APPM3021 Numerical Methods • Lab 5 • 26/5/2018
Tyson Cross 1239448

Exercises:

Exercise1.m :

```
% APPM3021 Lab 5 Exercise 1
% Tyson Cross 1239448
clear all;
syms x y;
x0 = 0;
xf = 3;
y0 = 1;
f(x,y) = y*(1 -(4*x)/3);
function_name = func2str(matlabFunction(f));
function_name = function_name(8:end);
N = 10;

% Numerical solution
[X,Y] = Euler(f,x0,y0,N,xf);

% Confirm exact solution
[X_solution,Y_solution,sol] = ExactODE(f,x0,y0,N,xf);

err = abs((Y_solution - Y)./Y_solution);

% Display results
Exercise1_Disp
```

When Exercise1.m is run in the workspace, the following output is displayed to the command window:

```
N = 10
h = 0.3
x is an element of [0,3]
f(x,y) = y.*(x.*(4.0./3.0)-1.0)
Exact Solution: y = exp(-(x*(2*x - 3))/3)

          Euler's Method:

     x         y          Exact          RE
    ___    ________    ________    ________

      0           1           1           0
    0.3         1.3      1.2712    0.022616
    0.6       1.534      1.4333    0.070235
    0.9       1.626      1.4333     0.13445
    1.2      1.5285      1.2712     0.20234
    1.5      1.2534           1     0.25335
    1.8     0.87735     0.69768     0.25753
    2.1     0.50886     0.43171     0.17871
    2.4     0.23408     0.23693    0.012037
    2.7    0.079586     0.11533      0.3099
      3    0.017509    0.049787     0.64832
```

Exercise2.m :

```
% APPM3021 Lab 5 Exercise 2
% Tyson Cross 1239448
clear all;
syms x y;
x0 = 0;
xf = 3;
y0 = 1;
f(x,y) = y*(1 -(4*x)/3);
function_name = func2str(matlabFunction(f));
function_name = function_name(8:end);
N = 10;
```

```
% Numerical solution
[X,Y] = Heun(f,x0,y0,N,xf);

% Confirm exact solution
[X_solution,Y_solution,sol] = ExactODE(f,x0,y0,N,xf);

err = abs((Y_solution - Y)./Y_solution);

% Display results
Exercise2_Disp
```

When Exercise2.m is run in the workspace, the following output is displayed to the command window:

```
N = 10
h = 0.3
x is an element of [0,3]
f(x,y) = y.*(x.*(4.0./3.0)-1.0)
Exact Solution: y = exp(-(x*(2*x - 3))/3)

            Heun's Method:

     x         y          Exact           RE
    ___    ________    ________    __________

      0           1           1             0
    0.3       1.267      1.2712     0.0033425
    0.6      1.4259      1.4333      0.005196
    0.9      1.4233      1.4333     0.0069867
    1.2      1.2602      1.2712      0.008689
    1.5     0.99178           1       0.00822
    1.8     0.69722     0.69768    0.00065216
    2.1     0.44162     0.43171      0.022954
    2.4     0.25534     0.23693      0.077732
    2.7     0.13722     0.11533       0.18987
      3    0.070121    0.049787       0.40841
```

Exercise3.m :

```
% APPM3021 Lab 5 Exercise 3
% Tyson Cross 1239448
clear all;
syms x y;
x0 = 0;
xf = 3;
y0 = 1;
f(x,y) = y*(1 -(4*x)/3);
function_name = func2str(matlabFunction(f));
function_name = function_name(8:end);
N = 10;

% Numerical solution
[X,Y] = Midpoint(f,x0,y0,N,xf);

% Confirm exact solution
[X_solution,Y_solution,sol] = ExactODE(f,x0,y0,N,xf);

err = abs((Y_solution - Y)./Y_solution);

% Display results
Exercise3_Disp
```

When Exercise3.m is run in the workspace, the following output is displayed to the command window:

```
N = 10
h = 0.3
x is an element of [0,3]
f(x,y) = y.*(x.*(4.0./3.0)-1.0)
Exact Solution: y = exp(-(x*(2*x - 3))/3)

            Midpoint Method:

     x         y          Exact          RE
```

0	1	1	0
0.3	1.276	1.2712	0.0037372
0.6	1.4429	1.4333	0.0066777
0.9	1.4429	1.4333	0.0066777
1.2	1.2749	1.2712	0.0029089
1.5	0.9965	1	0.0035013
1.8	0.69157	0.69768	0.0087522
2.1	0.42933	0.43171	0.0055218
2.4	0.24128	0.23693	0.018376
2.7	0.12489	0.11533	0.082916
3	0.060895	0.049787	0.22311

Exercise4.m :

```
% APPM3021 Lab 5 Exercise 4
% Tyson Cross 1239448
clear all;
syms x y;
x0 = 0;
xf = 3;
y0 = 1;
f(x,y) = y*(1 -(4*x)/3);
function_name = func2str(matlabFunction(f));
function_name = function_name(8:end);
N = 10;

% Numerical solution
[X,Y] = RK4(f,x0,y0,N,xf);

% Confirm exact solution
[X_solution,Y_solution,sol] = ExactODE(f,x0,y0,N,xf);

err = abs((Y_solution - Y)./Y_solution);

% Display results
Exercise4_Disp
```

When Exercise4.m is run in the workspace, the following output is displayed to the command window:

```
N = 10
h = 0.3
x is an element of [0,3]
f(x,y) = y.*(x.*(4.0./3.0)-1.0)
Exact Solution: y = exp(-(x*(2*x - 3))/3)

            Runge-Kutta Method:
```

x	y	Exact	RE
0	1	1	0
0.3	1.2712	1.2712	1.2232e-05
0.6	1.4333	1.4333	1.4336e-05
0.9	1.4333	1.4333	1.4336e-05
1.2	1.2712	1.2712	1.6542e-05
1.5	0.99998	1	1.9416e-05
1.8	0.69769	0.69768	2.0822e-05
2.1	0.43183	0.43171	0.00028245
2.4	0.23723	0.23693	0.0012564
2.7	0.11579	0.11533	0.0040528
3	0.05033	0.049787	0.0109

Exercise5a.m :

```
% APPM3021 Lab 5 Exercise 5a
% Tyson Cross 1239448
clc; clear all; set(0,'ShowHiddenHandles','on'); delete(get(0,'Children'));
syms x y;
x0 = 0;
xf = 3;
y0 = 1;
f(x,y) = (1 -(4/3)*x)*y;
% g = @(y1,y2) [y1;y2;-(4/3)*y1];

function_name = func2str(matlabFunction(f));
function_name = function_name(7:end);
N1 = 30;
N2 = 300;

%% Calculations
fprintf('Calculating...');                          tic;

[X1,Y1] = Euler(f,x0,y0,N1,xf);                     fprintf('.');
[X2,Y2] = Euler(f,x0,y0,N2,xf);                      fprintf('.');

% Confirm exact solution
[X_sol1,Y_sol1] = ExactODE(f,x0,y0,N1,xf);           fprintf('.');
[X_sol2,Y_sol2,sol] = ExactODE(f,x0,y0,N2,xf);       fprintf('.');

err1 = abs((Y_sol1 - Y1)./Y_sol1);
err1(isnan(err1)) = 0;

err2 = abs((Y_sol2 - Y2)./Y_sol2);
err2(isnan(err2)) = 0;

t = toc;                                             fprintf('\n');
disp(['Calculations complete in ',num2str(t)]);

% Display results
Exercise5a_Disp

% Plot results
Exercise5a_Plot
```

When Exercise5a.m is run in the workspace, the following output is displayed to the command window:

```
Calculating.......
Calculations complete in 6.4687

N1 = 30
h1 = 0.1
x is an element of [0,3]
f(x,y) = -y.*(x.*(4.0./3.0)-1.0)
Exact Solution: y = exp(-(x*(2*x - 3))/3)

            Euler's Method:

     x         y          Exact          RE
    ___    ________    ________    _________

      0           1           1           0
    0.1         1.1      1.0978   0.0019788
    0.2      1.1953      1.1893   0.0051047
    0.3       1.283      1.2712   0.0092366
    0.4        1.36      1.3409     0.01423
    0.5      1.4234      1.3956    0.019936
    0.6      1.4709      1.4333    0.026201
    0.7      1.5003      1.4526    0.032861
    0.8      1.5103      1.4526    0.039747
    0.9      1.5002      1.4333    0.046678
      1      1.4702      1.3956    0.053466
    1.1      1.4212      1.3409     0.05991
    1.2      1.3549      1.2712    0.065801
    1.3      1.2736      1.1893     0.07092
    1.4      1.1802      1.0978    0.075039
    1.5      1.0779           1    0.077922
    1.6     0.97013     0.89883    0.079331
    1.7     0.86018     0.79719    0.079022
    1.8     0.75123     0.69768    0.076754
    1.9     0.64605      0.6025    0.072288
      2     0.54699     0.51342    0.065396
    2.1     0.45583     0.43171    0.055863
    2.2     0.37378      0.3582    0.043493
```

2.3	0.30151	0.29327	0.028117
2.4	0.2392	0.23693	0.0095965
2.5	0.18658	0.18888	0.012169
2.6	0.14304	0.14857	0.037236
2.7	0.10776	0.11533	0.06561
2.8	0.079741	0.088331	0.097241
2.9	0.057945	0.066759	0.13202
3	0.041334	0.049787	0.16978

N2 = 300
h2 = 0.01
x is an element of [0,3]
f(x,y) = -y.*(x.*(4.0./3.0)-1.0)
Exact Solution: y = exp(-(x*(2*x - 3))/3)

Euler's Method:

x	y	Exact	RE
0	1	1	0
0.01	1.01	1.01	1.6998e-05
0.02	1.02	1.0199	3.5307e-05
0.03	1.0299	1.0298	5.4911e-05
0.04	1.0398	1.0397	7.5792e-05
0.05	1.0496	1.0495	9.7932e-05
0.06	1.0594	1.0593	0.00012132
0.07	1.0692	1.069	0.00014592
0.08	1.0789	1.0787	0.00017174
0.09	1.0885	1.0883	0.00019874
0.1	1.0981	1.0978	0.00022692
0.11	1.1076	1.1073	0.00025626
0.12	1.117	1.1167	0.00028673
0.13	1.1264	1.1261	0.00031832
0.14	1.1357	1.1353	0.00035102
0.15	1.145	1.1445	0.0003848
0.16	1.1541	1.1537	0.00041965
0.17	1.1632	1.1627	0.00045555
0.18	1.1722	1.1716	0.00049248
0.19	1.1811	1.1805	0.00053044
0.2	1.1899	1.1893	0.00056939
0.21	1.1987	1.1979	0.00060932
0.22	1.2073	1.2065	0.00065022
0.23	1.2158	1.215	0.00069206
0.24	1.2243	1.2234	0.00073483
0.25	1.2326	1.2316	0.00077851
0.26	1.2408	1.2398	0.00082309
0.27	1.2489	1.2478	0.00086855
0.28	1.2569	1.2558	0.00091486
0.29	1.2648	1.2636	0.00096202
0.3	1.2725	1.2712	0.00101
0.31	1.2802	1.2788	0.0010588
0.32	1.2877	1.2863	0.0011084
0.33	1.2951	1.2936	0.0011587
0.34	1.3023	1.3007	0.0012098
0.35	1.3094	1.3078	0.0012617
0.36	1.3164	1.3147	0.0013142
0.37	1.3233	1.3215	0.0013675
0.38	1.33	1.3281	0.0014214
0.39	1.3365	1.3346	0.0014761
0.4	1.3429	1.3409	0.0015313
0.41	1.3492	1.3471	0.0015872
0.42	1.3553	1.3531	0.0016437
0.43	1.3613	1.359	0.0017008
0.44	1.3671	1.3647	0.0017585
0.45	1.3727	1.3703	0.0018168
0.46	1.3782	1.3757	0.0018756
0.47	1.3836	1.3809	0.0019349
0.48	1.3887	1.386	0.0019947
0.49	1.3937	1.3909	0.0020551
0.5	1.3986	1.3956	0.0021159
0.51	1.4032	1.4002	0.0021771
0.52	1.4077	1.4046	0.0022388
0.53	1.412	1.4088	0.0023009
0.54	1.4162	1.4128	0.0023634
0.55	1.4201	1.4167	0.0024263
0.56	1.4239	1.4204	0.0024896

0.57	1.4275	1.4239	0.0025532
0.58	1.431	1.4272	0.0026172
0.59	1.4342	1.4304	0.0026815
0.6	1.4373	1.4333	0.002746
0.61	1.4401	1.4361	0.0028109
0.62	1.4428	1.4387	0.002876
0.63	1.4453	1.4411	0.0029414
0.64	1.4476	1.4433	0.0030069
0.65	1.4498	1.4453	0.0030727
0.66	1.4517	1.4472	0.0031387
0.67	1.4534	1.4488	0.0032049
0.68	1.455	1.4502	0.0032712
0.69	1.4563	1.4515	0.0033376
0.7	1.4575	1.4526	0.0034042
0.71	1.4585	1.4534	0.0034709
0.72	1.4593	1.4541	0.0035376
0.73	1.4598	1.4546	0.0036045
0.74	1.4602	1.4549	0.0036713
0.75	1.4604	1.455	0.0037382
0.76	1.4604	1.4549	0.0038052
0.77	1.4602	1.4546	0.0038721
0.78	1.4598	1.4541	0.003939
0.79	1.4593	1.4534	0.0040058
0.8	1.4585	1.4526	0.0040726
0.81	1.4575	1.4515	0.0041393
0.82	1.4563	1.4502	0.0042059
0.83	1.455	1.4488	0.0042725
0.84	1.4534	1.4472	0.0043388
0.85	1.4517	1.4453	0.0044051
0.86	1.4498	1.4433	0.0044711
0.87	1.4476	1.4411	0.004537
0.88	1.4453	1.4387	0.0046027
0.89	1.4428	1.4361	0.0046682
0.9	1.4401	1.4333	0.0047334
0.91	1.4372	1.4304	0.0047984
0.92	1.4342	1.4272	0.0048631
0.93	1.4309	1.4239	0.0049275
0.94	1.4275	1.4204	0.0049916
0.95	1.4239	1.4167	0.0050554
0.96	1.4201	1.4128	0.0051188
0.97	1.4161	1.4088	0.0051818
0.98	1.4119	1.4046	0.0052445
0.99	1.4076	1.4002	0.0053068
1	1.4031	1.3956	0.0053687
1.01	1.3984	1.3909	0.0054301
1.02	1.3936	1.386	0.0054911
1.03	1.3886	1.3809	0.0055516
1.04	1.3834	1.3757	0.0056116
1.05	1.378	1.3703	0.0056711
1.06	1.3725	1.3647	0.0057301
1.07	1.3668	1.359	0.0057885
1.08	1.361	1.3531	0.0058464
1.09	1.355	1.3471	0.0059037
1.1	1.3489	1.3409	0.0059604
1.11	1.3426	1.3346	0.0060164
1.12	1.3361	1.3281	0.0060719
1.13	1.3296	1.3215	0.0061267
1.14	1.3228	1.3147	0.0061808
1.15	1.3159	1.3078	0.0062342
1.16	1.3089	1.3007	0.0062869
1.17	1.3018	1.2936	0.0063389
1.18	1.2945	1.2863	0.0063902
1.19	1.2871	1.2788	0.0064407
1.2	1.2795	1.2712	0.0064904
1.21	1.2718	1.2636	0.0065393
1.22	1.264	1.2558	0.0065874
1.23	1.2561	1.2478	0.0066347
1.24	1.2481	1.2398	0.0066811
1.25	1.2399	1.2316	0.0067266
1.26	1.2316	1.2234	0.0067712
1.27	1.2233	1.215	0.006815
1.28	1.2148	1.2065	0.0068578
1.29	1.2062	1.1979	0.0068997
1.3	1.1975	1.1893	0.0069406
1.31	1.1887	1.1805	0.0069805
1.32	1.1799	1.1716	0.0070194
1.33	1.1709	1.1627	0.0070573
1.34	1.1618	1.1537	0.0070942
1.35	1.1527	1.1445	0.00713

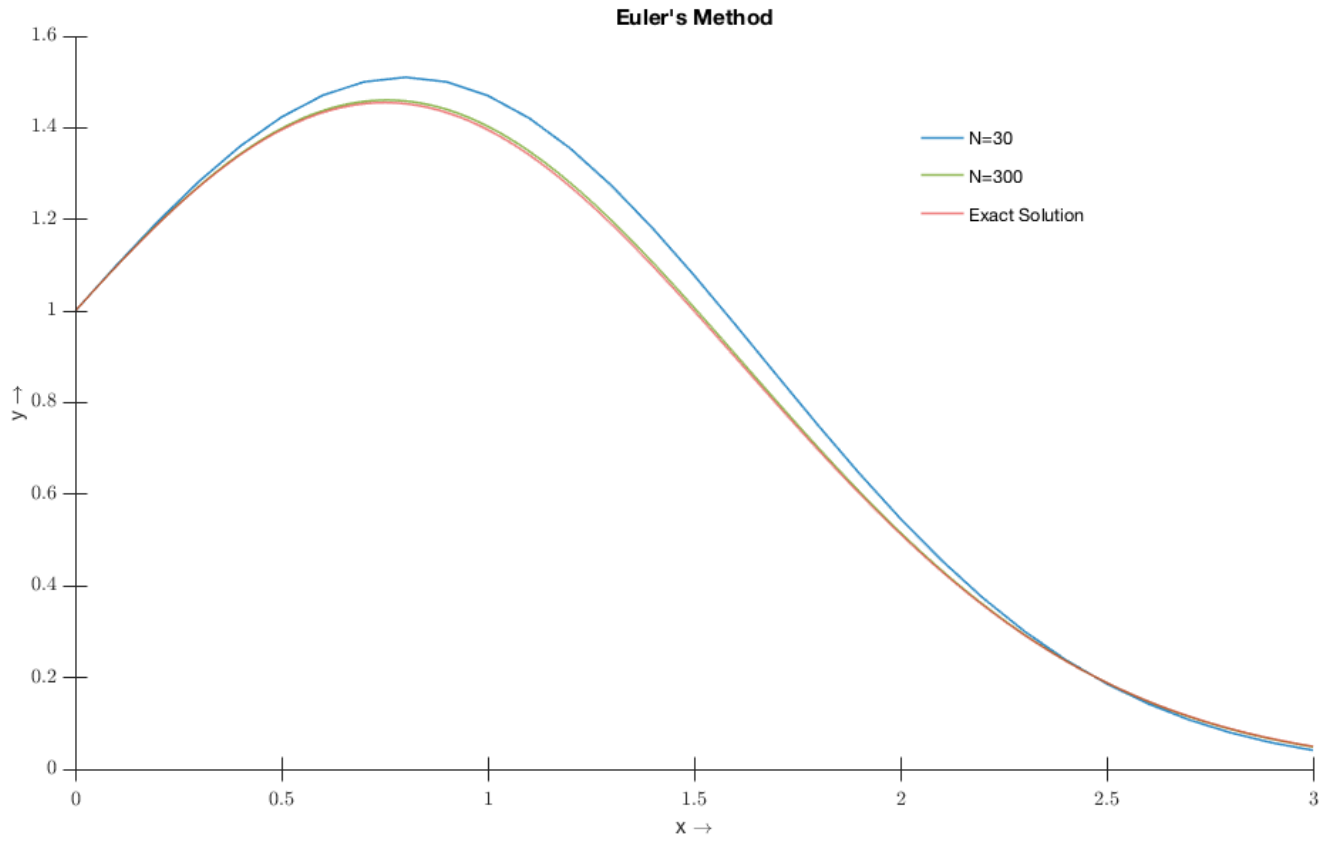
1.36	1.1435	1.1353	0.0071647
1.37	1.1342	1.1261	0.0071984
1.38	1.1248	1.1167	0.0072309
1.39	1.1154	1.1073	0.0072623
1.4	1.1058	1.0978	0.0072926
1.41	1.0962	1.0883	0.0073217
1.42	1.0866	1.0787	0.0073496
1.43	1.0769	1.069	0.0073764
1.44	1.0671	1.0593	0.0074019
1.45	1.0573	1.0495	0.0074261
1.46	1.0474	1.0397	0.0074491
1.47	1.0375	1.0298	0.0074709
1.48	1.0276	1.0199	0.0074913
1.49	1.0176	1.01	0.0075104
1.5	1.0075	1	0.0075282
1.51	0.99745	0.98998	0.0075447
1.52	0.98735	0.97994	0.0075598
1.53	0.97721	0.96986	0.0075735
1.54	0.96705	0.95977	0.0075858
1.55	0.95686	0.94965	0.0075967
1.56	0.94665	0.93951	0.0076061
1.57	0.93643	0.92935	0.0076141
1.58	0.92619	0.91919	0.0076206
1.59	0.91594	0.90901	0.0076256
1.6	0.90568	0.89883	0.0076291
1.61	0.89542	0.88864	0.0076311
1.62	0.88515	0.87845	0.0076315
1.63	0.87488	0.86826	0.0076303
1.64	0.86462	0.85807	0.0076276
1.65	0.85436	0.84789	0.0076233
1.66	0.84411	0.83772	0.0076173
1.67	0.83386	0.82757	0.0076097
1.68	0.82363	0.81742	0.0076005
1.69	0.81342	0.80729	0.0075895
1.7	0.80323	0.79719	0.0075769
1.71	0.79305	0.7871	0.0075625
1.72	0.7829	0.77704	0.0075465
1.73	0.77278	0.767	0.0075286
1.74	0.76268	0.75699	0.007509
1.75	0.75261	0.74702	0.0074876
1.76	0.74258	0.73707	0.0074645
1.77	0.73258	0.72717	0.0074394
1.78	0.72261	0.7173	0.0074126
1.79	0.71269	0.70747	0.0073839
1.8	0.70281	0.69768	0.0073533
1.81	0.69297	0.68793	0.0073208
1.82	0.68317	0.67823	0.0072864
1.83	0.67343	0.66858	0.00725
1.84	0.66373	0.65898	0.0072117
1.85	0.65408	0.64943	0.0071715
1.86	0.64449	0.63993	0.0071292
1.87	0.63495	0.63048	0.0070849
1.88	0.62547	0.6211	0.0070387
1.89	0.61605	0.61177	0.0069903
1.9	0.60668	0.6025	0.00694
1.91	0.59738	0.59329	0.0068875
1.92	0.58814	0.58415	0.0068329
1.93	0.57896	0.57507	0.0067762
1.94	0.56986	0.56605	0.0067174
1.95	0.56081	0.55711	0.0066565
1.96	0.55184	0.54823	0.0065933
1.97	0.54294	0.53942	0.006528
1.98	0.53411	0.53068	0.0064605
1.99	0.52535	0.52201	0.0063908
2	0.51666	0.51342	0.0063188
2.01	0.50805	0.5049	0.0062445
2.02	0.49952	0.49645	0.006168
2.03	0.49106	0.48808	0.0060892
2.04	0.48268	0.47979	0.006008
2.05	0.47437	0.47158	0.0059246
2.06	0.46615	0.46345	0.0058388
2.07	0.45801	0.45539	0.0057506
2.08	0.44995	0.44742	0.00566
2.09	0.44197	0.43952	0.005567
2.1	0.43407	0.43171	0.0054716
2.11	0.42626	0.42398	0.0053738
2.12	0.41853	0.41633	0.0052735
2.13	0.41088	0.40877	0.0051708
2.14	0.40332	0.40129	0.0050655

2.15	0.39585	0.3939	0.0049577
2.16	0.38846	0.38659	0.0048474
2.17	0.38116	0.37936	0.0047346
2.18	0.37394	0.37222	0.0046192
2.19	0.36681	0.36517	0.0045012
2.2	0.35977	0.3582	0.0043806
2.21	0.35281	0.35132	0.0042574
2.22	0.34594	0.34452	0.0041316
2.23	0.33916	0.33781	0.0040031
2.24	0.33247	0.33119	0.003872
2.25	0.32587	0.32465	0.0037381
2.26	0.31935	0.3182	0.0036016
2.27	0.31292	0.31184	0.0034623
2.28	0.30658	0.30556	0.0033203
2.29	0.30032	0.29937	0.0031756
2.3	0.29416	0.29327	0.0030281
2.31	0.28808	0.28725	0.0028777
2.32	0.28209	0.28132	0.0027246
2.33	0.27618	0.27547	0.0025687
2.34	0.27036	0.26971	0.0024099
2.35	0.26463	0.26404	0.0022482
2.36	0.25898	0.25845	0.0020837
2.37	0.25343	0.25294	0.0019162
2.38	0.24795	0.24752	0.0017459
2.39	0.24256	0.24218	0.0015726
2.4	0.23726	0.23693	0.0013964
2.41	0.23204	0.23176	0.0012172
2.42	0.2269	0.22667	0.0010351
2.43	0.22185	0.22166	0.00084994
2.44	0.21688	0.21674	0.00066177
2.45	0.21199	0.21189	0.00047058
2.46	0.20719	0.20713	0.00027633
2.47	0.20247	0.20245	7.9027e-05
2.48	0.19782	0.19785	0.00012136
2.49	0.19326	0.19332	0.00032485
2.5	0.18878	0.18888	0.00053145
2.51	0.18437	0.18451	0.00074118
2.52	0.18004	0.18022	0.00095406
2.53	0.17579	0.176	0.0011701
2.54	0.17162	0.17186	0.0013893
2.55	0.16753	0.1678	0.0016118
2.56	0.16351	0.16381	0.0018374
2.57	0.15956	0.15989	0.0020663
2.58	0.15569	0.15605	0.0022984
2.59	0.15189	0.15228	0.0025337
2.6	0.14816	0.14857	0.0027724
2.61	0.14451	0.14495	0.0030144
2.62	0.14092	0.14139	0.0032596
2.63	0.13741	0.13789	0.0035082
2.64	0.13397	0.13447	0.0037601
2.65	0.13059	0.13112	0.0040154
2.66	0.12728	0.12783	0.0042741
2.67	0.12404	0.12461	0.0045362
2.68	0.12087	0.12145	0.0048016
2.69	0.11775	0.11836	0.0050705
2.7	0.11471	0.11533	0.0053429
2.71	0.11173	0.11236	0.0056186
2.72	0.10881	0.10945	0.0058979
2.73	0.10595	0.10661	0.0061807
2.74	0.10315	0.10382	0.0064669
2.75	0.10041	0.1011	0.0067567
2.76	0.097737	0.098431	0.00705
2.77	0.095118	0.095822	0.0073468
2.78	0.092556	0.093269	0.0076472
2.79	0.090051	0.090772	0.0079512
2.8	0.087601	0.088331	0.0082588
2.81	0.085207	0.085943	0.00857
2.82	0.082866	0.083609	0.0088848
2.83	0.080579	0.081328	0.0092033
2.84	0.078345	0.079098	0.0095254
2.85	0.076161	0.076919	0.0098512
2.86	0.074029	0.07479	0.010181
2.87	0.071946	0.072711	0.010514
2.88	0.069913	0.070679	0.010851
2.89	0.067927	0.068696	0.011191
2.9	0.065989	0.066759	0.011536
2.91	0.064097	0.064868	0.011884
2.92	0.062251	0.063022	0.012236
2.93	0.06045	0.061221	0.012591

```
    2.94    0.058693    0.059463      0.012951
    2.95    0.056979    0.057748      0.013314
    2.96    0.055308    0.056075      0.013681
    2.97    0.053678    0.054443      0.014052
    2.98    0.052089    0.052852      0.014427
    2.99     0.05054      0.0513      0.014806
       3    0.049031    0.049787      0.015188
```

When Exercise5a.m is run in the workspace, the following figure is generated:

Figure 1. Exercise 5a



Exercise5b.m :

```
% APPM3021 Lab 5 Exercise 5b
% Tyson Cross 1239448

clc; clear all; set(0,'ShowHiddenHandles','on'); delete(get(0,'Children'));
syms x y;
x0 = 0;
xf = 3;
y0 = 1;
f(x,y) = y*(1 -(4*x)/3);
function_name = func2str(matlabFunction(f));
function_name = function_name(8:end);
N = 300;

%% Calculations
fprintf('Calculating...');                          tic;

[X,Y_Euler] = Euler(f,x0,y0,N,xf);                  fprintf('.');
[~,Y_Heun] = Heun(f,x0,y0,N,xf);                    fprintf('.');
[~,Y_Midpoint] = Midpoint(f,x0,y0,N,xf);            fprintf('.');
```

```
[~,Y_RK4] = RK4(f,x0,y0,N,xf);                    fprintf('.');

% Confirm exact solution
[~,Y_solution,sol] = ExactODE(f,x0,y0,N,xf);        fprintf('.');

err1 = abs((Y_solution - Y_Euler)    ./Y_solution);
err2 = abs((Y_solution - Y_Heun)     ./Y_solution);
err3 = abs((Y_solution - Y_Midpoint) ./Y_solution);
err4 = abs((Y_solution - Y_RK4)      ./Y_solution);

err1(isnan(err1)) = 0;
err2(isnan(err2)) = 0;
err3(isnan(err3)) = 0;
err4(isnan(err4)) = 0;

t = toc;                                                fprintf('\n');
disp(['Calculations complete in ',num2str(t)]);

% Display results
Exercise5b_Disp

% Plot results
Exercise5b_Plot
```

When Exercise5b.m is run in the workspace, the following output is displayed to the command window:

```
Calculating........
Calculations complete in 33.8954

N = 300
h = 0.01
x is an element of [0,3]
f(x,y) = y.*(x.*(4.0./3.0)-1.0)
Exact Solution: y = exp(-(x*(2*x - 3))/3)

            Comparison of Methods:

     Euler        Heun       Midpoint      RK4         Exact
    ________    ________    ________    ________    ________

         1           1           1           1           1
      1.01        1.01        1.01        1.01        1.01
      1.02      1.0199      1.0199      1.0199      1.0199
    1.0299      1.0298      1.0298      1.0298      1.0298
    1.0398      1.0397      1.0397      1.0397      1.0397
    1.0496      1.0495      1.0495      1.0495      1.0495
    1.0594      1.0593      1.0593      1.0593      1.0593
    1.0692       1.069       1.069       1.069       1.069
    1.0789      1.0787      1.0787      1.0787      1.0787
    1.0885      1.0883      1.0883      1.0883      1.0883
    1.0981      1.0978      1.0978      1.0978      1.0978
    1.1076      1.1073      1.1073      1.1073      1.1073
     1.117      1.1167      1.1167      1.1167      1.1167
    1.1264      1.1261      1.1261      1.1261      1.1261
    1.1357      1.1353      1.1353      1.1353      1.1353
     1.145      1.1445      1.1445      1.1445      1.1445
    1.1541      1.1537      1.1537      1.1537      1.1537
    1.1632      1.1627      1.1627      1.1627      1.1627
    1.1722      1.1716      1.1716      1.1716      1.1716
    1.1811      1.1805      1.1805      1.1805      1.1805
    1.1899      1.1893      1.1893      1.1893      1.1893
    1.1987      1.1979      1.1979      1.1979      1.1979
    1.2073      1.2065      1.2065      1.2065      1.2065
    1.2158       1.215       1.215       1.215       1.215
    1.2243      1.2234      1.2234      1.2234      1.2234
    1.2326      1.2316      1.2316      1.2316      1.2316
    1.2408      1.2398      1.2398      1.2398      1.2398
    1.2489      1.2478      1.2478      1.2478      1.2478
    1.2569      1.2557      1.2558      1.2558      1.2558
    1.2648      1.2636      1.2636      1.2636      1.2636
    1.2725      1.2712      1.2713      1.2712      1.2712
    1.2802      1.2788      1.2788      1.2788      1.2788
    1.2877      1.2862      1.2863      1.2863      1.2863
    1.2951      1.2936      1.2936      1.2936      1.2936
    1.3023      1.3007      1.3007      1.3007      1.3007
    1.3094      1.3078      1.3078      1.3078      1.3078
    1.3164      1.3147      1.3147      1.3147      1.3147
    1.3233      1.3215      1.3215      1.3215      1.3215
      1.33      1.3281      1.3281      1.3281      1.3281
```

1.3365	1.3346	1.3346	1.3346	1.3346
1.3429	1.3409	1.3409	1.3409	1.3409
1.3492	1.3471	1.3471	1.3471	1.3471
1.3553	1.3531	1.3531	1.3531	1.3531
1.3613	1.359	1.359	1.359	1.359
1.3671	1.3647	1.3647	1.3647	1.3647
1.3727	1.3703	1.3703	1.3703	1.3703
1.3782	1.3757	1.3757	1.3757	1.3757
1.3836	1.3809	1.3809	1.3809	1.3809
1.3887	1.386	1.386	1.386	1.386
1.3937	1.3909	1.3909	1.3909	1.3909
1.3986	1.3956	1.3956	1.3956	1.3956
1.4032	1.4002	1.4002	1.4002	1.4002
1.4077	1.4046	1.4046	1.4046	1.4046
1.412	1.4088	1.4088	1.4088	1.4088
1.4162	1.4128	1.4128	1.4128	1.4128
1.4201	1.4167	1.4167	1.4167	1.4167
1.4239	1.4204	1.4204	1.4204	1.4204
1.4275	1.4239	1.4239	1.4239	1.4239
1.431	1.4272	1.4272	1.4272	1.4272
1.4342	1.4304	1.4304	1.4304	1.4304
1.4373	1.4333	1.4333	1.4333	1.4333
1.4401	1.4361	1.4361	1.4361	1.4361
1.4428	1.4387	1.4387	1.4387	1.4387
1.4453	1.4411	1.4411	1.4411	1.4411
1.4476	1.4433	1.4433	1.4433	1.4433
1.4498	1.4453	1.4453	1.4453	1.4453
1.4517	1.4472	1.4472	1.4472	1.4472
1.4534	1.4488	1.4488	1.4488	1.4488
1.455	1.4502	1.4503	1.4502	1.4502
1.4563	1.4515	1.4515	1.4515	1.4515
1.4575	1.4526	1.4526	1.4526	1.4526
1.4585	1.4534	1.4535	1.4534	1.4534
1.4593	1.4541	1.4541	1.4541	1.4541
1.4598	1.4546	1.4546	1.4546	1.4546
1.4602	1.4549	1.4549	1.4549	1.4549
1.4604	1.455	1.455	1.455	1.455
1.4604	1.4549	1.4549	1.4549	1.4549
1.4602	1.4546	1.4546	1.4546	1.4546
1.4598	1.4541	1.4541	1.4541	1.4541
1.4593	1.4534	1.4535	1.4534	1.4534
1.4585	1.4526	1.4526	1.4526	1.4526
1.4575	1.4515	1.4515	1.4515	1.4515
1.4563	1.4502	1.4503	1.4502	1.4502
1.455	1.4488	1.4488	1.4488	1.4488
1.4534	1.4472	1.4472	1.4472	1.4472
1.4517	1.4453	1.4453	1.4453	1.4453
1.4498	1.4433	1.4433	1.4433	1.4433
1.4476	1.4411	1.4411	1.4411	1.4411
1.4453	1.4387	1.4387	1.4387	1.4387
1.4428	1.4361	1.4361	1.4361	1.4361
1.4401	1.4333	1.4333	1.4333	1.4333
1.4372	1.4304	1.4304	1.4304	1.4304
1.4342	1.4272	1.4272	1.4272	1.4272
1.4309	1.4239	1.4239	1.4239	1.4239
1.4275	1.4204	1.4204	1.4204	1.4204
1.4239	1.4167	1.4167	1.4167	1.4167
1.4201	1.4128	1.4128	1.4128	1.4128
1.4161	1.4088	1.4088	1.4088	1.4088
1.4119	1.4046	1.4046	1.4046	1.4046
1.4076	1.4002	1.4002	1.4002	1.4002
1.4031	1.3956	1.3956	1.3956	1.3956
1.3984	1.3909	1.3909	1.3909	1.3909
1.3936	1.386	1.386	1.386	1.386
1.3886	1.3809	1.3809	1.3809	1.3809
1.3834	1.3757	1.3757	1.3757	1.3757
1.378	1.3703	1.3703	1.3703	1.3703
1.3725	1.3647	1.3647	1.3647	1.3647
1.3668	1.359	1.359	1.359	1.359
1.361	1.3531	1.3531	1.3531	1.3531
1.355	1.3471	1.3471	1.3471	1.3471
1.3489	1.3409	1.3409	1.3409	1.3409
1.3426	1.3346	1.3346	1.3346	1.3346
1.3361	1.3281	1.3281	1.3281	1.3281
1.3296	1.3215	1.3215	1.3215	1.3215
1.3228	1.3147	1.3147	1.3147	1.3147
1.3159	1.3078	1.3078	1.3078	1.3078
1.3089	1.3007	1.3007	1.3007	1.3007
1.3018	1.2936	1.2936	1.2936	1.2936

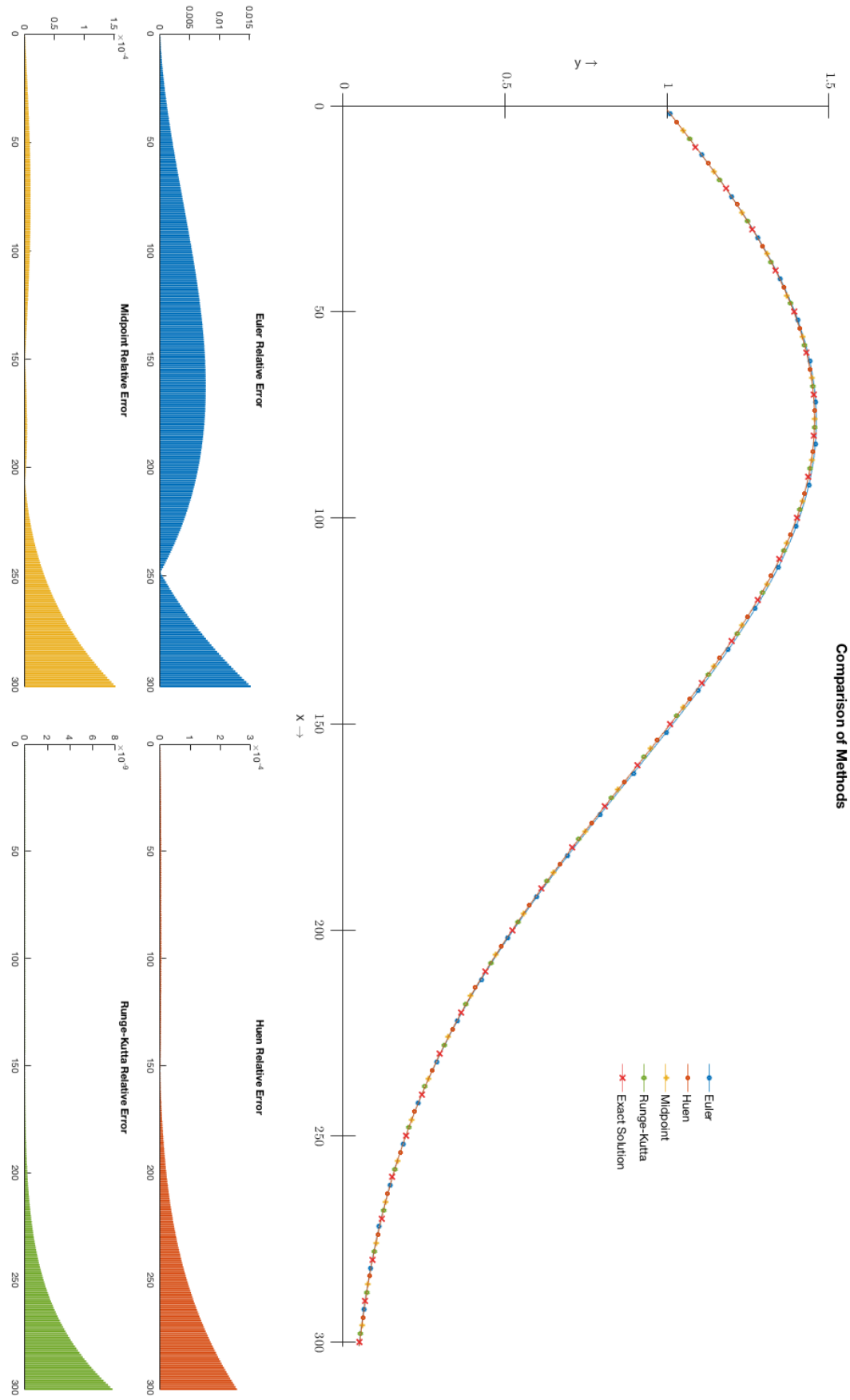
1.2945	1.2862	1.2863	1.2863	1.2863
1.2871	1.2788	1.2788	1.2788	1.2788
1.2795	1.2712	1.2713	1.2712	1.2712
1.2718	1.2636	1.2636	1.2636	1.2636
1.264	1.2557	1.2558	1.2558	1.2558
1.2561	1.2478	1.2478	1.2478	1.2478
1.2481	1.2398	1.2398	1.2398	1.2398
1.2399	1.2316	1.2316	1.2316	1.2316
1.2316	1.2234	1.2234	1.2234	1.2234
1.2233	1.215	1.215	1.215	1.215
1.2148	1.2065	1.2065	1.2065	1.2065
1.2062	1.1979	1.1979	1.1979	1.1979
1.1975	1.1893	1.1893	1.1893	1.1893
1.1887	1.1805	1.1805	1.1805	1.1805
1.1799	1.1716	1.1716	1.1716	1.1716
1.1709	1.1627	1.1627	1.1627	1.1627
1.1618	1.1537	1.1537	1.1537	1.1537
1.1527	1.1445	1.1445	1.1445	1.1445
1.1435	1.1353	1.1353	1.1353	1.1353
1.1342	1.1261	1.1261	1.1261	1.1261
1.1248	1.1167	1.1167	1.1167	1.1167
1.1154	1.1073	1.1073	1.1073	1.1073
1.1058	1.0978	1.0978	1.0978	1.0978
1.0962	1.0883	1.0883	1.0883	1.0883
1.0866	1.0787	1.0787	1.0787	1.0787
1.0769	1.069	1.069	1.069	1.069
1.0671	1.0593	1.0593	1.0593	1.0593
1.0573	1.0495	1.0495	1.0495	1.0495
1.0474	1.0397	1.0397	1.0397	1.0397
1.0375	1.0298	1.0298	1.0298	1.0298
1.0276	1.0199	1.0199	1.0199	1.0199
1.0176	1.01	1.01	1.01	1.01
1.0075	1	1	1	1
0.99745	0.98998	0.98998	0.98998	0.98998
0.98735	0.97994	0.97994	0.97994	0.97994
0.97721	0.96986	0.96986	0.96986	0.96986
0.96705	0.95977	0.95976	0.95977	0.95977
0.95686	0.94965	0.94964	0.94965	0.94965
0.94665	0.93951	0.93951	0.93951	0.93951
0.93643	0.92935	0.92935	0.92935	0.92935
0.92619	0.91919	0.91918	0.91919	0.91919
0.91594	0.90901	0.90901	0.90901	0.90901
0.90568	0.89883	0.89882	0.89883	0.89883
0.89542	0.88864	0.88864	0.88864	0.88864
0.88515	0.87845	0.87845	0.87845	0.87845
0.87488	0.86826	0.86826	0.86826	0.86826
0.86462	0.85807	0.85807	0.85807	0.85807
0.85436	0.8479	0.84789	0.84789	0.84789
0.84411	0.83773	0.83772	0.83772	0.83772
0.83386	0.82757	0.82756	0.82757	0.82757
0.82363	0.81743	0.81742	0.81742	0.81742
0.81342	0.8073	0.80729	0.80729	0.80729
0.80323	0.79719	0.79718	0.79719	0.79719
0.79305	0.7871	0.7871	0.7871	0.7871
0.7829	0.77704	0.77704	0.77704	0.77704
0.77278	0.76701	0.767	0.767	0.767
0.76268	0.757	0.75699	0.75699	0.75699
0.75261	0.74702	0.74702	0.74702	0.74702
0.74258	0.73708	0.73707	0.73707	0.73707
0.73258	0.72717	0.72716	0.72717	0.72717
0.72261	0.7173	0.71729	0.7173	0.7173
0.71269	0.70747	0.70746	0.70747	0.70747
0.70281	0.69768	0.69767	0.69768	0.69768
0.69297	0.68794	0.68793	0.68793	0.68793
0.68317	0.67824	0.67823	0.67823	0.67823
0.67343	0.66859	0.66858	0.66858	0.66858
0.66373	0.65898	0.65897	0.65898	0.65898
0.65408	0.64943	0.64942	0.64943	0.64943
0.64449	0.63994	0.63993	0.63993	0.63993
0.63495	0.63049	0.63048	0.63048	0.63048
0.62547	0.62111	0.6211	0.6211	0.6211
0.61605	0.61178	0.61177	0.61177	0.61177
0.60668	0.60251	0.6025	0.6025	0.6025
0.59738	0.5933	0.59329	0.59329	0.59329
0.58814	0.58416	0.58415	0.58415	0.58415
0.57896	0.57508	0.57507	0.57507	0.57507
0.56986	0.56606	0.56605	0.56605	0.56605
0.56081	0.55712	0.5571	0.55711	0.55711
0.55184	0.54824	0.54823	0.54823	0.54823

0.54294	0.53943	0.53942	0.53942	0.53942
0.53411	0.53069	0.53068	0.53068	0.53068
0.52535	0.52202	0.52201	0.52201	0.52201
0.51666	0.51343	0.51342	0.51342	0.51342
0.50805	0.50491	0.5049	0.5049	0.5049
0.49952	0.49646	0.49645	0.49645	0.49645
0.49106	0.4881	0.48808	0.48808	0.48808
0.48268	0.4798	0.47979	0.47979	0.47979
0.47437	0.47159	0.47158	0.47158	0.47158
0.46615	0.46346	0.46345	0.46345	0.46345
0.45801	0.4554	0.45539	0.45539	0.45539
0.44995	0.44743	0.44742	0.44742	0.44742
0.44197	0.43953	0.43952	0.43952	0.43952
0.43407	0.43172	0.43171	0.43171	0.43171
0.42626	0.42399	0.42398	0.42398	0.42398
0.41853	0.41635	0.41634	0.41633	0.41633
0.41088	0.40878	0.40877	0.40877	0.40877
0.40332	0.40131	0.40129	0.40129	0.40129
0.39585	0.39391	0.3939	0.3939	0.3939
0.38846	0.3866	0.38659	0.38659	0.38659
0.38116	0.37938	0.37936	0.37936	0.37936
0.37394	0.37224	0.37222	0.37222	0.37222
0.36681	0.36518	0.36517	0.36517	0.36517
0.35977	0.35821	0.3582	0.3582	0.3582
0.35281	0.35133	0.35132	0.35132	0.35132
0.34594	0.34454	0.34452	0.34452	0.34452
0.33916	0.33783	0.33781	0.33781	0.33781
0.33247	0.3312	0.33119	0.33119	0.33119
0.32587	0.32467	0.32466	0.32465	0.32465
0.31935	0.31822	0.31821	0.3182	0.3182
0.31292	0.31186	0.31184	0.31184	0.31184
0.30658	0.30558	0.30557	0.30556	0.30556
0.30032	0.29939	0.29938	0.29937	0.29937
0.29416	0.29328	0.29327	0.29327	0.29327
0.28808	0.28727	0.28725	0.28725	0.28725
0.28209	0.28133	0.28132	0.28132	0.28132
0.27618	0.27549	0.27548	0.27547	0.27547
0.27036	0.26973	0.26972	0.26971	0.26971
0.26463	0.26405	0.26404	0.26404	0.26404
0.25898	0.25846	0.25845	0.25845	0.25845
0.25343	0.25296	0.25295	0.25294	0.25294
0.24795	0.24754	0.24752	0.24752	0.24752
0.24256	0.2422	0.24219	0.24218	0.24218
0.23726	0.23694	0.23693	0.23693	0.23693
0.23204	0.23177	0.23176	0.23176	0.23176
0.2269	0.22669	0.22667	0.22667	0.22667
0.22185	0.22168	0.22167	0.22166	0.22166
0.21688	0.21675	0.21674	0.21674	0.21674
0.21199	0.21191	0.2119	0.21189	0.21189
0.20719	0.20715	0.20714	0.20713	0.20713
0.20247	0.20247	0.20246	0.20245	0.20245
0.19782	0.19786	0.19785	0.19785	0.19785
0.19326	0.19334	0.19333	0.19332	0.19332
0.18878	0.18889	0.18888	0.18888	0.18888
0.18437	0.18452	0.18451	0.18451	0.18451
0.18004	0.18023	0.18022	0.18022	0.18022
0.17579	0.17602	0.17601	0.176	0.176
0.17162	0.17188	0.17187	0.17186	0.17186
0.16753	0.16781	0.1678	0.1678	0.1678
0.16351	0.16382	0.16381	0.16381	0.16381
0.15956	0.15991	0.1599	0.15989	0.15989
0.15569	0.15606	0.15605	0.15605	0.15605
0.15189	0.15229	0.15228	0.15228	0.15228
0.14816	0.14859	0.14858	0.14857	0.14857
0.14451	0.14496	0.14495	0.14495	0.14495
0.14092	0.1414	0.14139	0.14139	0.14139
0.13741	0.13791	0.1379	0.13789	0.13789
0.13397	0.13449	0.13448	0.13447	0.13447
0.13059	0.13113	0.13112	0.13112	0.13112
0.12728	0.12784	0.12784	0.12783	0.12783
0.12404	0.12462	0.12461	0.12461	0.12461
0.12087	0.12146	0.12146	0.12145	0.12145
0.11775	0.11837	0.11836	0.11836	0.11836
0.11471	0.11534	0.11533	0.11533	0.11533
0.11173	0.11237	0.11237	0.11236	0.11236
0.10881	0.10947	0.10946	0.10945	0.10945
0.10595	0.10662	0.10662	0.10661	0.10661
0.10315	0.10384	0.10383	0.10382	0.10382
0.10041	0.10111	0.10111	0.1011	0.1011

```
0.097737    0.098447    0.098439    0.098431    0.098431
0.095118    0.095837     0.09583    0.095822    0.095822
0.092556    0.093285    0.093277    0.093269    0.093269
0.090051    0.090788     0.09078    0.090772    0.090772
0.087601    0.088346    0.088339    0.088331    0.088331
0.085207    0.085959    0.085951    0.085943    0.085943
0.082866    0.083624    0.083617    0.083609    0.083609
0.080579    0.081343    0.081336    0.081328    0.081328
0.078345    0.079113    0.079106    0.079098    0.079098
0.076161    0.076934    0.076927    0.076919    0.076919
0.074029    0.074805    0.074798     0.07479     0.07479
0.071946    0.072725    0.072719    0.072711    0.072711
0.069913    0.070694    0.070687    0.070679    0.070679
0.067927     0.06871    0.068704    0.068696    0.068696
0.065989    0.066773    0.066767    0.066759    0.066759
0.064097    0.064882    0.064876    0.064868    0.064868
0.062251    0.063036     0.06303    0.063022    0.063022
 0.06045    0.061235    0.061229    0.061221    0.061221
0.058693    0.059477    0.059471    0.059463    0.059463
0.056979    0.057761    0.057756    0.057748    0.057748
0.055308    0.056088    0.056083    0.056075    0.056075
0.053678    0.054456    0.054451    0.054443    0.054443
0.052089    0.052865    0.052859    0.052852    0.052852
 0.05054    0.051313    0.051308      0.0513      0.0513
0.049031      0.0498    0.049795    0.049787    0.049787
```

When Exercise5b.m is run in the workspace, the following figure is generated:

Figure 2. Exercise 5b



Exercise6.m :

```
% APPM3021 Lab 5 Exercise 6
% Tyson Cross 1239448

clc; clear all;

N = 30;
xspan = [0 5];
y0 = [0 0 1];

%{
    *Notes*

   y''' + y'' = 0
   {y' = dy/dx}

    which leads to
        y'   = y(2)
        y''  = y(3)
        y''' = -y(3)
%}

% function handles
f = @(x,y) [y(2);y(3);-y(3)];

function_name = 'y'''''' + y'''' = 0';

%% Calculations
fprintf('Calculating...');

% Numerical solution
[X,Y] = RK4system(f,xspan,y0,N);             fprintf('.');

% Confirm exact solution
tspan = linspace(xspan(1),xspan(end),N+1)';
[X_sol, Y_sol] = ode45(f, tspan, y0);           fprintf('.');

% Display results
Exercise6_Disp
```

When Exercise6.m is run in the workspace, the following output is displayed to the command window:

```
Calculating.....

N = 30
h = 0.16667
x is an element of [0  5]
f(x,y) = y''' + y'' = 0

            Runge-Kutta System Method:

       x          y          dy          d2y
    _______    _______    _______    _________

           0          0          0          1
     0.16667   0.013149    0.15352    0.84648
     0.33333   0.049866    0.28347    0.71653
         0.5    0.10653    0.39347    0.60653
     0.66667    0.18009    0.48658    0.51342
     0.83333    0.26793     0.5654     0.4346
           1    0.36788    0.63212    0.36788
      1.1667    0.47807    0.68859    0.31141
      1.3333    0.59693     0.7364     0.2636
         1.5    0.72313    0.77687    0.22313
      1.6667    0.85554    0.81112    0.18888
      1.8333    0.99322    0.84012    0.15988
           2     1.1353    0.86466    0.13534
      2.1667     1.2812    0.88544    0.11456
      2.3333     1.4303    0.90303   0.096974
         2.5     1.5821    0.91791   0.082087
      2.6667     1.7362    0.93052   0.069485
      2.8333     1.8922    0.94118   0.058818
           3     2.0498    0.95021   0.049788
      3.1667     2.2088    0.95786   0.042145
      3.3333      2.369    0.96433   0.035675
         3.5     2.5302     0.9698   0.030198
      3.6667     2.6922    0.97444   0.025562
```

```
    3.8333       2.855    0.97836     0.021638
         4      3.0183    0.98168     0.018316
    4.1667      3.1822     0.9845     0.015504
    4.3333      3.3465    0.98688     0.013124
       4.5      3.5111    0.98889     0.011109
    4.6667      3.6761     0.9906    0.0094039
    4.8333      3.8413    0.99204    0.0079602
         5      4.0067    0.99326    0.0067382
```

Exercise7a.m :

```
% APPM3021 Lab 5 Exercise 7a
% Tyson Cross 1239448

%{
Notes:
    f1 = system of equations (inhomogeneous)
        y'' = p*y' + p*y + r;
    f2 = system of equations (homogeneous)
        y'' = p*y' + p*y;
    x = [a b];
    y(x0) = y0;
    y(xf) = yf;

p = -3;
q = 2;
r = 2*x + 3;
x = [0 1];
y0 = 2;
yf = 1;

%}

clc; clear all; set(0,'ShowHiddenHandles','on'); delete(get(0,'Children'));

function_name = 'y'''' = -3*y'' + 2*y + 2*x + 3';

f1 = @(x,u) [u(2); -3*u(2) + 2*u(1) + 2*x + 3];
f2 = @(x,v) [v(2); -3*v(2) + 2*v(1)];
bcs = @(ya,yb) [ya(1)-2; yb(1)-1]; %y'(0) = 2, y(1) = 1
x0 = 0;
xf = 1;
y0 = 2;
yf = 1;
N = 30;
xspan = [x0 xf];

[X1,Y1] = shootingmethod(f1, f2, x0, y0, yf, N, xf);
[X2,Y2] = findDifference(x0,y0,yf,N,xf);

% Confirm exact solution
solinit = bvpinit([x0 xf],[y0 1]);
sol = bvp4c(f1,bcs,solinit);
x = linspace(x0,xf,N);
y = deval(sol,x);

% Display results
Exercise7a_Disp

% Plot results
Exercise7a_Plot
```

When Exercise7a.m is run in the workspace, the following output is displayed to the command window:

```
N = 30
h = 0.033333
x is an element of [0  1]
y(0) = 2
y(1) = 1
f(x,y) = y'' = -3*y' + 2*y + 2*x + 3
y1: @(x,u)[u(2);-3*u(2)+2*u(1)+2*x+3]
y2: @(x,v)[v(2);-3*v(2)+2*v(1)]
```

Linear Shooting Method (w/RK4system):

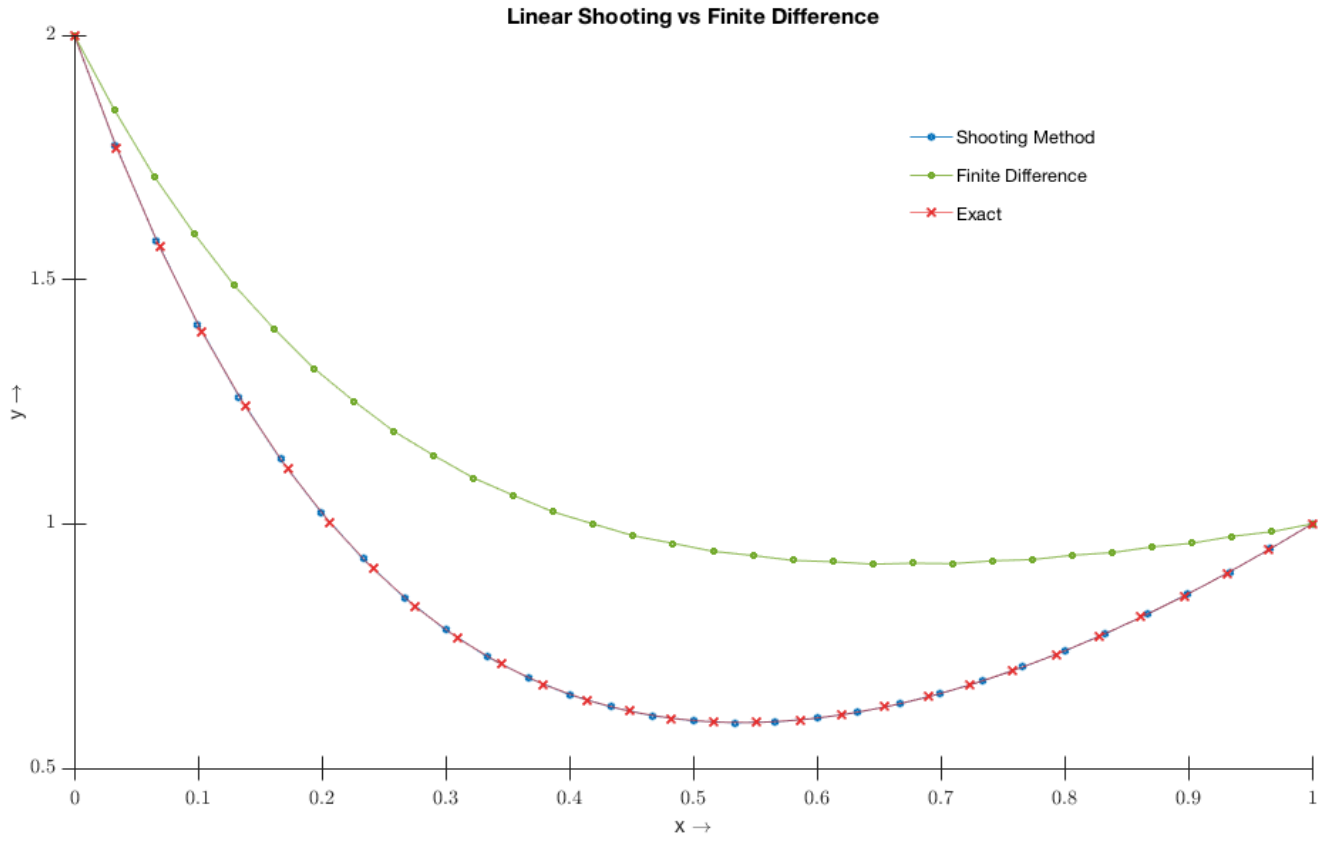
x	y	dy
0	2	-7.1961
0.033333	1.7754	-6.2956
0.066667	1.5792	-5.4919
0.1	1.4083	-4.7743
0.13333	1.2601	-4.1329
0.16667	1.1321	-3.5592
0.2	1.0221	-3.0455
0.23333	0.92844	-2.585
0.26667	0.84928	-2.1717
0.3	0.78319	-1.8002
0.33333	0.72886	-1.4658
0.36667	0.68511	-1.1641
0.4	0.65093	-0.89151
0.43333	0.62539	-0.64463
0.46667	0.6077	-0.42049
0.5	0.59714	-0.21646
0.53333	0.59307	-0.030181
0.56667	0.59495	0.14042
0.6	0.60228	0.2972
0.63333	0.61463	0.4418
0.66667	0.63162	0.57569
0.7	0.6529	0.70018
0.73333	0.6782	0.81641
0.76667	0.70725	0.92542
0.8	0.73983	1.0281
0.83333	0.77573	1.1254
0.86667	0.8148	1.2178
0.9	0.85688	1.3062
0.93333	0.90184	1.391
0.96667	0.94958	1.4728
1	1	1.5521

Finite Difference Method:

x	y
0	2
0.032258	1.848
0.064516	1.7112
0.096774	1.5944
0.12903	1.4886
0.16129	1.3996
0.19355	1.3184
0.22581	1.2514
0.25806	1.1897
0.29032	1.1402
0.32258	1.094
0.35484	1.0584
0.3871	1.0244
0.41935	0.99992
0.45161	0.97578
0.48387	0.96014
0.51613	0.94382
0.54839	0.93532
0.58065	0.92527
0.6129	0.92254
0.64516	0.91755
0.67742	0.91952
0.70968	0.91864
0.74194	0.92446
0.77419	0.92694
0.80645	0.93596
0.83871	0.94122
0.87097	0.95292
0.90323	0.96051
0.93548	0.97449
0.96774	0.98405
1	1

When Exercise7a.m is run in the workspace, the following figure is generated:

Figure 3. Exercise 7



Functions and Code:

Euler.m :

```
function [x,y] = Euler(f,x0,y0,N,xf)
%Euler() solves ODEs with Euler's Method. Outputs an (N+1)*2 matrix, with the
%first column represents the x values , and the second column containing the
%corresponding y values
% Tyson Cross 1239448

x(1,1) = x0;
y(1,1) = y0;
h = (xf-x0)/N;

for ii=2:N+1
    k = ii-1;
 x(ii,1) = x(k)+h;
 y(ii,1) = y(k) + h*f(x(k),y(k));
end

end
```

Heun.m :

```
function [x,y] = Heun(f,x0,y0,N,xf)
%Heun() solves ODEs with Heun's Method (Improved Euler Method)
% Outputs an (N+1)*2 matrix, with the first column represents the x values ,
% and second columns containing the corresponding y values
% Tyson Cross 1239448

x(1,1) = x0;
y(1,1) = y0;
h = (xf-x0)/N;

for ii=2:N+1
    k = ii-1;
 x(ii,1) = x(k)+h;
    fx = x(k) + h;
    fy = y(k) + h*f(x(k),y(k));
 y(ii,1) = y(k) + (h/2)*(f(x(k),y(k)) + f(fx,fy));
end

end
```

Midpoint.m :

```
function [x,y] = Heun(f,x0,y0,N,xf)
%Heun() solves ODEs with Heun's Method (Improved Euler Method)
% Outputs an (N+1)*2 matrix, with the first column represents the x values ,
% and second columns containing the corresponding y values
% Tyson Cross 1239448

x(1,1) = x0;
y(1,1) = y0;
h = (xf-x0)/N;

for ii=2:N+1
    k = ii-1;
 x(ii,1) = x(k)+h;
    fx = x(k) + h;
    fy = y(k) + h*f(x(k),y(k));
 y(ii,1) = y(k) + (h/2)*(f(x(k),y(k)) + f(fx,fy));
end

end
```

RK4.m :

```
function [x,y] = RK4(f,x0,y0,N,xf)
%RK4() solves ODEs with the Runge-Kutta Method
% Outputs an (N+1)*2 matrix, with the first column represents the x values ,
% and second columns containing the corresponding y values
% Tyson Cross 1239448

x(1,1) = x0;
y(1,1) = y0;
h = (xf-x0)/N;

for ii=2:N+1
    k = ii-1;
 x(ii,1) = x(k)+h;
    k1 = h*f(x(k),y(k));
    k2 = h*f(x(k) + (h/2), y(k) + (k1/2));
    k3 = h*f(x(k) + (h/2), y(k) + (k2/2));
    k4 = h*f(x(k) + h, y(k) + k3);
 y(ii,1) = y(k) + (1/6)*(k1 + 2*k2 + 2*k3 + k4);
end

end
```

RK4system.m :

```
function [X,Y] = RK4system(f,xspan,y0,N)
%RK4() solves a system of n ODEs with the Runge-Kutta Method
% Outputs a vector and a (N+1)xn matrix, with the first output column
% contains the x values and the matrix containing the corresponding y values
% and its derivatives y', y'', ... y^(n-1)
% Tyson Cross 1239448

x0 = xspan(1);
xf = xspan(end);
n = numel(y0);
h = (xf-x0)/N;

X = linspace(xspan(1),xspan(end),N+1)';
Y = zeros(N+1,n);
Y(1,:) = y0;

% solve with RK4, as matrix

for ii=2:N+1
    k = ii-1;
    k1 = h*f(X(k),Y(k,:))';
    k2 = h*f(X(k) + (h/2), Y(k,:) + (k1./2))';
    k3 = h*f(X(k) + (h/2), Y(k,:) + (k2./2))';
    k4 = h*f(X(k) + h, Y(k,:) + k3)';
 Y(ii,:) = Y(k,:) + (1/6)*(k1 + 2.*k2 + 2.*k3 + k4);
end

end
```

---

shootingmethod.m :

```
function [X,Y] = shootingmethod(f1,f2,x0,y0,yf,N,xf)
%shootingmethod() solves boundary value problems with the linear shooting method
% f1: system of ODE for the inhomogeneous equation
% f2: the homogoneous equation
% x0: initial x value
% xf: final x value
% y0: initial y value
% yf: final y value
% N: number of subintervals
% Returns X: an (N+1)*1 column vector and Y: an (N+1)*2 matrix
% Tyson Cross 1239448

xspan = [x0 xf];
y0_1 = [y0 0];  % y(a) = y0, y'(a) = 0;
y0_2 = [0 1];   % y(a) = 0,  y'(a) = 1;
[~,Y1] = RK4system(f1,xspan,y0_1,N);
[X,Y2] = RK4system(f2,xspan,y0_2,N);
val = (yf - Y1(N+1,1)) / Y2(N+1,1);
for ii=1:2
    Y(:,ii) = Y1(:,ii) + val*Y2(:,ii);
end

end
```

---

findDifference.m :

```
function [X,Y] = findDifference(x0,y0,yf,N,xf)
%findDifference() solves a BVP using the finite difference method
% x0: initial x value
% xf: final x value
% y0: initial y value
% yf: final y value
% N: number of interior points
% Returns X: an (N+2)*1 column vector and Y: an (N+2)*1 column vector
% Tyson Cross 1239448

A = zeros(N);
b = zeros(N,1);
```

```
Y = zeros(N,1);
X = linspace(x0,xf,N+2)';
h = (xf - x0)/N;

x_int = X(2:end-1);

% diagonals
for ii=1:N
    A(ii,ii)=2*(1+h^2);
end

% off-diagnonals
for ii=1:N-1
    A(ii,ii+1) = 1+(3/2)*h;
    A(ii+1,ii) = 1-(3/2)*h;
end

% b vector
   b(1,1) =  h^2*(2*x_int(1)+3) + (1 - (3/2)*h)*y0;
for ii=2:N-1
   b(ii,1) =  h^2*(2*x_int(ii)+3);
end
   b(N,1) =  h^2*(2*x_int(N)+3) - (1 + (3/2)*h)*yf;

   Y = A\b;
   Y = abs([y0;Y;yf]);

end
```