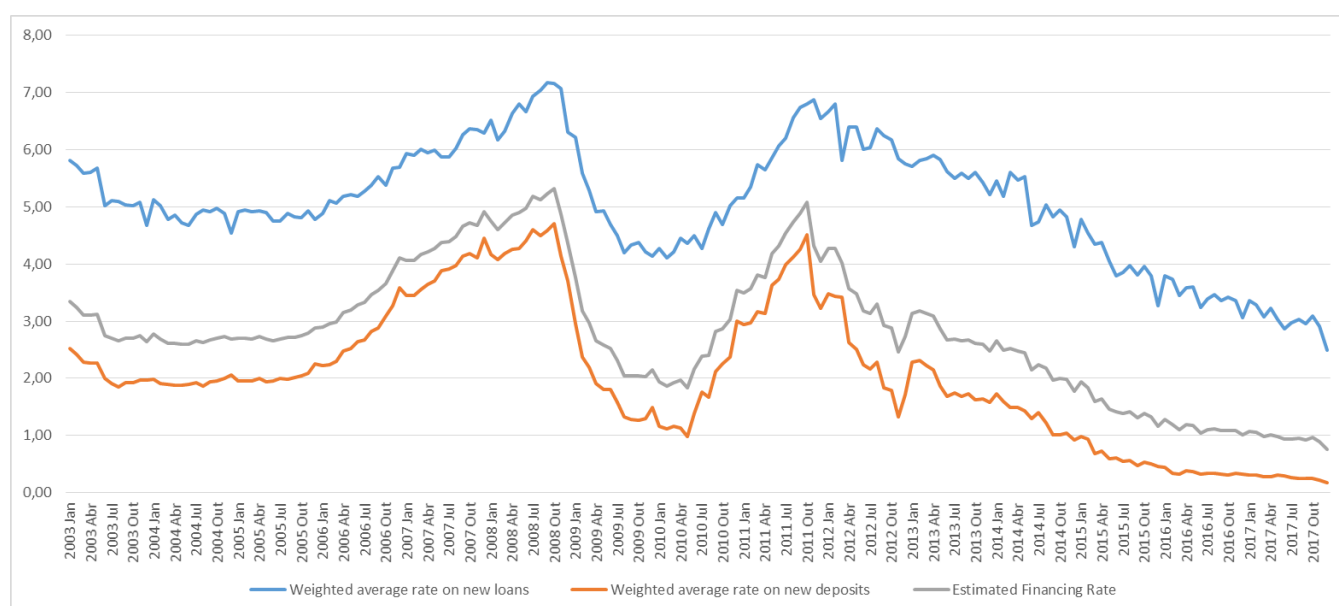

Figure 17 - Average interest rate on new loans and deposits and the estimated financing rate for the Financial Sector

The difference between this estimated funding rate and the 1-month Euribor gives us the Portugal Spread as shown in figure 18. Looking at its evolution we can identify the Lehman Brothers bankruptcy, the development of the European sovereign debt crisis, and the slow return to normality after the 'whatever it takes' speech of Mario Draghi.
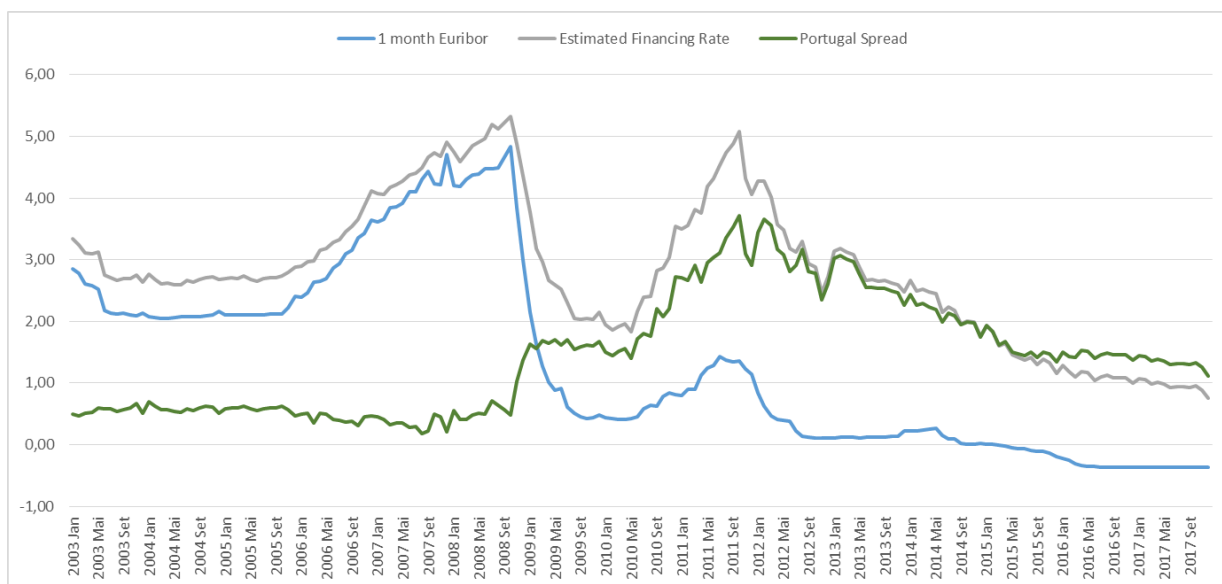
Figure 18 - Portugal Spread = Estimated financing rate - 1 month Euribor

In the next graphs in figure 19 we can see how the Portugal Spread relates with nominal GDP, private consumption, gross fixed capital formation and total new loans by private individuals. The correlations between the Portugal Spread and these variables are -0.76, -0.75, -0.57 and -0.83 respectively.
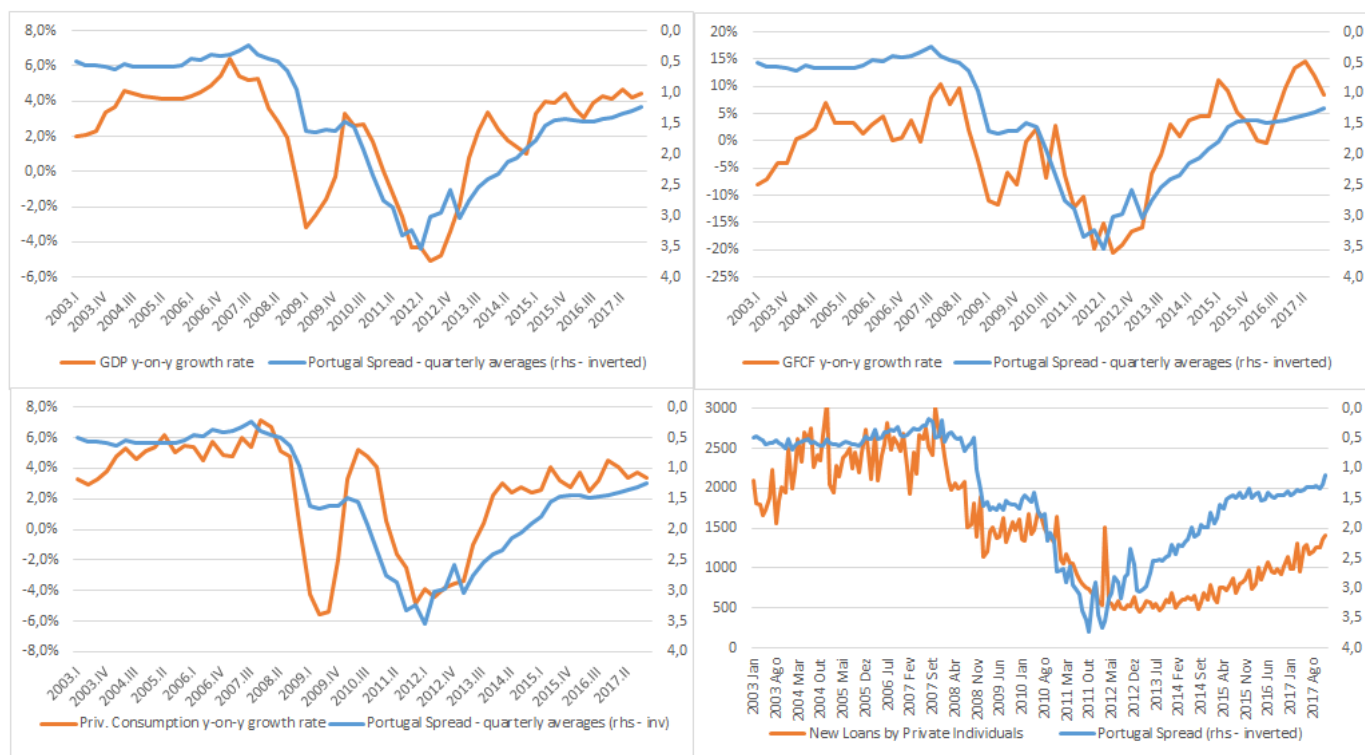

Figure 19 - Portugal Spread vs GDP, GFCF, Consumption and New Loans

The Portugal Spread will appear in several behavioural equations, like households' consumption, households' new housing and new consumption loans and on the definition of almost all interest rates of the model. The idea is to define (almost) all interest rates of the model from the benchmark interest rate (1-month Euribor) and the Portugal Spread. This is explained in the next section.

4.3 – Interest Rates

Interest payments in the model are obtained by multiplying an interest rate to the correspondent financial instrument. There are eight interest payments as shown in the transactions matrix in section 3, but only seven interest rates to be calculated, since we are assuming the same interest rate for government debt owned by residents and by non-residents. Of these seven interest rates, two of them – on private external debt and on external assets – are exogenously defined. The remaining five are defined as a function of the benchmark interest rate and the Portugal Spread.

The way we do this is based on the idea from Cadamagnani et all (2015), described in the previous section, that banks define the interest rates they offer by adding (in the case of loans) and subtracting (in the case of deposits) a spread to a rate that is strongly influenced by their funding costs. As described in the previous section, the Portugal Spread is the difference between a hypothetical financing rate for the financial sector and a risk-free benchmark interest rate. So the sum of the Portugal Spread with this benchmark interest rate - both exogenous variables of the model - will give us the hypothetical financing rate for the financial sector. From this we obtain interest rates on new loans (by adding a fixed spread) and deposits (by subtracting a fixed spread) which will then influence interest rates on stocks of loans and deposits. This is done differently for floating interest rates instruments and for fixed interest rate instruments as we detail next.

To illustrate the validity of this idea, in figure 20 we can see how interest rates on new loans and deposits compare with the estimate obtained by adding or subtracting a fixed spread to the estimated financing rate (which is equal to the one month Euribor plus the Portugal Spread). These are not the interest rates that will be used in the model - since the interest rates on stocks the model will try to replicate are given implicitly by the interest payments and the stock of the correspondent financial instrument – but they have very similar trajectories. The fact that from the estimated financing rate we can obtain a good approximation for interest rates on new loans and deposits is not surprising since these were used to obtain the estimated financing rate.
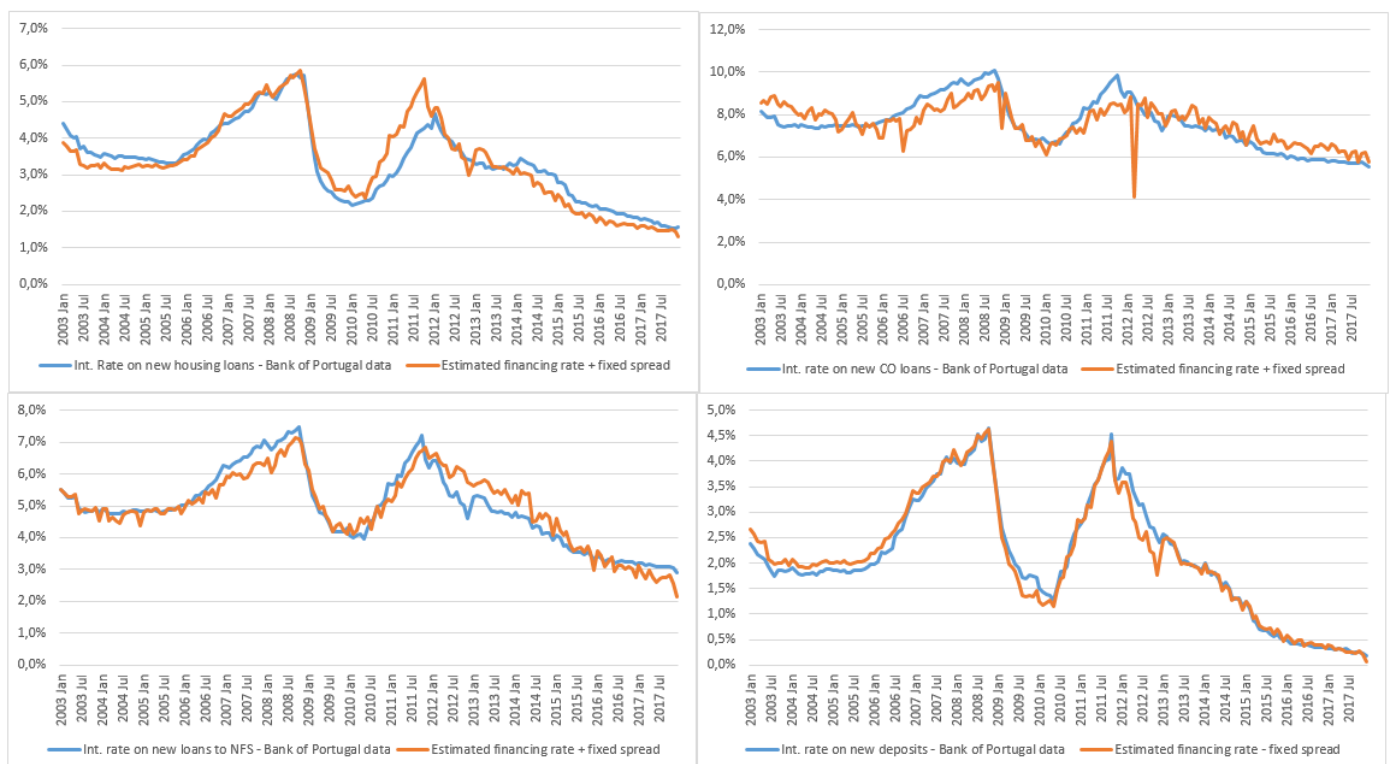

Figure 20 - Interest rates on new loans and deposits - Bank of Portugal data vs Estimated financing rate +/- fixed spread

Getting into the equations, we begin by describing the dynamics for fixed interest rates instruments, which in the model are households' consumption and other loans, households' other assets and government debt. For consumption and other loans, the interest rate on the stock of loans tends to the interest rate on new loans and the velocity which it does this depends on an adjustment factor (γ) and the weight of new loans on the stock of existing loans. The interest rate on new loans is obtained by adding a fixed spread (δ) to the hypothetical funding rate of the financial sector which is given by the sum of the one month Euribor with the Portugal Spread.

$$\frac{dIntRateCOLoans_{HH}}{dt} = \gamma * \frac{NewCOLoans}{LoansCO} * (IntRateCOLoans_{New} - IntRateCOLoans_{HH})$$

$$IntRateCOLoans_{New} = Euribor_{1m} + PortugalSpread + \delta$$

For government debt the logic is the same but there is a little twist. In the case of government debt we don't have new debt and debt repayment, we only have change in debt which is given by the government's net lending/borrowing. We can't make the adjustment being simply dependent on the weight of the deficit on total debt because when a surplus happened this would mean a change of signal to the adjustment taking place. Another point is that even when there is a small (close to zero) deficit, there is always a significant part of debt being rolled over. So in order to have the adjustment being dependent on the size of the government deficit while addressing these issues we simply added a constant (ρ) to the weight of the deficit on total debt which can be interpreted as the percentage of debt being rolled over.

$$\frac{dIntRateGovDebt}{dt} = \vartheta * \left(\frac{GovDeficit}{GovDebt} + \rho\right) * (IntRateGovDebt_{New} - IntRateGovDebt)$$

$$IntRateGovDebt_{New} = Euribor_{1m} + PortugalSpread + \alpha$$

In the case of households' other assets, the velocity of the adjustment is constant (β) and the spread defining interest rate on new deposits is subtracted instead of added.

$$\frac{dIntRateOthAssets_{HH}}{dt} = \beta * (IntRateOthAssets_{New} - IntRateOthAssets_{HH})$$

$$IntRateOthAssets_{New} = Euribor_{1m} + PortugalSpread - \mu$$

The dynamics for floating interest rate instruments - which in the model are housing loans and non-financial sector loans - is a little different. Instead of the stock interest rate tending to the interest rate on new loans, in this case it's the spread (total spread which includes the Portugal Spread) on the stock of loans that tends to the spread on new loans. This way, changes in the benchmark interest rate have an immediate effect on the stock interest rate. So for housing loans we have:

$$IntRateHLoans_{HH} = Euribor_{1m} + SpreadHLoans_{Stock}$$

$$\frac{dSpreadHLoans_{Stock}}{dt} = \gamma * \frac{NewLoansH}{LoansH} * (SpreadHLoans_{New} - SpreadHLoans_{Stock})$$

$$SpreadHLoans_{New} = PortugalSpread + \vartheta$$

In the case of non-financial sector loans the logic is the same but the velocity of adjustment is fixed.

$$IntRateLoans_{NFS} = Euribor_{1m} + SpreadLoansNFS_{Stock}$$

$$\frac{dSpreadLoansNFS_{Stock}}{dt} = \gamma * (SpreadLoansNFS_{New} - SpreadLoansNFS_{Stock})$$

$$SpreadLoansNFS_{New} = PortugalSpread + \vartheta$$

4.4 – Households

The complete transactions matrix for households can be seen in figure asxasxas. The top flows, which have an effect on households' net worth, were already in the transactions matrix shown in the beginning of section 3. Regarding these, it's worth pointing out the two flows that are not paid into Currency and transferable deposits but into Other Assets. This is to make clearer the distinction between current income, from which we define consumption, and other income which merely adds to households' assets. The bottom flows are financial flows between households' assets and liabilities with no effect on households' net worth. The grey coloured variables are exogenously defined, the green ones are described in this section and the other ones are described in another section.

	Households				
	Assets		Liabilities		Net Worth
	CurrTD	Other Assets	LoansH	LoansCO	
Wages and Mixed Income	+WagesMI				+WagesMI
Government Wages	+WagesGov				+WagesGov
Social Transfers	+SocTrsf				+SocTrsf
Current Transfers from abroad	+CurrTrsf				+CurrTrsf
Dividends	+Div$_{HH}$				+Div$_{HH}$
Interest on Other Assets	+IntDep$_{HH}$				+IntDep$_{HH}$
Taxes on Income and Social Contributions	-TaxIncSC				-TaxIncSC
Interest on Housing Loans	-IntLoansH$_{HH}$				-IntLoansH$_{HH}$
Interest on CO Loans	-IntLoansCO$_{HH}$				-IntLoansCO$_{HH}$
Consumption	-Cons				-Cons
Gross Capital Formation	-GCF$_{HH}$				-GCF$_{HH}$
Acquisitions less disposals on NPNFA		+Acq-DofNPNFA			+Acq-DofNPNFA
Income on Insurance		+InsIncome			+InsIncome
New Housing Loans	+NewLoansH		+NewLoansH		
Repayment of Housing Loans	-RepLoansH		-RepLoansH		
New CO Loans	+NewLoansCO			+NewLoansCO	
Repayment of CO Loans	-RepLoansCO			-RepLoansCO	
Portfolio Rebalancing	+PortReb	-PortReb			

Table 21 - All transactions of Households in the model

Taxes on income and social contributions (*TaxIncSC*) is simply given by the exogenously defined tax rate (*SCIncTaxHH$_{Rate}$*) times households' current income (*IncomeHH$_{CurrTD}$*) with this being equal to all income paid into households' Currency and transferable deposits.

$$TaxIncSC = SCIncTaxHH_{Rate} * IncomeHH_{CurrTD}$$

$$IncomeHH_{CurrTD} = WagesMI + WagesGov + SocTrsf + CurrTrsf + Div_{HH} + IntDep_{HH}$$

Households' consumption (Cons) is defined as a percentage of households' disposable income (DispIncHH) plus a percentage of their total assets (CurrTD+OtherAssets).The percentage of disposable income being used for consumption depends on the Portugal Spread, with a higher Portugal Spread meaning a lower percentage. We use the logarithmic function so that the same absolute change in the Portugal Spread has a higher impact for low levels of this variable. The value we add before applying the logarithmic function is to have the domain higher than one and consequently the function image to be positive.

$$Cons = \delta * \left(1 - \frac{\ln(PortugalSpread + 0.78)}{\gamma}\right) * DispIncHH + \vartheta * (CurrTD + OtherAssets)$$

$$DispIncHH = IncomeHH_{CurrTD} - (TaxIncSC - IntLoansH_{HH} - IntLoansCO_{HH})$$

The idea behind the function that determines gross capital formation of households is similar to what happens with the consumption function. But since this a more long-term and planned decision, the effects of changes in income and in the Portugal Spread are not immediate. The way this is modelled is by having gross capital formation of households (GCF$_{HH}$) tending to a certain desired level (GCF$_{Tgt}$) which is a function of total income and the Portugal Spread.

$$\frac{dGCF_{HH}}{dt} = \delta * (GCF_{Tgt} - GCF_{HH})$$

$$GCF_{Tgt} = \theta * \left(1 - \frac{\ln(PortugalSpread + 0.78)}{\gamma}\right) * (DispIncomeHH + AqDofNPNFA + InsIncome)$$

New consumption and other loans is defied using the same logic. There is a desired level of new loans (NewCOLoansTgt) that depends on the Portugal Spread and on disposable income, to which the actual level of new consumption and other loans tends to. The repayment of this type of loans is simply a fixed percentage of the stock of loans.

$$\frac{dNewCOLoans}{dt} = \delta * (NewCOLoans_{Tgt} - NewCOLoans)$$

$$NewCOLoans_{Tgt} = \theta * \left(1 - \frac{\ln(PortugalSpread + 0.78)}{\gamma}\right) * DispIncomeHH$$

$$RepLoansCO = \alpha * LoansCO$$

The functions that define the dynamics of housing loans are very similar to consumption and other loans except for two small differences. The first one is that the speed of the adjustment (π) depends on the sign of the adjustment. The idea is that "bad news" have a much faster effect on this kind of decisions then "good news". The second difference is that the repayment of housing loans includes the exogenously defined early repayment of housing loans.

$$\frac{dNewHLoans}{dt} = \pi * (NewHLoans_{Tgt} - NewHLoans)$$

$$NewHLoans_{Tgt} = \theta * \left(1 - \frac{\ln(PortugalSpread + 0.78)}{\gamma}\right) * DispIncomeHH$$

$$\pi = \begin{cases} \pi_1 & if \quad NewHLoans_{Tgt} \geq NewHLoans \\ \pi_2 & if \quad NewHLoans_{Tgt} < NewHLoans \end{cases}$$

$$RepLoansH = \alpha * LoansH + EarlyRepHLoans$$

The portfolio rebalancing flow is a function of the difference between a desired level of currency and transferable deposits and its actual level. The desired level is given by a weight target ($WgtCurrTD_{Tgt}$) which is a function of the interest rate on new deposits. The logarithmic function was used so that, for example, a change from 0.5% to 1.5% has a higher effect than a change from 4% to 5%.

$$PortReb = \mu * (WgtCurrTD_{tgt} * (CurrTD + OtherAssets) - CurrTD)$$

$$WgtCurrTD_{Tgt} = \tau - \vartheta * \ln(IntRateNewDep)$$

4.5 – Non-financial sector

In table 22 we can see all transactions of the non-financial sector. The only one that was not included in the transactions matrix of section 3 is the change in loans variable. As for the households case, variables in grey are exogenously defined and in green are explained in this section.

	Non-financial Sector			
	Assets		Liabilities	Net Worth
	CurrAct	CapAct	Loans	
Consumption	+Cons			+Cons
Other Consumption of Government	+OthConsGov			+OthConsGov
Gross Capital Formation of HH	+GCF$_{HH}$			+GCFHH
Gross Capital Formation of Gov	+GCF$_{Gov}$			+GCFGov
Gross Capital Formation of NFS	+GCF$_{NFS}$	-GCF$_{NFS}$		
Exports	+Exp			+Exp
Taxes on Production	-TaxProd			-TaxProd
Imports	-Imports			-Imports
Wages and Mixed Income	-WagesMI			-WagesMI
Operating Surplus of NFS	-OS$_{NFS}$	+OS$_{NFS}$		
Taxes on Income		-TaxInc$_{NFS}$		-TaxIncNFS
Dividends to HH		-Div$_{HH}$		-Div$_{HH}$
Dividends to RoW		-Div$_{RoW}$		-Div$_{RoW}$
Interest on Loans		-IntLoansNFS		-IntLoansNFS
Acquisitions less disposals on NPNFA		-Acq-DofNPNFA		-Acq-DofNPNFA
Change in Loans		+ChLoans$_{NFS}$	+ChLoans$_{NFS}$	

Table 22 - All transactions of the Non-financial Sector in the model

The way taxes on production and imports are obtained from final demand is the result of two ideas. The first one is that instead of having imports and taxes on production independent of each other, depending only on final demand, we want to have a formulation where taxes on production have an effect on the level of

imports. The way to do this is to first deduct taxes on production from final demand, and after that, apply an imported content coefficient to the remainder. The second idea is to have different imported content coefficients for different components of final demand. These coefficients are based on input-output tables published by Portugal Statistics. One issue in combining these two ideas is that we would need to have tax rates on production for every final demand component. To avoid this and since 1 - most taxes on production are levied on consumption and 2 - the tax rates on other components of final demand are fairly regular, we decided to have a varying tax rate on consumption ($TaxProd_{Rate}$) and fixed tax rates (θ, μ and ρ) for the remaining components. Regarding gross capital formation the same tax rate (μ) was used for all sectors.

$$TaxProd = TaxProd_{Rate} * Cons + \theta * Exports + \mu * GCF + \rho * OthConGov$$

So after deducting taxes on production, imports are calculated for each component. In the case of consumption, the imported content coefficient depends on two variables, negatively on the Portugal Spread and positively on the price of Brent. The reason for including the Portugal Spread is that the kind of consumption most affected by economic climate and credit market conditions is consumption on durables, which has a much higher imported content than total consumption. For exports, the price of Brent also influences the imported content coefficient, because a non-negligible part of exports (between 4% and 7% since 2010) is processed oil and lubricants which require significant imports of crude oil. In the imports equation θ, μ and ρ are the same as in the production taxes equation.

$$Imports = (\beta + \alpha * Brent - \gamma * PortugalSpread) * (1 - TaxProd_{Rate}) * Cons$$
$$+ \tau * (1 - \rho) * OthConsGov$$
$$+ \omega * (1 - \mu) * (GCF_{HH} + GCF_{Gov})$$
$$+ \epsilon * (1 - \mu) * GCF_{NFS}$$
$$+ (\pi + \delta * Brent) * (1 - \theta) * Exports$$

After deducting taxes on production and imports from total demand, wages and mixed income are determined with the exogenously defined wage and mixed income share. In the model there is a wage target to which wages and mixed income tend to. This is to avoid circular definitions of variables, because wages and mixed income directly enter the consumption function which without this delay would enter the wages and mixed income function directly as well. So in the mode the adjustment to the desired level is very fast.

$$\frac{dWagesMI}{dt} = \alpha * (WagesMI_{Tgt} - WagesMI)$$

$$WagesMI_{Tgt} = WageMIShare * (TotalDemand - TaxProd - Imports)$$

$$TotalDemand = Cons + OthConsGov + GCF_{HH} + GCF_{NFS} + GCF_{Gov} + Exports$$

Operating surplus of the non-financial sector is simply what is left of total demand once taxes, imports and wages are paid.

$$OS_{NFS} = TotalDemand - TaxProd - Imports - WagesMI$$

Taxes on income are defined by applying the exogenously defined income tax rate to the operating surplus of the non-financial sector.

$$TaxInc_{NFS} = IncTaxNFS_{Rate} * OS_{NFS}$$

The division of total dividends ($DivTotal$) into dividends paid to households (Div_{HH}) and to the rest of the world sector (Div_{RoW}) is given by the exogenously defined weight ($WgtDomDiv$). Total dividends are defined as tending to target level of dividends ($DivTgt$). This delay is introduced because dividends are not determined by present conditions but by what happened in a previous period. The target level of dividends

is a fraction of the non-financial sector's free cash flow, which is defined as what's left of the operating surplus after paying for taxes, interest and acquisitions less disposals of NPNFA. The fraction of free cash flow depends negatively on the general economic conditions (represented by the Portugal Spread) and by the available alternative financing rate via bank loans (represented by the interest rate on new loans which is given by the Euribor plus the spread on new loans).

$$Div_{HH} = WgtDomDiv * DivTotal$$

$$Div_{RoW} = (1 - WgtDomDiv) * DivTotal$$

$$\frac{dDivTotal}{dt} = \alpha * (Div_{Tgt} - DivTotal)$$

$$Div_{Tgt} = (\delta + \gamma * \ln(Euribor + SpreadNewLoans - \tau) - \theta * PortugalSpread) * FreeCashFlow_{NFS}$$

$$FreeCashFlow_{NFS} = OS_{NFS} - TaxInc_{NFS} - IntLoansNFS - AqDofNPNFA$$

The basic idea for the gross capital formation function is the "partial adjustment accelerator model" similar to the one found in Godley and Lavoie (2007). Gross capital formation is defined as the consumption of fixed capital (CFC) plus a fraction of the difference between the desired level of capital stock (*KapTgt*) and the actual level (*Kapital*). The desired level of capital stock is given by expected demand (*ExpDemand*) divided by the productivity of capital (ρ) and the desired utilization rate (μ) – we consider this a single parameter ($\rho*\mu$) in the model, with expected demand lagging total demand. The consumption of fixed capital is simply a fixed fraction (δ) of the stock of capital. Finally, the evolution of the stock of capital is given by gross capital formation minus the consumption of fixed capital.

$$GCF_{NFS} = \alpha * (KapTgt - Kapital) + CFC$$

$$KapTgt = \frac{ExpDemand}{\mu * \rho}$$

$$\frac{dExpDemand}{dt} = \theta * (TotalDemand - ExpDemand)$$

$$CFC = \delta * Kapital$$

$$\frac{dKapital}{dt} = GCF_{NFS} - CFC$$

The idea behind the change in loans flow is that there is a desired level (*CapAct_Tgt*) for the non-financial sector's capital account - which depends on total demand – and this flow is used to achieve it, either by increasing or decreasing the sector's indebtedness. The ratio of currency and deposits to total demand is relatively stable to be used like this, as shown in figure 21.

$$ChLoans_{NFS} = \vartheta * (CapAct_{Tgt} - CapAct)$$
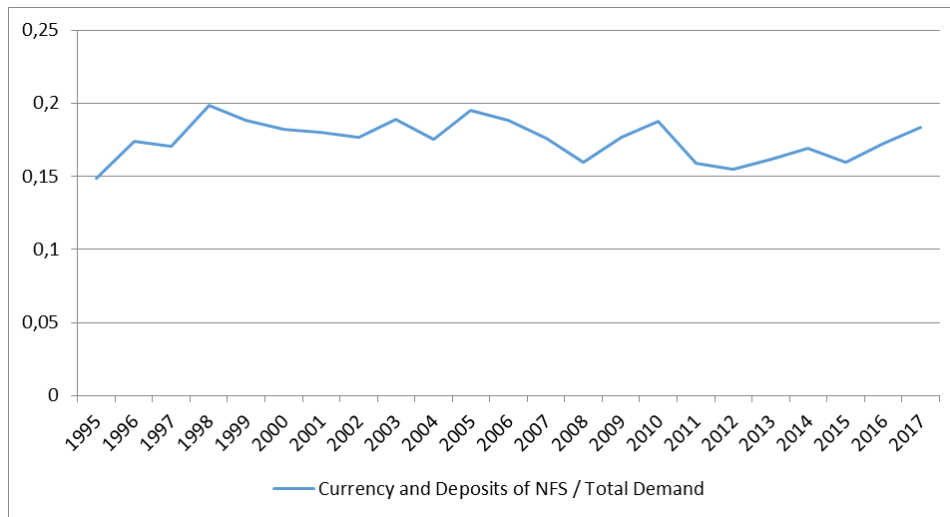
$$CapAct_{Tgt} = \mu * TotalDemand$$

Figure 21 - Currency and deposits of NFS divided by model's total demand

4.6 – Government

Table 23 shows the full transactions matrix for the government sector. As for the previous sectors, the first group of transactions were already in the transactions matrix in section 3, with the remaining variables having no effect on the sector's net worth. Grey coloured variables are exogenously defined and green coloured variables are defined in this section.

	Government			
	Assets	Liabilities		Net Worth
	GovDep	GovDebt$_{Dom}$	GovDebt$_{RoW}$	
Taxes on Production	+TaxProd			+TaxProd
Taxes on Income and Social Contributions	+TaxIncSC			+TaxIncSC
Taxes on Income NFS	+TaxInc$_{NFS}$			+TaxInc$_{NFS}$
Taxes on Income FS	+TaxInc$_{FS}$			+TaxInc$_{FS}$
Government Wages	-WagesGov			-WagesGov
Other Consumption of Government	-OthConsGov			-OthConsGov
Gross Capital Formation of Government	-GCF$_{Gov}$			-GCF$_{Gov}$
Social Transfers	-SocTrsf			-SocTrsf
Interest on Government Debt Domestic	-IntGovDebt$_{Dom}$			-IntGovDebt$_{Dom}$
Interest on Government Debt RoW	-IntGovDebt$_{RoW}$			-IntGovDebt$_{RoW}$
Government Net L/B Domestic	-GovNetL/B$_{Dom}$	-GovNetL/B$_{Dom}$		
Government Net L/B RoW	-GovNetL/B$_{RoW}$		-GovNetL/B$_{RoW}$	
Government Debt Redenomination		+GovDebtRed	-GovDebtRed	

Table 23 - All transactions of the Government in the model

Interest payments are simply obtained by multiplying the interest rate on government debt by the correspondent stock of debt.

$$IntGovDebt_{Dom} = IntRateGovDebt * GovDebt_{Dom}$$

$$IntGovDebt_{RoW} = IntRateGovDebt * GovDebt_{RoW}$$

Government net lending/borrowing is the difference between its revenues and expenditures, with half of the deficit being financed domestically and the remaining half abroad. When there is a surplus there is an equal reduction in both stocks of debt.

$$GovNetL/B_{Dom} = 0.5 * (GovRevenue - GovExpenditures)$$

$$GovNetL/B_{RoW} = 0.5 * (GovRevenue - GovExpenditures)$$

$$GovRevenue = TaxProd + TaxIncSC + TaxInc_{NFS} + TaxInc_{FS}$$

$$GovExpenditures = WagesGov + OthConsGov + GCF_{Gov} + SocTrsf + IntGovDebt_{Dom} + IntGovDebt_{RoW}$$

4.7 – Rest of the World

The full transactions matrix for the rest of the world sector can be seen in figure dsfvsv. Once again, the first group of transactions change the sector's net worth and were already in the transactions flow matrix of section 3, while the transactions in the bottom group are purely financial transactions. Grey coloured variables are exogenously defined and green coloured variables are defined in this section.

	Rest of the World				
	Assets		Liabilities		Net Worth
	GovDebt$_{RoW}$	PrvtExtDebt	Reserves	ExtAssets	
Exports			+Exports		-Exports
Imports			-Imports		+Imports
Dividends to RoW			-Div$_{RoW}$		+Div$_{RoW}$
Current Transfers from abroad			+CurrTrsf		-CurrTrsf
Interest on Government Debt RoW			-IntGovDebt$_{Dom}$		+IntGovDebt$_{Dom}$
Interest on Private External Debt			-IntPrvtExtDebt		+IntPrvtExtDebt
Interest on External Assets			+IntExtAss		-IntExtAss
Government Net L/B RoW	+GovNetL/B$_{RoW}$		+GovNetL/B$_{RoW}$		
Government Debt Redenomination	-GovDebtRed		-GovDebtRed		
Investment in External Assets			-InvExtAss	+InvExtAss	
Reserve adjustment		+ResAdj	+ResAdj		

Table 24 - All transactions of the Rest of the world sector in the model

The idea used to define investment in external assets is that this tends to accompany the evolution of the domestic private sector indebtedness. This means that the weight of external assets on total private assets of the financial sector is relatively stable, as shown in figure 23. In the model investment in external assets (or disinvestment if it's negative) adjusts the amount of external assets so that this weight is attained.

$$InvExtAss = \gamma * (\beta * (LoansH + LoansCO + LoansNFS + ExtAssets) - ExtAssets)$$
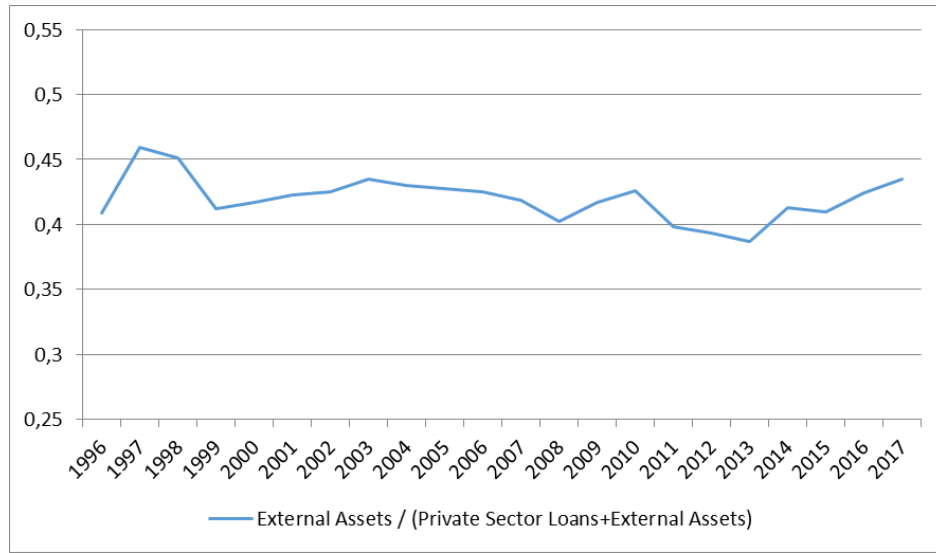
Figure 22 - Weight of External Assets on total private assets of the Financial Sector

All transactions with the rest of the world sector are settled with reserves, with inflows of money increasing reserves, and outflows of money decreasing them. The financial sector is required to maintain a certain level of reserves, defined as a fraction of their liabilities. To reach this required level it increases or decreases its indebtedness towards the rest of the world of the sector. When it has excess reserves, it uses them to reduce private external debt. On the other hand when the level of reserves is below the required level it must obtain them, financing this operation with an increase in private external debt.

$$ResAdj = \delta * (Reserves_{Tgt} - Reserves)$$

$$Reserves_{Tgt} = 0.005 * (CurrTD + OtherAssets + CapAct + PrivtExtDebt)$$

4.8 – Financial Sector

Since in the model all financial instruments, except one (government debt owed to non-residents), have the financial sector as a counterpart, all transactions of the model involve changes in the financial sectors' balance sheet. This means that full transactions flow matrix for this sector is a little bigger than the ones we've seen for other sectors, needing a full page (the next one) to be properly shown here. Once again the top group of transaction have an impact on the sectors' net worth, while the remaining transactions do not. The green coloured variables are described next.

Interest payments are simply determined by the interest rate - whose equations were described in section 4.3 - multiplied by the correspondent financial instrument.

$$IntDep_{HH} = IntRateOthAssets_{HH} * OtherAssets$$

$$IntLoansH_{HH} = IntRateHLoans_{HH} * LoansH$$

$$IntLoansCO_{HH} = IntRateCOLoans_{HH} * LoansCO$$

$$IntLoansNFS = IntRateLoans_{NFS} * LoansNFS$$

$$IntExtAss = IntRateEA * ExternalAssets$$

$$IntPrvtExtDebt = IntRatePrvtExtDebt * PrvtExtDebt$$

The income tax of the financial sector is given by the exogenously defined income tax rate times the difference between interest paid and received. Income on the investment of insurance technical reserves which is credited to households is not included in the calculation of taxable income since there is not an actual payment taking place.

$$TaxInc_{FS} = IncTaxFS_{Rate} * (IntLoansH_{HH} + IntLoansCO_{HH} + IntLoans_{NFS} + IntGovDebt_{Dom} + IntExtAss$$
$$- IntDep_{HH} - IntPrvtExtDebt)$$

| | Financial Sector | | | | | | | | | | | | |
| | **Assets** | | | | | | **Liabilities** | | | | | | **Net Worth** |
	LoansH	LoansCO	LoansNFS	GovDebt$_{Dom}$	ExtAssets	Reserves	CurrTD	Other Assets	CurrAct	CapAct	GovDep	PrvtExtDebt	Net Worth
Income on Insurance								+InsIncome					-InsIncome
Interest on Other Assets							+$IntDep_{HH}$						-$IntDep_{HH}$
Interest on Housing Loans							-$IntLoansH_{HH}$						+$IntLoansH_{HH}$
Interest on CO Loans							-$IntLoansCO_{HH}$						+$IntLoansCO_{HH}$
Interest on Loans NFS										-IntLoansNFS			+IntLoansNFS
Interest on Government Debt Domestic											-$IntGovDebt_{Dom}$		+$IntGovDebt_{Dom}$
Interest on Private External Debt						-IntPrvtExtDebt							-IntPrvtExtDebt
Interest on External Assets						+IntExtAss							+IntExtAss
Taxes on Income FS											+$TaxInc_{FS}$		-$TaxInc_{FS}$
Consumption							-Cons		+Cons				
Other Consumption of Government									+OthConsGov		-OthConsGov		
Gross Capital Formation of HH							-GCF_{HH}		+GCF_{HH}				
Gross Capital Formation of Gov									+GCF_{Gov}		-GCF_{Gov}		
Gross Capital Formation of NFS									+GCF_{NFS}	-GCF_{NFS}			
Exports						+Exports			+Exports				
Taxes on Production									-TaxProd		+TaxProd		
Imports						-Imports			-Imports				
Wages and Mixed Income							+WagesMI		-WagesMI				
Operating Surplus of NFS									-OS_{NFS}	+OS_{NFS}			
Government Wages							+WagesGov				-WagesGov		
Social Transfers							+SocTrsf				-SocTrsf		
Current Transfers from abroad						+CurrTrsf	+CurrTrsf						
Taxes on Income NFS										-$TaxInc_{NFS}$	+$TaxInc_{NFS}$		
Taxes on Income and Social Contributions							-TaxIncSC				+TaxIncSC		
Dividends to HH							+Div_{HH}			-Div_{HH}			
Dividends to RoW						-Div_{RoW}				-Div_{RoW}			
Acquisitions less disposals on NPNFA								+Acq-DofNPNFA		-Acq-DofNPNFA			
Interest on Government Debt RoW						-$IntGovDebt_{RoW}$					-$IntGovDebt_{RoW}$		
New Housing Loans	+NewLoansH						+NewLoansH						
Repayment of Housing Loans	-RepLoansH						-RepLoansH						
New CO Loans		+NewLoansCO					+NewLoansCO						
Repayment of CO Loans		-RepLoansCO					-RepLoansCO						
Portfolio Rebalancing							+PortReb	-PortReb					
Change in Loans			+$ChLoans_{NFS}$							+$ChLoans_{NFS}$			
Government Net L/B Domestic				-$GovNetL/B_{Dom}$							-$GovNetL/B_{Dom}$		
Government Net L/B RoW						-$GovNetL/B_{RoW}$					-$GovNetL/B_{RoW}$		
Government Debt Redenomination				+GovDebtRed			-GovDebtRed						
Investment in External Assets					+InvExtAss	-InvExtAss							
Reserve adjustment						+ResAdj						+ResAdj	

Table 25 - All transactions of the Financial Sector in the model