UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG
School of Computer Science and Applied Mathematics

Numerical Methods
Lab 3

9th March 2018

Some notes:

The bisection method is popular because it is robust. It always works, as long as there is a root in the interval. It is however, a very slow method, compared to the rest of the methods. In addition to being slow, it cannot find roots of multiplicity 2. The multiplicity of a root c is the power to which the factor $x-c$ is raised. Therefore, for functions like $f(x)=(x-c)^2$, the method fails since there are no such interval $[a\ b]$ for which $f(a)f(b)<0$.

Also, the **continuity of the function** over the entire interval is an important hypothesis for the bisection method to work. This is already stated in Theorem 1 in your note. For example, the function $f(x)=\frac{1}{x}$ satisfies $f(-1)=-1$ and $f(x)=1$. But, there is no root to the equation $\frac{1}{x}=0$. Also, Theorem 1 in your note does not say that there is a unique root in the interval $[a\ b]$. There may be many roots, or even, possible infinitely many roots. The Theorem only guarantees that there will be at least a root in the interval.

Instructions

- Your code should be able to communicate the appropriate message, in the case of a computational problem. Remember, you can always estimate the number of iterations needed, given a tolerance.

Questions 1

(a) Use the methods for finding roots of equations, namely the bisection method, regular falsi method and the Newton's fixed point method, which you have coded in questions 1, 2 and 3 of Lab 3 to find the root, accurate to within 10^{-5}, of the nonlinear equations

(I) $f(x)=e^x+2^{-x}+2\cos x-6, \quad 1\le x\le 2$

(II) $f(x)=1-\frac{2}{x^2-2x+2}, \quad -1\le x\le 1$

(III) For only Question (I), do plot the error at each iteration against the number of iterations, for each method, as given in Question 4 of Lab 3.

(b) Using any root finding method, find all the point of intersection of the function $f(x)=\tan x$ and $g(x)=x$ on the interval $0\le x\le 10$

Questions 2

(a) Use Newton's method for finding roots of nonlinear system of equations, which you have coded in Question 5 to find the root, to within 10^{-7} of the nonlinear system

$$\vec{f}(\vec{x}) = \begin{pmatrix} u^2 - 4uv - 2 \\ uv - u^2 + v^2 \\ u^2 + w \end{pmatrix}, \qquad \text{where} \quad \vec{x} = (u, v, w)$$

(b) Use Newton's method to find all points of intersection of the following pair of plane curves:

$$x^3 + y^3 = 3, \qquad x^2 - y^2 = 2$$