$$\mathbf{e}^{(k)} = \overline{\mathbf{x}} - \overline{\mathbf{x}}^{(k+1)} = \Big(\mathbf{B}\overline{\mathbf{x}} + \overline{\mathbf{c}}\Big) - \Big(\mathbf{B}\overline{\mathbf{x}}^{(k)} + \overline{\mathbf{c}}\Big) = \mathbf{B}\Big(\overline{\mathbf{x}} - \overline{\mathbf{x}}^{(k+1)}\Big) = \mathbf{B}\mathbf{e}^{(k)} \quad (1)$$

$$\therefore \mathbf{e}^{(k)} = \mathbf{B}^k \mathbf{e}^{(0)} \quad (2)$$

Taking the norm of both sides:

$$||\mathbf{e}^{(k)}|| = ||\mathbf{B}^k \mathbf{e}^{(0)}|| \leq ||\mathbf{B}^k|| ||\mathbf{e}^{(0)}|| \leq ||\mathbf{B}||^k ||\mathbf{e}^{(0)}|| \quad (3)$$

$$\text{We can also note that for convergence, } ||\mathbf{B}|| < 1 \quad (4)$$

$$\text{and that this is equivalent to the requirement } \rho(\mathbf{B}) = max|\lambda_j| < 1 \quad (5)$$

Note that for large value of k,

$$||\mathbf{e}^{(k+1)}|| = \rho ||\mathbf{e}^{(k)}||$$

We want to estimate the minimum number of iterations required to calculate, in order to reduce the relative error:

$$\rho^i < \epsilon \quad (6)$$

leading to

$$i \leq \frac{log(\epsilon)}{log(\rho)} \quad (7)$$

We will use equation 7 to estimate the number of iterations, noting the calculation of ρ for each method:

Jacobi: $\rho_j = max|\lambda_j|$

Gauss-Seidel: $\rho_{gs} = \rho_j^2$

SOR: $\rho_s = \omega - 1$, where $\omega = \dfrac{2}{1 + \sqrt{1 - \rho_j^2}}$