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Chapter 4

Realization of First- and Second-Order Functions Using Opamps

4.1 Introduction

All active elements, which were introduced in the previous chapter, are useful in the realization of filter functions, although some of them are more useful than others. Among these, the opamp is the most versatile active element in use today up to frequencies of the order of 100 kHz. As was shown in the previous chapter, it can be used to realize other active elements, e.g., all controlled-sources, GICs, etc., but it can also be used as a high (in theory infinite) gain amplifier on its own right. Both of these aspects of the opamp employment in filter design are exploited in this chapter.

As we shall see in the next chapter, one useful method of designing high-order filters is to cascade first- and second-order stages. Usually, one first-order stage will be required only when the filter function is of odd order, the remaining stages being of second-order. In another method of high-order filter design, multiple feedback is applied in the cascade connection of low-order stages. If the filter function is lowpass or highpass only first-order stages will be required whereas, if it is bandpass, bandreject, or allpass, all cascaded stages will be of second-order. It is therefore fully justified to study first- and second-order stages using the opamp(s) as their active element(s).

An abundance of circuits realizing, in particular, second-order functions have been proposed over the past 30 to 35 years, some of which are more "suitable" than others. Criteria of suitability are usually set by the filter designer according to the problem at hand; however, some are clearly objective. Among these, the following three will be of main concern to us in this chapter, namely, (a) the possibility that the circuit can realize the specific second-order function, (b) its sensitivity to component value variations (defined in Section 4.4), and (c) its cost (number and tolerance of its components both passive and active). Based on these criteria we have chosen to include in this chapter only a small number of second-order circuits. This does not necessarily imply that these circuits are the best in all cases, but the filter designer should not ignore their existence and usefulness.

4.2 Realization of First-Order Functions

Alongside its use in the integrator circuit in Fig. 3.9, the opamp can be also used in the realization of other first-order transfer functions, which are useful in filter design. Such functions are lowpass, highpass, and allpass. Although the lowpass and highpass functions can be realized using RC circuits only, the presence of the opamp in the circuit can provide it with gain and isolation from the circuit that follows it. Thus, the presentation of these circuits here is justifiable.

4.2.1 Lowpass Circuits

A first-order lowpass circuit is shown in Fig. 4.1(a). Its transfer voltage ratio can easily be shown to be

$$H(s) \equiv \frac{V_o}{V_i} = -\frac{1/CR_1}{s + 1/CR_2} \tag{4.1}$$

Thus, the circuit can realize the transfer function

$$F(s) = \frac{a}{s+b} \tag{4.2}$$

with
$$a = -\frac{1}{CR_1}, \; b = \frac{1}{CR_2}$$

The dc gain is

$$\frac{a}{b} = -\frac{R_2}{R_1}$$

and can be adjusted to any desired practical value.

An alternative circuit is that shown in Fig. 4.1(b), which does not introduce a phase inversion (sign reversal) as does that in Fig. 4.1(a). Its transfer function is

$$H(s) \equiv \frac{V_o}{V_i} = \frac{K/CR}{s + 1/CR} \tag{4.3}$$

where
$$K = 1 + \frac{R_2}{R_1}$$

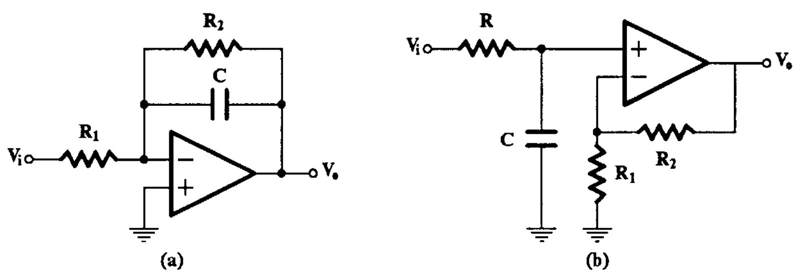

FIGURE 4.1
(a) A first-order lowpass circuit and (b) an alternative circuit.

Equating similar coefficients in Eqs. (4.2) and (4.3) we obtain the following design relationships:

$$a = \frac{K}{CR}, \quad b = \frac{1}{CR}, \quad \text{dc gain } \frac{a}{b} = K \tag{4.4}$$

4.2.2 Highpass Circuits

Both circuits in Fig. 4.2 can realize the first-order highpass function

$$F(s) = \frac{as}{s+b} \tag{4.5}$$

The transfer function of the circuit in Fig. 4.2(a) is

$$H(s) = -\frac{R_2}{R_1}\frac{s}{s+1/CR_1} \tag{4.6}$$

Therefore, following coefficient matching, we get from Eqs. (4.5) and (4.6) the design equations as follows:

$$a = -\frac{R_2}{R_1}, \quad b = \frac{1}{CR_1} \tag{4.7}$$

Clearly, the value of one component will have to be selected arbitrarily.

Similarly, the transfer function of the circuit in Fig. 4.2(b) is

$$H(s) = \frac{Ks}{s+1/(RC)} \tag{4.8}$$

and, after coefficient matching, we obtain

$$a = k = 1 + \frac{R_b}{R_a}, \quad b = \frac{1}{CR} \tag{4.9}$$

Here, two components should have their values arbitrarily selected.

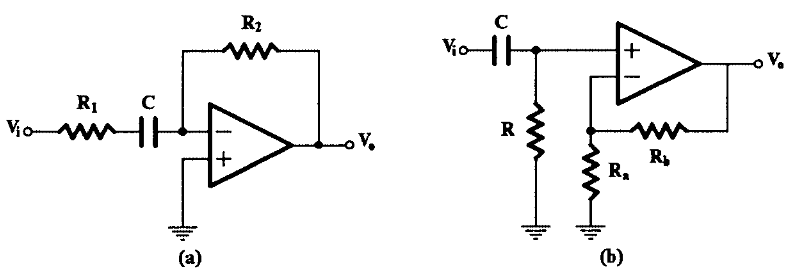

FIGURE 4.2
Two alternative first-order highpass circuits.

4.2.3 Allpass Circuits

Two circuits [1] suitable for the realization of the first-order allpass function,

$$F(s) = \frac{s-a}{s+a} \tag{4.10}$$

are shown in Fig. 4.3.

The transfer function of the circuit in Fig. 4.3(a) is the following:

$$H(s) = \frac{V_o}{V_i} = -\frac{s-1/CR}{s+1/CR} \tag{4.11}$$

The same holds for the circuit in Fig. 4.3(b), but without the negative sign in front. For both circuits, coefficient matching gives the following design relationship:

$$a = \frac{1}{CR} \tag{4.12}$$

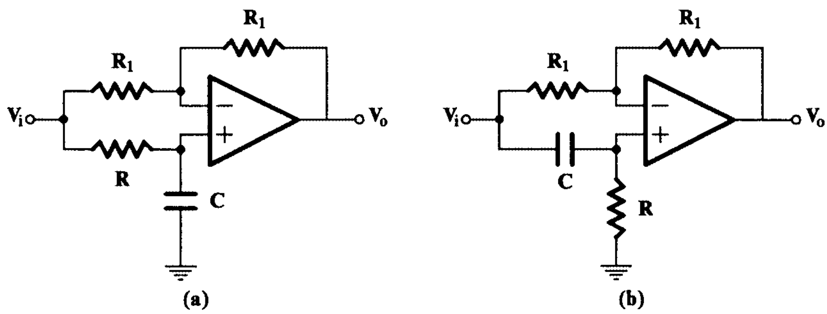

FIGURE 4.3
Two alternative first-order allpass circuits.

4.3 The General Second-Order Filter Function

The second-order filter function, in its general form, is the following:

$$F(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + \beta s + \gamma} \tag{4.13}$$

Realization of this function using active RC networks is of interest only in the case that

$$\sqrt{\gamma} > 0.5\beta$$

i.e., when the poles of $F(s)$ are complex conjugate. Otherwise, when the poles are negative real, the realization can be achieved using passive RC networks only.

$F(s)$ can be written alternatively in general "biquadratic" form as follows:

$$F(s) = K\frac{s^2 + \frac{\omega_{oz}}{Q_z}s + \omega_{oz}^2}{s^2 + \frac{\omega_{op}}{Q_p}s + \omega_{op}^2} \tag{4.14}$$

In Eq. (4.14), ω_{oz} and ω_{op} are the undamped natural frequencies of the zeros and poles respectively, while Q_z and Q_p are the corresponding quality factors, or Q factors. The zero or pole frequency is the magnitude of the zero or pole, respectively, while their quality is a measure of how near the $j\omega$-axis is the corresponding zero or pole in the s-plane.

Comparing Eq. (4.14) to Eq. (4.13), we get

$$\omega_{op} = \sqrt{\gamma} \tag{4.15}$$

$$Q_p = \frac{\sqrt{\gamma}}{\beta} \tag{4.16}$$

It is common to use ω_o instead of ω_{op} and Q instead of Q_p, and we will adopt these symbols here, too. In the case of the second-order bandpass filter, ω_o coincides with the filter center frequency (when the magnitude response is plotted on a log frequency scale), while the Q factor determines the relative width of the frequency response with respect to the center frequency.

Depending on the positions of the zeros on the *s*-plane, the second-order filter frequency responses of interest, obtained from Eq. (4.13), are as shown in Fig. 4.4. We will examine below the most useful practical active RC circuits realizing these functions, taking into consideration mainly the criterion of low sensitivity, which we introduce first.

4.4 Sensitivity of Second-Order Filters

The term *sensitivity* is used to express the degree of influence of a variation in the value of one (or more) component on the performance of the circuit in which it is embedded. Variations in the component values may be due to one or more of the following reasons:

- changes in the environmental conditions, e.g., temperature
- component aging
- component substitution due to failure
- component tolerances during the production of the circuit

The less sensitive the circuit is to component variations, the more stable its characteristics will be and, thus, the more likely it will be to remain within its specifications, regardless of these changes. Therefore, sensitivity is a major factor in determining how useful a circuit can be in practice.

Sensitivity measures of greatest interest in the case of second-order filters are introduced below. Other sensitivity measures, useful mainly in the case of high-order circuits, are presented in the next chapter.

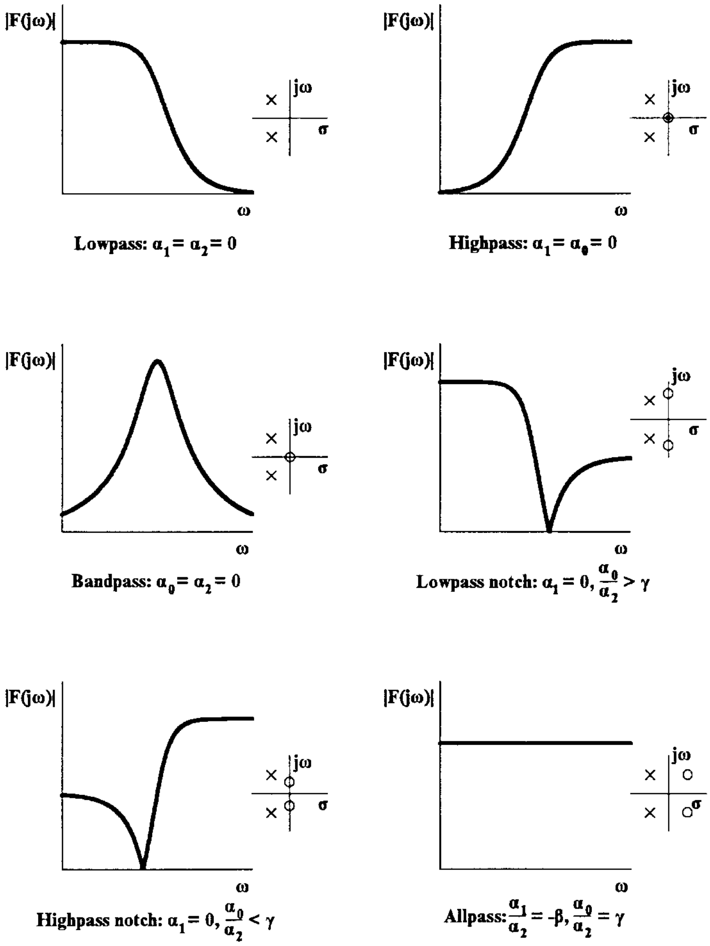

FIGURE 4.4
Frequency responses of various useful second-order filter functions.

Let us suppose that we are interested in the variation of the filter characteristics due to a change in the value of the circuit element x. Writing

$$H(s) = H(s, x) \tag{4.17}$$

we define the relative sensitivity of $H(s)$ to x as

$$S_x^H \equiv \frac{\partial \ln H}{\partial \ln x} \tag{4.18}$$

which can also be written as follows:

$$S_x^H \equiv \frac{\partial H/H}{\partial x/x} = \frac{x}{H}\frac{\partial H}{\partial x} \tag{4.19}$$

Clearly, S_x^H is a complex quantity. However, if H is replaced by $|H(j\omega)|$, we can obtain the sensitivity of the frequency response to variations in the element value and plot it against frequency. Note that the sensitivity $S_x^{|H(j\omega)|}$ is of considerable practical use in estimating the effect of element changes. Thus, if x changes by p percent, to a first approximation, $|H(j\omega)|$ will change by $S_x^{|H(j\omega)|} \cdot p$ percent.

It is also useful to note that at any frequency $s = j\omega$.

$$S_x^H = S_x^{|H|} + jQ_x^{\varphi}$$

where $Q_x^{\varphi} = x\dfrac{d\varphi}{dx}$ is the semi-relative phase sensitivity.

The sensitivity of H, as it was defined above, can be calculated for a filter function of any order; i.e., this definition is not restricted to the case of second-order functions. More suitable sensitivity measures for second-order filter functions are the pole, ω_o, and Q-factor sensitivities.

The pole p semi-relative sensitivity is defined by

$$S_x^p \equiv \frac{dp}{dx/x} \tag{4.20}$$

and expresses the pole displacement due to the variation in the value of the element x. It is also a complex quantity in general.

The Q factor and ω_o sensitivity measures are particularly useful in the case of 2nd-order bandpass filters. These are defined as follows:

$$S_x^Q \equiv \frac{dQ/Q}{dx/x} = \frac{x}{Q}\frac{dQ}{dx} \tag{4.21}$$

$$S_x^{\omega_o} \equiv \frac{d\omega_o/\omega_o}{dx/x} = \frac{x}{\omega_o}\frac{d\omega_o}{dx} \tag{4.22}$$

The element x in the previous considerations can be passive or active. In the case where it is active, it can be the open-loop gain of an opamp, the g_m of an OTA, or the gain of a controlled source of finite gain, obtained for example from an opamp using resistive feedback. In the latter case, its actual gain variation, due to a variation in the open-loop gain of the opamp, will greatly depend on the nominal value of x.

To show this let us consider the case depicted in Fig. 4.5, where an opamp is used to obtain the gain k of a VCVS. With

$$\beta = \frac{R_1}{R_1 + R_2} \tag{4.23}$$

we can easily find that

$$k \equiv \frac{V_o}{V_i} = \frac{A}{1 + \beta A} \tag{4.24}$$

and subsequently

$$dk = \frac{dA}{(1 + \beta A)^2} \tag{4.25}$$

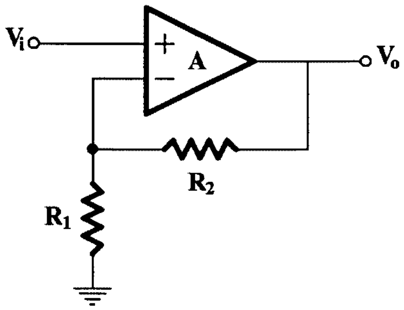

FIGURE 4.5
Opamp used to obtain gain k.

Thus, for given values of A and dA, the actual variation dk of k will depend on β. But,

$$\beta = \frac{1}{1 + \frac{R_2}{R_1}} = \frac{1}{k_n}$$

where k_n is the desired or nominal value of k. Substituting in Eq. (4.25), we get

$$dk = \frac{dA}{(1 + A/k_n)^2} \tag{4.26}$$

We conclude then that, independently of the sensitivity of Q or ω_o to k, their actual variations will greatly depend on the nominal value of k, being small for small values of k_n.

This observation has led to the introduction [1] of the *gain sensitivity product* as the sensitivity measure that takes into consideration the importance of the value of k, when examining the actual Q and ω_o variations. Under these circumstances, Eqs. (4.21) and (4.22) will be written as follows:

$$GS_k^Q = S_k^Q \cdot k \tag{4.27}$$

$$GS_k^{\omega_o} = S_k^{\omega_o} \cdot k \tag{4.28}$$

Since the open-loop gain of an opamp, and to a less extent k, are functions of frequency, one should take this into consideration in calculating the variations of Q and ω_o. It has been shown [2] that the percentage ω_o variations causes a much larger variation in the transfer function of an active RC filter (about $2Q$ times larger) than that caused by the same percentage Q variation. Therefore, it is important, when comparing second-order filter realizations, to determine the percentage change in ω_o

$$\delta\omega \equiv \frac{\delta\omega_o}{\omega_o} \tag{4.29}$$

which is caused by the finite bandwidth of the amplifier.

As has been mentioned, the sensitivity measures examined above are suitable for studying the sensitivity of active RC filters of second order. We have also introduced sensitivity measures concerning the overall transfer function variation when all circuit components, passive and active, are varying simultaneously. However, these do not give any important information concerning the sensitivity of second-order active RC filters in addition to the information given by the sensitivity measures introduced above. Such *multiparameter* sensitivity measures are suitable for studying the sensitivity of higher-order active RC filters and are introduced in Chapter 5, where the design of such filters is considered.

4.5 Realization of Biquadratic Functions Using SABs

An active RC circuit realizing a biquadratic transfer function is called a *biquad*. A single-amplifier biquad (SAB) is a biquad using one amplifier.

4.5.1 Classification of SABs

Most SABs can be classified [3] in one of the two general structures shown in Fig. 4.6. In Fig. 4.6(a) the enhanced positive feedback (EPF) configuration is shown, while in Fig. 4.6(b) the enhanced negative feedback (ENF) configuration is shown. The passive RC network is a complex zero-producing section, and it can be of second- or third-order. The input signal is fed to the circuit by inserting the signal voltage source between the terminal b and ground in Fig. 4.6(a) and, similarly, between the terminal c and ground in Fig. 4.6(b). It can be also fed as a current to one of the nodes of the RC passive network, either internal or external (not to terminals b and c). In some cases, when complex zeros should be realized, the input signal is also fed through a resistor connected to the common node of R_a and R_b (R_a' and R_b').

A SAB is called *canonic* if the RC network employs two capacitors only, since this is the minimum number to provide a biquadratic transfer function. However, it can employ three capacitors, as it is the case with the RC subnetwork being a twin-T.

The term *enhanced* stems from the fact that the resistive feedback by means of R_a, R_b or R_a', R_b' affects the position of the complex poles of the overall function, moving them on a circle towards the $j\omega$-axis.

An important property may be possessed by the two configurations as they appear in Fig. 4.6 if the following conditions are satisfied:

1. The RC networks are the same but with the respective connections of terminals b and c to earth and the opamp output interchanged, as is suggested in the figure.
2. If R_b is written as

$$R_b = (n-1)R_a \qquad \text{[Fig. 4.6(a)]} \tag{4.30}$$

then

$$R_a' = (n-1)R_b' \qquad \text{[Fig. 4.6(b)]} \tag{4.31}$$

where n is a positive number greater than one.

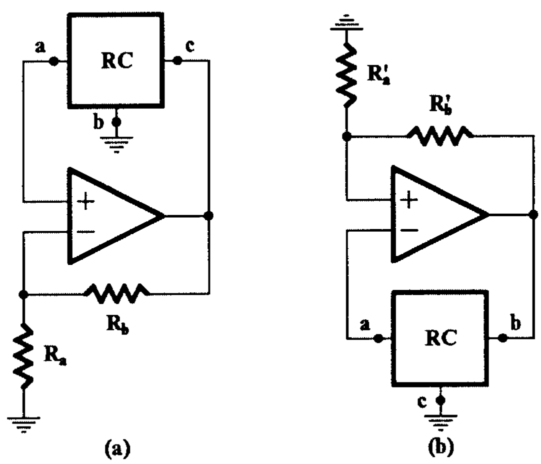

FIGURE 4.6
(a) The SAB EPF and (b) ENF configurations.

Under these conditions, the two configurations are said to be related by the complementary transformation (CT) [4]. The important characteristic of this situation is that the two circuits have identical poles and same sensitivities.

The process of applying the CT is the following: the terminals of the feedback network, which are connected to the output terminal of the opamp in the initial circuit, are connected to ground, and those connected to ground in the initial network are now connected to the opamp output terminal. At the same time, the opamp input terminals are interchanged. This is shown clearly in Fig. 4.7. It should be mentioned that with the application of the CT an ENF biquad is transformed to an EPF, and vice versa.

Application of the complementary transformation to a useful circuit gives rise to another that may also be useful. This idea has been successfully used [3, 4], and many SABs have been obtained that have interesting properties.

The SABs we shall present below will belong to one or other of the configurations in Fig. 4.6, and mostly they will be canonic. These are among the most useful ones in practice.

4.5.2 A Lowpass SAB

The most popular second-order lowpass circuit is that of Sallen-Key's [5] shown in Fig. 4.8, which employs an opamp in an arrangement of a VCVS with gain G. Clearly, it belongs to the EPF class of SABs. Its transfer voltage ratio, obtained by analyzing the circuit, is as follows:

$$H(s) = \frac{V_0}{V_i} = \frac{G/(C_1C_2R_1R_2)}{s^2 + \left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1-G}{R_2C_2}\right)s + \frac{1}{R_1R_2C_1C_2}} \tag{4.32}$$

where

$$G = 1 + \frac{R_b}{R_a}$$

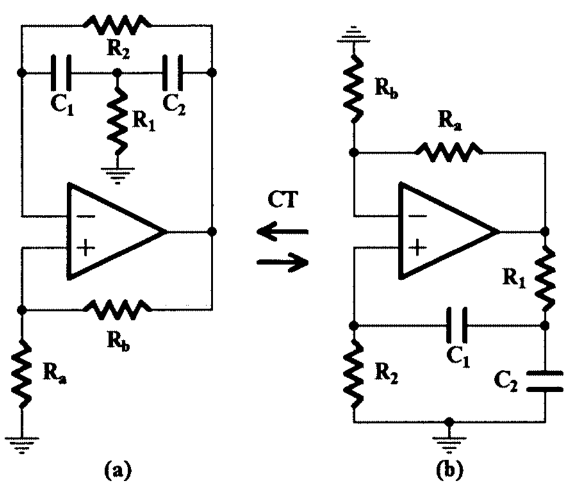

FIGURE 4.7
Application of the complementary transformation.

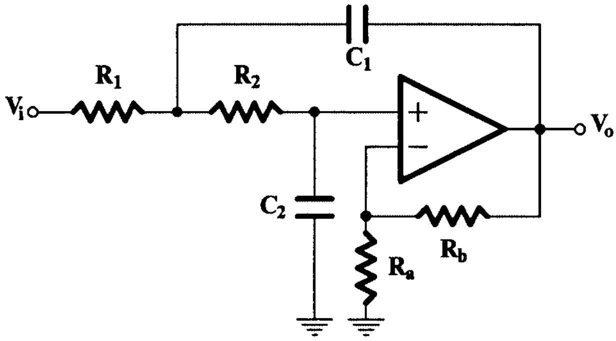

FIGURE 4.8
The Sallen-Key second-order lowpass circuit.

The realization of the second-order lowpass function,

$$F(s) = \frac{K}{s^2 + \beta s + \gamma} \tag{4.33}$$

by this circuit can be obtained by matching the coefficients of equal powers of s in Eqs. (4.32) and (4.33). Following this, one obtains the following:

$$\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1-G}{R_2C_2} = \beta \tag{4.34a}$$

$$\frac{1}{R_1R_2C_1C_2} = \gamma \tag{4.34b}$$

$$\frac{G}{R_1R_2C_1C_2} = K \tag{4.34c}$$

Since the number of unknowns is larger than the number of the equations, some components will have to be selected *arbitrarily.* One popular choice is the following:

$$R_1 = R_2 = R \quad \text{and} \quad G = 1(R_a = \infty) \tag{4.35}$$

With this selection, one takes advantage of the whole bandwidth of the opamp, i.e., up to frequency f_T. Substituting in Eqs. (4.34), we get

$$\frac{2}{RC_1} = \beta \tag{4.36a}$$

$$\frac{1}{R^2C_1C_2} = \gamma \tag{4.36b}$$

with the restriction that $K = \gamma$, which is not usually a problem in filter design.

From these equations, the following capacitance values are obtained:

$$C_1 = \frac{2}{R\beta} \tag{4.37a}$$

$$C_2 = \frac{\beta}{2\gamma R} \tag{4.37b}$$

As an example, consider the realization of the second-order Butterworth lowpass filter function

$$F(s) = \frac{1}{s^2 + \sqrt{2}s + 1} \tag{4.38}$$

Selecting $R = 1$ the (normalized) values of the passive components of the circuit in Fig. 4.7 will be as follows:

$$R_1 = R_2 = 1 \tag{4.39a}$$

$$C_1 = \sqrt{2} \tag{4.39b}$$

$$C_2 = \frac{1}{\sqrt{2}} \tag{4.39c}$$

The circuit can be denormalized to a cutoff frequency ω_c and an impedance level R_o. Two sets of passive component values can be the following:

Set 1	Set 2
$R_1 = R_2 = R_o$	$R_1 = R_2 = \frac{R_o}{\omega_c}$
$C_1 = \frac{\sqrt{2}}{\omega_c R_o}$	$C_1 = \frac{\sqrt{2}}{R_o}$
$C_2 = \frac{1}{\sqrt{2}R_o\omega_c}$	$C_2 = \frac{1}{\sqrt{2}R_o}$

In both cases, a proper value for R_b (in order to avoid dc offset voltage in the opamp output) will be

$$R_b = 2R_o$$

while $R_a = \infty$.

For example, if $R_o = 10$ kΩ, and the desired cutoff frequency is 1 kHz, one finds the following:

Set 1	Set 2
$R_1 = R_2 = 10\ \text{k}\Omega$	$R_1 = R_2 = \dfrac{10^4}{2\pi\times 10^3} = 1.59\ \Omega$
$C_1 = \dfrac{\sqrt{2}}{2\pi\times 10^3\times 10^4} = 22.5\ \text{nF}$	$C_1 = \dfrac{\sqrt{2}}{10^4} = 141.4\ \mu\text{F}$
$C_2 = \dfrac{1}{2\sqrt{2}\pi\times 10^3\times 10^4} = 11.25\ \text{nF}$	$C_2 = \dfrac{1}{\sqrt{2}\times 10^4} = 70.71\ \mu\text{F}$
$R_b = 20\ \text{k}\Omega$	$R_b = 3.18\ \Omega$

Clearly, the values in set 2 are not practical, but they can become so by raising the impedance level to 10 Ω. Then we will have:

$$R_1 = R_2 = 15.9\ \text{k}\Omega$$

$$C_1 = 14.14\ \text{nF}$$

$$C_2 = 7.071\ \text{nF}$$

$$R_b = 31.8\ \text{k}\Omega$$

Using Eqs. (4.27) and (4.28) the ω_o and Q sensitivities with respect to variations in the passive component values (passive sensitivities) are found to be as follows ($\omega_o = \sqrt{\gamma}$, $Q = \sqrt{\gamma}/\beta$):

$$S^{\omega_o}_{R_1, R_2, C_1, C_2} = -\frac{1}{2} \qquad S^{\omega_o}_{R_a} = S^{\omega_o}_{R_b} = 0 \qquad S^{Q}_{R_1} = S^{Q}_{R_2} = 0$$

$$S^{Q}_{C_1} = -S^{Q}_{C_2} = \frac{1}{2} \qquad S^{Q}_{R_a} = S^{Q}_{R_b} = 0$$

This design is optimum with respect to passive sensitivities, but it is not optimum when variations of the open-loop gain of the opamp are taken into account.

The sensitivity of this circuit has also been studied for other design approaches. For example, selecting

$$R_1 = R_2 \quad \text{and} \quad C_1 = C_2$$

requires

$$G = 3 - \frac{1}{Q} = 3 - \frac{\beta}{\sqrt{\gamma}}$$

Although the ω_o sensitivities remain the same, the corresponding Q sensitivities increase considerably, becoming proportional to Q [6]. Thus, this design is not useful in practice.

Another design approach, followed by Saraga [7], starts with the requirement to minimize the sensitivity of Q with respect to variations in the open-loop gain of the opamp (active sensitivity). This approach leads to the following component values:

$$C_1 = \frac{\sqrt{3\gamma}}{\beta}, \qquad C_2 = 1, \qquad R_1 = \frac{\beta}{\gamma}, \qquad R_2 = \frac{1}{\sqrt{3\gamma}}, \qquad G = \frac{4}{3}$$

Although this design is optimum with respect to Q active sensitivity, it is not optimum with reference to Q passive sensitivities. We can find an overall optimum design with G between 1 and 4/3 taking into account the expected maximum variations both of passive and active components [6, 7].

4.5.3 A Highpass SAB

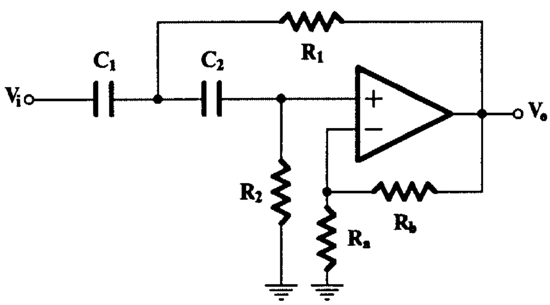

FIGURE 4.9
The Sallen-Key highpass SAB.

A useful circuit is the Sallen-Key [5] highpass SAB shown in Fig. 4.9, which belongs to the EPF class also.

Clearly, this circuit has the same topology as the Sallen-Key lowpass SAB, and it can be obtained from the latter by applying the RC:CR transformation to the RC section of the lowpass circuit. According to this transformation, each resistance R_L in the lowpass circuit is replaced by a capacitance C_H of value

$$C_H = \frac{1}{R_L}$$

and each capacitance C_L in the lowpass circuit is replaced by a resistance R_H of value

$$R_H = \frac{1}{C_L}$$

Normalized values are considered to apply in this transformation. Substituting R_{iH} for $1/sC_{iL}$ and sC_{iH} for $1/R_{iL}$, i = 1,2 in Eq. (4.32), we obtain the following transfer voltage ratio for the highpass circuit in Fig. 4.9:

$$H(s) \equiv \frac{V_0}{V_i} = \frac{Gs^2}{s^2 + \left(\frac{1}{R_2C_2} + \frac{1}{R_2C_1} + \frac{1-G}{R_1C_1}\right)s + \frac{1}{R_1R_2C_1C_2}} \tag{4.40}$$

In Eq. (4.40) R_i, C_i, $i = 1,2$ are those in Fig. 4.9.

This result can be easily verified by a straightforward analysis of the circuit in Fig. 4.9. Clearly, the two networks, lowpass and highpass, have the same poles and therefore the same Q and ω_o sensitivities.

As an example, the normalized component values of the Sallen-Key highpass circuit realizing the second-order Butterworth highpass filter function

$$F(s) = \frac{s^2}{s^2 + \sqrt{2}s + 1} \tag{4.41}$$

will be the following:

$$C_{1H} = C_{2H} = 1$$

$$R_{1H} = \frac{1}{\sqrt{2}}$$

$$R_{2H} = \sqrt{2}$$

These values are obtained by applying the RC:CR transformation to the corresponding component values of the Sallen-Key lowpass filter, which are given by Eqs. (4.39).

Of course, the value of G remains unchanged, i.e., $G = 1$. The highpass circuit can be denormalized to an impedance level R_o and a cutoff frequency ω_c in exactly the same way as it was explained for the lowpass circuit.

4.5.4 A Bandpass SAB

An active RC circuit [8], useful in the realization of a second-order bandpass function is shown in Fig. 4.10. Clearly, this circuit belongs to the ENF class of SABs.

Assuming that the opamp is ideal, the transfer voltage ratio of the SAB is as following:

$$H(s) \equiv \frac{V_0}{V_i} = -\frac{hs}{s^2 + \left(\frac{C_1 + C_2}{R_2 C_1 C_2} - K\frac{1}{R_1 C_2}\right)s + \frac{1}{R_1 R_2 C_1 C_2}} \tag{4.42}$$

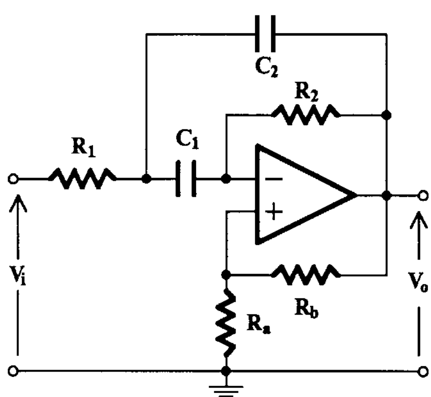

FIGURE 4.10
A bandpass SAB.

where

$$h = \frac{1+K}{R_1 C_2}, \qquad K = \frac{R_a}{R_b} \tag{4.43}$$

The second-order bandpass transfer function

$$F(s) = \frac{\alpha s}{s^2 + \beta s + \gamma} \tag{4.44}$$

can be realized by this SAB to within a constant multiplier as the voltage ratio $-V_o/V_i$, if we rewrite $F(s)$ as follows:

$$F(s) = \frac{\alpha s}{s^2 + [(n+1)\beta - n\beta]s + \gamma} \tag{4.45}$$

where n is real and positive.

Let

$$r = \frac{R_1}{R_2} \qquad q = \frac{C_1}{C_2}$$

Then, coefficient matching between Eqs. (4.42) and (4.45), after some simple mathematical manipulations, gives the following:

$$n = Q(q+1)\sqrt{\frac{r}{q}} - 1 \tag{4.46a}$$

$$K = \frac{n}{Q}\sqrt{\frac{r}{q}} \tag{4.46b}$$

$$R_2 C_1 = \sqrt{\frac{q}{\gamma r}} \tag{4.46c}$$

$$R_1 C_2 = \sqrt{\frac{r}{\gamma q}} \tag{4.46d}$$

$$R_2 C_2 = \frac{1}{\sqrt{\gamma r q}} \tag{4.46e}$$

Clearly, the Q-factor sensitivity of the SAB is affected by the choice of n, which has to be as low as possible for low Q-factor sensitivity.

In an attempt to minimize the Q-factor sensitivity, we may show, using Eq. (4.46a), that the value of n is minimal for any r, if

$$q = 1$$

i.e., when

$$C_1 = C_2$$

Then,

$$n = 2Q\sqrt{r} - 1 \tag{4.47a}$$

which, depending on r, can be (within the practical limitations) as small as it is desired. With this choice of q, the rest of the design Eqs. (4.46) become as follows ($C_1 = C_2 = C$):

$$K = \frac{n}{Q}\sqrt{r} \tag{4.47b}$$

$$R_1C = \sqrt{\frac{r}{\gamma}} \tag{4.47c}$$

$$R_2C = \frac{1}{\sqrt{\gamma r}} \tag{4.47d}$$

Since Eqs. (4.47) are not sufficient to give unique component values, the designer should select r and one of R_1, R_2, or C as well as R_a or R_b taking into consideration the opamp specifications.

The ω_o and Q-factor sensitivities to variations in component values, assuming an ideal opamp, are given in Table 4.1.

TABLE 4.1

ω_o and Q-Factor Sensitivities ($C_1 = C_2$)

Component x_i	$S_{x_i}^{\omega_o}$	$S_{x_i}^{Q}$
R_1	$-\frac{1}{2}$	$\frac{1}{2} - 2Q\sqrt{r}$
R_2	$-\frac{1}{2}$	$-\frac{1}{2} + 2Q\sqrt{r}$
C_1	$-\frac{1}{2}$	$Q\sqrt{r} - \frac{1}{2}$
C_2	$-\frac{1}{2}$	$-\left(Q\sqrt{r} - \frac{1}{2}\right)$
K	0	$2Q\sqrt{r} - 1$

Clearly, the value of r has to be very small, but since n in Eq. (4.47a) should be positive for stability, the following condition must hold:

$$\sqrt{r} > \frac{1}{2Q} \tag{4.48}$$

Fleischer [9] has shown that, when taking into consideration the effect of variation of the open-loop gain $A(s)$ of the opamp on ω_o and Q variations [assuming $A(s) \approx \omega_T/s$], the approximately optimum value of r is given by

$$r \approx 0.25 \frac{\omega_o}{\omega_T} \frac{\sigma_{\omega_T}}{\sigma_{R,C}} \tag{4.49}$$

where $\sigma_{R,C}$ and σ_{ω_T} are the standard deviations of the passive elements (assumed equal) and of the gain bandwidth product ω_T of the opamp, respectively. In fact, the value of r need not be less than 1/60, which results in practical values of the components R_1, R_2, R_α, and R_b. An analogous optimization approach followed by Daryanani [6] leads to a value of r very close to that given by Eq. (4.49).

Some important features of this SAB are the following:

a. *Q factor*, and hence bandwidth, can be varied independently of ω_o by varying K.
b. Successive stages can be cascaded without the need for isolating stages. This also holds for the SABs in Figs. 4.8 and 4.9.
c. All capacitors in all cascaded stages can be designed to be of the same value.
d. Any source resistance can be absorbed by R_1 to avoid errors in the frequency response. This is true for the SAB in Fig. 4.8 but not for the SAB in Fig. 4.9.
e. It is easy to reduce the output voltage at the center frequency to avoid bringing the opamp to the nonlinear region of its characteristics, i.e., to saturation, even for small input signals. Clearly, at the center frequency, the output voltage will be, from Eq. (4.42),

$$V_o\big|_{\omega_o} = \frac{hQ}{\omega_o} V_i\big|_{\omega_o}$$

where h is given by Eq. (4.43). This value can be quite high, even for moderate Q. The way to reduce this voltage is to split R_1 into two others, R'_1, R''_1, as shown in Fig. 4.11, such that

$$R_1 = \frac{R'_1 R''_1}{R'_1 + R''_1}$$

This will not affect the shape of the frequency response, i.e., ω_o and Q.

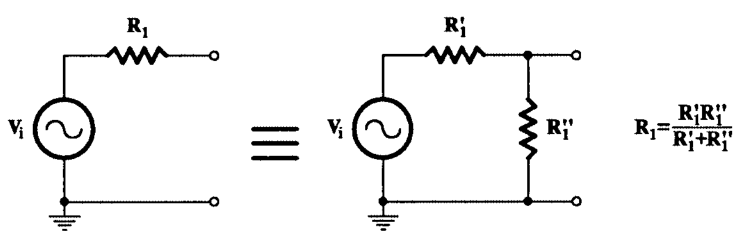

FIGURE 4.11
Some transformation to reduce the output voltage at the center frequency without altering the shape of the frequency response.

f. The ratio ω_T/ω_o in Eq. (4.49) gives the value of the amplifier gain at the center frequency of the filter. Since the sensitivities depend on the value of r, amplifiers with different ω_T will affect differently the performance of the circuit. Thus, a two-pole one-zero frequency compensated opamp will extend the useful range of the filter [9] further than that corresponding to the use of the 741-type opamps.

The form of the transfer function remains unchanged if R_1 and R_2 are interchanged with C_1 and C_2 respectively. However, in this case, feature d explained above is not applicable, if the signal source impedance is not zero. The same is true for feature e.

Example 4.1

Consider the use of this SAB to realize the function

$$F(s) = \frac{0.1s}{s^2 + 0.1s + 1}$$

with the center frequency at 10 krad/s and impedance level at 10 kΩ.

Clearly, the normalized ω_o and the Q-factor values are as following:

$$\omega_o = 1, \quad Q = 10$$

Selecting the practical resistance spread to be 1/100 results in $r = 10^{-2}$, which leads, through Eq. (4.47a) to

$$n = 2 \times 10 \times 10^{-1} - 1 = 1$$

Therefore,

$$K = \frac{R_a}{R_b} = \frac{1}{10} \times 10^{-1} = 10^{-2}$$

Selecting $C = 1$, we get from Eqs. (4.47)

$$R_2 = 10, \quad R_1 = 10^{-1}$$

and from Eq. (4.43),

$$h = \frac{1 + 10^{-2}}{10^{-1} \times 1} = 10.1$$

One set of denormalized component values can then be as follows:

$$C_1 = C_2 = 10 \text{ nF}$$

$$R_1 = 1 \text{ k}\Omega,\ R_2 = 100 \text{ k}\Omega,\ R_a = 1 \text{ k}\Omega,\ R_b = 100 \text{ k}\Omega$$

To achieve unity gain at the center frequency, we observe that since

$$\frac{hQ}{\omega_o} = 10.1 \times 10 = 101$$

we should arrange that

$$\frac{R''_1}{R'_1 + R''_1} = \frac{1}{101} \quad \text{and} \quad \frac{R'_1 R''_1}{R'_1 + R''_1} = R_1 = 1\ \text{k}\Omega$$

Solving then for R'_1 and R''_1, we get

$$R'_1 = 101\ \text{k}\Omega, \quad R''_1 = 1.01\ \text{k}\Omega$$

Let us now look for the optimum design taking into consideration the effect of the finite gain bandwidth product ω_T of the opamp. Assuming

$$\sigma_{\omega_T} = 0.25 \qquad \sigma_{R,C} = 0.005$$

and $\omega_T = 2\pi \times 10^6$ rad/s, we obtain from (4.49)

$$r \approx 0.25 \frac{10^4 \times 0.25}{2\pi \times 10^6 \times 5 \times 10^{-3}} \approx \frac{1}{50}$$

To simplify the calculations a little, we may choose, with no problem,

$$r = \frac{1}{49}$$

Then, following the same procedure as before, we obtain the following set of denormalized values:

$$C_1 = C_2 = 10\ \text{nF}, \quad R_1 = 10/7\ \text{k}\Omega\ (R'_1 = 102.7\ \text{k}\Omega, \quad R''_1 = 1.449\ \text{k}\Omega),$$

$$R_2 = 70\ \text{k}\Omega,\ R_a = 2\ \text{k}\Omega,\ R_b = 75.4\ \text{k}\Omega$$

Note that R_a and R_b can be denormalized at a different impedance level from that for C_i, R_i $i = 1,2$.

4.5.5 Lowpass- and Highpass-Notch Biquads

Friend [10] has introduced additional resistors to the SAB in Fig. 4.10, by means of which the input signal is added directly to the input terminals of the opamp as is shown in Fig. 4.12. Clearly, the circuit remains canonic, but it is now possible to realize complex zeros in addition to the complex poles. Depending on the values of these additional components all types of SABs except the lowpass are obtained, namely:

Bandpass when	$R_6, R_7, R_c = \infty$
Highpass when	$R_7 = \infty$

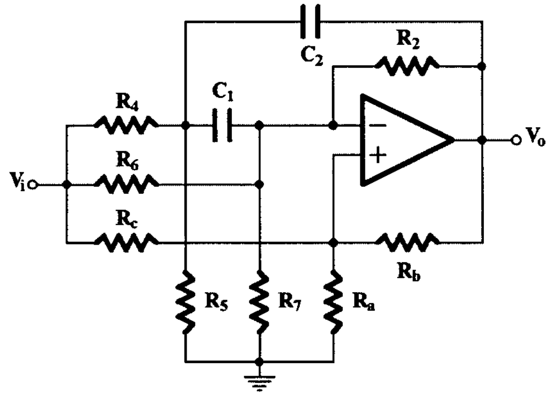

FIGURE 4.12
The generalized SAB developed by Friend.

Lowpass notch when	$R_6 = \infty$
Highpass notch when	$R_7 = \infty$
Allpass when	$R_6, R_7 = \infty$

Friend's generalization of the circuit in Fig. 4.10 has led to the design of the popular STAR building block [11]. It should also be mentioned that, if in addition to $R_6, R_7 = \infty$, also $R_b = \infty$ and $R_5 = \infty$, the circuit is reduced to that shown in Fig. 4.13 [12], which is studied separately below. Of the other four cases, the bandpass has already been studied above. The highpass is less economical than that of Sallen and Key, which we have examined already, and therefore we will not elaborate on it. This leaves us with the two notch cases, which are now examined a little further.

4.5.6 Lowpass Notch ($R_6 = \infty$)

For convenience in the analysis of the circuit we use conductances (G_i) instead of resistances (R_i). Assuming that

$$R_6 = \infty \tag{4.50a}$$

$$G_1 = G_4 + G_5 \tag{4.50b}$$

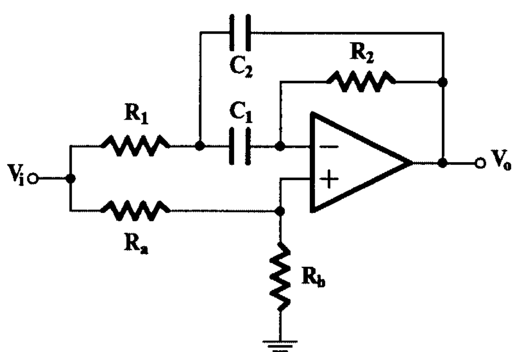

FIGURE 4.13
A simple allpass biquad.

$$K_c = \frac{G_c}{G_a + G_c} \tag{4.50c}$$

$$K_b = \frac{G_b}{G_a + G_c} \tag{4.50d}$$

$$C_1 = C_2 = C \tag{4.50e}$$

and an ideal opamp, straightforward analysis gives the following transfer function:

$$\frac{V_o}{V_i} = \frac{K_c s^2 + \frac{K_c G_1 (G_7 + G_2)}{C^2}}{s^2 + \left[\frac{2G_2}{C} - \frac{K_b 2(G_7 + G_1)}{C}\right] s + \frac{G_1}{C^2}(G_2 - K_b G_7)} \tag{4.51}$$

under the conditions

$$G_c(2G_7 + 2G_2 + G_5) = G_4(G_a + G_b) \tag{4.52a}$$

$$\frac{G_7}{G_2} = \frac{G_a}{G_b + G_c} \tag{4.52b}$$

Clearly, this transfer function is of the form

$$F(s) = \frac{ds^2 + e}{s^2 + \beta s + \gamma} \tag{4.53}$$

which corresponds to a lowpass notch filter, provided that $d < e/\gamma$. This latter condition is in fact satisfied since, from the circuit transfer function, Eq. (4.51),

$$K_c < K_c \frac{G_2 + G_7}{G_2 - K_b G_7}$$

Equating coefficients of equal powers of s in Eqs. (4.51) and (4.53), we get the following four equations:

$$d = K_c \tag{4.54a}$$

$$e = \frac{K_c G_1 (G_7 + G_2)}{C^2} \tag{4.54b}$$

$$\beta = \frac{2G_2}{C} - \frac{K_b(2G_7 + G_1)}{C} \tag{4.54c}$$

$$\gamma = \frac{G_1}{C^2}(G_2 - K_b G_7) \tag{4.54d}$$

$$G_1 = G_4 + G_5$$

With the addition of Eqs. (4.52a) and (4.52b), we have only six equations—not enough to determine the values of all passive components of the circuit. We may then make the usual choices, as in the bandpass case.

$$C = 1 \tag{4.55a}$$

$$r = \frac{G_2}{G_1} \approx 0.25\frac{\omega_p}{\omega_T}\frac{\sigma_{\omega_T}}{\sigma_{R,C}} \tag{4.55b}$$

where ω_p is the magnitude of the pole frequency.

Thus, the set of the necessary equations to determine all component values is completed.

4.5.7 Highpass Notch ($R_7 = \infty$)

A highpass notch SAB is obtained from the general circuit in Fig. 4.12 if $R_7 = \infty$. Working with conductances as previously, and assuming ideal opamp, the transfer voltage ratio of the circuit is found to be

$$\frac{V_o}{V_i} = \frac{K_c s^2 + \frac{G_1}{C^2}(K_c G_2 - G_6)}{s^2 + \left[\frac{2G_2}{C} - \frac{K_b}{C}(G_1 + 2G_6)\right]s + \frac{G_1}{C^2}(G_2 - K_b G_6)} \tag{4.56}$$

under the condition

$$K_b\frac{G_c}{G_b} = \frac{G_4 + 2G_6}{2G_2 + G_5} \tag{4.57}$$

where G_1, K_c, K_b, and C are given again by Eqs. (4.50b, c, d, e).

This transfer voltage ratio is again of the form of Eq. (4.53), but with $d > e/\gamma$, since

$$K_c > \frac{K_c G_2 - G_6}{G_2 - K_b G_6} = K_c\frac{G_2 - \frac{G_6}{K_c}}{G_2 - K_b G_6}$$

Therefore, the SAB can realize a highpass notch filter function. The design equations can be found as it was explained above, in the case of the lowpass notch function.

4.5.8 An Allpass SAB

Allpass biquads are useful in the realization of a high-order allpass function as the cascade connection of second-order sections, the transfer functions of which have the form

$$F(s) = \frac{s^2 - \beta s + \gamma}{s^2 + \beta s + \gamma} \tag{4.58}$$

Such a function is the (n,n) Padè approximation of the function e^{-s} [13].

A simple biquad [12] suitable for such a realization is shown in Fig. 4.13. Its transfer voltage ratio, assuming ideal opamp is as follows:

$$H(s) \equiv \frac{V_o}{V_i} = K \frac{s^2 - \left[\frac{R_a}{R_b}\frac{1}{R_1 C_2} - \frac{1}{R_2}\frac{C_1 + C_2}{C_1 C_2}\right]s + \frac{1}{R_1 R_2 C_1 C_2}}{s^2 + \frac{1}{R_2}\frac{C_1 + C_2}{C_1 C_2}s + \frac{1}{R_1 R_2 C_1 C_2}} \tag{4.59}$$

where

$$K = \frac{R_b}{R_a + R_b}$$

It can be shown that as the pole Q increases, so does the ratio $r = R_2/R_1$. For a given resistance ratio r, the maximum value of ω_i/σ_i, where $-\sigma_i \pm j\omega_i$ are the poles of $F(s)$ in Eq. (4.58), achieved when $C_1 = C_2$, is the following:

$$\left(\frac{\omega_i}{\sigma_i}\right)_{max} = \sqrt{r - 1} \tag{4.60}$$

Thus, depending on the maximum acceptable range of resistance values, a limit is set on the position of the poles of $F(s)$ which are realized by this network.

Letting $C_1 = C_2 = C$ in Eq. (4.59), the transfer function becomes

$$H(s) = \frac{V_o}{V_i} = K \frac{s^2 - \left[\frac{R_a}{R_b}\frac{1}{R_1 C} - \frac{2}{R_2 C}\right]s + \frac{1}{R_1 R_2 C^2}}{s^2 + \frac{2}{R_2 C}s + \frac{1}{R_1 R_2 C^2}} \tag{4.61}$$

Equating coefficients of equal powers of s in Eqs. (4.58) and (4.61) we obtain the following component values:

$$R_1 = \frac{\beta}{2\gamma C}, \quad R_2 = \frac{2}{\beta C}, \quad \frac{R_a}{R_b} = \frac{\beta^2}{\gamma} = \frac{4R_1}{R_2} \tag{4.62}$$

with

$$K = \frac{R_b}{R_a + R_b}$$

Functions with higher ω_i/σ_i ratios can be realized by this network by connecting an additional resistor from the noninverting input of the opamp to the output, as was mentioned above in connection with Friend's work. Although this change makes the circuit more flexible, at the same time it increases its sensitivity to component values.

With or without this additional resistance, the network cannot realize a function whose poles and zeros are not equidistant from the origin in the s-plane. In such a case, other biquads [12, 14] should be used.

Another observation is the following: if the positions of resistors R_1 and R_2 are interchanged with those of capacitors C_1 and C_2, respectively, the resulting network is also an allpass biquad with a similar transfer function. As the reader can similarly show, the highest ω_i/σ_i ratio is also given by Eq. (4.60), achieved when $R_1 = R_2$ with $r = C_1/C_2$ this time.

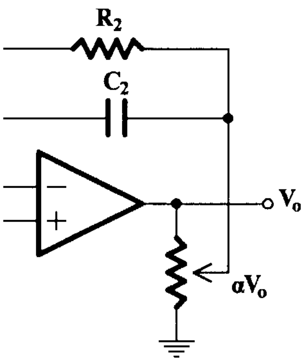

FIGURE 4.14
A practical way to enhance the gain of the SAB in Fig. 4.13.

A drawback of the circuit is its gain, which, given by K in Eq. (4.59), is lower than one. One way to increase the value of K is to reduce the amount of the output voltage V_o fed back to the input by means of R_2 and C_2. In practice, this may be achieved by means of a potentiometer as shown in Fig. 4.14. Thus, the value of K becomes

$$K' = \frac{K}{\alpha}$$

where α is the potentiometer setting, this being between zero and one. Depending on the value of α, the circuit gain can be even greater than one. Naturally, the potentiometer resistance must be low with respect to R_2 and $1/\omega C_2$.

Finally, as can be easily seen from Eq. (4.59), if the coefficient of s in the numerator is made equal to zero, the circuit can be used as a bandstop *symmetrical notch* biquad realizing the function

$$F(s) = K\frac{s^2 + \omega_o^2}{s^2 + \beta s + \omega_o^2}$$

displaying the same behavior at zero and infinity.

As an example, let us use the circuit in Fig. 4.13 to obtain a delay of $\tau = 10$ ms by means of the realization of the second-order Padè approximation written for our purpose as follows:

$$F(s) = \frac{\tau^2 s^2 - 6\tau s + 12}{\tau^2 s^2 + 6\tau s + 12} = \frac{s^2 - 600s + 12\times10^4}{s^2 + 600 + 12\times10^4}$$

Selecting a convenient value for C, e.g., $C = 0.1\ \mu$F, and using Eqs. (4.62), we get the following:

$$R_1 = \frac{600}{2 \times 12 \times 10^4 \times 10^{-7}} = 25\ \text{k}\Omega$$

$$R_2 = \frac{2}{600 \times 10^{-7}} = 33.3\ \text{k}\Omega$$

$$\frac{R_a}{R_b} = \frac{600^2}{12 \times 10^4} = 3$$

The actual values of R_a, R_b can be obtained either by assuming a suitable value of K or by optimizing some other circuit characteristic, e.g., the opamp offset. In the latter case, selecting $R_a // R_b = R_2$ we can obtain

$$R_a = 120 \text{ k}\Omega \qquad R_b = 40 \text{ k}\Omega$$

resulting in a $K = 0.25$. However, this value of K can become unity by means of a 1 kΩ potentiometer connected as shown in Fig. 4.14.

4.6 Realization of a Quadratic with a Positive Real Zero

In realizing certain lowpass non-minimum phase delay functions [14] as the cascade connection of second-order stages, one is involved with the realization of a quadratic with a positive real zero, i.e., with the function

$$F(s) = \frac{k(\alpha - s)}{s^2 + \beta s + \gamma} \tag{4.63}$$

An RC active network, which can realize $F(s)$ using one operational amplifier [15] and which has very low sensitivity to variations in the values of its components, is shown in Fig. 4.15. Its transfer voltage ratio is

$$\frac{V_o}{V_i} = \frac{\dfrac{g_1}{C_2}\left(\dfrac{g_a}{g_b}\dfrac{g_2}{C_1} - s\right)}{s^2 + g_2\dfrac{C_1 + C_2}{C_1 C_2}s + \dfrac{g_2}{C_1 C_2}\left[\left(1 + \dfrac{g_a}{g_b}\right)g_1 + g_a\right]} \tag{4.64}$$

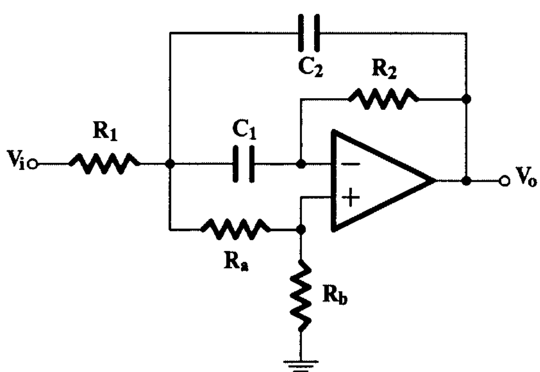

FIGURE 4.15
An active RC network for the realization of $F(s)$, Eq. (4.63).

The values of the components of the network realizing $F(s)$, Eq. (4.63), can be found by equating coefficients of equal powers of s in Eqs. (4.63) and (4.64). Since the number of the unknowns is larger than the number of the resulting equations, the values of any two components can be selected arbitrarily. Selecting, for example,

$$C_1 = C_2 = 1$$

coefficient matching in Eqs. (4.63) and (4.64) gives the following:

$$\alpha = \frac{g_a}{g_b} g_2 \qquad \beta = 2g_2 \qquad \gamma = g_2\left[\left(1 + \frac{g_a}{g_b}\right) g_1 + g_a\right] \qquad k = g_1$$

Solving for the unknown component values gives the following:

$$g_1 = k \tag{4.65a}$$

$$g_2 = \frac{\beta}{2} \tag{4.65b}$$

$$g_a = \frac{2\gamma - (\beta + 2\alpha)k}{\beta} \tag{4.65c}$$

$$g_b = \frac{2\gamma - (\beta + 2\alpha)k}{2\alpha} \tag{4.65d}$$

Clearly, for positive values of g_α and g_b, the following conditions should hold between the coefficients of the transfer function:

$$2\gamma = k(2\alpha + \beta)$$

On the other hand, if $C_1 \neq C_2$, one can show that, for the component values to be positive, k should be

$$k < \frac{\gamma}{\alpha + \beta \dfrac{C_2}{C_1 + C_2}} \tag{4.66}$$

It can also be shown that the Q-factor sensitivities are as follows:

$$S^Q_{g_1}, S^Q_{g_a}, (-S^Q_{g_b}) < \frac{1}{2}$$

$$S^Q_{g_2} = -\frac{1}{2}$$

$$S^Q_{C_1} = S^Q_{C_2} = 0 \text{ if } C_1 = C_2$$

It can be seen that the Q-factor sensitivities are extremely low and independent of the Q factor. Also, the Q factor is insensitive to variations in the values of the two capacitances, if $C_1 = C_2$. Therefore, this condition can be the starting point in the design of the stage.

As an example, consider the realization of the following function:

$$F(s) = \frac{0.5628(5.902 - s)}{s^2 + 4.117s + 6.9963} \tag{4.67}$$

which is part of a delay function. Selecting

$$C_1 = C_2 = 1$$

we have inequality (4.66) satisfied. The values of the other components are calculated by means of Eqs. (4.65). The circuit, denormalized to an impedance level of $10^6/3\ \Omega$ and a 0.1 s delay, is shown in Fig. 4.16.

4.7 Biquads Obtained Using the Twin-T RC Network

The twin-T (TT) RC network is shown in its simplest version in Fig. 4.17a. Its transfer voltage ratio V_2/V_1 is as follows:

$$\frac{V_2}{V_1} = \frac{s^2 + \dfrac{1}{T^2}}{s^2 + \dfrac{4}{T}s + \dfrac{1}{T^2}} \tag{4.68}$$

where $T = RC$.

Thus, the TT in Fig. 4.17(a) processes a pair of transmission zeros on the $j\omega$-axis or, in other words, it is a bandstop circuit. This pair of zeros can be moved out of the $j\omega$-axis, if a

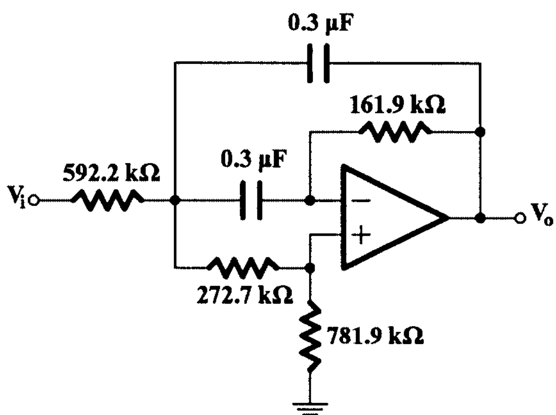

FIGURE 4.16
Realization of $F(s)$, Eq. (4.67), denormalized to $10^6/3\ \Omega$ and 0.1 s delay.

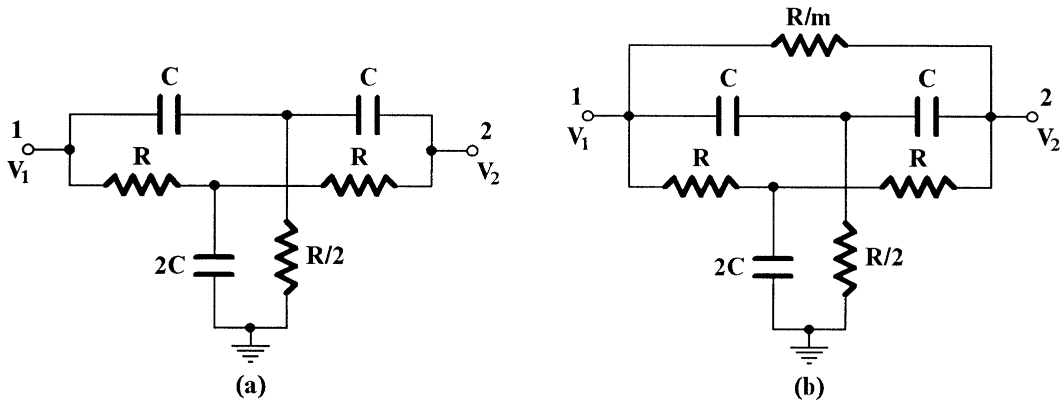

FIGURE 4.17
(a) The TT network and (b) the Bridged-TT network.

resistor is used to bridge the terminals 1 and 2, as shown in Fig. 4.17(b). In this case, the new transfer voltage ratio will be the following:

$$\frac{V_2}{V_1} = \frac{s^2 + \frac{2m}{T}s + \frac{1+2m}{T^2}}{s^2 + \frac{4+2m}{T}s + \frac{1+2m}{T^2}} \tag{4.69}$$

It can be seen from Eq. (4.69) that the transmission zeros have moved inside the LH of the s-plane. Either form of the TT shown in Fig. 4.17 (and, of course, more complicated ones [16]) can be used in conjunction with active elements to realize various biquadratic functions.

As an example, consider the circuit in Fig. 4.18. If

$$\frac{1}{R_3} = \frac{1}{R_1} + \frac{1}{R_2}$$

analysis of the circuit, assuming ideal opamp, gives the following voltage ratio:

$$\frac{V_o}{V_i} = \frac{G(s^2 + \omega_o^2)}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2} \tag{4.70}$$

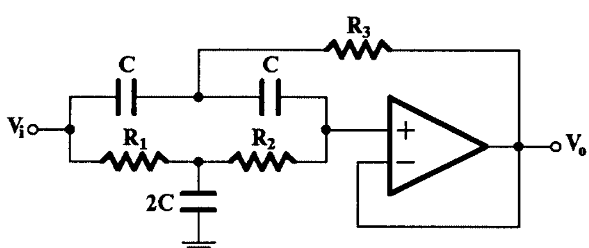

FIGURE 4.18
Example circuit.

where

$$G = 1, \quad \omega_o^2 = \frac{1}{R_1 R_2 C^2}, \quad \frac{\omega_o}{Q} = \frac{2}{R_2 C} \tag{4.71}$$

Thus, a useful bandstop circuit with zero output impedance has been obtained using the TT. A number of other biquads can be obtained using other forms of TT, and the interested reader is advised to refer to other sources [1, 16] for details.

4.8 Two-Opamp Biquads

A large number of two-opamp biquads can be obtained by a technique introduced below for the enhancement of the Q of certain SABs. Other useful two opamp biquads have also been suggested which possess interesting characteristics. In addition, very low sensitivity biquads may be obtained if the inductance of the LCR biquad is simulated by the two-opamp GIC [17, 18]. Although inductance simulation is explained in detail in Chapter 6, we include this possibility here because of its simplicity. In what follows, we examine first this possibility and then some other useful two-opamp biquads.

4.8.1 Biquads by Inductance Simulation

In Fig. 4.19, three possible biquads, obtained by combining an inductance, a capacitance, and a resistance, are shown. All of them have the same poles with $\omega_o = 1/\sqrt{LC}$ and $Q = R\sqrt{C/L}$.

Substituting the component L in Fig. 4.19(a) by its GIC equivalent (see Sections 3.5.2 and 6.4), the corresponding active RC circuit in Fig. 4.20 is obtained without the buffer amplifier of gain k.

If the signal source is removed and placed at node b feeding R in series, while node a is earthed the bandpass biquad equivalent to that in Fig. 4.19(b) is obtained. Finally, to obtain the highpass equivalent, the signal source is placed between a broken node at c and earth, while nodes a and b are earthed.

It can be shown by referring to Fig. 4.20 that the value of the equivalent inductance is ideally as follows:

$$L_{eq} = C\frac{R_1 R_3 R_L}{R_2} \tag{4.72}$$

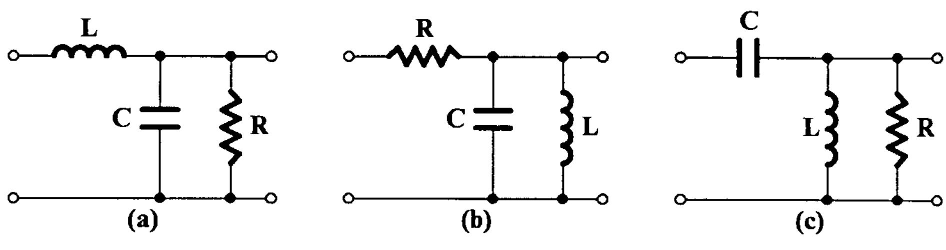

FIGURE 4.19
RLC biquads: (a) lowpass, (b) bandpass, and (c) highpass.

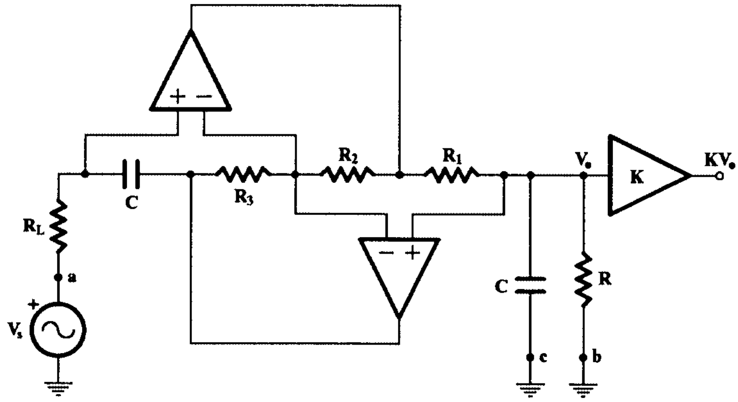

FIGURE 4.20
Lowpass biquad.

Substituting for L, we obtain the following ω_o and Q values:

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{C(R_1R_3R_L/R_2)^{1/2}} \tag{4.73}$$

$$Q = \omega_o CR = \frac{R}{(R_1R_3R_L/R_2)} \tag{4.74}$$

In designing these filters, it is usual to select

$$R_1 = R_2 = R_3 = RL = r \tag{4.75}$$

when, from Eqs. (4.73) and (4.74),

$$rC = \frac{1}{\omega_o} \tag{4.76}$$

and

$$R = Qr \tag{4.77}$$

The Q and ω_o sensitivities to variation in the passive component values are the following:

$$S_R^Q = -2S_{R_1}^Q = -2S_{R_3}^Q = 2S_{R_2}^Q = -2S_{R_L}^Q = 1$$

$$S_R^{\omega_o} = 0,\, S_c^{\omega_o} = 2S_{R_1}^{\omega_o} = -2S_{R_2}^{\omega_o} = 2S_{R_3}^{\omega_o} = 2S_{R_L}^{\omega_o} = -1$$

On the other hand, if we consider matched opamps with the one pole model describing their frequency response, we can determine that the error of ω_o is approximately [16] the following:

$$\frac{\Delta\omega_o}{\omega_o} \cong -2\frac{\omega_o}{\omega_T} \tag{4.78}$$

One important drawback of these biquads is that their output is not taken from a zero impedance node. Therefore in order to cascade such stages for realizing higher order filters isolation amplifiers should be inserted between successive stages, as shown in Fig. 4.20.

Inductance simulation can also be applied to obtain notch biquads, both lowpass (LPN) and highpass (HPN) notch. Suitable LCR biquads are those shown in Fig. 4.21. For the *all-pass* notch, i.e., with $\omega_{oz} = \omega_{op}$, C_2 and L_2 in these networks should be deleted. The substitution of L by its GIC equivalent in Fig. 4.21(a) is straightforward, as it was achieved in the case of the lowpass biquad in Fig. 4.19(a). However, in the case of the HPN in Fig. 4.21(b), both L_1 and L_2 are simulated using the same GIC, as explained in Chapter 6. We do not include the active equivalents of the LPN and the HPN here, leaving them to the reader as an exercise.

Finally, the allpass biquad using inductance simulation is clearly a three-opamp biquad, and consequently it does not belong to the two opamp class of biquads. A simulation is given below.

4.8.2 Two-Opamp Allpass Biquads

The allpass biquadratic function

$$F(s) = \frac{s^2 - \beta s + \gamma}{s^2 + \beta s + \gamma} \tag{4.79}$$

is written in the following form:

$$F(s) = 1 - 2\frac{\beta s}{s^2 + \beta s + \gamma} = 1 - 2F_1(s) \tag{4.80}$$

where

$$F_1(s) = \frac{\beta s}{s^2 + \beta s + \gamma} \tag{4.81}$$

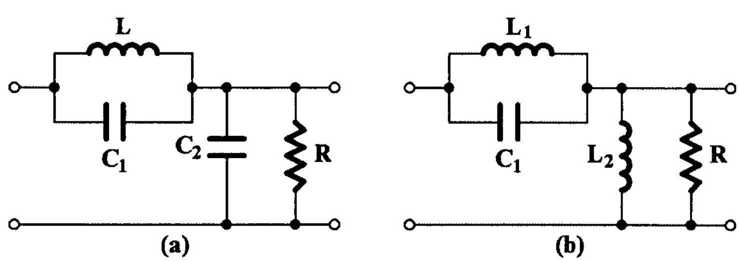

FIGURE 4.21
LCR notch biquads: (a) LPN and (b) HPN.

Then, $F_1(s)$ is realized by any bandpass SAB, and $F(s)$ is formed by using a second opamp to perform the summation in Eq. (4.80). Such an allpass active circuit is shown in Fig. 4.22, where the SAB in Fig. 4.10 has been used to realize $F_1(s)$.

The merits of the SAB in Fig. 4.10 can be used advantageously in the realization of the high-Q biquadratics in a high-order allpass function.

4.8.3 Selectivity Enhancement

There is a category of active RC networks with inherently low sensitivities but requiring excessive spread in component values, even when they realize relatively low (~10) Q values. These networks employ negative feedback exclusively around a finite- or infinite-gain amplifier.

To improve the selectivity of such circuits, one could employ positive feedback [8, 19], but this leads to sensitivity degradation of the circuits. We describe another method here, by means of an example, which does not lead to severe sensitivity degradation if it is applied carefully.

Consider the bandpass circuit in Fig. 4.23. It can be shown that the Q value of this circuit is the following:

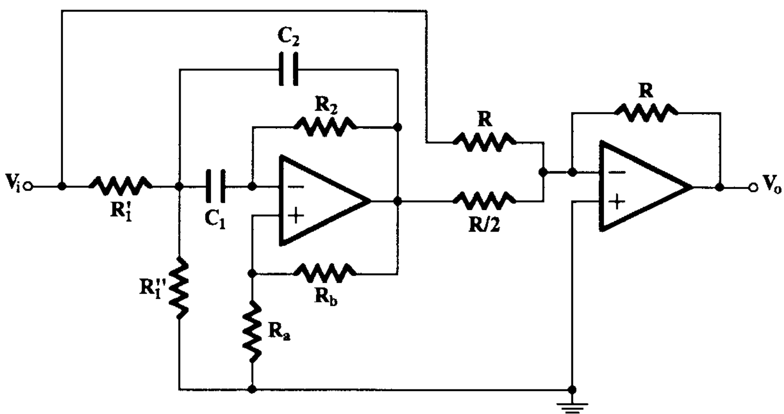

FIGURE 4.22
A two-opamp allpass biquad.

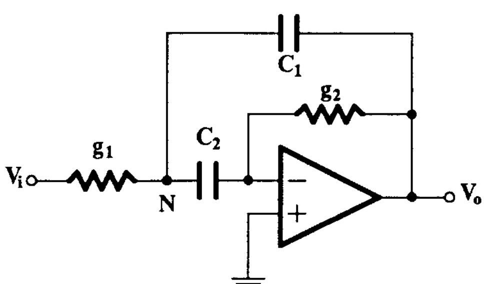

FIGURE 4.23
A low-Q bandpass circuit.

$$Q = \sqrt{\frac{g_1}{g_2}\frac{C_1C_2}{(C_1+C_2)^2}} \tag{4.82}$$

We now introduce a VCVS of gain K at node N as shown in Fig. 4.24. Note that there are now available two zero impedance outputs.

By straightforward analysis, we can find that

$$\frac{V_o}{V_i} = -\frac{\left(\frac{g_1}{C_1}\right)s}{s^2 + \frac{g_2}{KC_2}s + \frac{g_1g_2}{KC_1C_2}} \tag{4.83}$$

and

$$\frac{V_a}{V_i} = -\frac{g_1g_2/C_1C_2}{s^2 + \frac{g_2}{KC_2}s + \frac{g_1g_2}{KC_1C_2}} \tag{4.84}$$

i.e., the simultaneous realization of a bandpass and a lowpass function both having the same poles. From Eq. (4.83), the new Q factor will be

$$Q' = \sqrt{\frac{KC_2g_1}{C_1g_2}} \tag{4.85}$$

It can be seen that for the same component values in the two circuits, there is an enhancement in Q since

$$\frac{Q'}{Q} = \left(1 + \frac{C_2}{C_1}\right)\sqrt{K} \tag{4.86}$$

Since the introduction of the VCVS affects both the Q and ω_o values, their corresponding sensitivities will be increased. However, if the VCVS is realized by means of an opamp, and the value of K is very low (e.g., unity), its effect on the sensitivity will not be significant.

The Q enhancement technique suggested above can be applied to a large number of circuits [20–22]. In all cases, the availability of the second zero-impedance output is an advan-

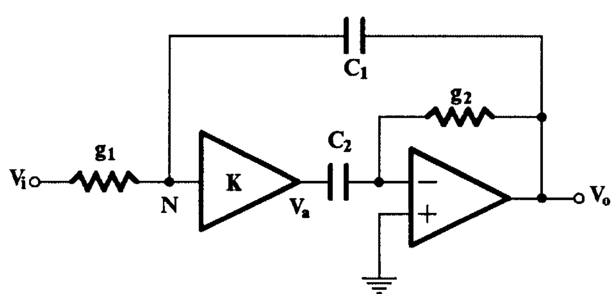

FIGURE 4.24
The enhanced-Q circuit.

tage. Note that, if the positions of resistors and capacitors in Fig. 4.24 are interchanged, the second transfer function of the network will be highpass instead of lowpass.

It is interesting to note that a circuit due to Bach [23] can be obtained from the Sallen and Key lowpass circuit using the Q enhancement technique. Thus, assuming that the gain of the voltage amplifier in the Sallen and Key filter is unity, Fig. 4.25(a), introducing an additional unity gain VCVS at node A, Bach's circuit in Fig. 4.25(b) is obtained.

Of course, this does not imply that the respective components in the two circuits have the same values. Applying the RC:CR transformation, the corresponding highpass circuits are obtained.

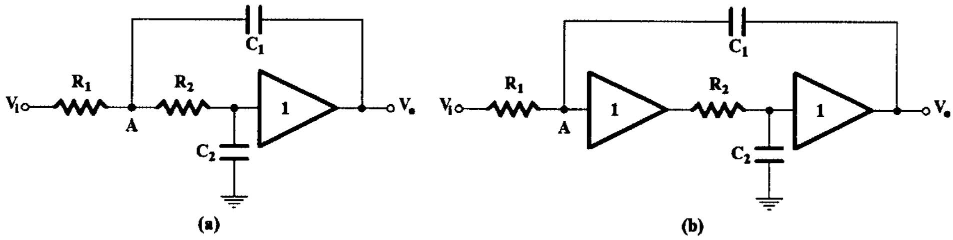

FIGURE 4.25
(a) Sallen and Key and (b) Bach's circuits.

4.9 Three-Opamp Biquads

The use of three opamps to realize second-order filter functions leads to multiple-output biquads with the additional advantage of versatility in that ω_o, Q, and the filter gain can be independently adjusted. Although they are not without problems, as we shall see later, technology has produced three opamp chips ready for use in realizing biquadratics.

The poles of the circuits we present here are obtained by means of two integrators in a feedback loop. That is why these are often referred to as the two *integrator-loop biquads.*

We may develop such a three-opamp biquad following the old analog computing technique for solving a differential equation. To show this, let us consider the second-order lowpass function realized as the ratio of two voltages V_o and V_i, i.e.,

$$\frac{V_o}{V_i} = \frac{K}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2} \tag{4.87}$$

We can write this equation as follows:

$$\frac{s^2}{\omega_o^2}V_o = \frac{K}{\omega_o^2}V_i - \frac{1}{Q}\frac{s}{\omega_o}V_o - V_o \tag{4.88}$$

Clearly, V_o may be obtained from $(s^2/\omega_o^2)V_o$ by two successive integrations with proper time constants. On the other hand, the term $(s^2/\omega_o)V_o$ can be obtained as the sum of the three terms on the right-hand side of Eq. (4.88). We may thus realize Eq. (4.88) as it is shown in block diagram form in Fig. 4.26.

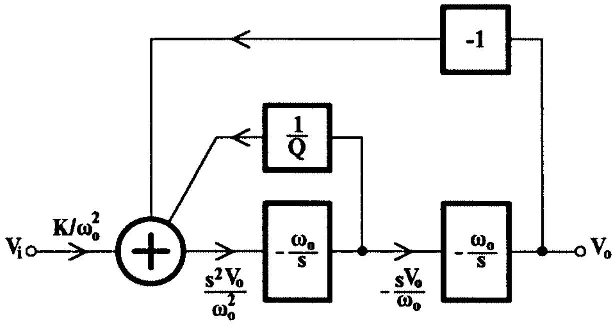

FIGURE 4.26
Block diagram for realizing V_o/V_i, Eq. (4.88).

Using opamps to perform the summing and integration operations, we obtain the circuit shown in Fig. 4.27. Straightforward analysis of this circuit shows that the function in Eq. (4.87) is realized with a minus sign. To avoid this, and also the use of a fourth amplifier, we make use of the circuit in Fig. 3.8, giving the difference of two voltages.

Following this, the circuit in Fig. 4.27 becomes as is shown in Fig. 4.28. Again, straightforward analysis of this circuit gives the following:

$$\frac{V_o}{V_i} = \frac{V_{LP}}{V_i} = \frac{K'/(RC)^2}{s^2 + \frac{1}{RC}\frac{1}{Q}s + \frac{1}{R^2C^2}} \tag{4.89}$$

$$\frac{V_{BP}}{V_i} = -\frac{(K'/RC)s}{s^2 + \frac{1}{RC}\frac{1}{Q}s + \frac{1}{R^2C^2}} \tag{4.90}$$

$$\frac{V_{HP}}{V_i} = \frac{K's^2}{s^2 + \frac{1}{RC}\frac{1}{Q}s + \frac{1}{R^2C^2}} \tag{4.91}$$

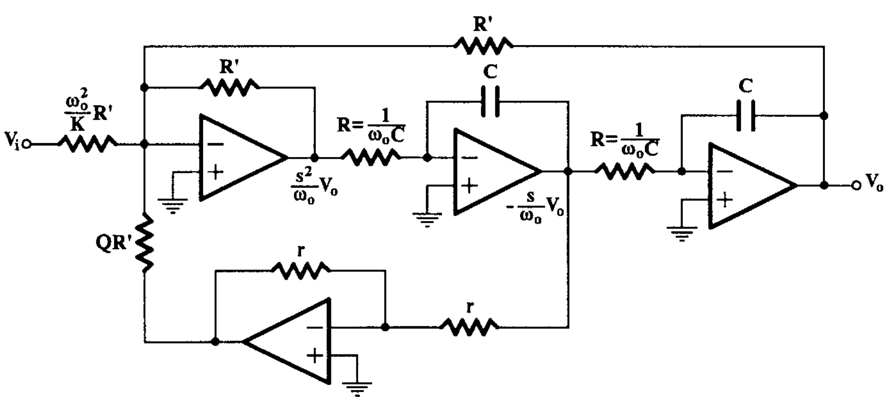

FIGURE 4.27
Implementation of the block diagram in Fig. 4.26.

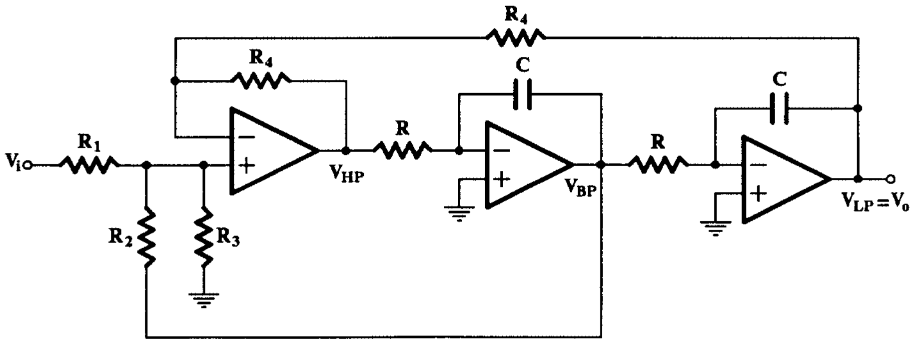

FIGURE 4.28
The KHN biquad.

where

$$K' = \frac{2R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3} \tag{4.92a}$$

$$1/Q = \frac{2R_1R_3}{R_1R_2 + R_1R_3 + R_2R_3} \tag{4.92b}$$

and

$$\frac{K'}{R^2C^2} = K \tag{4.92c}$$

From these, we can obtain the following design equations:

$$RC = \frac{1}{\omega_o} \tag{4.93a}$$

$$R_2 = R_1\frac{KQ}{\omega_o^2} \tag{4.93b}$$

$$R_3 = \frac{R_1}{\frac{2\omega_o^2}{K} - 1 - \frac{\omega_o^2}{KQ}} \tag{4.93c}$$

with R_1 and R_4 taking suitably chosen values.

This circuit is referred to as the KHN (Kerwin, Huelsman, Newcomb) biquad [24], and it simultaneously displays lowpass, bandpass, and highpass behavior. All three filters have the same poles, of course.

A notch response can be obtained by adding the lowpass and highpass outputs using an extra opamp. The ω_0 and Q passive sensitivities are very low. However, the excess phase shift introduced by each integrator and the summer, due to the finite GB product of the opamps, leads to Q enhancement with undesirable effects in the filter response. This is examined below, but first let us see another three-opamp biquad with some additional interesting characteristics.

4.9.1 The Tow-Thomas [25–27] Three-Opamp Biquad

A more versatile three-opamp biquad is shown in Fig. 4.29. Its transfer voltage ratio (V_o/V_i) is as follows:

$$\frac{V_o}{V_i} = -\frac{\dfrac{C_1}{C}s^2 + \dfrac{1}{CR}\left(\dfrac{R}{R_1} - \dfrac{r}{R_3}\right)s + \dfrac{1}{C^2RR_2}}{s^2 + \dfrac{1}{CR_4}s + \dfrac{1}{C^2R^2}} \tag{4.94}$$

Clearly, it is possible to obtain any kind of second-order filter function by a proper choice of the component values. Thus we may have the following:

LP: if $C_1 = 0$, $R_1 = R_3 = \infty$

BP: if $C_1 = 0$, $R_1 = R_2 = \infty$ (positive sign)

BP: if $C_1 = 0$, $R_2 = R_3 = \infty$ (negative sign)

HP: if $C_1 = C$, $R_1 = R_2 = R_3 = \infty$

Notch: if $C_1 = C$, $R_1 = R_3 = \infty$

Allpass: if $C_1 = C$, $R_1 = \infty$, $r = R_3/Q$

This circuit was initially proposed by Tow [25] and studied by Thomas [26, 27]. Its passive sensitivities are similar to those of the KHN three-opamp network. However, it is more versatile, realizing all kinds of second-order filter function without the use of an extra opamp, which is required in the case of the KHN circuit. It suffers, though, from the results of excess phase shift on Q enhancement, as explained below.

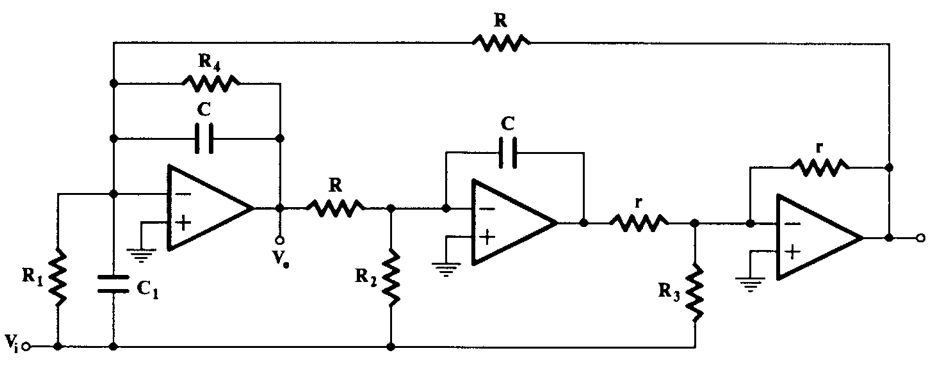

FIGURE 4.29
The Tow-Thomas three-opamp biquad.

4.9.2 Excess Phase and Its Compensation in Three-Opamp Biquads

Because of the finite gain-bandwidth product of the opamps, excess phase appears in the summer (beyond the 180°) and the integrators (beyond the 90°) in the three-opamp biquads, which finally leads to Q enhancement and to instabilities of the circuits. We may calculate this excess phase in the case of the simple sign-reversing amplifier in Fig. 4.30 as follows. Assuming that the opamp gain may be approximated with a model having a single pole at the origin (that is, at zero frequency), i.e.,

$$A = -\frac{\omega_T}{s}$$

simple analysis of the circuit in Fig. 4.30 gives the following transfer voltage ratio V_o / V_i:

$$\frac{V_o}{V_i} = -\frac{1}{1 + \frac{2s}{\omega_T}} \tag{4.95}$$

Thus, at frequency ω, there is an extra phase shift beyond the 180°,

$$\varphi_{ex} = -\tan^{-1}\frac{2\omega}{\omega_T} \cong \frac{2\omega}{\omega_T} \quad \text{for} \quad \omega << \omega_T \tag{4.96}$$

This extra phase shift φ_{ex}, which in the case of the ideal opamp would be zero, is called the *excess phase*.

In a similar way, it can easily be shown that the excess phase for an integrator is approximately

$$\varphi_{ex} \cong -\frac{\omega}{\omega_T} \tag{4.97}$$

Since the two integrators and the summer are connected in cascade inside the loop, the overall excess phase becomes substantial. It can be shown that this leads to Q enhancement, which is undesirable. Thus, assuming that the opamps are identical (i.e., they have the same ω_T), the enhanced Q_{enh} of the biquad in Fig. 4.29 is approximately [28]

$$Q_{enh} \cong Q\frac{1}{1 - KQ\frac{\omega_o}{\omega_T}} \tag{4.98}$$

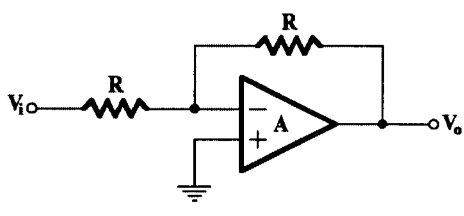

FIGURE 4.30
Sign-reversing amplifier.

where the value of Q is that for ideal opamps, ω_o is the pole frequency of the biquad, and K typically is 4. A similar Q enhancement is obtained in the case of the biquad in Fig. 4.28.

The excess phase, being a phase lag, can be cancelled by introducing an equal phase lead inside the loop. Passively, this can be easily achieved by connecting a capacitor of suitable value in parallel with one of resistors R in Fig. 4.28 or the resistor R in the integrator in Fig. 4.29. However, since the temperature coefficients of this capacitor and ω_T are not the same, this compensation cannot be perfect at different temperatures.

More successful is the active compensation as employed in the Åkerberg-Mossberg three-opamp biquad, which we examine next.

Clearly, to reverse the sign of an integrator, a second opamp should be employed as shown in Fig. 4.31(a). The excess phase for this non-inverting or *positive integrator* is approximately, according to Eqs. (4.96) and (4.97),

$$\varphi_{exc} \cong -\frac{3\omega}{\omega_T}$$

However, if the sign reversal of the integrator is achieved according to Fig. 4.31(b), then it can be easily shown that the excess phase of the integrator becomes

$$\varphi'_{exc} \cong \frac{\omega}{\omega_T}$$

i.e., phase lead instead of phase lag.

Thus, this phase lead will cancel out the phase lag of the other integrator inside the loop of the three-opamp biquads, and the overall excess phase will be zero. This scheme of excess phase compensation is called *active compensation*, and it is less temperature dependent than the passive compensation of excess phase that was mentioned above.

4.9.3 The Åkerberg-Mossberg Three-Opamp Biquad [29]

The Åkerberg-Mossberg three-opamp biquad is a modified version of the Tow-Thomas three-opamp biquad. To simplify matters we set the following values to some of the passive components in the circuit in Fig. 4.29:

$$C_1 = 0, \; R_2 = R_3 = \infty$$

Then, the Tow-Thomas biquad becomes as shown in Fig. 4.32(a). By actively compensating the integrator as in Fig. 4.31, the Åkerberg-Mossberg biquad is obtained as shown in Fig. 4.32(b).

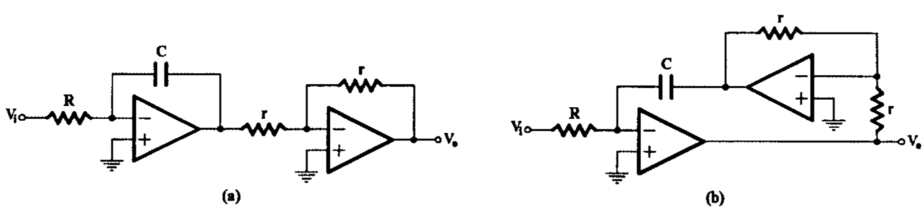

FIGURE 4.31
(a) A non-inverting integrator and (b) a non-inverting integrator with active compensation of excess phase.

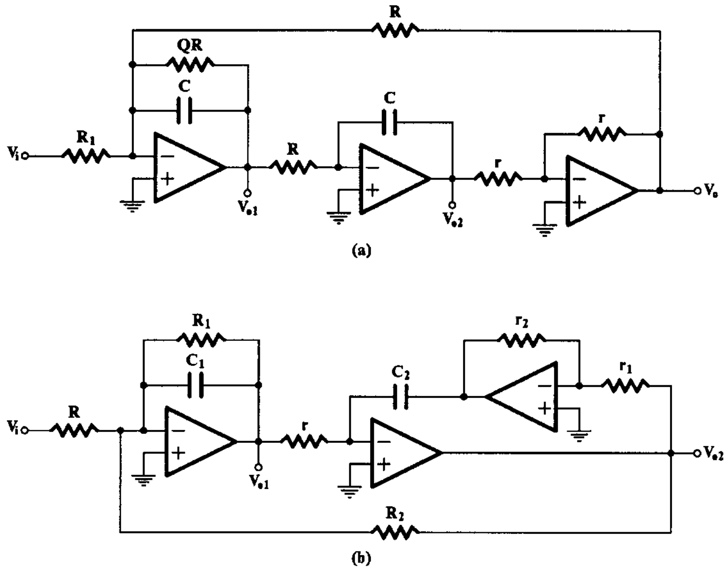

FIGURE 4.32
(a) The Tow-Thomas biquad actively compensated for excess phase, giving the Åkerberg-Mossberg biquad in (b).

Straightforward analysis of the biquad in Fig. 4.32(b) gives the following:

$$\frac{V_{o1}}{V_i} = -\frac{s/RC_1}{s^2 + s\dfrac{1}{R_1C_1} + \dfrac{r_1}{C_1C_2R_2rr_2}} \tag{4.99}$$

$$\frac{V_{o2}}{V_i} = -\frac{r_1/C_1C_2Rrr_2}{s^2 + s\dfrac{1}{R_1C_1} + \dfrac{r_1}{C_1C_2R_2rr_2}} \tag{4.100}$$

The Åkerberg-Mossberg biquad, in its simplified form, can thus simultaneously realize lowpass and bandpass functions with the same poles. In its more general version, it can realize any type of biquadratic function, if the input signal is fed properly weighted to the inputs of all amplifiers. In all cases, the poles are not affected.

It is clear from Eqs. (4.99) and (4.100) that both the Q factor and the pole frequency ω_o can be independently adjusted. Depending on the component values, voltages at internal nodes may differ substantially. This inevitably will lead to nonlinear operation of some amplifiers and, therefore, to reduced dynamic range even for small signals. For the minimization of such effects, r_1 should be equal to r_2, but this will make the active compensation ineffective, and thus Q enhancement will not be avoided.

4.10 Summary

Low-order (first and second) filter circuits were presented in this chapter, which have been chosen among an abundance that have appeared in the literature over the years. These are mostly canonic and have been proven useful in practice.

The biquads can be classified as SABs (single-amplifier biquads), two-opamp biquads, and three-opamp biquads. Also, the SABs can be classified as enhanced positive feedback (EPF) and enhanced negative feedback (ENF) biquads, depending on the feedback paths, positive or negative, in which the frequency dependent passive network is connected. By means of the application of the complementary transformation to one SAB, another SAB is obtained which has the same poles as the first one and, consequently, same pole sensitivities. This has led to the discovery of a number of useful biquads as well as to the rediscovery of others, previously suggested.

Among the important merits of a biquad, its sensitivity to variations in component values and economy are the most prominent. Sensitivity of Q, ω_o, and the gain-sensitivity product are the most useful measures that characterize the value of a biquad. It is because of this that high-Q biquadratic functions could better be realized by two- or three-opamp biquads instead of using SABs. This, of course, increases the cost and power consumption, and it is a matter of priority for the designer in solving a particular filtering problem to make the appropriate choice.

First- and second-order filter circuits are very useful in the realization of high-order filter functions. Some methods for such realizations are based on the use of lower-order sections, particularly biquads. Therefore, knowledge of the most suitable biquad in a particular case is of fundamental importance for achieving the "best" design. This will become apparent in the next chapter, where the design of high-order filters is considered.

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