



## Numerical Methods

### Lab 4

15 March 2018

#### Instructions

- Read all the instructions carefully.
- MATLAB has a help file for every function if you get stuck.
- There are also numerous sources available on the internet, Google is your friend!

#### SECTION 1

#### Introduction

This week, we will continue programming some actual numerical methods. Each new lab will have all of the previous labs' material and append new information where necessary.

#### SECTION 2

#### Vectors and Matrices

Vectors and matrices are entered in square brackets. Row vectors are written with their elements separated by spaces or commas (e.g. `[1 2 3]` or `[1,2,3]`), column vectors have their elements separated by semicolons (e.g. `[1;2;3]`). Matrices are entered similarly, e.g. `[1 2 3; 4 5 6]`. A vector can be transposed by using the `'` operator: e.g. `[1 2 3]'`. Matrices and vectors can be added (+) and multiplied (\*) just as in mathematics. A matrix or vector also be pairwise-multiplied and divided (`.*` and `./` respectively) by a matrix or vector of the same dimensions. The linear system  $Ax = b$  can be solved by typing `A\b`.

## 2.1

### Indexing

An element of a vector is referenced by an index in round brackets. Indices start from 1 (not 0 as in PYTHON) and can also be specified by a vector, e.g. `x(2)` returns the second element in the vector `x`, while `x([2 4])` returns the second and fourth elements.

Elements of a matrix are referred to by two comma separated indices, the first refers to the row and the second refers to the column, e.g. `A(1,2)`. An index colon may be used to refer to whole rows or columns of a matrix. E.g. `A(:,1)` refers to the first column of the matrix `A`, while `A(2,:)` refers to the second row of the matrix `A`.

## 2.2

### Manipulating Matrices

o By using the relevant index, elements in vectors and matrices can be changed or deleted. To change the 4th element in a vector, `x` to the value 3, use:

```
1 >> x(4) = 3
```

and to delete it, use:

```
1 >> x(4) = []
```

To change or delete more than one element use a vector of indices to indicate the positions of the elements that you wish to remove. E.g.

```
1 >> x([1 3 6]) = []
```

Vectors and matrices can also be concatenated (joined). E.g. if `y` and `z` are vectors, then

```
1 >> x = [y z]
```

creates a new vector `x` with all the elements of `y` and `z` in order.

Below is an example of a function that will convert four binary digits to a decimal number:

```
1 function [x] = FromBinary4(a,b,c,d)
2     % FromBinary4 converts 4 binary digits to a decimal number
3     % INPUTS: a,b,c,d (0 or 1) are the digits of the binary number
4     % OUTPUTS: x, the decimal number corresponding to (abcd)_2
5     x = a*8+b*4+c*2+d;
6 end
```

## 2.3

### Script Files

It is also possible in MATLAB to create a `.m` file without the function header. Such a file is known as a script file and serves the purpose of collecting together a number of commands you might otherwise have typed in the command window. You can run a script file from the editor by pressing the play button (or typing F5). A script can also be run from the Command Window by typing its name.

A good example of how to use a script is to call a function with specific values of its input variables. For example:

```
1 % BinaryTester.m
2 % This script calls FromBinary4.m with different inputs and displays
3 % the results neatly on the screen
4
5 d1 = 1; d2 = 0; d3 = 1; d4 = 0; % binary digits
6 result1 = FromBinary4(d1,d2,d3,d4); % result 1 is (1010)_2
7 d4 = 1; % change a digit
8 result2 = FromBinary4(d1,d2,d3,d4); % result 2 is (1011)_2
9 result3 = FromBinary4(1,1,1,1); % result 3 is (1111)_2
10
11 disp('First_Answer: ');
12 disp(result1);
13 disp('——');
14 disp('Second_Answer: ');
15 disp(result2);
16 disp('——');
17 disp('Third_Answer: ');
18 disp(result3);
19 disp('——');
```

If you run the script above, you'll notice that the workspace window updates itself to contain the new variables created in the script: d1, d2, d3, d4, result1, result2 and result3. This is because running the script is exactly the same thing as typing each of the commands in the script, one at a time, in the command window.

#### A script is very different from a function:

- Functions **cannot** access or edit any variables created in the command window. Variables created or modified within a function are **not** created or modified in the workspace outside of that function. The only way that the outside world can communicate with the function is by passing it INPUTS, and the only way a function can communicate with the outside world is by returning OUTPUTS.
- Any variable created in a script **will** be on the workspace, and any variable modified in a script **will also** be modified in the workspace. A script cannot take inputs and produce outputs. Especially as programs get larger, this is extremely undesirable behaviour. You should be very cautious about creating a script to do the job of a function!

## Loop Constructs

### 3.1

#### For Loops

The for loop is used to repeat a collection of statements a fixed number of times. The most common form (used in a script file, not on the command line) is:

```
1 for indexvar = j:m:k
2     statements
3 end
```

This will execute the statements a number of times with the index variable taking on the values from  $j$  to  $k$ , in increments of  $m$ . MATLAB uses a default increment of 1 if you leave out the increment  $m$ , i.e.  $j:k$  is equivalent to using  $j:1:k$ . A more general form of the for loop is given by:

```
1 for indexvar = v
2     statements
3 end
```

where  $v$  is any vector. In this case, the index will take on the values of each vector element in turn. Using either form, the index variable can be used by the statements section of the loop, but changing its value will not cause the loop to skip steps.

### 3.2

#### While Loops

The while loop is a construct that repeats statements while a condition remains true. It is used to repeat a collection of statements a variable number of times. The general form of the while loop is:

```
1 while condition
2     statements
3 end
```

**Note** that the variables that make up the condition being tested must be changed by the statements, otherwise the while loop will **run forever!**

### 3.3

#### Nested Loops

A nested loop is a loop within a loop: an inner loop within the body of an outer one. These loops may be for or while loops. For example, the following construction shows two nested for loops:

```
1 for indexvar1 = j:k
2     for indexvar2 = l:m
3         statements
4     end
5 end
```

Notice that within each loop, indentation (extra spacing) is used to indicate the level of nesting - this enhances readability of the code and is good coding practice. The index ranges of the inner loops may depend on the index values of the outer loops.

### 3.4

#### If-Else Loops

The if-elseif-else statement allows conditional execution of code fragments depending on one or more conditional expressions. The construction of such loops like the following:

```

1  if condition1
2      .
3      statementsA
4      .
5  elseif condition2
6      .
7      statementsB
8      .
9  else
10     .
11     statementsC
12     .
13 end

```

Both the elseif and else clauses are optional and more than one elseif clause is allowed. Remember - as always the construction should terminate with an end.

#### SECTION 4

### Some Useful MATLAB Built-in Commands

Command	Result
<code>zeros(m,n)</code>	Creates an $m \times n$ matrix consisting of all zeros.
<code>ones(m,n)</code>	Creates an $m \times n$ matrix consisting of all ones.
<code>eye(m,n)</code>	Creates an $m \times n$ Identity matrix.
<code>X = diag(v,k)</code>	Creates a diagonal matrix with elements of $v$ on the $k$ th diagonal. $k = 0$ is the main diagonal.
<code>v = diag(X)</code>	Returns a vector of the main diagonal of matrix $X$ .
<code>transpose(X)</code>	Returns the transpose of matrix $X$ .
<code>trace(X)</code>	Returns the sum of the elements on the main diagonal of $X$ .
<code>inv(X)</code>	Returns the inverse of the square matrix $X$ .
<code>det(X)</code>	Returns the determinant of the square matrix $X$ .
<code>[V, D] = eig(X)</code>	Returns the eigenvalues and eigenvectors of matrix $X$ .
<code>fliplr(X)</code>	Returns $X$ with rows preserved and columns flipped in the left/right direction.
<code>flipud(X)</code>	Returns $X$ with columns preserved and rows flipped in the up/down direction.
<code>tril(X,k)</code>	Extracts the lower triangular part of the matrix $X$ , consisting of the elements on and below the $k$ th diagonal.
<code>triu(X,k)</code>	Extracts the upper triangular part of the matrix $X$ , consisting of the elements on and above the $k$ th diagonal.
<code>linspace(a,b,n)</code>	Returns a vector of $n$ equally spaced points between (and including) $a$ and $b$ .

## Main Exercises

The due date for the lab submission is the 6th of April at 12 pm. The block ends next week Friday 23/03/2018 and it is APPM3021 test next week Thursday. It just makes sense to defer the submission. Tomorrow at 10:15am, we can have a lab session for a period and do lecture in the second. Friday will be kept as tutorial time.

### 5.1

#### Exercise 1

Program the Newton's Divided Difference for Interpolation. Your function should take as two inputs, some  $x$  column vector and some  $y$  column vector (the respective  $y$  measurement of  $x$ ). Your function should also output two variables. The variable  $co$  as a row vector giving the coefficients of the Newton Polynomial and  $T$ , a matrix containing the divided difference values - you may assume this to be lower or upper triangular in nature and thus pad the matrix with zeros where necessary. The first line of your function should look like:

```
1 function [co,T] = NewtonInterp(x,y)
```

### 5.2

#### Exercise 2

Modify your code in Exercise 1 so that it can take in vector  $xq$  whose coordinates are the query points. That is, interpolate at certain values of  $x$ , and return the approximate values  $yq$  at the query points  $xq$ . The first line of your function should look like:

```
1 function yq = NewtonInterp(x,y,xq)
```

### 5.3

#### Exercise 3

Generate  $N$  random numbers for the variable  $x$  from the continuous uniform distribution on the interval  $(0,5)$  using the `unifrnd` or `rand` function. Sort your data accordingly. The corresponding  $y$  data is obtained from the relation  $y = f(x)$ , where  $f(x)$  is any smooth function of  $x$ . Use the interpolation function in Exercise 2, based on Newton's divided difference method, to obtain the interpolating polynomial for the  $(x,y)$  data set, and then evaluate the interpolating polynomial at the set of points  $xq$ , where  $xq$  is a vector of  $\frac{N}{2}$  random numbers, sorted after being generated from the continuous uniform distribution on the interval  $(0,5)$  using the `unifrnd` or `rand` function.

Plot  $y$  against  $x$ . Also, do plot  $yq$  against  $xq$ . Remember to use a circle marker in your plot to differentiate between the two graphs.