UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG
School of Computer Science and Applied Mathematics

Numerical Methods
Lab 2

1st March 2018

Instructions

- Your code should be able to communicate the appropriate message, in the case of a computational problem.

Questions 1

Using your code for Exercise 1, 2 and 3 of Lab 2(Iterative techniques for linear systems), solve the linear system where

(a)

$$A = \begin{pmatrix} 2 & 1 & -1 & 2 \\ 4 & 5 & -3 & 6 \\ -2 & 5 & -2 & 6 \\ 4 & 11 & -4 & 8 \end{pmatrix}, \quad \text{and} \quad b = \begin{pmatrix} 5 \\ 9 \\ 4 \\ 2 \end{pmatrix}$$

(b)

$$A = \begin{pmatrix} 3 & 1 & -1 \\ 1 & -4 & 2 \\ -2 & -1 & 5 \end{pmatrix}, \quad \text{and} \quad b = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

Use the zero vector as initial approximation.

Questions 2

The matrix $A = \begin{pmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 4 & -1 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 4 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 4 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 4 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 4 \end{pmatrix}$ arises in the finite difference discretization of the Poisson equation on a nine point grid.

(a) Is matrix A diagonally dominant?

(b) Find the spectral radius of the Jacobi and Gauss-Seidel iteration matrices.

(c) Using the formula

$$\omega = \frac{2}{1+\sqrt{1-\rho_J^2}}, \qquad (\star)$$

fix the optimal value of the relaxation parameter.

(d) If $\rho_J = \max|\lambda|$ denotes the spectral radius of the Jacobi method, then the Gauss-Seidel iterative method has spectral radius $\rho_{\mathrm{GS}} = \rho_J^2$, while the relaxation method with optimal relaxation parameter given in $(\star)$ has spectral radius

$$\rho = \omega - 1. \qquad (\Upsilon)$$

Verify that the spectral radius of the relaxation method, obtained from its iterative matrix agrees with that obtained in (Υ).

(e) For each iterative scheme, predict how many iterations are needed to solve the linear system $A\mathbf{x} = \mathbf{e_1}$. Then verify your prediction by direct computation, using the zero vector as initial approximation. Vector $\mathbf{e_1}$ is a unit column vector of dimension compatible with matrix A, which has 1 in the first entry and zero everywhere else.