Nomenclature

Of the very large number of symbols required by the subject, many have more than one meaning. Usually the meaning is clear from the context in which the symbol is used.

a	Wing or wing-body lift curve slope: Acceleration: Local speed of sound
a'	Inertial or absolute acceleration
a_0	Speed of sound at sea level: Tailplane zero-incidence lift coefficient
a_1	Tailplane lift curve slope
a_{1_f}	Canard foreplane lift curve slope
a_{1_F}	Fin lift curve slope
a_2	Elevator lift curve slope
a_{2_A}	Aileron lift curve slope
a_{2_R}	Rudder lift curve slope
a_3	Elevator tab lift curve slope
a_∞	Lift curve slope of infinite-span wing
a_h	Local lift curve slope at coordinate h
a_y	Local lift curve slope at spanwise coordinate y
$a_{z_{cg}}$	Normal acceleration at the cg
a_{z_p}	Normal acceleration at the pilot
ac	Aerodynamic centre
A	Aspect ratio
A_F	Effective aspect ratio of fin
A_T	Effective aspect ratio of tailplane
$\mathbf{A}$	State matrix
b	Wing span
b_1	Elevator hinge moment derivative with respect to α_T
b_2	Elevator hinge moment derivative with respect to η
b_3	Elevator hinge moment derivative with respect to β_η
b_T	Tailplane span
$\mathbf{B}$	Input matrix
c	Chord: Viscous damping coefficient: Command input
$\bar{c}$	Standard mean chord (*smc*)
$\bar{\bar{c}}$	Mean aerodynamic chord (*mac*)
$\bar{c}_\eta$	Mean elevator chord aft of hinge line
c_h	Local chord at coordinate h
c_r	Root chord

• Control system to control an unmanned fixed-wing aircraft •

model: airframe / actuators / propulsion

design: controller

simulate: closed-loop controlled system (in Matlab)

flight regulations → sensible specifications

environmental factors → health risks
green design
pollution

flight stability

energy efficiency

*identify simplifications.

Elen 4011A

Information Engineering Design II

Dr. Nicholas West

Ms. Yu-Chieh (Jessie) Yen

Tyson Cross

1239448

Prof. Van Wyk

Tues 17th initial meeting
Wed 18th meet with Prof. van Wyk
specifications & assumptions [prestudy → scope]
Mon 23rd Platform: airframe, actuators, propulsion
initial model
Mon 30th refine model [regulations, environment, safety]
Mon 7th Controller & Sim
Mon 14th Start Report *Van Wyk abs
Mon 21st Finish Report * Van Wyk abs
↓
Due Fri 25th Report due

spec by 21st
model by 25th
outline of Report: 30th ToC

Van Wyk → leaves 4th Fri
return 16th (am) Wed

- get list of names & student no.s
- nominate facilitator
- schedule meeting chair/secretary
- secretary : Darrion
- facilitator : Daniel
- assistants : Tyson & Sidwell

UAV: RPA

Aim Class I B for risk cost compliance simplification

Energy @ impact <15kJ
Height (max) 121.92m (400ft)
Weight (max) < 7 kg

Civil Aviation Act 2009
act 13 of 2009
SA-CATS 101

SA-CAA : SA civil aviation authority
ICAO : International Aviation org.

Airframe
· start with 2D model

actuators
reduce to linear component

propulsion
· gain or 1st order
τ_c from throttle → thrust

controller

simulation
(closed-loop)
Matlab.

regulations

environmental

safety
· failure plan
* cut engine, deploy parachute

Report : 15 pages, size 11
excluding. cover, ToC, appendices (post A)
Non-technical report : 2 pages [appendix A]
check ELOs *particulary 7a, 7b

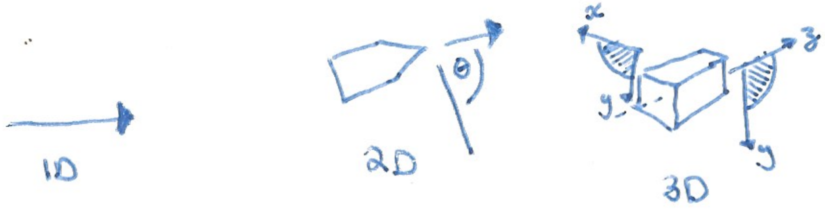
∘ cross coupling between systems

Big Problem, research, scale, options

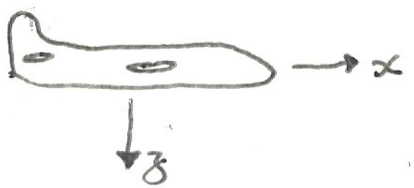
scoped down, assumptions, simplest

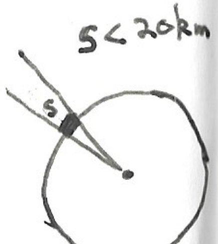
build up

reduce assumptions, complexify model



Flat earth:





flat, non-rotating

Report: mathematical model derivation unneccesary?

$\dot{X} = f(X, u)$

control vector

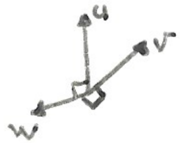
state variables

non-linear functions

$\dot{x} = Ax + Bu$ { perturbations from linear equilibrium

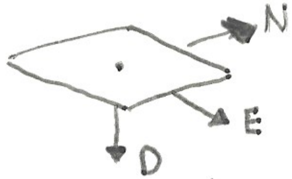
$\underline{u} \cdot \underline{v} = |\underline{u}||\underline{v}| \cos \theta$

$|\underline{u} \times \underline{v}| = |\underline{u}||\underline{v}| \sin \theta$



Euler angles: x, y, z (or z, y, x)

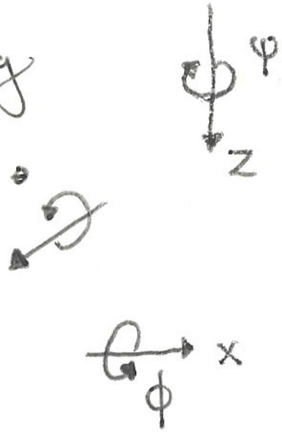
NED:



ψ compass heading (yaw)

θ pitch

ϕ roll



$Z\ Y\ X \rightarrow 3, 2, 1$

yaw, pitch, roll

$$\left.\begin{array}{l} -\pi < \psi < \pi \\ -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \\ -\pi < \phi < \pi \end{array}\right\} \text{ranges for rotation angles}$$

$$D = \begin{bmatrix} c\theta s\psi & c\theta s\psi & -s\theta \\ -c\phi s\psi + s\phi s\theta c\psi & c\phi c\psi + s\phi s\theta s\psi & s\phi c\theta \\ s\phi s\psi + c\phi s\theta c\psi & -s\phi c\psi + c\phi s\psi & c\phi c\theta \end{bmatrix}$$

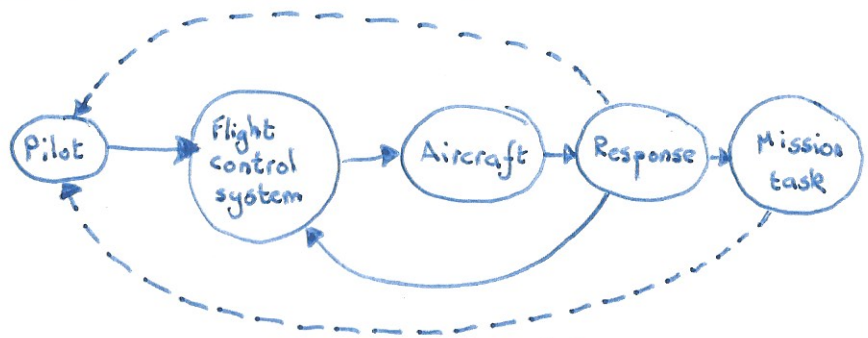
→ direction cosine matrix

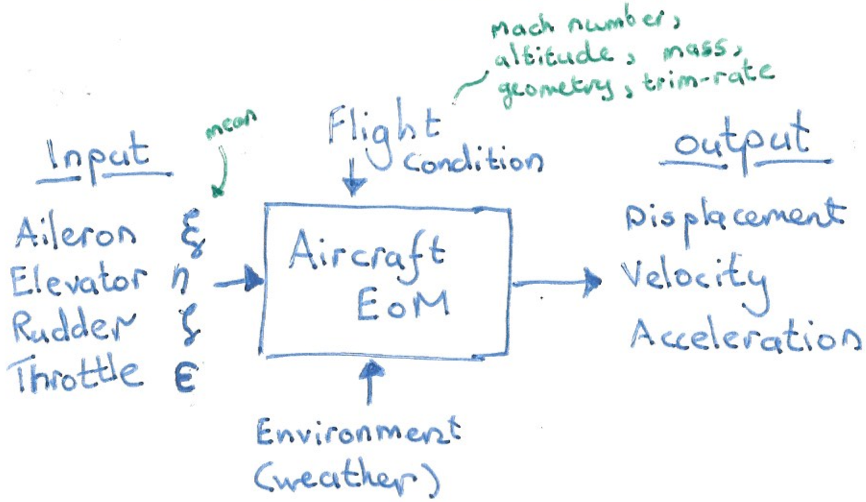
c : cos
s : sin

{forward / NED}

transforms from wind → body

Fly-by-wire flying & handling:





control displacement
control force } static characteristics

response to controls – dynamic characteristic

ξ xi ζ zeta ψ psi ρ rho

η eta ε epsilon ϕ phi ω omega

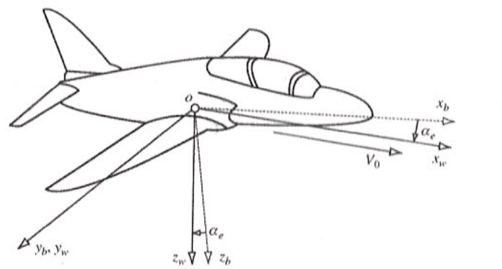


FIGURE 2.2 Moving-axis systems.

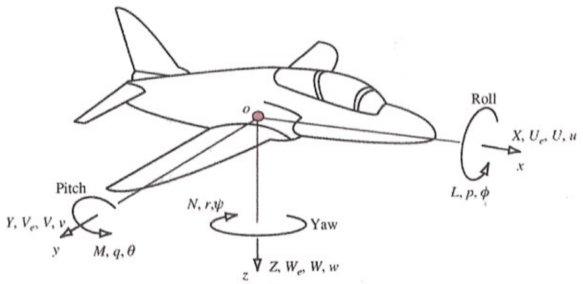


FIGURE 2.3 Motion variables notation.

Table 2.2 Perturbation Variables

X	Axial "drag" force	Sum of the components of aerodynamic, thrust and weight forces
Y	Side force	
Z	Normal "lift" force	
L	Rolling moment	Sum of the components of aerodynamic, thrust and weight moments
M	Pitching moment	
N	Yawing moment	
p	Roll rate	Components of angular velocity
q	Pitch rate	
r	Yaw rate	
U	Axial velocity	Total linear velocity components of the cg
V	Lateral velocity	
W	Normal velocity	

Table 2.1 Summary of Motion Variables

	Trimmed Equilibrium			Perturbed		
Aircraft axis	ox	oy	oz	ox	oy	oz
Force	0	0	0	X	Y	Z
Moment	0	0	0	L	M	N
Linear velocity	U_e	V_e	W_e	U	V	W
Angular velocity	0	0	0	p	q	r
Attitude	0	θ_e	0	ϕ	θ	ψ

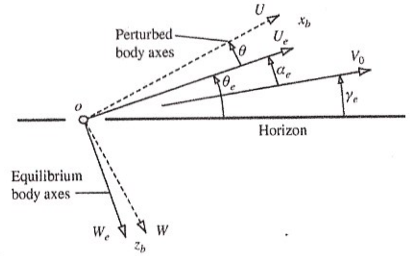
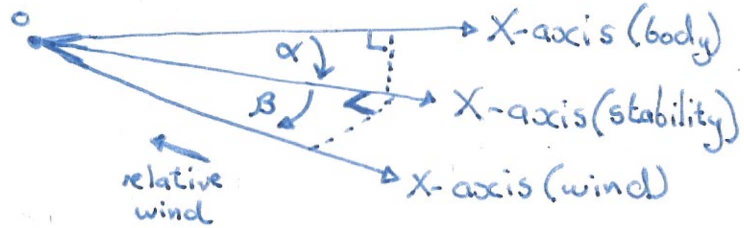


FIGURE 2.4 Generalised body axes in symmetric flight.



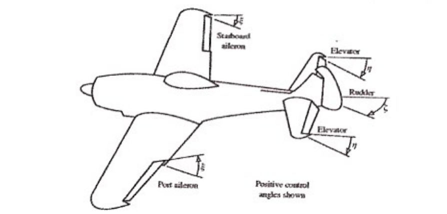


FIGURE 2.11 Aerodynamic controls notation.

trim stability

Subsonic speeds: < mach 1.0

linear (almost)

$$C_L \underset{\sim}{\propto} \alpha$$

$$\frac{dC_M}{d\alpha} < 0 \quad \text{or} \quad \frac{dC_m}{dC_L} < 0$$

$$C_m(\alpha=0) > 0$$

$$C_m(C_L=0) > 0$$

pitching moment C_M

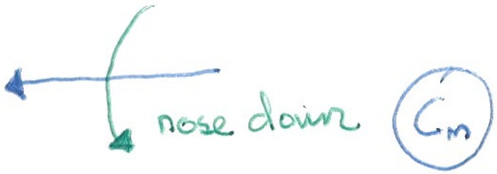
lift coefficient C_L

angle of attack α

Too stable : difficult to change/control

Unstable : too responsive to change

High τ :



Low τ :

Assume Rigid structure (no elasticity)

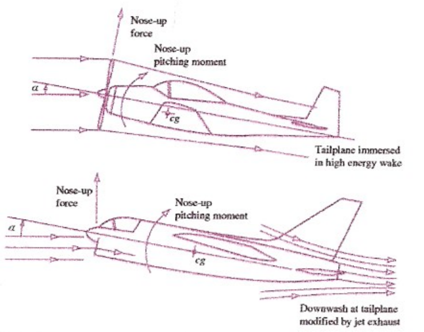


FIGURE 3.5 Typical induced-flow effects on pitching moment.

Max height 400 ft = 120 m

chosen height 100 m

air density 1.2 kg/m^3 [JHB 0.98 kg/m^3]

airspeed 20 m/s

$\phi = 0$

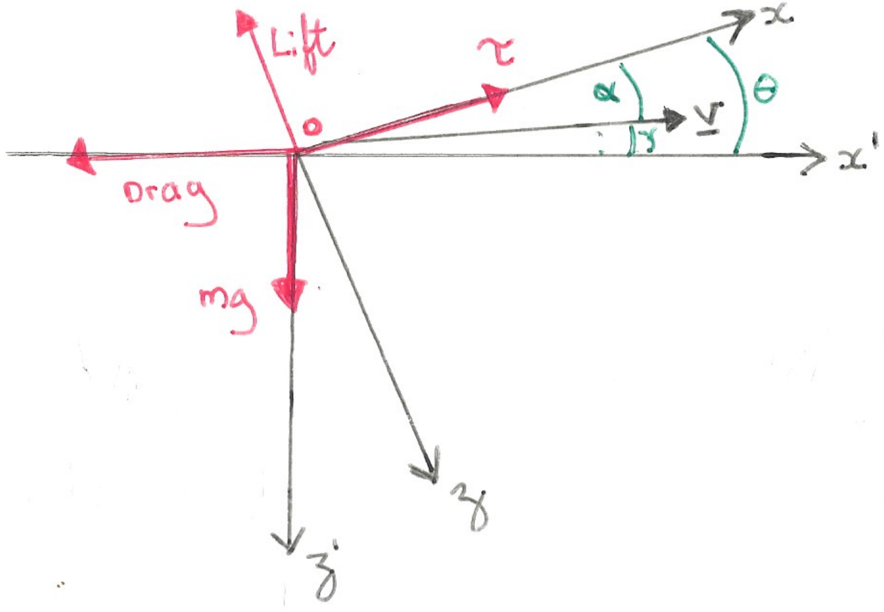
$\boxed{\text{Airframe}}$ → $\boxed{\text{ARF60}}$ → ~~Ma~~

{ Daniel calculate co-efficients

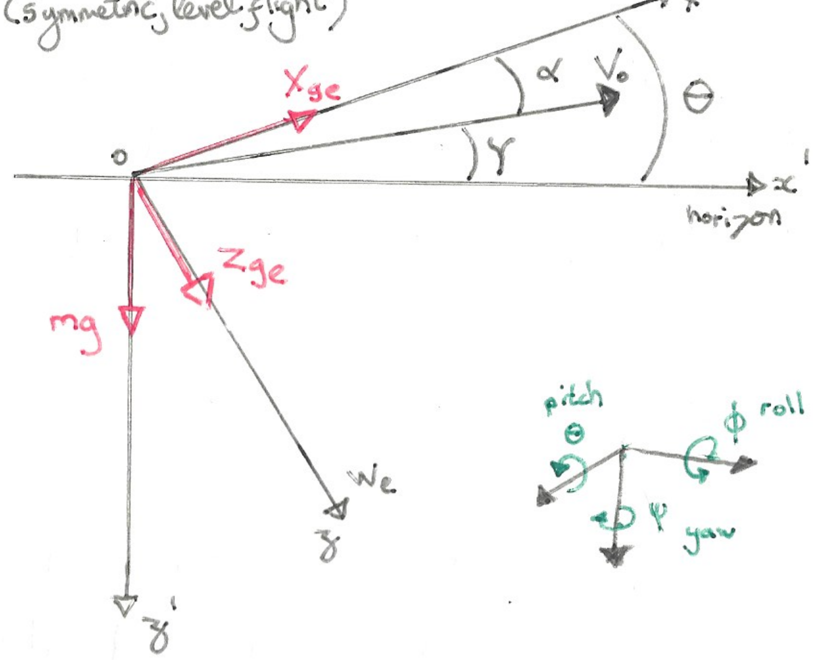
$\boxed{\text{electrical}}$

TF → SS model

$\boxed{\text{ARF60}}$ → vs. Ultra-flight (smaller, lighter, easier to classify as class 1B)



Weight / gravity:
(symmetric, level flight)



$$X_{ge} = -mg\sin\Theta_e$$

$$Y_{ge} = 0$$

$$Z_{ge} = mg\cos\Theta_e$$

$$\begin{bmatrix} X_g \\ Y_g \\ Z_g \end{bmatrix} = \begin{bmatrix} 1 & \psi & -\theta \\ -\psi & 1 & \phi \\ \theta & -\phi & 1 \end{bmatrix} \begin{bmatrix} X_{ge} \\ Y_{ge} \\ Z_{ge} \end{bmatrix}$$

$$X_g = -mg\sin\Theta_e - mg\theta\cos\Theta_e$$

$$Y_g = mg\psi\sin\Theta_e + mg\phi\cos\Theta_e$$

$$Z_g = mg\cos\Theta_e - mg\theta\sin\Theta_e$$

Forces:

yaw rate, gravity

$$m(\dot{U} - rV + qW) = X_a + X_g + X_c + X_p + X_d$$

aerodynamic, gravitational, control, power, external disturbance

roll rate

$$m(\dot{V} - pW + rU) = Y_a + Y_g + Y_c + Y_p + Y_d$$

pitch rate

$$m(\dot{W} - qU + pV) = Z_a + Z_g + Z_c + Z_p + Z_d$$

Moments:

rolling moment

$$I_x\dot{p} - (I_y - I_z)qr - I_{xz}(pq + \dot{r}) = L_a + L_g + L_c + L_p + L_d$$

inertial moment

pitching moment

$$I_y\dot{q} + (I_x - I_z)pr + I_{xz}(p^2 - r^2) = M_a + M_g + M_c + M_p + M_d$$

yawing moment

$$I_z\dot{r} - (I_x - I_y)pq + I_{xz}(qr - \dot{p}) = N_a + N_g + N_c + N_p + N_d$$

Assume steady, trimmed, symmetric, rectilinear flight with no roll, sideslip or yaw angles.
stable, undisturbed atmosphere.

$$X_d = Y_d = Z_d = L_d = M_d = N_d = 0$$

u, v, w : small peturbations in $oxyz$
p, q, r : small angular peturbation velocities.

so
$$U = U_e + u$$
$$V = V_e + v = v \qquad (V_e = 0)$$
$$W = W_e + w$$

ignoring small values that are squared or multiplied:

aerodynamic

$$m(\dot{u} + qW_e) = X_a + X_g + X_c + X_p$$
$$m(\dot{v} + rU_e) = Y_a + Y_g + Y_c + Y_p$$
$$m(\dot{w} - qU_e) = Z_a + Z_g + Z_c + Z_p$$

$$I_x\dot{p} - I_{xz}\dot{r} = L_a + L_g + L_c + L_p$$
$$I_y\dot{q} = M_a + M_g + M_c + M_p$$
$$I_z\dot{r} + I_{xz}\dot{p} = N_a + N_g + N_c + N_p$$

Aerodynamic

const.

$$X_a = X_{a_e} + \left(\frac{\partial X}{\partial u}u + \frac{\partial^2 X}{\partial u^2}\frac{u^2}{2!} + \dots\right) + \left(\frac{\partial X}{\partial v}v + \frac{\partial^2 X}{\partial v^2}\frac{v^2}{2!} + \dots\right)$$

$\dots$ [terms in $w, p, q, r, \dot{u}, \dot{v}, \dot{w}, \dot{p}, \dot{q}, \dot{r}$
+ higher order derivative terms

$$\approx X_{a_e} + \frac{\partial X}{\partial u}u + \frac{\partial X}{\partial v}v + \frac{\partial X}{\partial w}w + \frac{\partial X}{\partial p}p + \frac{\partial X}{\partial q}q + \frac{\partial X}{\partial r}r + \frac{\partial X}{\partial \dot{w}}\dot{w}$$

$\mathring{X}$ ← dimensional

or:

aerodynamic stability derivatives

$$X_q = X_{q_e} + \mathring{X}_u u + \mathring{X}_v v + \mathring{X}_w w + \mathring{X}_p p + \mathring{X}_q q + \mathring{X}_r r + \mathring{X}_{\dot{w}} \dot{w}$$

etc. for Y, Z, L, M, N

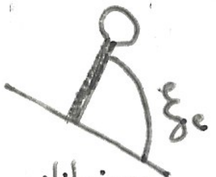
Control terms:

elevator angle

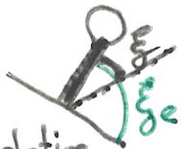
$$M_c = \frac{\partial M}{\partial \xi}\xi + \frac{\partial M}{\partial \eta}\eta + \frac{\partial M}{\partial \zeta}\zeta$$

aerilon angle

rudder angle



equilibrium angle (trim setting)



relative angle

$$\Rightarrow M_c = \mathring{M}_\xi \xi + \mathring{M}_\eta \eta + \mathring{M}_\zeta \zeta$$

Power terms:

τ: thrust peturbation from τ_e

ε : throttle lever angle from ε_e

$$\frac{\tau(s)}{\varepsilon(s)} = \frac{k_\tau}{(1+sT_\tau)}$$

(1st order TF of jet engine)

e.g. $Z_p = \mathring{Z}_\tau \tau$ (normal force due to thrust)

Substitute into e.o.m:

$$m(\dot u + qW_e) = \boxed{X_{a_e}} + \mathring{X}_u u + \mathring{X}_v v + \mathring{X}_w w + \mathring{X}_p p + \mathring{X}_q q + \mathring{X}_r r + \mathring{X}_{\dot w}\dot w \boxed{-mg\sin\Theta_e} - mg\cos\Theta_e$$
$$+ \mathring{X}_\xi \xi + \mathring{X}_\eta \eta + \mathring{X}_\zeta \zeta + \mathring{X}_\tau \tau$$

$$m(\dot v - pW_e + rU_e) = \cancel{Y_{a_e}} + \mathring{Y}_u u + \dots + \mathring{Y}_{\dot w}\dot w + mg\psi\sin\Theta_e + mg\phi\cos\Theta_e + \mathring{Y}_\xi \xi + \dots + \mathring{Y}_\zeta \zeta + \mathring{Y}_\tau \tau$$

$$m(\dot w - qU_e) = \boxed{Z_{a_e}} + \mathring{Z}_u u + \dots + \mathring{Z}_{\dot w}\dot w + \boxed{mg\cos\Theta_e} - mg\theta\sin\Theta_e + \mathring{Z}_\xi \xi + \dots + \mathring{Z}_\zeta \zeta + \mathring{Z}_\tau \tau$$

$$I_x\dot p - I_{xz}\dot r = \cancel{L_{a_e}} + \mathring{L}_u u + \dots + \mathring{L}_{\dot w}\dot w + \mathring{L}_\xi \xi + \dots + \mathring{L}_\zeta \zeta + \mathring{L}_\tau \tau$$

$$I_y\dot q = \cancel{M_{a_e}} + \mathring{M}_u u + \dots + \mathring{M}_{\dot w}\dot w + \mathring{M}_\xi \xi + \dots + \mathring{M}_\zeta \zeta + \mathring{M}_\tau \tau$$

$$I_z\dot r - I_{xz}\dot p = \cancel{N_{a_e}} + \mathring{N}_u u + \dots \mathring{N}_{\dot w}\dot w + \mathring{N}_\xi \xi + \dots + \mathring{N}_\zeta \zeta + \mathring{N}_\tau \tau$$

no gravity terms.

but in trimmed flight: $X_{a_e} = mg\sin\Theta_e$ $\quad L_{a_e} = 0$

$Y_{a_e} = 0$ $\quad M_{a_e} = 0$

$Z_{a_e} = -mg\cos\Theta_e$ $\quad N_{a_e} = 0$

To decouple Lateral & Longitudinal:

assume disturbances limited to

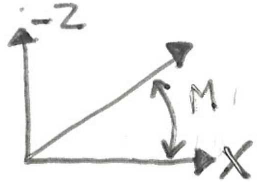
oxz plane $\Rightarrow X, Z, M$ only

$v, p, r, \dot v, \dot p, \dot r = 0$

also, $\mathring{X}_v = \mathring{X}_p = \mathring{X}_r = \mathring{Z}_v = \mathring{Z}_p = \mathring{Z}_r = \mathring{M}_v = \mathring{M}_p = \mathring{M}_r = 0$ (coupled forces & moments negligibly small)

and $\mathring{X}_\xi = \mathring{X}_\zeta = \mathring{Z}_\xi = \mathring{Z}_\zeta = \mathring{M}_\xi = \mathring{M}_\zeta = 0$

(aileron & rudder perturbations don't affect longitudinal plane motion)



$$m\dot u - \mathring{X}_u u - \mathring{X}_{\dot w}\dot w - \mathring{X}_w w - (\mathring{X}_q - mW_e)q + mg\theta\cos\Theta_e$$
$$= \mathring{X}_\eta \eta + \mathring{X}_\tau \tau$$

$$-\mathring{Z}_u u + (m - \mathring{Z}_{\dot w})\dot w - \mathring{Z}_w w - (\mathring{Z}_q + mU_e)q + mg\theta\sin\Theta_e$$
$$= \mathring{Z}_\eta \eta + \mathring{Z}_\tau \tau$$

$$-\mathring{M}_u u - \mathring{M}_{\dot w}\dot w - \mathring{M}_w w + I_y\dot q - \mathring{M}_q q$$
$$= \mathring{M}_\eta \eta + \mathring{M}_\tau \tau$$

$\Rightarrow$

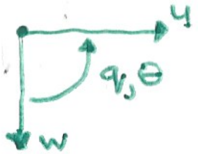
$$m\dot{u} - \mathring{X}_{\dot{w}}\dot{w} = \mathring{X}_u u + \mathring{X}_w w + (\mathring{X}_q - mW_e)q - mg\theta\cos\theta_e + \mathring{X}_\eta \eta + \mathring{X}_\tau \tau$$

$$m\dot{w} - \mathring{Z}_{\dot{w}}\dot{w} = \mathring{Z}_u u + \mathring{Z}_w w + (\mathring{Z}_q + mU_e)q - mg\theta\sin\theta_e + \mathring{Z}_\eta \eta + \mathring{Z}_\tau \tau$$

$$I_y\dot{q} - \mathring{M}_{\dot{w}}\dot{w} = \mathring{M}_u u + \mathring{M}_w w + \mathring{M}_q q + \mathring{M}_\eta \eta + \mathring{M}_\tau \tau$$

state variables: control:

$$\mathbb{X}(t) = \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix}$$



$$\mathbb{u}(t) = \begin{bmatrix} \eta \\ \tau \end{bmatrix}$$

4th equation: $\dot{\theta} = q$ for small petubations

$$\mathbb{M}\dot{\mathbb{x}}(t) = \mathbb{A}'\mathbb{x}(t) + \mathbb{B}'\mathbb{u}(t)$$

$$\mathbb{A} = \mathbb{M}^{-1}\mathbb{A}' = \begin{bmatrix} X_u & X_w & X_q & X_\theta \\ Z_u & Z_w & Z_q & Z_\theta \\ m_u & m_w & m_q & m_\theta \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

aerodynamic stability derivatives

$$\mathbb{B} = \mathbb{M}^{-1}\mathbb{B}' = \begin{bmatrix} X_\eta & X_\tau \\ Z_\eta & Z_\tau \\ m_\eta & m_\tau \\ 0 & 0 \end{bmatrix}$$

control derivatives

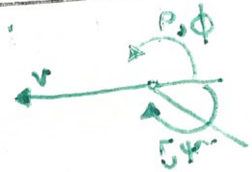
$$y(t) = \mathbb{I}\mathbb{x}(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix}$$

---

roll, yaw, sideslip

Lateral:

$$\mathbb{X}_{lat}(t) = \begin{bmatrix} v \\ p \\ r \\ \phi \\ \psi \end{bmatrix}$$



$$\mathbb{u}(t) = \begin{bmatrix} \xi \\ \zeta \end{bmatrix}$$

4th equation: $\dot{\phi} = p$ for small perturbations

5th equation: $\dot{\psi} = r$ for " "

$$\mathbb{A} = \begin{bmatrix} Y_v & Y_p & Y_r & Y_\phi & Y_\psi \\ L_v & L_p & L_r & L_\phi & L_\psi \\ N_v & N_p & N_r & N_\phi & N_\psi \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

values?

$$\mathbb{B} = \begin{bmatrix} Y_\xi & Y_\zeta \\ L_\xi & L_\zeta \\ N_\xi & N_\zeta \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

η: elevator
ε: throttle
ξ: aerilorons
ζ: rudder

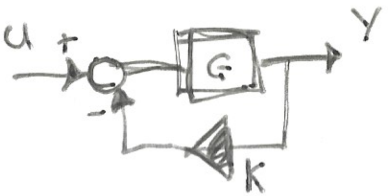
u: petubation in x
w: petubation in z
q: pitching rate
θ: pitch angle
X: axial force
Z: normal force
M: pitching moment

v: petubation in y
p: roll rate
r: yaw rate
ϕ: roll angle
ψ: yaw angle
Y: sideslip force
L: rolling moment
N: yawing moment

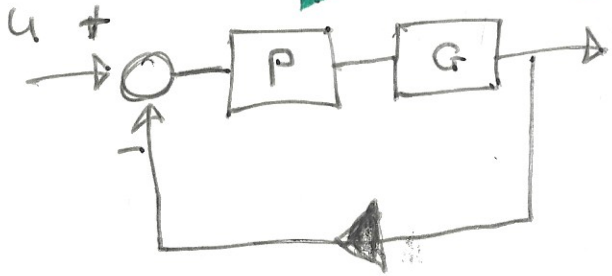
Control: η elevator

output : θ pitch angle

~~Assume thrust unaffected by pitch?~~
~~(for simplicity) i.e. T = const?~~



PID: affordable & easy to tune



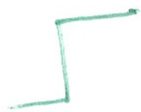
But not optimal.

2 loops in previous project: will three loops work.

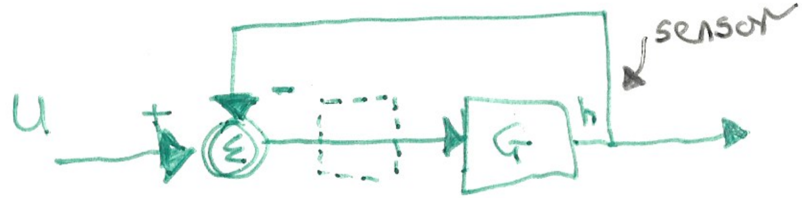
q → pitch rate responsiveness

θ → pitch angle smooth & dampen

h → regulate by external measurement?



altitude change.



State observer?

Non-linear system?