UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG
School of Computer Science and Applied Mathematics

Numerical Methods
Lab 4

19th March 2018

Some notes: In class, a number of methods for obtaining interpolating polynomials that fit a given data are discussed. Two such methods are Lagrange method and Newton divided difference method.

In general, an nth degree interpolating polynomial can be fitted to the $n+1$ data points $(x_i, f(x_i))$, $i = 0, 1, \cdots, n$ in the form

$$P_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \cdots + a_n(x - x_0)(x - x_1)\cdots(x - x_{n-1}).$$

The $a_i's$ are the divided differences discussed in the note. If $n = 1$, we can fit a 1 degree polynomial to the 2 data points (x_0, y_0) and (x_1, y_1) in the form

$$P_n(x) = a_0 + a_1(x - x_0).$$

This is the linear interpolant for Newton's divided difference method. It is exactly the same as the linear interpolating formula that was discussed in section 1.2.1 of the note.

If $n = 2$, we fit a 2 degree polynomial to the 3 data points (x_0, y_0), (x_1, y_1) and (x_2, y_2) in the form

$$P_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1).$$

This is the quadratic interpolant for Newton's divided difference method. It is exactly the same as the quadratic interpolating formula that was discussed in section 1.2.2 of the note.

Some disadvantages of Lagrange interpolation formula are that:

- When the number of data points are changed, be it a decrease or an increase in the number of data points, the results of previous computations cannot be used. This means that a change of degree involves recomputation of all terms.
- For a polynomial of high degree the process involves a large number of multiplications, whence it may be quite slow as the amount of computations become large.

Some advantages of Lagrange interpolation formula are that:

- The formula is simple and easy to remember.
- There is no need to construct the divided difference table.

Some advantages of Newton's divided difference interpolation formula are that:

- When the number of data points are changed, the result of the previous computation can be used. Each higher order interpolant builds on the previously computed terms in the series.
- It takes less computational time.

Finally, an observation, based on the example done in class is that an approximation function $p(x)$ gives a good approximation to $f(x)$ if we increase the number of data points used to generate the interpolating function. This is generally true for most functions.

Instructions

- Your code should be able to communicate the appropriate message, in the case of a computational problem. Remember, you can always estimate the number of iterations needed, given a tolerance.

Questions 1

Thermistors are used to measure the temperature of bodies. Thermistors are based on material's change in resistance with temperature. To measure temperature, manufacturers provide you with a temperature versus resistance calibration curve. If you measure resistance, you can find the temperature. A manufacturer of thermistors makes several observations with a thermistor, which are given below.

R(ohm)	T($\circ C$)
1101.0	25.113
911.3	30.131
636.0	40.120
451.1	50.128
233.5	60.136

1. Using your function in Exercise 1 of lab 4, obtain the divided difference table, T and the Newton's polynomial coefficients.
2. Using your function in Exercise 2 of lab 4, Obtain the values of the interpolating polynomial, Tq, at the query points Rq = [1050.1, 901.56, 875.11, 711.40, 545.27, 333.1, 200]. Plot T versus R from the table. Also, plot Tq versus Rq on same graph. Remember to use a marker in your plot to differentiate between the two graphs.

Questions 2

Consider interpolating the Runge function $f(x) = \frac{1}{1+25x^2}$ on $x \in [-5, 5]$. Define a set of n = 5 data values xdata which are evenly spaced from -5 to 5, and set ydata to the value of the Runge function at these points. Now, construct the interpolating polynomial to this data using your function in Exercise 2. Define a set of 101 query values xq also evenly spaced between -5 and 5. Evaluate the runge function and the polynomial at each of these points, and call the vectors of results respectively, y and yq. Find the maximum absolute value of the difference between the entries of y and yq. Repeat the process for n = 10, 20 and 30. Make a table containing four lines

n=5	Max difference =
n=10	Max difference =
n=20	Max difference =
n=30	Max difference =

Isn't there a theorem that says that on a given closed interval, any continuous function can be uniformly well approximated by a polynomial? What is the relationship between your evidence, and this fact?

Questions 3

Attempt Exercise 3 of Lab 4 using $f(x) = \ln x$. Now, use data values n= 10, 20 and 30 on (0,10]. For the query points xq, define a set of 101 query points.

How does this function compare to the Runge function in Question 2?