School of Electrical and Information Engineering
University of the Witwatersrand, Johannesburg
ELEN3016 Control I

Control Laboratory

Analogue Computers – Modelling and Control

Contents

1 Preamble

This laboratory experiment is to be conducted in groups of three. Simulation code has to be written and circuits have to be designed before the actual laboratory session commences. Demonstration of your working system and assessments will be done during each laboratory session and therefore attendance of every laboratory session is compulsory. The persons who are assigned to act as lab assistants/demonstrators for Control I are listed on the last page of this document.

2 Purpose

The purpose of this laboratory experiment is to introduce the concept of system modelling and control using analogue computers. The objective is to use an analogue computer to model the response of a given physical system. In addition a controller has to be designed in order for the compensated system to meet the given design specifications. A second analogue computer then has to be used to model the controller. Combining the two analogue computers in a closed-loop configuration will then model the compensated system and if the designs and implementations have been done correctly the analogue computer model of the compensated system will then meet the prescribed specifications. The system which has to be controlled is often referred to as the plant.

3 Objectives

On completion you will:

- Understand what analogue computers are and how to use them for modelling and control.
- Appreciate the preference of integration over differentiation used in analogue computers.
- Design and implement a classical controller.
- Experiment with gain compensation, system stability and closed-loop control.

4 References

4.1 Review topics

- Transfer functions of continuous-time feedback systems.
- Continuous-time response of first-order and second-order systems.
- Poles, zeros, and characteristic roots.
- Gain and stability.

4.2 Exploratory topics

- Different feedback control concepts/
- Analogue computers, magnitude scaling and time scaling.

5 Overview

An *analogue computer* [8] is defined to be any form of computer that uses energy storage devices/systems such as electrical, mechanical, or hydraulic devices/systems to model the problem to be solved. In this experiment we shall focus on electronic analogue computers which use operational amplifiers configured as integrators or differentiators. In an electronic analogue computer the voltages at various places within the computer are proportional to the variables at the corresponding places in the actual system.

In a digital computer the programming of the problem to be solved uses numerical methods and algorithms whereas in an analogue computer the problem has to be decomposed into summing, amplification and integration/differentiation operations by means of decomposition/programming techniques. The decomposition technique we shall be using is called *direct decomposition/programming*. Then by implementing the resulting decomposition using operational amplifiers we obtain an analogue computer designed to solve the problem.

6 Background

6.1 Linear Operational Amplifier Building Blocks

The following presents a very brief summary of basic circuit configurations of interest for this laboratory experiment.

Summing amplifier with weighted inputs

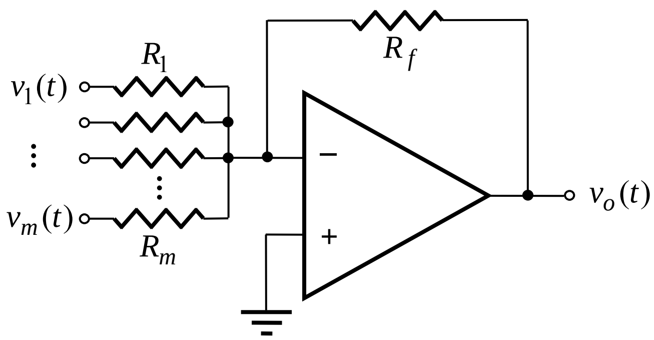

Figure 1. Weighted-input summing amplifier.

Transfer characteristic: $V_o(s) = -\left(\frac{R_f}{R_1}V_1(s) + \frac{R_f}{R_2}V_2(s) + \ldots + \frac{R_f}{R_m}V_m(s)\right)$

Integrator with weighted inputs

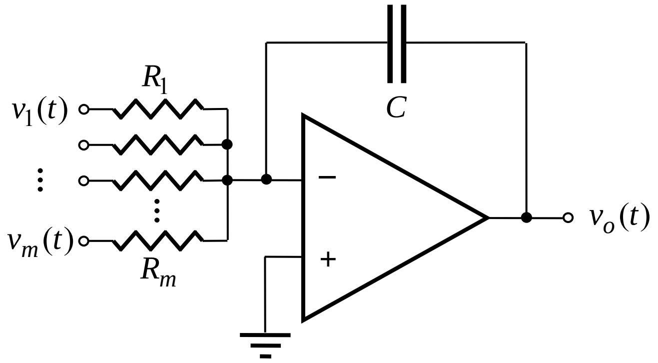

Figure 2. Integrator.

Transfer characteristic: $V_o(s) = -\left(\frac{1}{R_1 C} s^{-1} V_1(s) + \frac{1}{R_2 C} s^{-1} V_2(s) + \ldots + \frac{1}{R_m C} s^{-1} V_m(s) \right)$

Differentiator

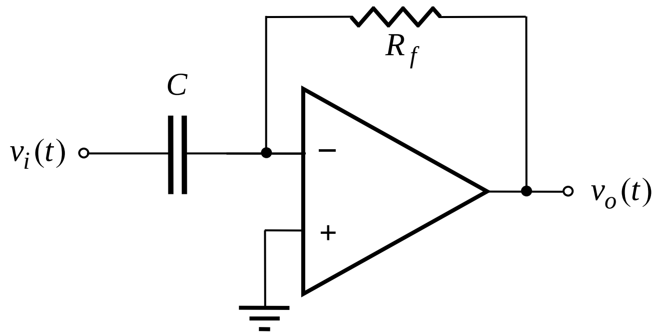

Figure 3. Differentiator.

Transfer characteristic: $V_o(s) = -RC\,sV_i(s)$

6.2 Direct Decomposition [3]

Consider the system with transfer function

$$\frac{Y(s)}{U(s)} = \frac{b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \ldots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0}. \tag{1}$$

Multiplying the numerator and denominator of the right hand side of (1) by $s^{-n}X(s)$, where $X(s)$ is just a dummy variable, yields

$$\frac{Y(s)}{U(s)} = \frac{\left(b_{n-1}s^{-1} + b_{n-2}s^{-2} + \ldots + b_1 s^{-n+1} + b_0 s^{-n}\right)X(s)}{\left(1 + a_{n-1}s^{-1} + \ldots + a_1 s^{-n+1} + a_0 s^{-n}\right)X(s)}. \tag{2}$$

Next we equate the numerators and denominators across the equality to obtain

$$Y(s) = \left(b_{n-1}s^{-1} + b_{n-2}s^{-2} + \ldots + b_1 s^{-n+1} + b_0 s^{-n}\right)X(s) \tag{3}$$

and

$$U(s) = \left(1 + a_{n-1}s^{-1} + \ldots + a_1 s^{-n+1} + a_0 s^{-n}\right)X(s)\,. \tag{4}$$

Equation (3) is in the proper cause-effect form but not (4) and to achieve this we express (4) as

$$X(s) = U(s) - \left(a_{n-1}s^{-1} + \ldots + a_1 s^{-n+1} + a_0 s^{-n}\right)X(s)\,. \tag{5}$$

The state diagram of the direct decomposition (i.e. (3) and (5)) is depicted in Figure 4. Notice that direct decomposition used summers and integrators to represent the system transfer function.

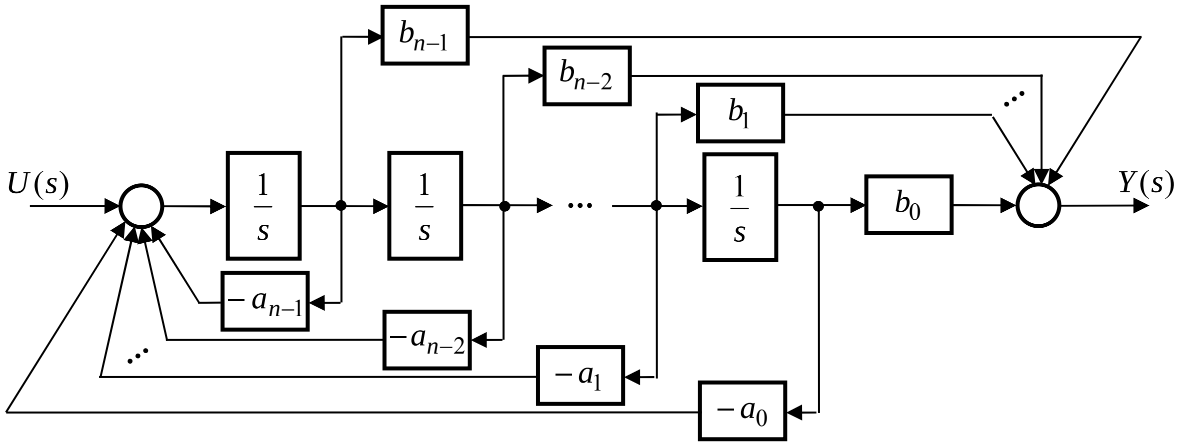

Figure 4. State diagram depicting direction decomposition of a transfer function.

Notes:

- Direct decomposition can be performed in the time domain. Therefore it can be applied to non-linear systems in which case the mathematical techniques based on Laplace transform are not particularly helpful. Current research focuses on generalising the concept of a transfer function to non-linear system [7].
- In order to make measuring the time excursions of variables of the analogue computer more practical it might be necessary to introduce either *magnitude scaling* or *time scaling* or both at design time. See section 6.6 below.

6.3 Control system design

Given the transfer function $G(s)$ for a system one of the usual controller design methodologies must then be used to design a controller to satisfy the user's specifications.

6.4 Closed-loop system

Figure 5 shows the block diagram of the closed-loop system also called the compensated system. The output or process variable as it is also called is fed back and subtracted from the control setpoint. This difference (or error) is then fed to the controller which transforms the error into a compensated input to the model/system.

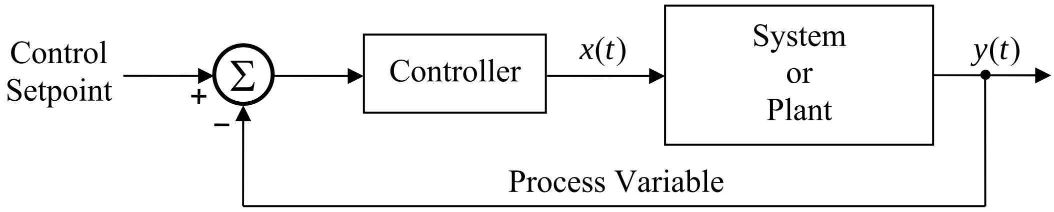

Figure 5. Simplified block diagram of the closed-loop system.

6.5 Magnitude and time scaling

Often the problem being simulated on an analogue computer exceeds the regions of operation of electronic components and instruments. In these circumstances magnitude and/or time scaling becomes necessary. Magnitude and time scaling are discussed in sections 2.6, 8.2 (p. 242) and 8.3 of [2].

6.6 Incorporating initial conditions

From the properties of the Laplace transform it is evident how to incorporate initial conditions. Refer to section 8.1(p. 238) of [2]. Alternative approaches are discussed in Problem 2.35 on p. 95 of [6].

7 Laboratory Experiment

7.1 Experiments

Question 1 (*Week 1*)

Design an analogue computer using operational amplifiers for modelling the plant described by the transfer function

$$G(s) = \frac{K}{s(s+4)(s+6)}\,.$$

Now, with unity negative feedback applied around the plant, use an oscilloscope to display the system's step response. Measure the rise time, percentage overshoot, time of first peak, settling time and steady state error. Compare these with the theoretically calculated values as well as with a Matlab implementation using Simulink (completed prior to the lab session). Write a brief summary and discussion of your findings.

Question 2 (*Week 2*)

Draw the root locus diagram for the theoretical system, using Matlab beforehand and, during the lab session, verify it for the corresponding analogue computer implementation by considering the cases when the closed-loop system is critically damped, and marginally stable and at least another two or three points in between these. Write a brief summary and discussion of your findings.

Question 3 (*Week 3*)

Design a phase-lead compensator/controller for the above system that will reduce the settling time by a factor of 2 and will yield an overshoot of 30%. Design and build an analogue computer to implement the compensator and then test the closed-loop system (compensator and system) in the lab. Write a brief summary and discussion of your findings.

7.2 Controller Analogue Computer Implementation (Optional – for bonus marks!!)

Develop a simple Matlab/Scilab/Octave program to simulate the open-loop system as well as the closed-loop system and compare these results with the results obtained from the analogue computer simulations. Write a brief summary and discussion of your findings.

Note: Alternatives to Matlab to consider include Scilab/Octave/FreeMat and others.

8 Bibliography

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