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Decoupled Altitude and Direction Autopilot Controller Design for a Fixed-wing Unmanned Aerial Vehicle

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Abstract

The design of two separate pitch and yaw flight control systems for a small fixed-wing, unmanned aerial vehicle using successive loop closure and PID controllers is presented. A linearised model of trimmed flight from the system's simplified and decoupled equations of motion was used to model longitudinal and lateral flight behaviour in state space form. The airframe is based on the Ultra Stick 25e hobby plane, using published aerodynamic stability derivative coefficients from literature. The altitude control has a rise time of 5.78 seconds, with 0.52% overshoot. The yaw heading control has a rise time of 2.93 seconds and and overshoot 0.54%. The aircraft conforms to the requirements for a class 1B UAV from section 101 of the South African Civil Aviation Authority's SA-CATS legislation. The MATLAB codebase and Simulink simulations of the control system has been made available at `https://github.com/TysonCross/ELEN4011`

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Nomenclature

a'	Inertial acceleration
$\mathbf{A}$	State matrix
b	Wing span
$\mathbf{B}$	Input matrix
$\bar{C}$	Mean wing chord
$\mathbf{C}$	Output matrix
$\mathbf{D}$	Feedforward (direct) matrix
h	Height (altitude)
I_x	Moment of inertia in x
I_y	Moment of inertia in y
I_z	Moment of inertia in z
I_{xz}	Inertial moment about *ox* and *oz* axes
L	Rolling moment
m	Gross weight
M	Pitching moment
N	Yawing moment
p	Roll rate
q	Pitch rate
r	Yaw rate
S	Wing reference area
u	Perturbation along x
$\mathbf{u}$	Input (control) vector
U	Axial velocity
v	Perturbation along y
V	Lateral velocity
w	Perturbation along z
W	Normal velocity
$\mathbf{x}$	State vector
X	Axial force
Y	Side force
Z	Normal force
α	Angle of attack
β	Sideslip angle
η	Elevator
ε	Throttle
ξ	Ailerons
ζ	Rudder
θ	Pitch angle
ϕ	Roll angle
ψ	Yaw angle
NED	North, East, Down external earth reference frame
RC	Radio Control
RPA	Remotely Piloted Aircraft
PID	Proportional Integral Derivative
SACAA	South African Civil Aviation Authority
UAV	Unmanned Aerial Vehicle

1 Introduction

Unmanned Aerial Vehicles (UAVs) are a popular class of airborne vehicles, and the area of much recent academic research. Ranging from small radio-controlled hobby kits, to larger crafts for surveillance, research and payload delivery, UAVs have multiple commercial, scientific and industrial applications. The design of aircraft for successful and safe flight involves complex interactions between multiple engineering disciplines, and the requirements for stability, robust control and safe operation of an UAV puts several important design issues firmly within the realm of control engineering.

2 Background

The chosen vehicle is the Ultra Stick 25e, a small manoeuvrable fixed wing UAV. The accurate analysis or the dynamics and physics of flight, and the modelling of a moving aircraft is a complex and highly coupled, non-linear problem requiring expert knowledge and experience, with a vast body of literature, theory and applied science. In order to build a control system to allow the steering of an unmanned fixed wing vehicle, the system must be modelled, consisting of an airframe, control surfaces, means of propulsion, actuators. In order to design control systems around complex interrelated physical mechanics approximations, simplifications, and assumptions can be made to produce an initial working framework.

After developing the system of equations that describe aerodynamic forces and moments acting on the plane in motion, the system will have to be reduced down in order or complexity, using approximations and decoupling assumptions. Linear systems are solvable, and classical control theory depends on linear relationships to apply many of the basic techniques of transformation and comprehensible mathematical representation.

2.1 Literature Review

The body of work in aerodynamics and flight control law is extremely prolific. [1] demonstrates the development of the equations of motion that can be used to construct a detailed mathematical system to represent the physics that governs flight and flight control. Similarly authoritative, and including detailed appendices regarding the calculation of stability derivatives, [2].

[3] explored a frequency-based system identification technique to model and compare with measured time domain flight data sets, and quantifying the accuracy between decoupled, baseline linearised systems with more complex and coupled precise dynamic models. [4] explains the dynamic modes of flight in different degrees of freedom, examining directional and lateral stability. [5] presented a detailed working of the technique of successive loop closure and tuning methods of PID controllers. [6] develops the longitudinal model with the Ultra Stick 25e. [7] develops a dynamic control system with both lateral and longitude using PID controllers. [8] is a detailed and complete MATLAB toolbox which implements a complete autopilot wayfinding solution, with several plant models of different complexity.

2.2 Project management

This project was undertaken as part of fulfilment for an undergraduate electrical engineering degree, as an individual assignment with weekly meetings as part of a larger group. The project assumed a client/customer relationship between the supervising professor and the individual candidate candidate engineers, to emulate commercial and industrial practices in the workplace. The short six week time frame required careful scoping to determine a useful but achievable outcomes in the complex area of aerospace control design. The project included planning, research, initial design and computer simulation, with several stages of design iteration in the Simulink implementation. The initial project plan is included in Appendix E

2.3 Success Criteria

The project requires the development of a system to control an unmanned, fixed wing aircraft. No specific requirements were provided for the size, intention or budgetary concerns of the vehicle. The relevant South African legislation governing UAVs was consulted to determine the specifications. Existing airframe models with known aerodynamic specifications were compared to select a suitable model to design the control system for. Within the available project time, a reduced scope was adopted to limit development of the control system to two separate decoupled systems each with 3DoF. The control system must be stable.

The control system should be able to correctly and safely allow a desired change in altitude to be sent to the autopilot, and the vehicle should then move to this new altitude within an acceptable period of time that does not attempt to exceed safe climbing rates. Once the new altitude is achieved, within the airframe's safety and legal height restrictions and rate-of-climb/descent, the plane should maintain this fixed altitude. Additionally, if a change in compass direction is sent to the plane, then the control system should safely turn the vehicle towards this new direction, within the safe limits of the aircraft's maximum bank angle and then maintain this new heading. The control system should be capable of maintaining stability in the plant during normal flight operations.

2.4 Legislation

The South African Civil Aviation Authority (SACAA) is an agency of the South African Department of Transport, and has the legislative authority over civil aviation. The relevant Act governing UAVs, referred to more generally as Remotely Piloted Aircrafts (RPAs) is Section 101 of the Civil Aviation Act, referred to as SA-CATS-101. Unmanned aircraft are permitted to possess autopilot capabilities, as long as a mechanism to take over flight manually is provided. Classification of vehicles are by weight, size and energy capabilities, with larger and more powerful planes subject to greater stringency for safety, testing and licensing. For the purpose of the project, the hobby-class Class 1B RPA, for vehicles weighing less than 7 kg and flying lower than 120 m is selected as the target airframe limits. This class of aircraft is permitted to fly up to 500 m around the remote pilot maintaining line-of-sight, and up to 1 km with extended line-of-sight (with an additional observer participating).

2.5 Requirements

A Class 1B RPA with autonomous capability only has to conform to safety sections 1 and 2 of SA-CATS-101. The extended safety sections require a complete profile of performance capabilities and limitations, and plans for emergency system failures. The system selected to conform to the Class 1B airframe requirements is the Ultra Stick 25e [9], a popular hobby kit that has been widely used for UAV control design in literature [3, 10, 7, 6, 11, 12, 13, 14]. The plane is considered to have a single propeller for thrust, driven by a battery-powered electric motor (Power 25 BL Outrunner with an 11.1 V lithium-polymer 4200 mAh battery). The control system should send signals to operate the control surfaces of the plane, namely: ηs to control pitch and altitude, Ailerons (ξ) to control the roll, and hence the yaw angle (direction). The other two controls are the Throttle (ε) which determines the amount of propulsion, and the Rudder (ζ) which is mainly used for taxiing, take-off and landing. The reduce project scope, the throttle and rudder were not specifed to be controlled, except to stabilise or dampen if neccesary.

2.6 Assumptions

The following assumptions are made to reduce the complexity of the non-linear equations of motions, and to help develop a linear plant suitable for a initial control system design:

- At a maximum range of 1 km and height limit of <120 m, a flat earth reference frame is suitable.
- The aeroplane is initially in steady, trimmed level flight, at equilibrium with all forces summing to zero.
- The aeroplane is in rectilinear flight, with the oxz plane vertically symmetrical to the NED reference frame.
- Initially, the weather is completely still and stable, with negligible external influence on the plane.
- Lateral and longitudinal response to input controls are considered to be decoupled.
- The vehicle is initially moving at a constant 40 km/h (11.11 m/s).
- The plane only makes small changes in directions and altitude.
- Drag is considered to be balanced out by the thrust.
- The aeroplane is symmetrical along its y-axis.
- Drag and propeller turbulence are neglected.
- STP with air pressure ρ = 1.225 kg/m^3.
- The plane is flying 100m above level ground.
- The vehicle is a single rigid body.

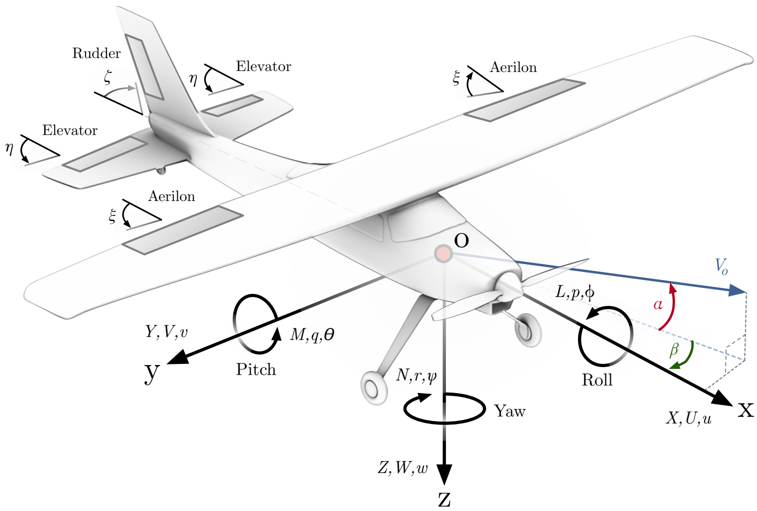


Figure 1: Notation and directional conventions for control surfaces and airframe body axes

3 Modelling

Modelling flight is difficult, due to its highly non-linear nature. Using the methods demonstrated by [2], [15] and explicated in detail in [1], the equations of motion were developed, followed by a reduction of complexity and decoupling into linear longitude and lateral systems with a neat use of novel notation to represent the summed stability derivatives. For details, consult [1] and [2].

The reference frame used is North, East, Down external earth reference frame (NED), with a flat earth approximation. The body axes orientation, directions, and notation are shown in Figure 1. The full 6DoF equations of motion for a rigid, fixed wing UAV starts with a Newtonian analysis of the forces, moments, kinematics and inertial components and then relates the physical geometry of the plane to these forces to recover stability derivatives which can be used to describe the aircraft's interactions with physical forces as it moves through the air. It is common to arrange the linearised equation in standard state space form.

3.1 Airframe

Relevant airframe information for the Ultra Stick 25e is shown in Table 1, with data compiled from the plane's manual [9] and measurements reported by literature from the University of Minnesota [13, 3]

Table 1: Ultra Stick 25e information

Parameter	Symbol	Value
Wing span	b	1.27 m
Wing area	S	0.31 m^2
Gross weight	m	1.91 kg
Length	l	1.05 m
Mean aerodynamic chord	$\bar{C}$	0.25 m
Moment of inertia in x	I_x	0.0895 kg·m^2
Moment of inertia in y	I_y	0.1444 kg·m^2
Moment of inertia in z	I_z	0.1620 kg·m^2
Planar inertial moment in *oxz*	I_{xz}	0.0140 kg·m^2

3.2 Propulsion

Thrust is produced in a Ultra Stick 25e in the standard configuration by a electrical motor rotating a propeller shaft, with airfoils which are shaped to produce pressure differences on either side of the surface as there move through the air, which accelerates the fluid. This produces thrust, drag and lift forces. The propulsion was initially modelled as a simple transfer function (Equation 1) between the throttle position as demand, and forward output thrust as a first order system. With an electrical motor, propulsive efficiency is the ratio of thrust power to electrical power. Lift and drag were neglected on the initial assumption of constant speed in a level line at a fixed height above a flat earth model. The plane was modelled to be flying at 40 km/s (11 m/s) with a small disturbance model of trimmed rectilinear flight. A feedback system was partially developed to improve speed of responsiveness to throttle input, but the primary focus of the project was on automatic height control with η input, and directional Roll angle (ϕ) autopilot using the ailerons.

$$\frac{\tau(s)}{\varepsilon(s)} = \frac{60}{2\pi} \frac{1}{1 - 0.01s} \tag{1}$$

3.3 Actuators

The actuators were all modelled as 2nd order systems with a natural frequency of 25, and a damping ratio of 0.75. MATLAB's DroneSimulink and Aerospace Blockset toolbox was used as reference for these values [8], using the provided 2nd order Actuator model block, with values taken from simulations of a similar sized UAV model (The Multiplex Mentor, a slightly larger Radio Control (RC) plane) [16, 17] The transfer function used for all actuators in the system is shown in equation 2.

$$E(s) = \frac{1}{s^2 + 52.5s + 1225} \tag{2}$$

The actuators are servos that move the control surfaces of the plane. Physical saturation limits from the plane manuals were used to limit the range of available angles, and the actuator transfer functions were put in cascade with the plant and each control input (η, ε, ζ, ξ.)

3.4 Control Surfaces

The input controls for the UAV are the elevator, throttle rudder and ailerons. The control signals are received by servos which actuate the signal, producing a physical alteration in the control surface (a change in angle in radians except for the throttle). This produces a change in the aerodynamic properties of the aircraft. Controlling the signal by measuring output from the plant and using an electronic or mechanical mechanism to alter the provided signal through feedback from the measurement causes an alteration in the angle and hence the facing direction of the control surfaces, allowing the vehicle to steer, adjusting the orientation of the craft with six degrees of freedom.

The physical limits of the control surfaces are provided in the Ultra Stick 25e manual [9], and confirmed by [18]. These saturation values are shown in Table 2. These limits were implemented in Simulink at clamped saturation values in the PID controllers for the control inputs before the actuators interface. The values are converted to radians within the simulation.

Table 2: Ultra Stick 25e control surface limits

Control	Upper Limit	Lower Limit
Ailerons	23°	-23°
Elevator	20°	-20°
Rudder	25°	-25°

3.4.1 Equations of Motion

The **Inertial Equations** provide the components of inertial accelerations of an arbitrary point of the rigid aircraft:

$$\begin{aligned} a'_x &= \dot{U} - rV + qW - x(q^2 + r^2) + y(pq - \dot{r}) + z(pr + \dot{q}) \\ a'_y &= \dot{V} - pW + rU + x(pq + \dot{r}) - y(p^2 + r^2) + z(qr - \dot{p}) \\ a'_z &= \dot{W} - qU + pV + x(pr - \dot{q}) + y(qr + \dot{q}) - z(p^2 + q^2) \end{aligned} \tag{3}$$

The **Force Equations** describe the resultant components of the total sum of forces acting up on the rigid aircraft:

$$\begin{aligned} m(\dot{U} - rV + qW) &= X \\ m(\dot{V} - pW + rU) &= Y \\ m(\dot{W} - qU + pV) &= Z \end{aligned} \tag{4}$$

The **Moment Equations** give the moments of the forces acting across the integrated total mass on incremental masses at every point in the rigid aircraft body. Because the plane is symmetric on either side of the *oxz* plane, these equations can be simplified and expressed as:

$$\begin{aligned} I_x\dot{p} - (I_y - I_z)qr - I_{xz}(pq + \dot{r}) &= L \\ I_y\dot{q} + (I_x - I_z)pr + I_{xz}(p^2 - r^2) &= M \\ I_z\dot{r} - (I_x - I_y)pq + I_{xz}(qr - \dot{p}) &= N \end{aligned} \tag{5}$$

If these equations are combined and rearranged, they can be written as:

$$\begin{aligned} m(\dot{U} - rV + qW) &= X_a + X_g + X_c + X_p + X_d \\ m(\dot{V} - pW + rU) &= Y_a + Y_g + Y_c + Y_p + Y_d \\ m(\dot{W} - qU + pV) &= Z_a + Z_g + Z_c + Z_p + Z_d \\ I_x\dot{p} - (I_y - I_z)qr - I_{xz}(pq + \dot{r}) &= L_a + L_g + L_c + L_p + L_d \\ I_y\dot{q} + (I_x - I_z)pr + I_{xz}(p^2 - r^2) &= M_a + M_g + M_c + M_p + M_d \\ I_z\dot{r} - (I_x - I_y)pq + I_{xz}(qr - \dot{p}) &= N_a + N_g + N_c + N_p + N_d \end{aligned} \tag{6}$$

The subscripts meaning is: a refers to aerodynamic effects, g is gravitational, c is any forces resulting from the controls, p is the power effects, and d refers to atmospheric disturbances. To reduce these equations to a linear set, we assume linear trimmed rectilinear flight, and adopt the approximations of a small perturbance model where all the forces acting on the rigid aircraft are in equilibrium. This allows setting several terms to zero, and assuming that some other terms are negligible small and so can be discarded. If there is no atmospheric disturbance, constant speed, and only the relevant vector components acting in specific directions are retained, the terms are reduced to:

$$\begin{aligned} m(\dot{u} + qW_e) &= X_a + X_g + X_c + X_p \\ m(\dot{v} - pW + rU_e) &= Y_a + Y_g + Y_c + Y_p \\ m(\dot{w} - qU_e) &= Z_a + Z_g + Z_c + Z_p \\ I_x\dot{p} - I_{xz}\dot{r} &= L_a + L_g + L_c + L_p \\ I_y\dot{q} &= M_a + M_g + M_c + M_p \\ I_z\dot{r} - I_{xz}\dot{p} &= N_a + N_g + N_c + N_p \end{aligned} \tag{7}$$

These terms can be further reduced by assuming the decoupling of lateral and longitudinal equations of motion, producing two sets of equations describing 3DoF each. Substituting in the gravitational, control, thrust terms, and then expressing the aerodynamic terms as a sum of partial derivatives (aerodynamic stability derivatives), and then discarding any small terms. These equations can then be rearranged and expressed with coefficients calculated from the stability derivatives.

3.4.2 State Space Form

Arranging the equations of motion, with the given assumptions of trimmed flight in equilibrium with respect to the body axis, gives a state space representation in the standard form as shown in Equation 8.

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)\end{aligned} \tag{8}$$

The state variables for the decoupled lateral state space system x_{long} are: perturbation from equilibrium along the x-axis u; perturbation along the z-axis w; changes in the pitch rate's angular velocity q; and a change in the pitching angle θ. We also augment the **A** matrix with an additional bottom row, to allow the inclusion of additional state variable, perturbation in Height (altitude) (h). The augmented input vector is shown in equation 9.

$$x_{long}^T = \begin{bmatrix} u & w & q & \theta & h \end{bmatrix} \tag{9}$$

The Longitudinal state space form of linear, small perturbation model of the airframe is shown in equation 10, augmented with a row for height.

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} X_u & X_w & 0 & -g & 0 \\ Z_u & Z_w & u_0 & 0 & 0 \\ M_u + M_w Z_u & M_w + M_w Z_w & M_q + M_w u_0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -sin(\theta_0) & -cos(\theta_0) & 0 & -u_0 cos(\theta_0) & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \\ h \end{bmatrix} + \begin{bmatrix} X_\eta & X_\varepsilon \\ Z_\eta & Z_\varepsilon \\ M_\eta + M_w Z_\eta & M_\varepsilon + M_w Z_\varepsilon \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \eta \\ \varepsilon \end{bmatrix} \tag{10}$$

Equation 11 presents the linearised, small perturbation lateral state space model:

$$\begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} Y_v & Y_p & -(u_0 - Y_r) & -g cos\theta_0 & 0 \\ L_v & L_p & L_r & 0 & 0 \\ N_v & N_p & N_r & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & sec(\theta_0) & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ r \\ \phi \\ \psi \end{bmatrix} + \begin{bmatrix} 0 & Y_\zeta \\ L_\xi & L_\zeta \\ N_\xi & N_\zeta \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \xi \\ \zeta \end{bmatrix} \tag{11}$$

3.4.3 Longitudinal model

The linearised state space equations coefficients, taken from the baseline Ultra Stick 25e model presented in [3] are shown in equation 12. This linearised system of equations are decoupled from the lateral dynamics, and are for trimmed flight in equilibrium in a small perturbation model. The system incorporates the simplifying assumptions listed above. The baseline model is considered adequate for the basic development of a control system, allowing the use of linear algebra and classical control techniques, or state space tools of analysis and computer simulation.

$$A_{long} = \begin{bmatrix} -0.15 & 0.15 & 0 & -9.81 & 0 \\ -0.53 & -5.30 & 11.11 & 0 & 0 \\ 2.67 & -3.38 & -32.91 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -0.52 & -1 & 0 & 11.11 & 0 \end{bmatrix} B_{long} = \begin{bmatrix} 8.49 & 0 \\ 0 & -1.66 \\ 0.25 & -49.79 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{12}$$

Examining the pole/zero map of the longitudinal state space reveals the dynamic flight modes, shown in Figure 3. The values of the the roots and the associated dynamic modes are given in Table 3.

The presence of two real poles for the short-period mode is likely a result of the linearisation of the equations of motion. This is unusual in reality, and usually the SSPO mode is characterised by more highly damped but still oscillatory, complex conjugate poles with a shorter period that the Phugoid mode [4]. However, the model is sufficiently accurate under the given assumptions to adequately represent the system dynamics. Figures 2 and 4 show the isolated poles of the modes, excited by an impulse input. The short-mode is stable, and the system

Table 3: Longitudinal system poles

Pole	Damping	Frequency	Time Constant	Dynamic mode
0	-1.0	0	∞	-
-0.0656 $\pm$ 0.856i	0.0764	0.8858 rad/s	15.2 s	Phugoid
-6.73	1.0	6.73 rad/s	0.149 s	Short-Period
-31.5	1.0	31.5 rad/s	0.0318 s	Short-Period

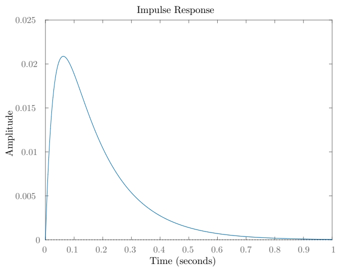


Figure 2: Short-period mode

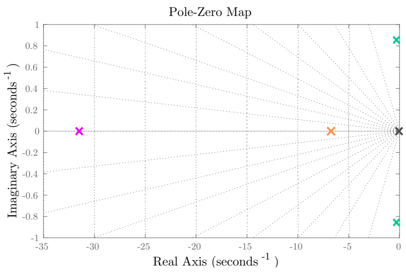


Figure 3: Longitude system poles

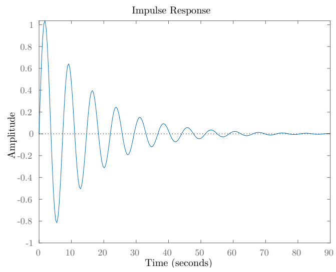


Figure 4: Phugoid mode

dynamics quickly die out within a second. The phugoid complex poles result in stronger, decaying oscillations that take over a minute to settle.

3.4.4 Lateral model

The state space equations representing the linearised, decoupled lateral model of an airframe in steady trimmed flight is shown in equation 13. These coefficients are also taken from the same system identification study as the longitude values [3]. The control inputs are the rudder and the ailerons. The primary method of turning the plane to a new heading is with the ailerons: the rudder is mainly used when the plane is on the ground, and for takeoff and landing. The state vector control inputs (equation 14) are: perturbation from equilibrium in y v; roll rate p; yaw rate r; roll angle ϕ; and yaw angle ψ.

$$A_{lat} = \begin{bmatrix} -0.95 & 0 & -1 & 0.88 & 0 \\ -12.02 & -7.47 & 5.87 & 0 & 0 \\ 5.04 & -1.94 & -6.60 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} B_{lat} = \begin{bmatrix} 0 & 0.22 \\ 21.65 & 4.49 \\ -0.25 & -5.71 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{13}$$

$$x_{lat}^T = \begin{bmatrix} v & p & r & \phi & \psi \end{bmatrix} \tag{14}$$

The poles of this system are found by solving the characteristic equation of the A matrix, which gives the eigenvalues, which can be easily done in Matlab. Examining the system's poles shows the lateral dynamic modes as shown in Figure 5. Ignoring the real root at the origin, there are two real poles and a pair of complex poles, which is expected. Spiral mode is the lowest frequency pole, and Dutch roll mode is indicated by oscillatory motion coupling yaw and roll. These latter dynamics will be the primary behaviour to mitigate in the lateral control system. The roll subsidence mode is activated by excitation near the pole with highest frequency. The values are shown in Table 4.

4 Controller Design

Two control systems were developed, for the decoupled lateral and longitude models. The first is an altitude control, using the elevator control as input, to regulate the pitching angle θ. In order to smooth the dynamics, and speed up the responsiveness of the controls, the elevator control was also used to control the pitching rate through closed loop feedback, and then the control system was progressively built up to eventually use the measured output from the change in height, to control the vehicle's altitude.

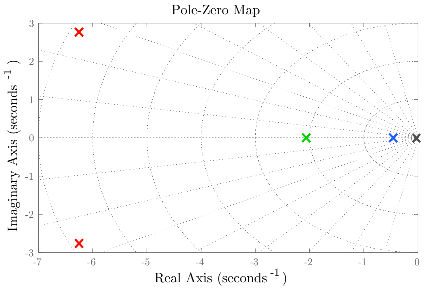


Figure 5: Lateral system poles

Table 4: Lateral Poles

Pole	Damping	Frequency	τ_c	Mode
0	-1	0	∞	
-0.457	1	0.457e	2.19	Spiral mode
-2.06	1	2.06	0.485	Roll subsidence
-6.25 $\pm$ 2.76i	0.915	6.83	0.160	Dutch roll

The lateral system aims to control the heading of the vehicle, with an input to the ailerons affecting a desired change to the yaw angle ψ. This is accomplished by feedback loops from the measured outputs of yaw rate, roll rate and roll angle. The dutch mode poles couple roll and yaw, so the system requires damping in addition to augmenting responsiveness to control input by reducing the rise and settling time of the dynamic responses.

4.1 PID control

Proportional Integral Derivative (PID) controllers are prolific, widely used controllers that combine an integrator with gain and (in theory) a derivative component. They are used in closed feedback loops to regulate and control industrial and electric systems. Often in practice the derivative introduces noise or rapid discontinuities, and is substituted in implementation with other means of zero placement such lag/lead compensators. The mathematical expression of the controller is a second order function with three terms. The design of such a controller aims to find suitable coefficients for these terms to reduce error with the integral and derivative components, and influence gain to increase stability and speed up system rise times with the proportional term. P, I, D, PI, and PD controllers are all simpler reductions without some of the components. The basic form of a PID can be expressed as shown in equation 15.

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt} \tag{15}$$

Although PID controllers are not guaranteed optimality, and are often tuned by trial and error, a major advantage is that unlike more sophisticated or advanced controllers and control methods, a PID only relies upon the measurements of the process it is used to control, and therefore an engineer designing a feedback system with PIDs, does not have to posses specialised or expert domain knowledge over the processes being controlled. For this project, involving the enormously complex, detailed and complicated area of flight and the control laws of aeroplanes, PIDs and simple feedback loops seem an appropriate choice to apply. Matlab has excellent tools to help reduce the task of searching the parameter space for the three terms, with graphical tool-sets and the ability to parameterize the coefficient terms or to prioritise disturbance rejection or reference tracking, in both the time and frequency domains.

4.2 Successive loop enclosure

Successive loop enclosure is a design method in which a coupled system has feedback loops designed sequentially building up inner loops which target specific aspects of a system to regulate. The overall system is built up with each loop that is closed becoming Incorporated in the new plant model to be regulated. [19] discusses the technique as applied to lateral-directional autopilot and roll autopilot. Using Matlab and the PID block in Simulink, the desired control systems were built up successively and iteratively, targeting specific outputs in turn, tuning a PID controller and then moving to a new controller in a separate outer loop. The basic process followed for the height autopilot design is graphically shown in Figure 6. The PID values often had to be slightly adjusted or re-tuned, as the loops were closed.

The lateral controller design was developed slightly differently, with a third PID being inserted after the basic design was complete, in the innermost loop. The innermost PID for the lateral was inserted and tuned to improve the response of the roll rate after the design was essentially finished. The overall steps for both control systems are shown in Figure 7.

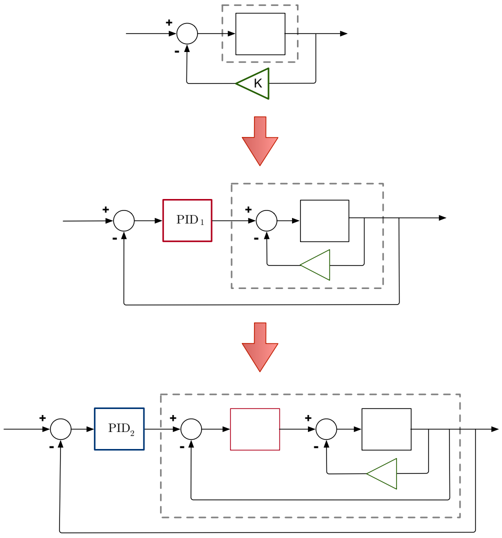


Figure 6: Longitudinal control design steps

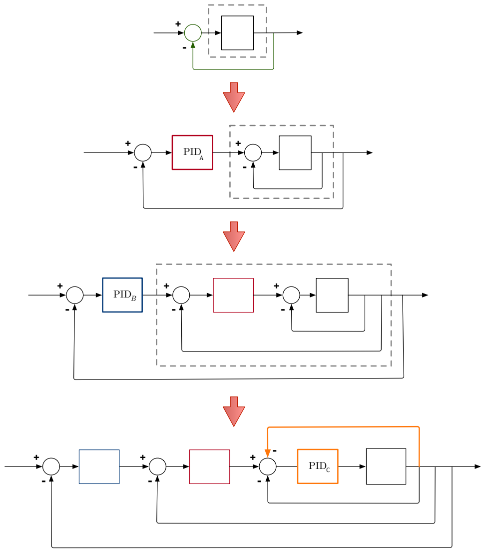


Figure 7: Lateral control design steps

4.3 Longitudinal height controller

4.3.1 Inner loop

Figure 8 shows the aeroplane plant G(s) cascaded with the elevator actuator block E(s).

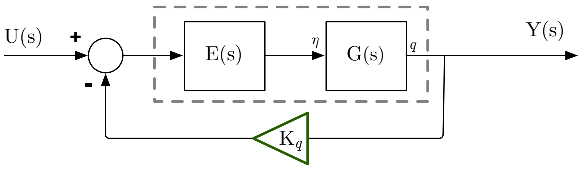


Figure 8: Elevator η to q feedback loop

The transfer function for G(s) (in equation 16) describes the relationship from input (η to output q.

$$G_q(s) = \frac{q(s)}{\eta(s)} = \frac{0.2546s^3 + 24.02s^2 + 135.3s}{s^4 + 38.35s^3 + 217.7s^2 + 55.95s + 156.1} \tag{16}$$

Combining E(s) and G(s) as a single plant, and closing the feedback loop with negative gain K_q gives equation 17.

$$\text{open loop} : E(s)G_q(s) \tag{17}$$

$$\text{closed loop} : \frac{E(s)G_(s)}{1 + E(s)G_(s)K_q} \tag{18}$$

$$\text{closed loop poles} : -0.656 \pm 0.856i, 6.73, -26.3 \pm 23.2i, -31.5 \tag{19}$$

$K_p = 2.0753$ was chosen through trial and error, using the feeback from Matlab's tuning tool, with the result that the closed loop had better damping, reducing oscillations and was also faster, improving the responsiveness in the pitch angle step response. The simple gain produced steady state error, but this disappeared after the next loop was closed. Figures 9 10 and compares the open and close-loop performance for pitch angle and pitch rate output, with a step response applied to the elevator control input. With the closed loop and added gain, q corrected the rrro signal via feedback down to zero, maintaining a steady state output θ. The step response has a rise time of 3.1 seconds with a step input applied to η.

4.3.2 Middle loop

The next loop targeted θ with η input. Equation 20 shows the transfer function from input η to output θ. When closed, the loop introduced an undershoot error of 8.15%. A PID controller was inserted into the feed-forward

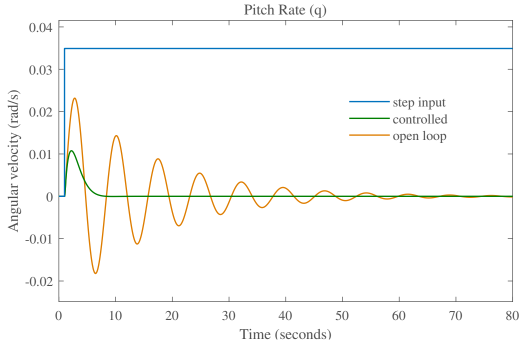


Figure 9: Pitch rate step response

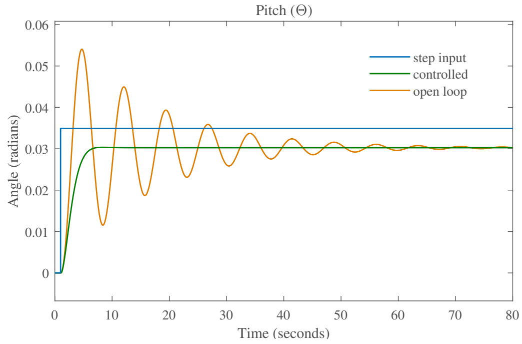


Figure 10: Pitch angle step response

path before the augmented plant, as shown in Figure 11.

$$G_{\theta}(s) = \frac{\theta(s)}{\eta(s)} = \frac{0.2546s^2 + 24.02s + 135.3}{s^4 + 38.35s^3 + 217.7s^2 + 55.95s + 156.1} \tag{20}$$

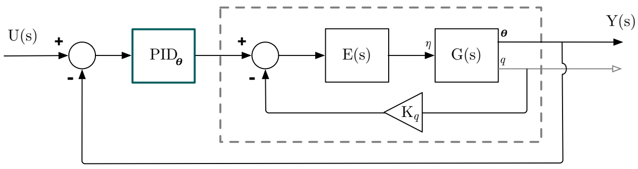


Figure 11: Middle feedback loop η to θ

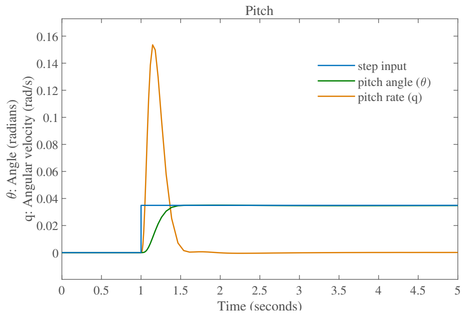


Figure 12: θ step response (middle loop)

The step response for θ shown in Figure 12 demonstrated that the steady state error had been reduced by the integral and derivative terms of the PID controller. The signal rise time was measured at 234 ms with an overshoot of 1.5 %. The signal undershoot was measured a at 0.47%. The MATLAB PID tuning (with "Reference Tracking prioritisation") resulted in K_p = 11.85, K_I = 5.205, and K_D = 6.625 with a filter order f ∼52.

4.3.3 Outer loop

The final outer loop targets the regulation of the measured change in height. The output h was fed back to the feed-forward path at a summer, and a PID controller inserted in cascade with the augmented plant.

A diagram of the completed outer loop with feedback from the altitude output is shown in Figure 13. A third PID in placed in the feed forward path after the altitude measurement signal is subtracted from the input signal in a summer in a negative feedback loop. After some iterative tuning, the selected PID values for the longitudinal system are presented in Table 5.

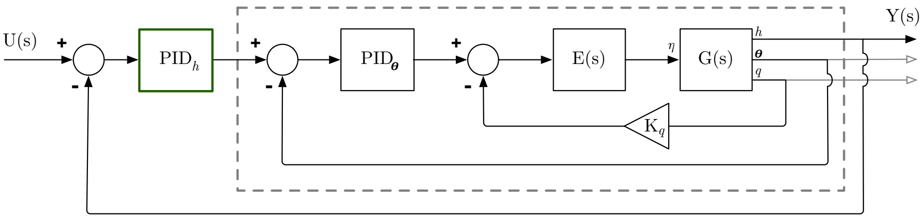


Figure 13: System diagram of the altitude control (longitude)

The step response for the outputs are shown in Figures 14, and 15. The input step had a magnitude of 10 (representing an input signal intended to indicate the desire to raise the plane by pitching upwards smoothly to reach 10 m higher then the equilibrium altitude, and them automatically level the plane back out to level rectilinear flight at the new altitude which must be maintained. The elevator control surface produces a rapid change in pitch rate, the pitch angle increased smoothly with a rise time of 918 ms. The height output rise time was measured at 5.877 seconds with an overshoot of 0.513%. However, there is also a noticeable undershoot of 1.536%. This means that the plane briefly pitches forward and moves down before ascending, when the control system requested a rise in altitude. This behaviour is undesirable.

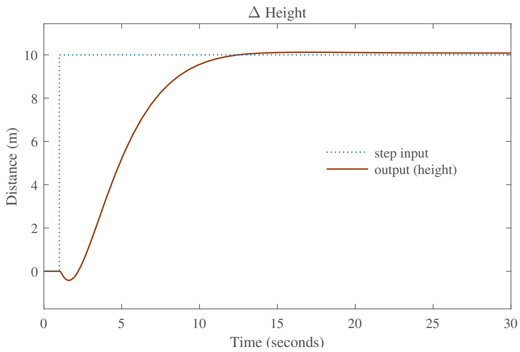


Figure 14: Height step response

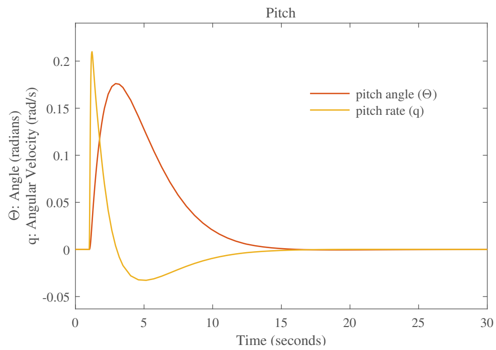


Figure 15: Pitch step response

The final PID and the gain block values for the longitudinal altitude autopilot system are shown in Table 5. The outermost controller has very small values, which may not be realistic to implement. However, the autopilot system works smoothly under 10% over- or undershoot. If a large height change is requested, that would cause the pitch angle to exceed the physical limit of the elevator control surface, then the system simulates this, limiting η to its maximum value, which then causes the rise time of the height output to be much slower. However, large changes in altitude also violate the assumption of the small pterbaton model.

Table 5: Selected Altitude Control PID parameters

	$\mathbf{K}_q$ gain	$\mathbf{PID}_\theta$ inner	$\mathbf{PID}_h$ outer
Proportional gain	2.075	11.48	0.020375
Integrator value	-	3.734	0.00039879
Derivative value	-	6.131	-0.014098
Filter Order	-	∼502	1

4.4 Lateral Heading Controller

For brevity, the entire design process is not discussed for the Lateral controller. This system was developed in a similar method to the longitude system, with successive loop closure. The final design is depicted, along with stages of the successive loop development in Figures C.5, C.6, C.7, C.8 and C.9 in Appendix C. The system structure is shown in Figure 16,and the full simulink system diagram developed for relative heading control autopilot is shown in Figure C.10a. The final PID values for the heading autopilot system are tabulated in Table 6.

5 Safety

5.1 Failure Plan

A class 1B UAV is not required by law to provide a full set of failure modes, but it would be responsible to ensure the safety of the vehicle, and to ensure that injury or damage are mitigated or prevented. At the current project scope, the physical implementation of the UAV falls outside the project parameters. However, it is vital that a full and complete failure plan is drawn up and adhered to in response to some of the issues to follow.

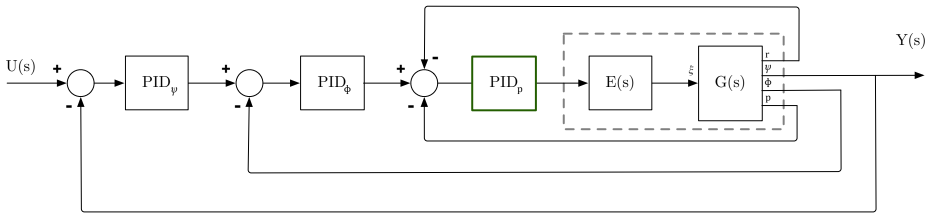


Figure 16: System diagram of the heading autopilot (lateral) system

Table 6: Selected Yaw Control PID parameters

	PID$_\phi$ rate	**PI$_r$** roll	**PID$_\psi$** heading
Proportional gain	0.59015	13.835	0.89733
Integrator value	1.5725	2.4411	0.0028477
Derivative value	0.026301	-	-0.23606
Filter Order	$\sim$1927	-	$\sim$4

A mechanism such as a parachute being deployed in case of critical failure of engine components or if the structural integrity of the aircraft is compromised is important, to limit the energy the falling vehicle can release upon impact. Guided by the legislation [20], important considerations to be developed if the project proceeds to physical implementation should include some of the following point of discussion.

- Environmental limitations of the aircraft, to limit any meteorological impacts such as rain, snow, high humidity, and temperature or pressure variations.
- Specified minimum distances for take off and landing areas.
- Detailed measurements and theoretical analysis of the control actuators and surfaces.
- Detailed plans and blueprints, detailing all the locations of the plane components and subsystems. This should include documentation for:
    - the safe handling and operation of the control system
    - any radio equipment used to communicate and interface with the plane in flight.
    - all sensor and controller data and the location on the plane
    - specifications of all replaceable components
    - details of commercially available equipment installed on the plane

A failure plan should follow the relevant regulations, and in particular should consider the danger posed by the batteries used to power the engine, communication and control circuity. The manufacturer-provided batteries are lithium-polymer, which are at risk of catastrophic failure subject to excessive force, or if the battery is punctured or heated. Particular care should be taken when designing the physical layout of the control system on the aircraft, to limit the exposure to the corrosive and reactive gel contained in the power source in case of component failure.

6 Impact

6.1 Social

The remote sensing capability of UAVs means that the potential role of surveillance could have ethical, social and privacy concerns. The legislation regarding drones differs between different nations and localities, and there is a possibility that the engineers behind the design of some components of an unmanned flight vehicle could be liable for serious privacy violations. Engineers also have ethical responsibilities to ensure a high standard to maintain safety, particularly with the risk of a plane falling out of the sky and causing damage or death in collision.

The sound and sight of large, loud or dangerous vehicles can be a source of pollution in cities and rural contexts. Unmanned vehicles could also be exciting ways for people to interact and learn. Unmanned deliveries of gifts, food or valuable resources could be make cheaper and more convenient with the ubiquity of self-flying vehicles. Rural and isolated communities could use fixed wing UAVs for vital supplies and communications, wither supplementary or

The growth of UAVs also offer the possibility of democratising air-travel transportation of individuals, for work or useful payloads with social benefits. The freedom of flight also means that special care should be taken with private property, air rights and legislative concerns regarding the altitude, location and speed of flight, landing and take-off. Piloting and operating planes is a highly stressful and fatiguing job. Unmanned vehicles present significant opportunities to improve safety and save lives, reducing costs and providing value and service to individuals and communities.

6.2 Environmental

The manufacture, operation, repair and disposal of fixed wing UAVS have important environmental impacts. The use of fossil fuels in combustion engines contribute to climate change, and inject heated carbon dioxide and other fuel byproducts directly into the atmosphere. Electrical systems also have to manufactured and the energy stored by batteries must be generated, aside from the cost and complexity of battery disposal. Hydrogen powered fixed-wing UAVs offer a compelling commercial alternative, but at raised costs and additional safety and production concerns.

The noise generated by low flight UAVs can contribute to noise pollution, and the sight of flying machines in the air could also interrupt sporting, scientific or commercial concerns. UAVs could also pose a threat to birds or other animals. The requirements for safe and environmentally considerate autonomous operation imposes special requirements for robust and safe control. Self-flying vehicles powered by AI could being about radical shifts in our daily living and operating environment as biological creatures, and many unseen consequences and opportunities may arise with the possibility of entirely autonomous vehicles capable of rapid manoeuvrability with six-degrees of freedom at high speeds.

6.3 Economic

The role of autonomous or unmanned aeroplanes also offers many commercial, industrial, academic and social opportunities. Media, science, education and commerce could all benefit hugely from the development, design, operation, exploration and manufacture of fixed wing UAVs. The control systems required for the safe and comfortable operation or autonomous flight could provide significant economic advantages and entrepreneurial opportunities. However, there is also the strong possibility that the fourth industrial revolution also brings employment reductions and increased individual obsolescence. Manual pilots, air controllers, delivery companies, agricultural pesticide pilots and other jobs could be put at risk with the rise of affordable, safe and fast unmanned fixed wing flight. UAVs could also contribute to efficient delivery to remote locations to save fuel and time costs associated with road travel and delivery.

6.4 Sustainability

The choice of materials and manufacturing process of UAVs is important to ensure the sustainability of industrial manufacturing processes. Components designed without sufficient care for the mechanical and electrical controller effort lead to wear and failure, producing waste and costing time and lost opportunities. High performance requirements such as rapid responsiveness and high manoeuvrability impose requirements of more regular maintenance and more rigorous specifications. An electrical system should carefully consider the timeline, timescale and lifetime considerations including disposal and component disassembly. The use of "greener", ecologically friendly materials just as corn-derived or other biodegradable starch-based foams and composites could replace plastic, balsa, metals and other substances which are environmentally damaged to produce or dispose of. However, regular unmanned flights for payload delivery into the form of medicine, information, or technology in electric planes powered by renewable energies are a tantalising and emerging possibility, that could disrupt and replace wasteful routes that are too difficult, dangerous or damaging to travel to by current means.

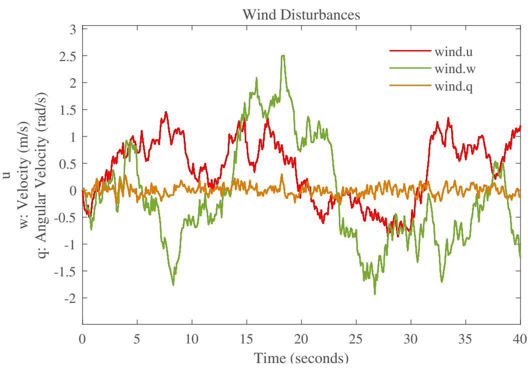
In the presented design, the choice to use PID controllers was partially motivated by the ubiquity and wide availability. A major advantage is the ability to re-tune the controllers during flight sessions. This means less waste and a simpler process for individual aircraft. At a manufacturing scale poorly tuned PIDs are far too common according to [21].

7 Evaluation

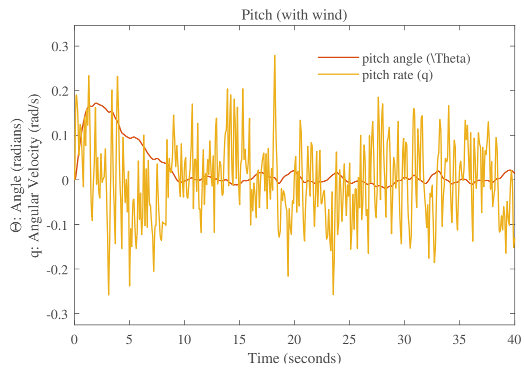
The scope and complexity of the project did not permit time for the full development of a decoupled or non-linear model of flight to evaluate the presented control system with. Introducing a turbulent Dryden wind model as a source of non-linear disturbance allowed a small component of non-linearity to be introduced into the decoupled baseline model used for the development of the autopilot systems.

7.1 Weather simulation

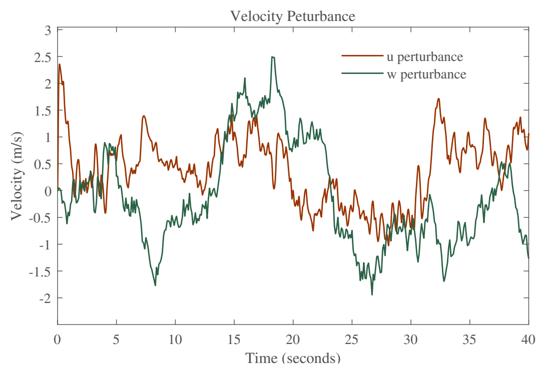
To help evaluate the performance of the control system, a Dryden Wind Turbulence Model based on [22] was applied as external disturbance on the feedback from the output from the plant. The wind model emulates turbulent gust of varying pressure applying forces and inertial moments to the airframe by filtering white-band noise in a stochastic process.



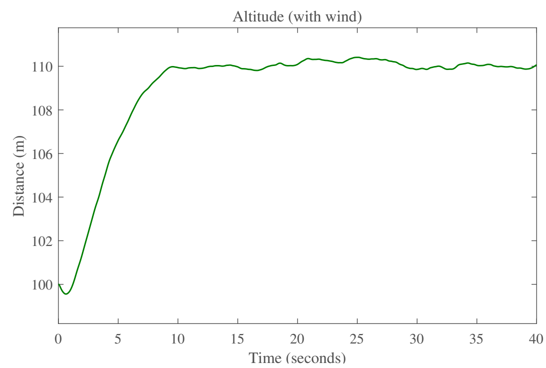
(a) Wind model output



(b) Pitch step response in relative wind



(c) Velocity changes along x and z body axes



(d) Altitude output with wind

Figure 17: Wind turbulence simulation

7.2 Results

The measured results of the output variables for each autopilot system was measured, and shown in Table 7. Both systems exhibit more undershoot that overshoot. The immediate motion of the vehicle is briefly in the opposite direction of the requested direction, which might disconcerting or confusing if it wee in response to manual control. However, the rapid rise times and the nature of an automated control system mean that the flaw is considered minor and acceptable in the context of an UAV.

Similarly, in a manual piloting scenario, human pilots often with for banking controls from the ailerons to not return the zero error signal in response to input control signal, finding it desirable to not maintain have to exert pressure on the lever position in order to bank the plane. Controls have different requirement in different contexts: a remote pilot vehicle who sends electronics signals by wireless communication, or programs the remote commands via flight computer interface has very different requirements in responsiveness to a live pilot in a larger vehicle.

Table 7: Signal measurements

	Rise time	Overshoot	Undershoot
Altitude Autopilot	5.778 s	0.521 %	1.770 %
Yaw Autopilot	2.927 s	0.538 %	1.968 %

The presented control system for longitudinal altitude autopilot performs well in the presence of small gusts of wind, when a fairly large change in altitude is requested, greater than 1 m. For small values, the plane cannot maintain its altitude within the bounds of the amplitude of the applied wind noise disturbance. In the presented plot in Figure 17d, with a step input of 10, representing a height in metres to increase elevation by, the plane performs well with the decoupled linear model.

As the system is able to simulate successfully turning to a specified heading in a reasonable amount of time, and can self correct in the presence of noise or small amounts of turbulence, the project can be considered to have met the basic success criteria.

7.3 Critical Discussion

The presented system has some crucial flaws in the evaluation methodology. The logical progression for an initial control syste, design based on linearisation, is to substitute the plant for a non-linear model, or a model with more detailed and realistic dynamics. There was insufficient time to implement a more complex, coupled or non–linear system to evaluate the control system on. The use of limited resources is a valuable and realistic constraints in engineering projects, which often require compromise or reduction in scope to make meaningful accomplishments in the assailable time. While the addition of a turbulent wind scenario is a useful way to evaluate the system's noise rejection, it does not not adequately capture the required range and rigour of more complete flight dynamics. The control system presented is useful for a demonstration of the flexibility and utility of PID controllers as a tool for control engineers to design and implement control systems in domains of specialist knowledge outside their experience. The undershoot exhibited in the system was difficult to eliminate, and may be an indication of excessive integer winding, or indicative of poor methodology in tuning the controllers. PIDs require experience and practice to master a trial and error approach.

8 Recommendations

The control system would benefit from more rigorous evaluation around a more complex and coupled plant. It would be useful to vary the plane architecture and parameters to determine the generality of the system. Extending the existing system to have a succession of automated instructions for the system to perform automatically would be the next development stage. Testing general sensor noise, or combining the noisy measurements from multiple sensors with Kalman filters would be a useful way to extend the control system's applicability. Future work would benefit from access to a physical build of the plane airframe for measurements and in person testing.

9 Conclusion

An autopilot system to separately control the altitude and heading of a small UAV was designed and simulated with a decoupled linear set of dynamic equations, arranged in a state space form. The system was designed by successive loop closure and used PID controllers and tuning in Matlab and Simulink. The system successfully demonstrates a speedy response to desired control signals, but was not tested on a more fully developed non-linear flight behaviour. The presented system meets the basic requirements of the success criteria.

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Thank you to my wife and children for their love and support throughout my undergraduate career.

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A Non-Technical Report

Droning on about control theory...

Unmanned aerial vehicles, control engineering and managing complexity with the art of approximation

by Tyson Cross

Introduction

Drones, quad-copters, remotely piloted aircraft and unmanned aerial vehicles (UAVs) are an increasingly common and visible vehicle across a wide variety of industries and disciplines. The convenience, speed, small-size and low fuel requirements (compared to large commercial aircraft) have all helped lead to the recent explosion of autonomous, semi-autonomous or remotely controlled planes and 'copters. From hobbyists to military and scientific applications, Amazon drone deliveries to silent, distant surveillance from the upper atmosphere where sleek fixed-wing planes scour the edge of our world collecting clues to our climate and tomorrow's weather, autonomous flying vehicles are forecast to continue to become a major source of technological innovation, investment opportunities and exciting new develpments.

Control Engineering and Unmanned Aircraft

Control engineering is the science and practice of applying control theory for the automation, regulation and augmentation of automatic processes and technologies. Control engineers use mathematical models of electrical, mechanical and industrial activities to apply structured processes and mathematical techniques to correct errors, regulate devices and factories, and generally ensure the safe and efficient operation of technology across a variety of disciplines. Robotics, AI, machine learning, transportation, and drones all fall under the wide umbrella of control engineering.

In the case of UAVs, control engineers work with with other specialist aeronautical, aerospace, electrical, robotic and mechanical engineers to fulfil their professional responsibilities. An important concept in all disciplines of engineering is obviously mathematics, that formal system of logic and scientific symbols that powers almost every aspect of our daily lives. A central idea in engineering mathematics is the notion of "linearity". This is a mathematical property that ensures predictable and solvable relationships between entities. A relationship is said to be linear, if the input to a system can be predictably and repeatedly related in a strictly proportional relationship to the output. In other words, the same consequence must occur every time an action is performed. If you go eating out, with a group of friends, and say that you all keep track of the cost of each ordered meal together. Then in an hour's time when you came to pay the bill, the total should definitely be the sum of the individual meal prices, not some random or unpredictable price with no clear relationship to the circumstances, or the action of ordering a known number of meal at a known price.

The problem is that many, many processes in the world are decidedly non-linear, chaotic and highly unpredictable. The hidden or complex interaction of physical processes or complex interconnected events sometimes makes it very difficult to predict with certainty what is going to happen next in all kind of circumstances. The advent of massive computational power, and ever vaster amounts of information mean that there are lots of ways that computers can help solve problems that were very difficult to solve just a few decades ago, but the role of linear mathematics is still of central importance to the science of engineering and control theory. One of the primary techniques used by engineers is called "linearisation" the process by which a natural, or complex non-linear system is *modelled* as a linear system, within strict conditions or clear assumptions.

As long as the system (or process) being simulated stays within the boundaries of these assumptions or conditions, then we can use the powerful tools of linear algebra and control theory to solve, predict and control very complicated processes and interactions in the real world.. One such highly complicated area of non-linear complexity is the physics of flight. There are 6 degrees of freedom for a powered aircraft in the air: translational (moving in straight lines in 3-dimensional space: up/down, back/forward, side-to-side) and rotational (there are also three rotational directions, that describes curved motion around a fixed point). Working out the mathematics and relationships of moving *and* rotating simultaneously, at high speeds in the air without human intervention or direct control, under the forces imposed by the atmosphere, climate, air pressure, and the highly complicated forces of turbulence... it becomes very clear, very fast, of the need to make some initial simplifications, in order to make the whole process more comprehensible, measurable and manageable.

Instead of considering all the complex non-linear aspects of flight, we can assume that the plane is already flying, at a fixed height and a constant speed. Further, we can say that just to make the model simpler, that the earth could be considered flat, not round (at least at the scale we are interested in.) We can also assume that there is no inclement weather: it's a perfect still day at a very reasonable temperature. Once we start making these small simplifications, we can strip away the difficult-to-solve bits of the problem of modelling flight, and come up with a simplified but still robust approach that allows the use of powerful mathematical techniques to control and automate the operation of an unmanned vehicle (or to augment the abilities of a pilot, making the plane easier to fly, for example.)

Once a simple model has been developed to a working standard, it can be modified or improved, with small adjustments to make the model closer and closer to representing the full non-linear world that the invention, device, vehicle or process has to operate in, eventually. The skill at making the right amount of approximation, the correct number of assumption while still having a useful model that has some correlation with reality is a genuine skill that only comes with experience and practice, as a result of discipline, learning, careful thought, and collaboration.

Conclusion

Drones are an example of a domain that involves extremely complex engineering problems. Simplifying and approximating until a working model that can use the power of linear algebra is a crucial skill for engineers to develop. The rapid expansion of unmanned aerial vehicles and other automated aircraft means that control engineers will have many opportunities and responsibilities to exercise their skill-set to ensure the safety and stability of control processes in automated vehicle flight.

B ECSA ELO Table

Table B.8 highlights the relevant sections of the presented report as they apply the the assessment rubric for the Engineering Learning Outcomes (ELO) as defined by the Engineering Council of South Africa (ESCA).

Table B.8: ELO sections in report

ELO	**Relevant sections**
	Part I
ELO3	§ 4
i.	§§ 2.4 to 2.6
ii.	§ 4
iii.	§ 7
ELO6a	Main Report
ELO6b	Appendix A
	Part II
ELO1	§§ 2.6, 4, 4.2 and 7.1
ELO2	§§ 3 and 4
ELO5	§§ 3 and 4
ELO7a	§ 6
ELO7b	§ 6.4
ELO8a	Appendices E and F

C Simulink

Longitudinal Model

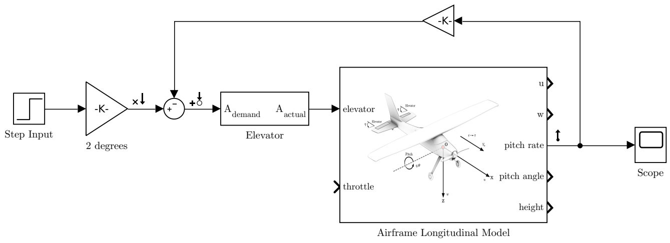


Figure C.1: Elevator η to pitch rate q inner feedback loop

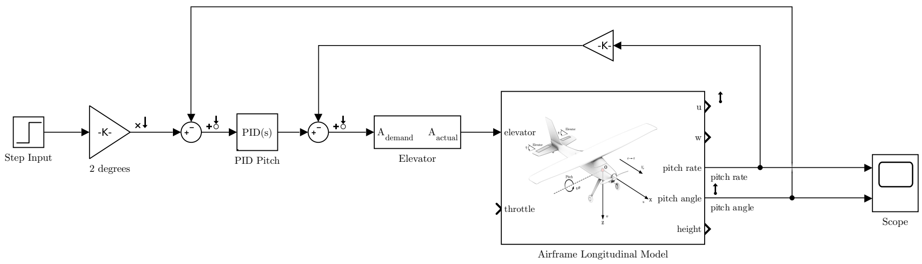


Figure C.2: Elevator η to pitch angle θ middle feedback loop

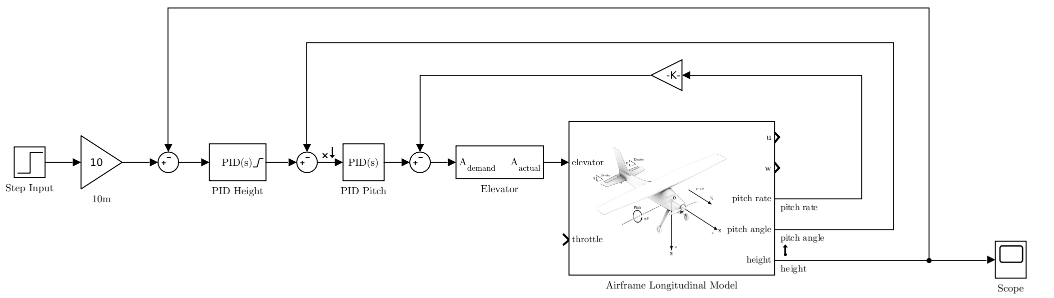


Figure C.3: Elevator η to height h outer feedback loop

Longitudinal Wind Model

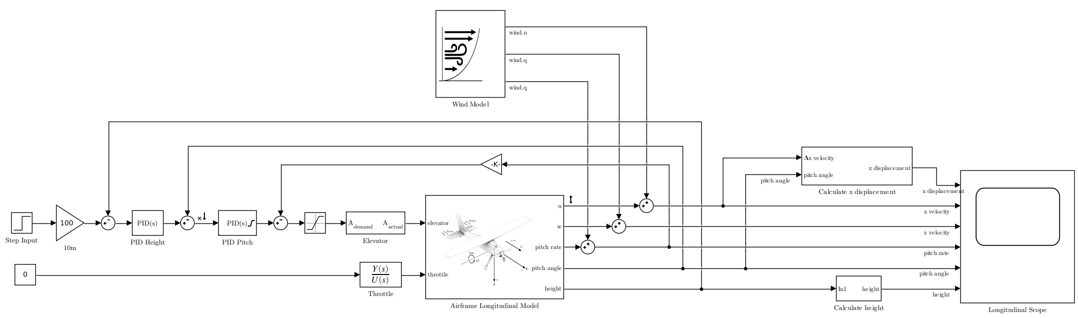


Figure C.4: Altitude autopilot flight controller with turbulent wind

Lateral Model

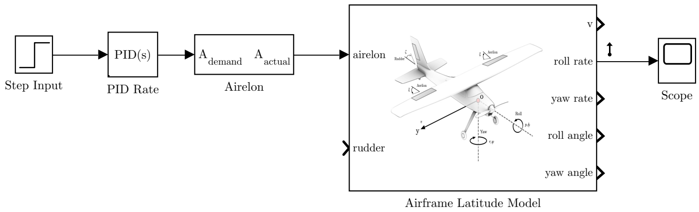


Figure C.5: Aileron ξ to roll rate p open loop

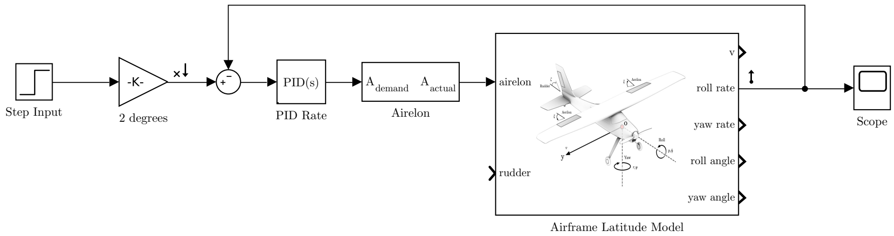


Figure C.6: Aileron ξ to roll rate p inner feedback loop

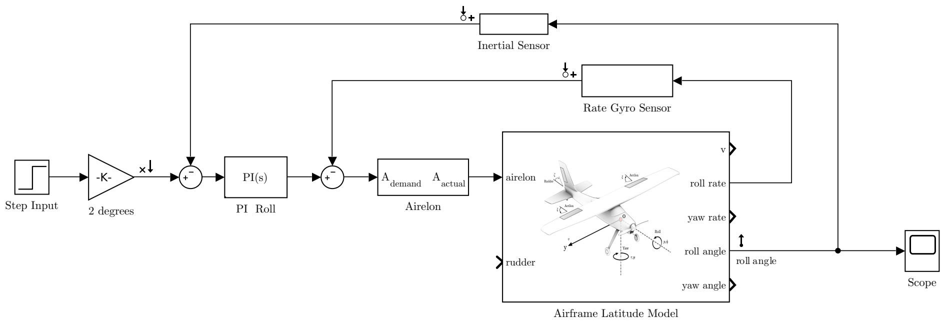


Figure C.7: Aileron ξ to roll angle q middle feedback loop

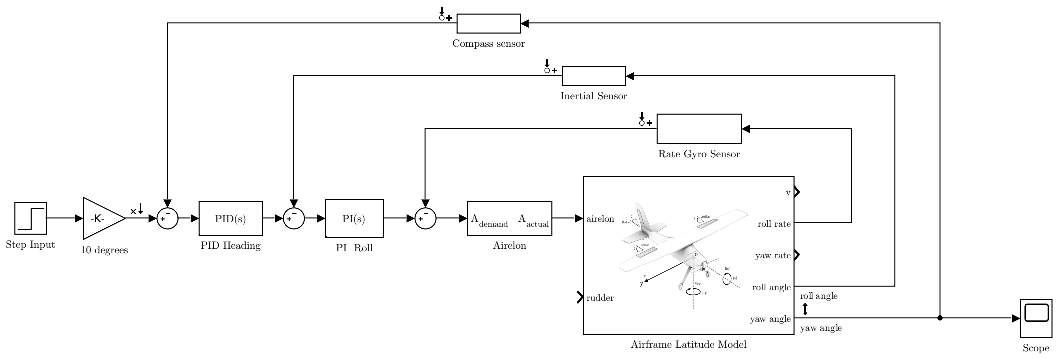


Figure C.8: Aileron ξ to yaw angle ψ outer feedback loop

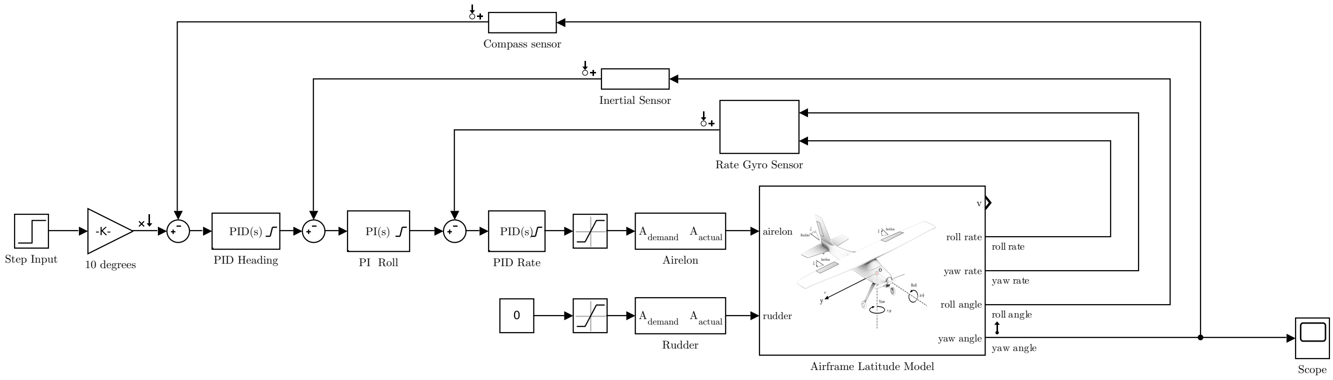
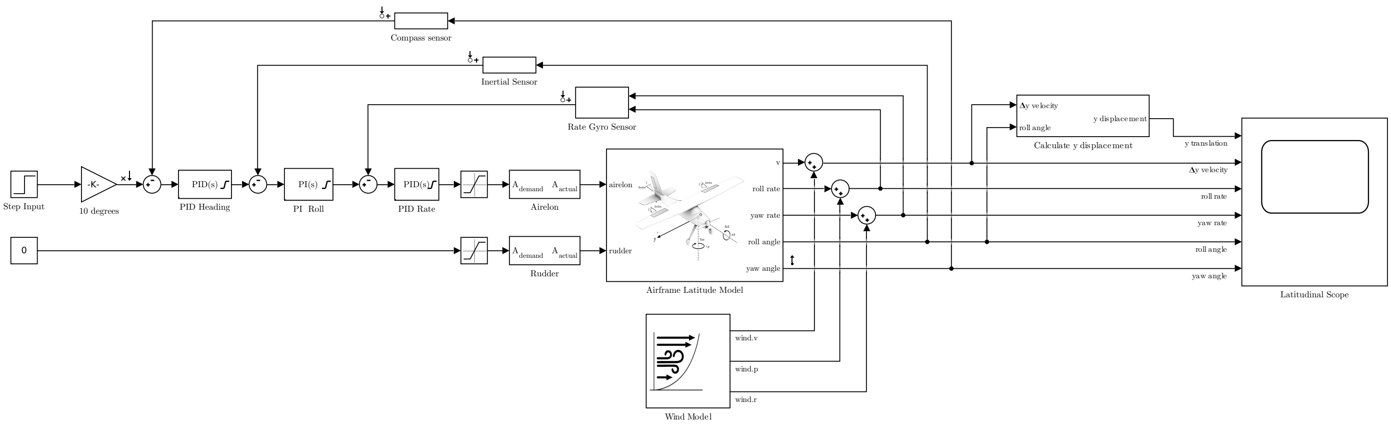


Figure C.9: Aileron ξ to yaw rate q inner feedback loop

Lateral Wind Model



(a) Lateral heading autopilot flight controller with turbulent wind

D MATLAB Code

Code 1: code/control_system_setup.m

```
%% ELEN4011 Engineering Design
% Control system for UAV
% Tyson Cross 1239448

clc; clear all; close all;

system_setup;

%% State Space equations of motion (small peturbations)
% ---------------------------------------------------------------------------

%%% Longitudinal
disp('==================== Longitudinal EOM =============================')
sys.long.A = [ deriv.x_u,  deriv.x_w,  deriv.x_q,  deriv.x_theta,  0 ;
               deriv.z_u,  deriv.z_w,  deriv.z_q,  deriv.z_theta,  0 ;
               deriv.m_u,  deriv.m_w,  deriv.m_q,  deriv.m_theta,  0 ;
               0,          0,          1,          0,              0 ;
               deriv.th_u, deriv.th_w, deriv.th_q, env.V_0,        0 ];

sys.long.B = [ deriv.x_eta,     deriv.x_tau     ;
               deriv.z_eta,     deriv.z_tau     ;
               deriv.m_eta,     deriv.m_tau     ;
               0,               0               ;
               0,               0               ];

sys.long.C = eye(5);

sys.long.D = zeros(size(sys.long.C,1),size(sys.long.B,2));

sys.long.SS = ss(sys.long.A, sys.long.B, sys.long.C, sys.long.D, ...
    'Name', 'Longitude model', ...
    'StateName', {'u'; 'w'; 'q'; 'theta'; 'h'}, ...
    'InputName', {'elevator'; 'throttle'}, ...
    'OutputName', {'x'; 'z'; 'pitch rate'; 'pitch angle'; 'height'});

sys.long.TF = tf(sys.long.SS);

sys.long.poles = sort(eig(sys.long.SS));
figs.long.fig1 = figure('Position',[1240,120,650,700], ...
    'Name', 'Longitudinal Step Response');
step(sys.long.TF,50);

figs.long.fig2 = figure('Position',[680,490,550,325], ...
    'Name', 'Longitudinal Pole Zero Map');
pzmap(sys.long.SS);
grid on;

% check observability/controlability
if size(sys.long.A,1)==rank(ctrb(sys.long.A, sys.long.B))
    disp('The LTI system is controllable ')
else
    warning('The LTI system is not controllable !')
end

if size(sys.long.A,1)==rank(obsv(sys.long.A, sys.long.C))
        disp('The LTI system is observable')
else
    warning('The LTI system is not observable!')
end

sys.long.num = sys.long.TF.Numerator;
sys.long.den = sys.long.TF.Denominator;
```

```
sys.long.SS
damp(sys.long.SS)
disp(' ')
disp('Phugoid mode: low damping, low frequency, long period')
disp('Short-Period mode: higher damping, short period')
disp(' ')
sys.long.modes.phugoid = tf(zpk([], [sys.long.poles(2) sys.long.poles(3)], 1));
sys.long.modes.sp = tf(zpk([], [sys.long.poles(4) sys.long.poles(5)], 1));

disp('===================== Lateral EOM ==============================')
% ------------------------------------------------------------------------------

%%% Lateral
sys.lat.A = [   deriv.y_v,  deriv.y_p,  deriv.y_r,  deriv.y_phi,    deriv.y_psi ;
                deriv.l_v,  deriv.l_p,  deriv.l_r,  deriv.l_phi,    deriv.l_psi ;
                deriv.n_v,  deriv.n_p,  deriv.n_r,  deriv.n_phi,    deriv.n_psi ;
                0,          1,          0,          0,              0           ;
                0,          0,          1,          0,              0           ];

sys.lat.B = [   deriv.y_xi, deriv.y_zeta ;
                deriv.l_xi, deriv.l_zeta ;
                deriv.n_xi, deriv.n_zeta ;
                0,          0            ;
                0,          0            ];

sys.lat.C = eye(5);

sys.lat.D = zeros(size(sys.lat.C, 1), size(sys.lat.B, 2));

sys.lat.SS = ss(sys.lat.A, sys.lat.B, sys.lat.C, sys.lat.D, ...
    'Name', 'Lateral model', ...
    'StateName', {'v'; 'p'; 'r'; 'phi'; 'psi'}, ...
    'InputName', {'aerilons'; 'rudder'}, ...
    'OutputName', {'v'; 'roll rate'; 'yaw rate'; 'roll angle'; 'yaw angle'});

sys.lat.TF = tf(sys.lat.SS);

sys.lat.poles = sort(eig(sys.lat.SS));

figs.lat.fig1 = figure('Position', [1900,120,650,700], ...
    'Name', 'Latitudinal Step Response');
step(sys.lat.TF, 50);

figs.lat.fig2 = figure('Position',[680,60,550,325], ...
    'Name', 'Latitudinal Pole Zero Map');
pzmap(sys.lat.SS);
grid on;

% check observability/controlability
if size(sys.lat.A, 1)==rank(ctrb(sys.lat.A, sys.lat.B))
    disp('The LTI system is controllable ')
else
    warning('The LTI system is not controllable !')
end

if size(sys.lat.A, 1)==rank(obsv(sys.lat.A, sys.lat.C))
        disp('The LTI system is observable')
else
    warning('The LTI system is not observable!')
end

sys.lat.num = sys.lat.TF.Numerator;
sys.lat.den = sys.lat.TF.Denominator;

sys.lat.SS
```

```
damp(sys.lat.SS)
disp(' ')
disp('Spiral mode: lowest frequency')
disp('Dutch roll mode: oscilatory motion coupling yaw and roll')
disp('Roll subsidence mode: highest frequency')
disp(' ')
sys.lat.modes.spiral = tf(zpk([], sys.lat.poles(2), 1));
sys.lat.modes.roll = tf(zpk([], sys.lat.poles(3), 1));
sys.lat.modes.dutch = tf(zpk([], [sys.lat.poles(4) sys.lat.poles(5)], 1));

pzmap(sys.lat.SS)
grid on;

% -------------------------------------------------------------------------
% Actuators (all modelled as 2nd order systems with the same coefficents)

actuators.naturalFreq = 35;
actuators.dampingRatio = 0.75;
[actuators.num, actuators.den]= ord2(35,0.75);
actuators.TF = tf(actuators.num,actuators.den);

actuators.thrust.gain = 1;
actuators.thrust.T_tau = 0.01;
actuators.thrust.TF = tf(actuators.thrust.gain,[-actuators.thrust.T_tau 1]);


% -------------------------------------------------------------------------
% PIDs (values taken from Simulink tuned controllers)
pids.pitch = pid( 7.25749605779279, 2.78399941784119, 4.69900154341756);
pids.height = pid( 0.0394284340423091, 0.000147333496020652, -0.0144298382924662);
pids.yaw = pid(1.21069845200907,0.0055970895188787,0.229410917426781,5.54692256878503);
pids.roll = pid(13.8346091340675,2.4410545354894);

sys.long.elevator_pitch = sys.long.TF(3,1);
sys.lat.airelons_yaw = sys.lat.TF(3,1);
sys.lat.rudder_yaw = sys.lat.TF(3,2);
```

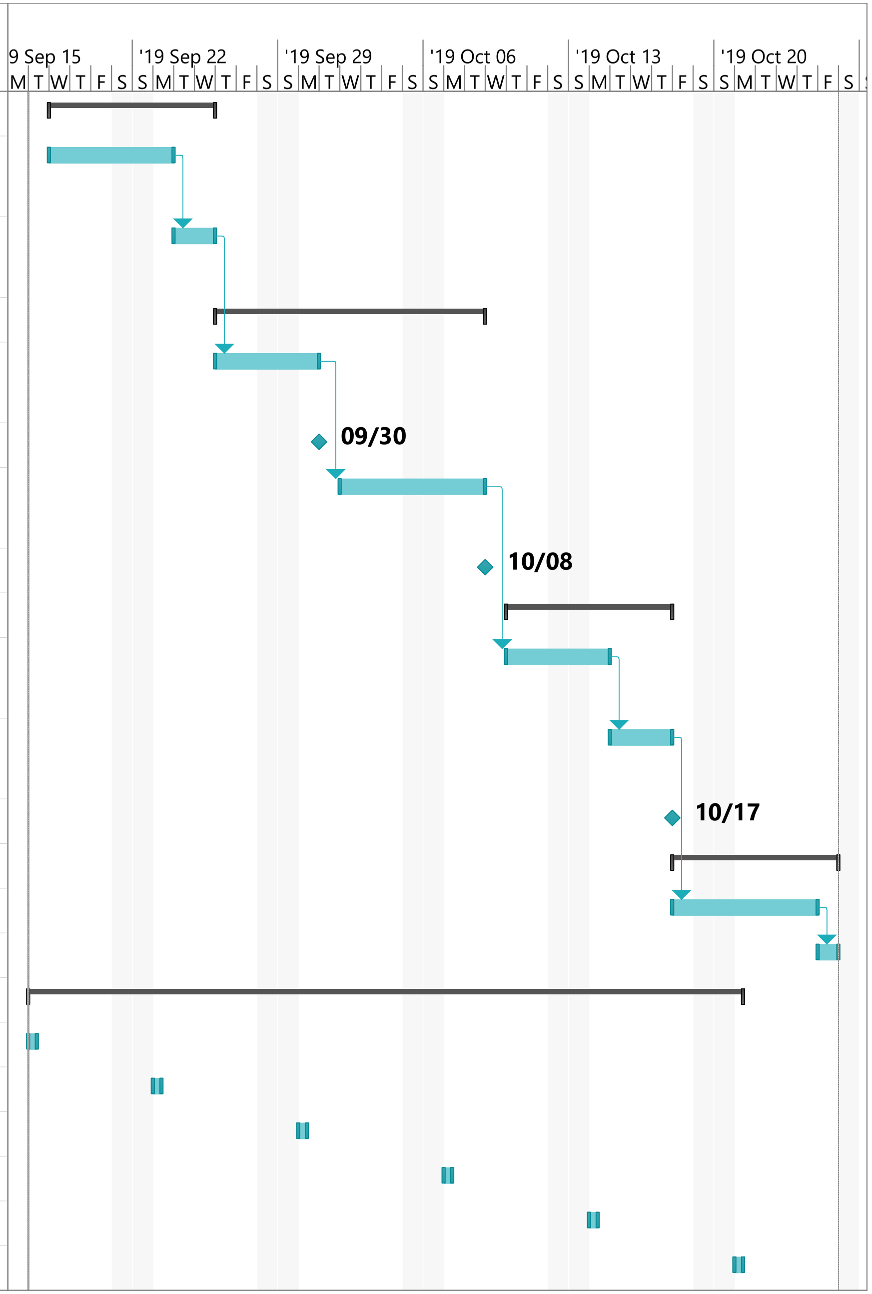
Code 2: code/system_setup.m

```
%% ELEN4011 Engineering Design
% Data for UAV airframe and modelling
% Tyson Cross 1239448

%% Data
%%% Environment properties
env.rho = 1.225;              % Air Density (at Sea Level, STP) [kg/m^3]
env.g = 9.81;                 % Gravity [m/s^2]
env.V_0 = 40/3.6;             % Speed [m/s]
env.h = 100;                  % Height [m]

%%% UltraStick 25e geometric properties
geometry.m = 1.943;           % Weight (fixed mass w/electric motor) [kg]
geometry.b = 1.27;            % Wing Span [m]
geometry.s = 0.3097;          % Wing Area [m^2]
geometry.c = 0.25;            % Chord (mean aerodynamic) [m]
geometry.cg = [0;0;0];        % Centre of gravity (x,y,z)
geometry.inertia.Ix = 0.0895; % Inertial Moment (x) [kg.m^2]
geometry.inertia.Iy = 0.1444; % Inertial Moment (y) [kg.m^2]
geometry.inertia.Iz = 0.1620; % Inertial Moment (x) [kg.m^2]
geometry.inertia.Ixz = 0.0140; % Inertial Moment (planar) [kg.m^2]

%%% Stability derivative coefficients
```

```
% Dimensionless derivatives (with ref to Body axes)
% Longitudinal
deriv.x_u = -0.1492;
deriv.x_w = 0.1490;
deriv.x_q = 0;
deriv.x_theta = -env.g;
deriv.x_h = 0;
deriv.x_eta = 8.4872;
deriv.x_tau = 0;

deriv.z_u = -0.5272;
deriv.z_w = -5.2988;
deriv.z_q = 11.1100;
deriv.z_theta = 0;
deriv.z_h = 0;
deriv.z_eta = 0;
deriv.z_tau = -1.6553;

deriv.m_u = 2.6669;
deriv.m_w = -3.3818;
deriv.m_q = -32.9054;
deriv.m_theta = 0;
deriv.m_h = 0;
deriv.m_eta = 0.2546;
deriv.m_tau = -49.7923;

deriv.th_u = -0.523;
deriv.th_w = -0.9986;
deriv.th_q = 0;

%Lateral
deriv.y_v = -0.9512;
deriv.y_p = 0;
deriv.y_r = -1;
deriv.y_phi = 0.8830;
deriv.y_psi = 0;
deriv.y_xi = 0;
deriv.y_zeta = 0.2189;

deriv.l_v = -12.0240;
deriv.l_p = -7.4665;
deriv.l_r = 5.8687;
deriv.l_phi = 0;
deriv.l_psi = 0;
deriv.l_xi = 21.6477;
deriv.l_zeta = 4.4873;

deriv.n_v = 5.0421;
deriv.n_p = -1.9428;
deriv.n_r = -6.6032;
deriv.n_phi = 0;
deriv.n_psi = 0;
deriv.n_xi = -0.2506;
deriv.n_zeta = -5.7111;

% Initial conditions (stable, trimmed flight)


% syms u w q theta;   % state variables: perturbance along x, z, pitch rate, pitch angle
% syms eta tau;       % control variables: peturbance in elevator angle, thrust
% x = [ u; w; q; theta ]
% u = [ eta; tau ]
% y = eye(4)*x
```

E Project Plan

ID	Task Name	Duration	Start	Finish	Pre(	Notes
1	**Prestudy**	**6 days**	**2019/09/18**	**2019/09/25**		
2	Research	4 days	2019/09/18	2019/09/23		Define Specifications and Assumptions
3	Regulations and specifications	2 days	2019/09/24	2019/09/25	2	Legislation, environment and safety
4	**Iteration 1**	**9 days**	**2019/09/26**	**2019/10/08**		
5	Platform modelling	3 days	2019/09/26	2019/09/30	3	Airframe, actuators, propulstion
6	Initial Model	1 day	2019/09/30	2019/09/30		
7	Initial controller design and sim	5 days	2019/10/02	2019/10/08	5	Implement in MATLAB/Simulink
8	Initial Controller	1 day	2019/10/08	2019/10/08		
9	**Iteration 2**	**6 days**	**2019/10/10**	**2019/10/17**		
10	Refine Model	3 days	2019/10/10	2019/10/14	7	Adjustments or improvements
11	Controller refinements and	3 days	2019/10/15	2019/10/17	10	Advanced controller design
12	Design Complete	1 day	2019/10/17	2019/10/17		
13	**Project Completion**	**6 days**	**2019/10/18**	**2019/10/25**		
14	Report	5 days	2019/10/18	2019/10/24	11	
15	Report Due	1 day	2019/10/25	2019/10/25	14	12:00 submission
16	**Meetings**	**24.25 d**	**2019/09/17**	**2019/10/21**		
17	Initial Meeting	2 hrs	2019/09/17	2019/09/17		
18	Weekly Meeting	2 hrs	2019/09/23	2019/09/23		
19	Weekly Meeting	2 hrs	2019/09/30	2019/09/30		
20	Weekly Meeting	2 hrs	2019/10/07	2019/10/07		
21	Weekly Meeting	2 hrs	2019/10/14	2019/10/14		
22	Weekly Meeting	2 hrs	2019/10/21	2019/10/21		



F Meeting Minutes

ELEN4000/4011 CONTROL RESEARCH GROUP MEETING MINUTES

The following are always in attendance unless specified in 'Changes to Attending'.

Students:

Jacob Riba (1442672)
Sidwell Nkosi (1497963)
Chizeba Maulu (900986)
Tyson Cross (1239448)
Sean Janse Van Rensburg (1073682)
Darrion Singh (1056673)
Haroon Rehman (1438756)
Daniel de Barros (1036613)
Malebo Maboko (672871)
Lloyd Patsika (1041888)
Thapelo Makhalanyane (875691)
Sello Molele (0604606x)
Nkululeko Sikhosana (1135124)

Lecturers:

Prof. Anton Van Wyk

The following document contains the minutes of all meetings for the ELEN4000/4011 Control Research Group for 2019.

Date: 17 Sep. 2019
Start Time: 9 AM
End Time: 10:30 AM
Venue: CM5, Chamber of Mines Building
Chair: Daniel De Barros
Secretary: Darrion Singh
Approval of Minutes: Daniel De Barros
Changes to Attending:

Prof. Anton Van Wyk (Absent)
Thapelo Makhalanyane (Absent)
Nkululeko Sikhosana (Absent)
Sello Molele (Absent)

Proceedings

1. Physical appearance of plane with assumed variables.
2. Clarification of the dynamics we are modelling?
3. What constitutes fixed wing?
4. We should choose a standard system and any extra common final model preliminary final scope by next monday.

Key Notes

1. Bring questions to Prof. Van Wyk regarding proceedings of meeting.
2. Specifications are required in more detail.
3. D. Barros has offered to put together a preliminary project plan for the next meeting.
4. Group Google Drive has been set up by T. Cross.
5. Online meeting documentation set up by D. Singh.

Announcements

1. The next meeting will take place at Seminar Room, EIE Reception, 18 September 2019

Date: 18 Sep. 2019
Start Time: 8 AM
End Time: 9:30 AM
Venue: Seminar Room, EIE Reception, Chamber of Mines Building
Chair: Daniel De Barros
Secretary: Darrion Singh
Approval of Minutes: Daniel De Barros
Changes to Attending:
Sello Molele (Absent)
Malebo Maboko (Absent)

Proceedings

General:

1. ELO 7A/B noted as commonly unvisited sections in report, but regarded as important by external examiners.
2. Purpose of Design II is to bridge the gap between University and Industry.
3. Design process should start from high-level understanding of important processes, followed by a specific design choices that meets the problem criteria.
4. Design complexity should increase with time. Primary objective is to create a model that meets reasonable assumptions, and better design entails removing assumptions and catering to them.
5. Focus on how changes in one subsystem affects another subsystem i.e. cross-coupling of systems.
6. Prof. Van Wyk to confirm that Sello Molele is still part of Control Research Group.
7. D. Singh volunteers to be secretary for remnant of project.
8. T. Cross and S. Nkosi to facilitate meetings.
9. T. Cross proposes that D. Barros as lead of group; D. Barros accepts role.

Regarding previous meeting:

1. Choose the simplest possible variables when designing.
2. Quote regulations as motivation for design choices.
3. Minimum specifications of design as per the email from Prof. Van Wyk titled "Design Project 2019 - Control Group", 16 Sep. 2019.
4. Advised to scale down model as far as possible as to be viable in the timeframe given.
5. Acceptable to split group that subdivides workload into a single model per group.
6. Solutions **should not** be the same for more than one student.
7. Even if the results are the same, there must be distinct differences in the critical analysis.
8. Airframe ~ 6 degrees of freedom, propulsion system should be catered to various issues such as loss of remote control, weight distribution of frame is critical.
9. Be aware of research on propulsion; we may decide to simplify this as necessary.

Current Meeting:

1. Start drafting the outline as soon as possible. The outline provides context to project scope as well as helps in removing ambiguity.
2. Think of short non-technical report should address the concerns of the layman, and meaningfully explain the aspects of the project.
3. Review "Communications for the Engineer" if possible regarding the non-technical report.
4. Format can be informative, for marketing (e.g. press release), educational.
5. Consider the environment, sustainability, economic factors and their associated processes, not just the end outcome.

Key Notes

1. Common model to be confirmed by Monday 23 Sep.
2. Simple modelling to take place before complex decisions.
3. Sub-divide groups by Monday 23 Sep after everyone has researched the entire system.

Announcements

1. The next meeting will take place at Seminar Room, EIE Reception, 23 September 2019 at 8 AM.

Date: 23 Sep. 2019
Start Time: 11:30 AM
End Time: 12:10 AM
Venue: Control Lab, Chamber of Mines Building
Chair: Daniel De Barros
Secretary: Tyson Cross
Approval of Minutes: Daniel De Barros
Changes to Attending:
None

Proceedings

1. MATLAB model of airframe and equations of motion:
   a. Simulink toolbox good base for project
2. Report Structure
3. High level approach to project

Key Notes

1. Use of MATLAB toolbox discussed and approved
2. Report needs clear explanation, cannot rely entirely on the MATLAB drone-simulink "black box"
3. Report must demonstrate understanding
4. Report Structure: 15 pages technical report @ 11pt: ~3000 words with figures/tables/plots
5. Project must demonstrate experimentation and "tinkering" as evidence of engineering
6. High level approach:
   "In front of you is an impossible task…"
   a. make reasonable compromises and find appropriate scope
7. Find MATLAB alternatives to be rigorous
   a. Aerospace industry standards?
8. Appropriate avoidance of unnecessary over-complexity
   a. (not new physics research i.e. turbulence)
9. Modelling Propulsion
   a. No thermodynamics expert in our group
   b. justification for constant mass and avoiding complexity of propeller/turbulence
   c. Abstract t simple representative sub-system
10. Energy Source
    a. time of flight/weight
    b. avoid hybrid system due to added complexity
11. Assumption of steady trimmed flight acceptable
12. Non-technical report: worth writing a preliminary draft already, before implementation inevitably focuses each individual engineer' attention on specific areas of the project.

Announcements

1. The next meeting will take place at Seminar Room, EIE Reception, 30 September 2019
2. Prof. van Wyk will be away from October 4th to the 16th. His attendance to meetings during this time will be via Skype.
3. Future meetings to be made as calendar invites by T. Cross

Date: 30 Sep. 2019
Start Time: 8 AM
End Time: 9:30 AM
Venue: EIE Seminar Room, Chamber of Mines Building
Chair: Tyson Cross
Secretary: Darrion Singh
Approval of Minutes: Tyson Cross
Changes to Attending:

- Jacob Riba (Absent)
- Sidwell Nkosi (Absent)
- Daniel de Barros (Absent)
- Malebo Maboko (Absent)
- Sello Molele (Absent)

Proceedings

1. Publish list of sub-groups to group drive.
2. Tyson has mentioned having completed a model that he is willing to share to be used as a base model, and requests that controller design with regards to the actuator be shared.
3. Find a specification sheet for the motor that shows its thrust response.
4. Sean has mentioned MAV research that shows decoupled actuator control.
5. For first cycle, assume step input, and assume them to be deterministic.
6. Thrust - 1st order, Actuators - gain/1st order, PID controllers for first cycle.
7. Two A/F models - longitudinal (normal force, axial force, pitch), lateral (yaw, roll, side slip)
8. Normal force and Side slip decided to be internal/remnant dynamics that need not be controlled.
9. Using body axes for frame of reference (instead of wind axes). Note the pro's and con's of using either.
10. Find previous reports to see Table of Contents.

Key Notes

1. Use the simplified models proposed for the first design cycle.
2. If time permits, have two or three cycles.

Announcements

1. The next meeting will take place at Seminar Room, EIE Reception, 7 October 2019.
2. Prof. van Wyk will not be attending this meeting, and may Skype into the meeting.

Date: 7 Oct. 2019
Start Time: 8 AM
End Time: 9:30 AM
Venue: EIE Seminar Room, Chamber of Mines Building
Chair: Daniel de Barros
Secretary: Tyson Cross
Approval of Minutes: Malebo Maboko
Changes to Attending:

- Prof van Wyk (Absent)
- Darrion -- (Absent)
- Lloyd -- (Absent)
- Malebo Maboko (Absent)

Proceedings

1. Airframe: propose use of cited reference for SS matrix values (after derivation of EOM and simplifications/assumptions)
   a. ARF60 with known coefficients cited from MA thesis
2. Regarding report: in terms of contents (derivation of equations of motion)
   a. Shall the entire derivation be included in main report or in appendix?
      i. Summary in main report, with full derivation in appendix
      ii. Main salient points explicitly included in main report
3. Non-technical report: does it need to specifically be written as a marketing release?
4. Ensure all ELOs are in main report
5. Actuators -> external block, or included into the plant (i.e. in SS matrix)
6. Electric Motor (cited reference uses combustion engine and fuel, i.e. not fixed mass) For simpler design, we propose using electric motor and batteries (i.e. fixed mass)
7. Propellor modelling: non-linear, research mentions that it is complex to represent.
8. Gift will put past papers onto Google drive for reference
9. Sidwell proposal of thrust modelling as 2nd order function, with voltage -> propulsion
10. Sidwell will publish the proposed model for propulsion as a 1st order system

Key Notes

1. Get approval from Prof van Wyk for using cited values for ARF60 airframe model from MA thesis (on Google Drive)
2. Past year papers onto Google Drive as reference
3. Sidwell to publish 1st order propulsion model

Announcements

1. The next meeting will take place at Seminar Room, EIE Reception, 14 October 2019.
2. Prof. van Wyk will not be attending this meeting, and may Skype into the meeting.

Date: 14 Oct. 2019
Start Time: 8 AM
End Time: 9:30 AM
Venue: EIE Seminar Room, Chamber of Mines Building
Chair: Daniel de Barros
Secretary: Tyson Cross
Approval of Minutes:
Changes to Attending:
Prof van Wyk (Absent)
Malebo Maboko (Absent)

Proceedings

1. Discussion of last weeks work:
2. Finding coefficients for lateral state space model: missing PSI (yaw) values for the ARF60
3. Suggestion to use the ICENS model: both lateral and longitudinal coefficients provided. Also meets Class 1B UAV SA legal requirements.
4. Focus on Longitudinal model emphasised
5. Actuator time constant: 0.1s realistic from reference (Cook)
6. Sean tried Pole Placement: pitch-angle to elevator (dominant pole theory).
    i. Results in new TF, how to justify this new model?
    ii. Daniel suggests a comparison to show the small change
7. Propellor thrust modelling - TBD
8. Non-linear model to test the controller on after development:
    a. We do not have the data to construct a non-linear model
9. Social/environmental aspect: use of UAVs in disaster management
    a. Scientific mapping
10. Ethical aspect: improving safety: not involved in the actual design of the airframe (using existing model). Usage of the hobby-level craft
11. Success criteria: appeal to authority (citation) or proposed/evaluated as an engineer to be realistic.

Key Notes

- Questions to ask Prof van Wyk:
    - How to get a non-linear model?
    - Should be be focusing on this non-linear model? (Time aspect with remaining period before deadline for report)

Announcements

1. The last meeting will take place at Seminar Room, EIE Reception, 21 October 2019.

Date: 21 Oct. 2019
Start Time: 8 AM
End Time: 9:30 AM
Venue: EIE Seminar Room, Chamber of Mines Building
Chair: Daniel de Barros
Secretary: Tyson Cross

Proceedings

1. 6DOF freedom model: decoupled into two 3DOF model, linearized around small perturbation model in trimmed flight conditions
2. Need to reapply the control system back to the non-linear model, to compare performance and evaluate the success of the design
    a. How to get the required coefficients for this non-linear model?
3. If unable to implement the Nonlinear system block (given limited access to full parameters of flight conditions and aircraft measurements):
    a. Report should detail explanations of time estimates that would be required for developing/implementing Non-linear system that can be swapped out
4. Process completed so far:
    a. mathematical development of trimmed stable flight for linearization
    b. choice of aircraft suggested by legislation
        i. UltraStick 25e (or ARF60)
    c. citation from MA or equivalent journal report/paper for actual numerical State Space matrix values for A, B
    d. development of control system(s) for decoupled lateral/longitud. systems
5. Engineering techniques: not just the use of papers, formulas, coefficients.
    a. Must involve the investigation of the meaning of the values, and the iteration through design methods
6. PID controllers:
    a. showing insight into the behaviour of the chosen values (gain, derivative, integral values) could be a good example of engineering.
7. Report should discuss what was possible, what was not possible (why, justify choices, explain context and prioritisation)
    a. Time/project management analysis useful
    b. Time breakdown per phase of the project (appendix?)
    c. Projected estimations to show required time to complete the unfinished components
8. Emphasise value of the process, achievements. Critical analysis of the problems and limitations, assumptions must be discussed.
9. Submission links: Must check with the Course Coordinator
10. MATLAB / code submission
    a. GitHub repo
    b. Flash drive submission
    c. Codebase in appendix (if not too long)
11. ELO appendix: a mapping to indicate the location and applicability of the individual ELO marking rubric within the report.
12. Non-technical report: marketing material/interview/press release. A broad explanation of the project, addressed to non-technical readers.

Announcements

Report Due at 12:00 on Friday 25th October

Thanks to Prof. van Wyk for all his help and advice

G Engineering Notebook

Nomenclature

Of the very large number of symbols required by the subject, many have more than one meaning. Usually the meaning is clear from the context in which the symbol is used.

a	Wing or wing-body lift curve slope: Acceleration: Local speed of sound
a'	Inertial or absolute acceleration
a_0	Speed of sound at sea level: Tailplane zero-incidence lift coefficient
a_1	Tailplane lift curve slope
a_{1_f}	Canard foreplane lift curve slope
a_{1_F}	Fin lift curve slope
a_2	Elevator lift curve slope
a_{2_A}	Aileron lift curve slope
a_{2_R}	Rudder lift curve slope
a_3	Elevator tab lift curve slope
a_∞	Lift curve slope of infinite-span wing
a_h	Local lift curve slope at coordinate h
a_y	Local lift curve slope at spanwise coordinate y
$a_{z_{cg}}$	Normal acceleration at the cg
a_{z_p}	Normal acceleration at the pilot
ac	Aerodynamic centre
A	Aspect ratio
A_F	Effective aspect ratio of fin
A_T	Effective aspect ratio of tailplane
$\mathbf{A}$	State matrix
b	Wing span
b_1	Elevator hinge moment derivative with respect to α_T
b_2	Elevator hinge moment derivative with respect to η
b_3	Elevator hinge moment derivative with respect to β_η
b_T	Tailplane span
$\mathbf{B}$	Input matrix
c	Chord: Viscous damping coefficient: Command input
$\bar{c}$	Standard mean chord (*smc*)
$\bar{\bar{c}}$	Mean aerodynamic chord (*mac*)
$\bar{\bar{c}}_\eta$	Mean elevator chord aft of hinge line
c_h	Local chord at coordinate h
c_r	Root chord

• Control system to control an unmanned fixed-wing aircraft •

model: airframe / actuators / propulsion

design: controller

simulate: closed-loop controlled system (in Matlab)

flight regulations → sensible specifications

environmental factors → health risks
green design
pollution

flight stability

energy efficiency

*identify simplifications.

Elen 4011A

Information Engineering Design II.

Dr. Nicholas West

Ms. Yu-Chieh (Jessie) Yen

Tyson Cross

1239448

Prof. Van Wyk

Tues 17th initial meeting
Wed 18th meet with Prof. van Wyk
specifications & assumptions [prestudy → scope]
Mon 23rd Platform: airframe, actuators, propulsion
initial model
Mon 30th refine model [regulations, environment, safety]

Mon 7th Controller & Sim
Mon 14th Start Report *Van Wyk abs
Mon 21st Finish Report *Van Wyk abs
↓
Due Fri 25th Report due

spec by 21st
model by 25th
outline of Report: 30th ToC

Van Wyk → leaves 4th Fri
return 16th (am) Wed

- get list of names & student no.s
- nominate facilitator
- schedule meeting chair/secretary
- secretary : Darrion
- facilitator : Daniel
- assistants : Tyson & Sidwell

UAV: RPA

Aim Class IB for risk / cost / compliance / simplification

Energy @ impact	< 15kJ
Height (max)	121.92m (400ft)
Weight (max)	< 7kg

Civil Aviation Act 2009
act 13 of 2009
SA-CATS 101

SA-CAA : SA civil aviation authority
ICAO : International Aviation org.

Airframe
- start with 2D model

actuators
reduce to linear component

propulsion
- gain or 1st order
τ_c from throttle → thrust

controller

simulation
(closed-loop)
Matlab.

regulations

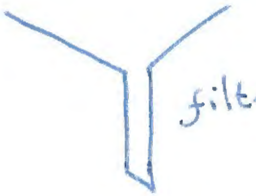
environmental

safety
- failure plan
- cut engine, deploy parachute

Report : 15 pages, size 11
excluding. cover, ToC, appendices (post A)
Non-technical report : 2 pages [appendix A]
check ELOs *particulary 7a, 7b

- cross coupling between systems

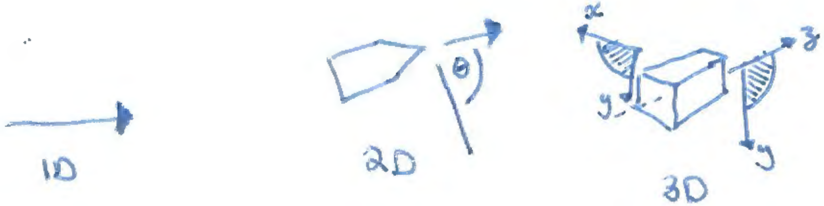
Big Problem, research, scale, options



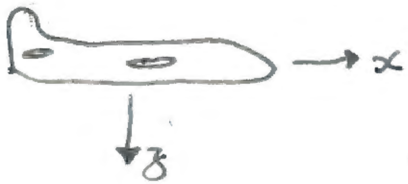
scoped down, assumptions, simplest

$\Downarrow$ build up

reduce assumptions, complexify model



Flat earth:



flat, non-rotating

Report: mathematical model derivation unneccesary?

$\dot{X} = f(X, u)$

control vector

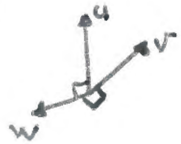
state variables

non-linear functions

$\dot{x} = Ax + Bu$ { perturbations from linear equilibrium

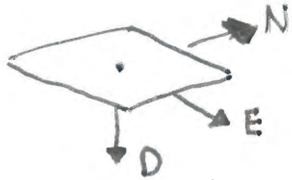
$\underline{u} \cdot \underline{v} = |\underline{u}||\underline{v}| \cos\theta$

$|\underline{u} \times \underline{v}| = |\underline{u}||\underline{v}| \sin\theta$



Euler angles: x, y, z (or z, y, x)

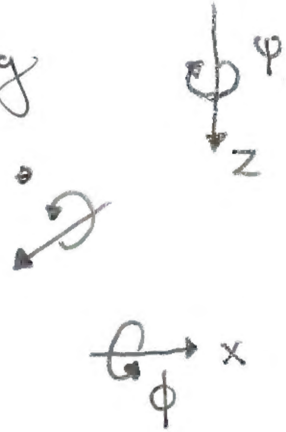
NED:



ψ compass heading (yaw)

θ pitch

ϕ roll



Z Y X → 3, 2, 1

yaw, pitch, roll

$$-\pi < \psi < \pi$$
$$-\frac{\pi}{2} \leq \theta \leq \pi/2$$
$$-\pi < \phi < \pi$$

} ranges for rotation angles

$$D = \begin{bmatrix} c\theta s\psi & c\theta s\psi & -s\theta \\ -c\phi s\psi + s\phi s\theta c\psi & c\phi c\psi + s\phi s\theta s\psi & s\phi c\theta \\ s\phi s\psi + c\phi s\theta c\psi & -s\phi c\psi + c\phi s\psi & c\phi c\theta \end{bmatrix}$$

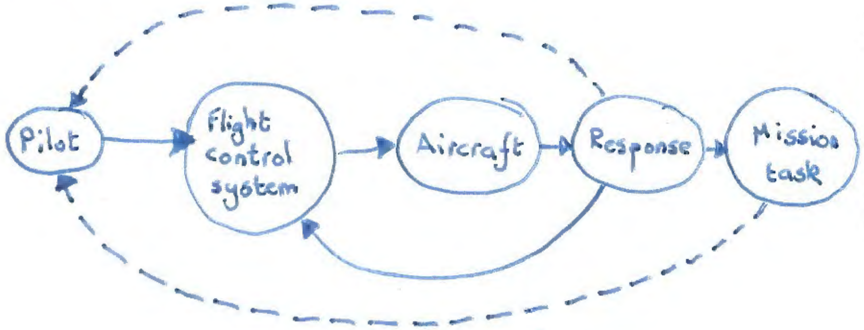
↳ direction cosine matrix

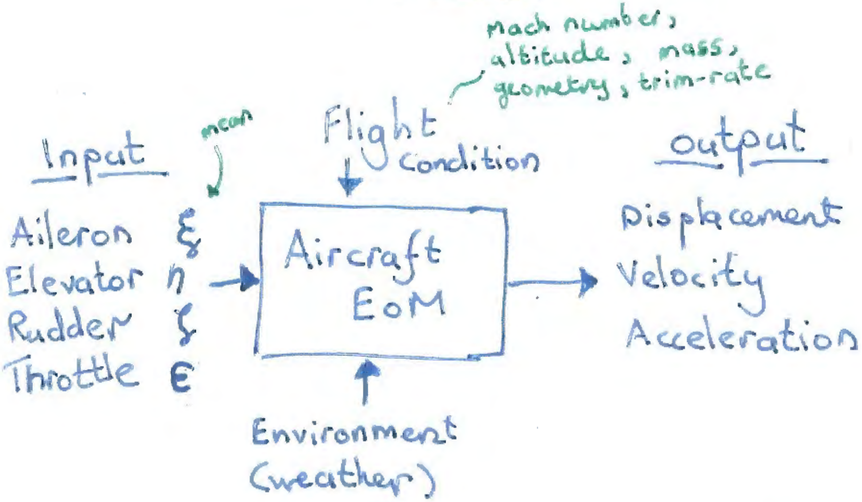
c : cos
s : sin

{ forward / NED }

transforms from wind → body

Fly-by-wire flying & handling:





control displacement
control force } static characteristics

response to controls - dynamic characteristic

ξ	xi	ζ	zeta	ψ	psi	ρ	rho
η	eta	ϵ	epsilon	ϕ	phi	ω	omega

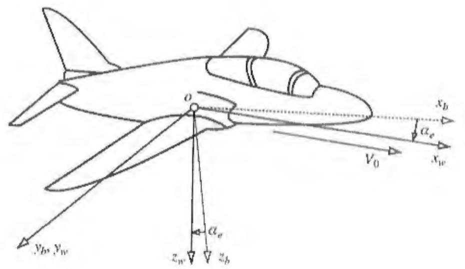


FIGURE 2.2 Moving-axis systems.

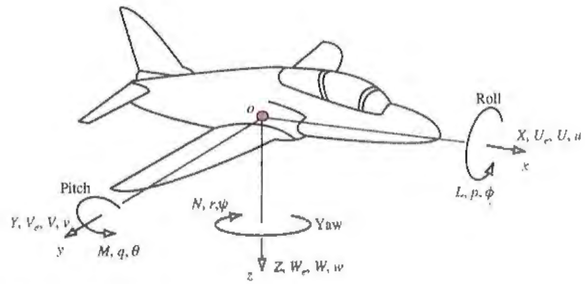


FIGURE 2.3 Motion variables notation.

Table 2.2 Perturbation Variables

X	Axial "drag" force	Sum of the components of aerodynamic, thrust and weight forces
Y	Side force	
Z	Normal "lift" force	
L	Rolling moment	Sum of the components of aerodynamic, thrust and weight moments
M	Pitching moment	
N	Yawing moment	
p	Roll rate	Components of angular velocity
q	Pitch rate	
r	Yaw rate	
U	Axial velocity	Total linear velocity components of the cg
V	Lateral velocity	
W	Normal velocity	

Table 2.1 Summary of Motion Variables

	Trimmed Equilibrium			Perturbed		
Aircraft axis	ox	oy	oz	ox	oy	oz
Force	0	0	0	X	Y	Z
Moment	0	0	0	L	M	N
Linear velocity	U_e	V_e	W_e	U	V	W
Angular velocity	0	0	0	p	q	r
Attitude	0	θ_e	0	ϕ	θ	ψ

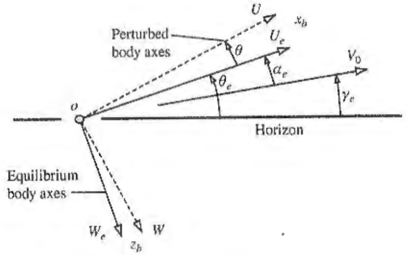
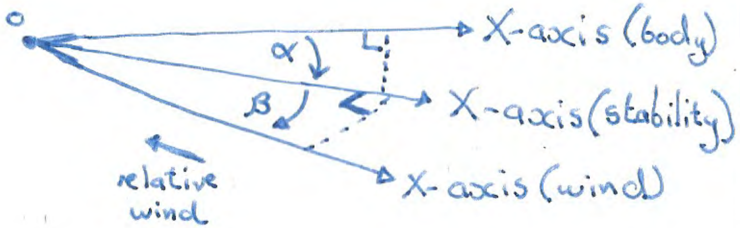


FIGURE 2.4 Generalised body axes in symmetric flight.



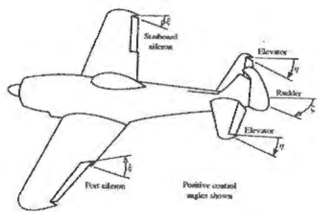


FIGURE 2.11 Aerodynamic controls notation.

trim stability

Subsonic speeds: $< \text{mach } 1.0$

linear (almost)

$C_L \underset{\sim}{\propto} \alpha$

$\frac{dC_M}{d\alpha} < 0$ or $\frac{dC_m}{dC_L} < 0$

$C_m(\alpha = 0) > 0$

$C_m(C_L = 0) > 0$

pitching moment C_M

lift coefficient C_L

angle of attack α

Too stable: difficult to change/control

Unstable: too responsive to change

High τ:

nose down C_m

Low τ:

nose up C_m

Assume Rigid structure (no elasticity)

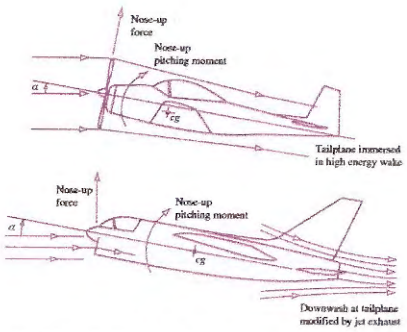


FIGURE 3.5 Typical induced-flow effects on pitching moment.

Max height 400ft = 120m

chosen height 100m

air density 1.2 kg/m^3 [JHB 0.98 kg/m^3]

airspeed 20 m/s

$\phi = 0$

ARF60 → Ma...

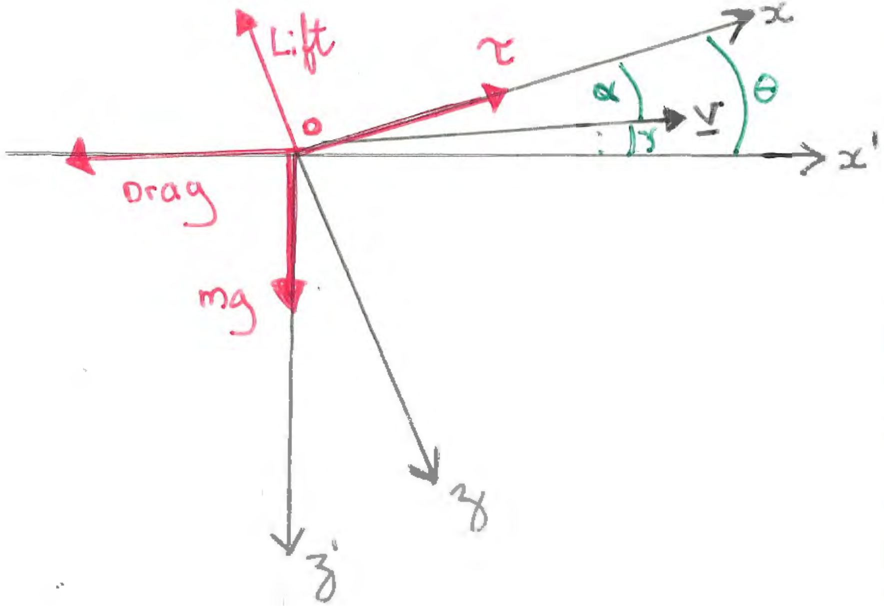
Airframe

{Daniel calculate co-efficients}

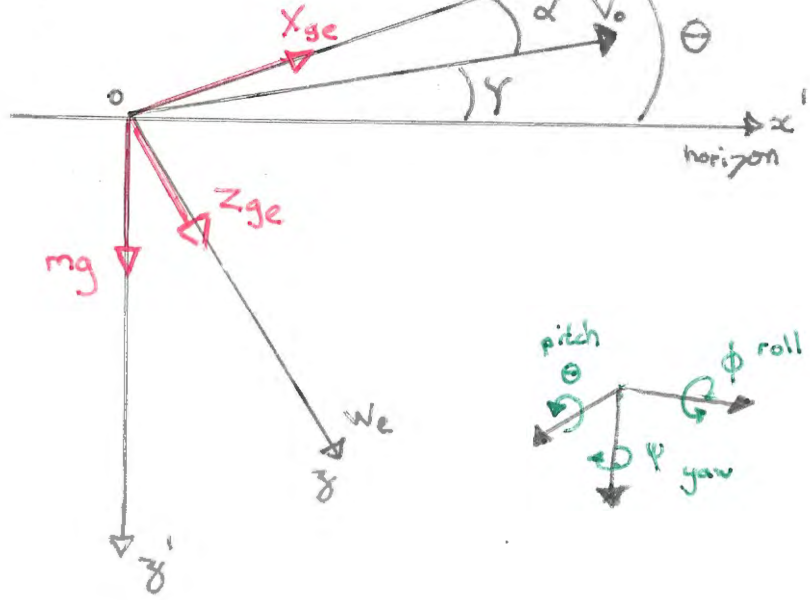
electical.

TF → SS model.

ARF60 → vs. Ultra-flight (smaller, lighter, easier to classify as class 1B)



Weight / gravity :
(symmetric, level flight)



$$X_{ge} = -mg\sin\Theta_e$$
$$Y_{ge} = 0$$
$$Z_{ge} = mg\cos\Theta_e$$

$$\begin{bmatrix} X_g \\ Y_g \\ Z_g \end{bmatrix} = \begin{bmatrix} 1 & \psi & -\theta \\ -\psi & 1 & \phi \\ \theta & -\phi & 1 \end{bmatrix} \begin{bmatrix} X_{ge} \\ Y_{ge} \\ Z_{ge} \end{bmatrix}$$

$$X_g = -mg\sin\Theta_e - mg\theta\cos\Theta_e$$
$$Y_g = mg\psi\sin\Theta_e + mg\phi\cos\Theta_e$$
$$Z_g = mg\cos\Theta_e - mg\theta\sin\Theta_e$$

Forces:

$$m(\dot{u} - rV + gW) = X_a + X_g + X_c + X_p + X_d$$

yaw rate, gravity, aerodynamic, gravitational, control, power, external disturbance

roll rate

$$m(\dot{v} - pW + rU) = Y_a + Y_g + Y_c + Y_p + Y_d$$

pitch rate

$$m(\dot{w} - qU + pV) = Z_a + Z_g + Z_c + Z_p + Z_d$$

Moments:

rolling moment

$$I_x\dot{p} - (I_y - I_z)qr - I_{xz}(pq + \dot{r}) = L_a + L_g + L_c + L_p + L_d$$

inertial moment

pitching moment

$$I_y\dot{q} + (I_x - I_z)pr + I_{xz}(p^2 - r^2) = M_a + M_g + M_c + M_p + M_d$$

yawing moment

$$I_z\dot{r} - (I_x - I_y)pq + I_{xz}(qr - \dot{p}) = N_a + N_g + N_c + N_p + N_d$$

Assume steady, trimmed, symmetric, rectilinear flight with no roll, sideslip or yaw angles.

stable, undisturbed atmosphere.

$$X_d = Y_d = Z_d = L_d = M_d = N_d = 0$$

u, v, w : small peturbations in $oxyz$

p, q, r : small angular peturbation velocities.

so $U = U_e + u$

$V = V_e + v = v \quad (V_e = 0)$

$W = W_e + w$

ignoring small values that are squared or multiplied:

aerodynamic

$$m(\dot{u} + qW_e) = X_a + X_g + X_c + X_p$$
$$m(\dot{v} + rU_e) = Y_a + Y_g + Y_c + Y_p$$
$$m(\dot{w} - qU_e) = Z_a + Z_g + Z_c + Z_p$$

$$I_x\dot{p} - I_{xz}\dot{r} = L_a + L_g + L_c + L_p$$
$$I_y\dot{q} = M_a + M_g + M_c + M_p$$
$$I_z\dot{r} + I_{xz}\dot{p} = N_a + N_g + N_c + N_p$$

Aerodynamic

$$X_a = \underbrace{X_{a_e}}_{\text{const.}} + \left(\frac{\partial X}{\partial u}u + \frac{\partial^2 X}{\partial u^2}\frac{u^2}{2!} + \dots\right) + \left(\frac{\partial X}{\partial v}v + \frac{\partial^2 X}{\partial v^2}\frac{v^2}{2!} + \dots\right)$$

... [terms in $w, p, q, r, \dot{u}, \dot{v}, \dot{w}, \dot{p}, \dot{q}, \dot{r}$

+ higher order derivative terms

$$\approx X_{a_e} + \frac{\partial X}{\partial u}u + \frac{\partial X}{\partial v}v + \frac{\partial X}{\partial w}w + \frac{\partial X}{\partial p}p + \frac{\partial X}{\partial q}q + \frac{\partial X}{\partial r}r + \frac{\partial X}{\partial \dot{w}}\dot{w}$$

$\mathring{X}$ ← dimensional

or:

aerodynamic stability derivatives

$$X_a = X_{a_e} + \mathring{X}_u u + \mathring{X}_v v + \mathring{X}_w w + \mathring{X}_p p + \mathring{X}_q q + \mathring{X}_r r + \mathring{X}_{\dot{w}} \dot{w}$$

etc. for Y, Z, L, M, N

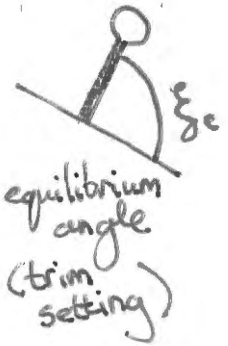
Control terms:

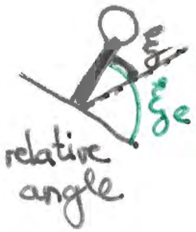
elevator angle

$$M_c = \frac{\partial M}{\partial \xi}\xi + \frac{\partial M}{\partial \eta}\eta + \frac{\partial M}{\partial \zeta}\zeta$$

aerilon angle

rudder angle





$$\Rightarrow M_c = \mathring{M}_\xi \xi + \mathring{M}_\eta \eta + \mathring{M}_\zeta \zeta$$

Power terms:

τ: thrust peturbation from τ_e

ε: throttle lever angle from ε_e

$$\frac{\tau(s)}{\varepsilon(s)} = \frac{k_\tau}{(1+sT_\tau)}$$

(1st order TF of jet engine)

e.g. $Z_p = \mathring{Z}_\tau \tau$ (normal force due to thrust)

Substitute into e.o.m:

$$m(\dot{u}+qW_e) = \boxed{X_{a_e}} + \mathring{X}_u u + \mathring{X}_v v + \mathring{X}_w w + \mathring{X}_p p + \mathring{X}_q q + \mathring{X}_r r + \mathring{X}_{\dot{w}} \dot{w} \boxed{- mg\sin\Theta_e} - mg\cos\Theta_e$$
$$+ \mathring{X}_\xi \xi + \mathring{X}_\eta \eta + \mathring{X}_\zeta \zeta + \mathring{X}_\tau \tau$$

$$m(\dot{v} - pW_e + rU_e) = \cancel{Y_{a_e}} + \mathring{Y}_u u + \ldots + \mathring{Y}_{\dot{w}} \dot{w} + mg\psi\sin\Theta_e + mg\phi\cos\Theta_e + \mathring{Y}_\xi \xi + \ldots + \mathring{Y}_\zeta \zeta + \mathring{Y}_\tau \tau$$

$$m(\dot{w} - qU_e) = \boxed{Z_{a_e}} + \mathring{Z}_u u + \ldots + \mathring{Z}_{\dot{w}} \dot{w} + \boxed{mg\cos\Theta_e} - mg\theta\sin\Theta_e + \mathring{Z}_\xi \xi + \ldots + \mathring{Z}_\zeta \zeta + \mathring{Z}_\tau \tau$$

$$I_x\dot{p} - I_{xz}\dot{r} = \cancel{L_{a_e}} + \mathring{L}_u u + \ldots + \mathring{L}_{\dot{w}} \dot{w} + \mathring{L}_\xi \xi + \ldots + \mathring{L}_\zeta \zeta + \mathring{L}_\tau \tau$$

$$I_y\dot{q} = \cancel{M_{a_e}} + \mathring{M}_u u + \ldots + \mathring{M}_{\dot{w}} \dot{w} + \mathring{M}_\xi \xi + \ldots + \mathring{M}_\zeta \zeta + \mathring{M}_\tau \tau$$

$$I_z\dot{r} - I_{xz}\dot{p} = \cancel{N_{a_e}} + \mathring{N}_u u + \ldots \mathring{N}_{\dot{w}} \dot{w} + \mathring{N}_\xi \xi + \ldots + \mathring{N}_\zeta \zeta + \mathring{N}_\tau \tau$$

no gravity terms.

but in trimmed flight: $X_{a_e} = mg\sin\Theta_e$ $\quad L_{a_e} = 0$

$Y_{a_e} = 0$ $\quad M_{a_e} = 0$

$Z_{a_e} = -mg\cos\Theta_e$ $\quad N_{a_e} = 0$

To decouple Lateral & Longitudinal:

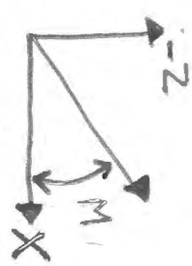
assume disturbances limited to oxz plane $\Rightarrow X, Z, M$ only

$$v, p, r, \dot{v}, \dot{p}, \dot{r} = 0$$

also, $\mathring{X}_v = \mathring{X}_p = \mathring{X}_r = \mathring{Z}_v = \mathring{Z}_p = \mathring{Z}_r = \mathring{M}_v = \mathring{M}_p = \mathring{M}_r = 0$ (coupled forces & moments negligibly small)

and $\mathring{X}_\xi = \mathring{X}_\zeta = \mathring{Z}_\xi = \mathring{Z}_\zeta = \mathring{M}_\xi = \mathring{M}_\zeta = 0$

(aileron & rudder perturbations don't affect longitudinal plane motion)



$$m\dot{u} - \mathring{X}_u u - \mathring{X}_{\dot{w}} \dot{w} - \mathring{X}_w w - (\mathring{X}_q - mW_e)q + mg\theta\cos\Theta_e = \mathring{X}_\eta \eta + \mathring{X}_\tau \tau$$

$$-\mathring{Z}_u u + (m - \mathring{Z}_{\dot{w}})\dot{w} - \mathring{Z}_w w - (\mathring{Z}_q + mU_e)q + mg\theta\sin\Theta_e = \mathring{Z}_\eta \eta + \mathring{Z}_\tau \tau$$

$$-\mathring{M}_u u - \mathring{M}_{\dot{w}} \dot{w} - \mathring{M}_w w + I_y \dot{q} - \mathring{M}_q q = \mathring{M}_\eta \eta + \mathring{M}_\tau \tau$$

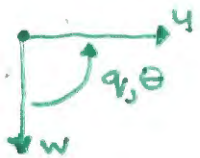
$$m\dot{u} - \mathring{X}_{\dot{w}}\dot{w} = \mathring{X}_u u + \mathring{X}_w w + (\mathring{X}_q - mW_e)q - mg\theta\cos\theta_e + \mathring{X}_\eta \eta + \mathring{X}_\tau \tau$$

$$m\dot{w} - \mathring{Z}_{\dot{w}}\dot{w} = \mathring{Z}_u u + \mathring{Z}_w w + (\mathring{Z}_q + mU_e)q - mg\theta\sin\theta_e + \mathring{Z}_\eta \eta + \mathring{Z}_\tau \tau$$

$$I_y\dot{q} - \mathring{M}_{\dot{w}}\dot{w} = \mathring{M}_u u + \mathring{M}_w w + \mathring{M}_q q + \mathring{M}_\eta \eta + \mathring{M}_\tau \tau$$

state variables: control:

$$\mathbb{X}(t) = \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} \qquad \mathbb{u}(t) = \begin{bmatrix} \eta \\ \tau \end{bmatrix}$$



4th equation: $\dot{\theta} = q$ for small peturbations

$$\mathbb{M}\dot{\mathbb{x}}(t) = \mathbb{A}'\mathbb{x}(t) + \mathbb{B}'\mathbb{u}(t)$$

$$\mathbb{A} = \mathbb{M}^{-1}\mathbb{A}' = \begin{bmatrix} X_u & X_w & X_q & X_\theta \\ Z_u & Z_w & Z_q & Z_\theta \\ m_u & m_w & m_q & m_\theta \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

aerodynamic stability derivatives

$$\mathbb{B} = \mathbb{M}^{-1}\mathbb{B}' = \begin{bmatrix} X_\eta & X_\tau \\ Z_\eta & Z_\tau \\ m_\eta & m_\tau \\ 0 & 0 \end{bmatrix}$$

control derivatives

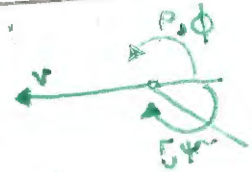
$$\mathbb{Y}(t) = \mathbb{I}\mathbb{x}(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix}$$

---

roll, yaw, sideslip

Lateral:

$$\mathbb{X}_{lat}(t) = \begin{bmatrix} v \\ p \\ r \\ \phi \\ \psi \end{bmatrix} \qquad \mathbb{u}(t) = \begin{bmatrix} \xi \\ \zeta \end{bmatrix}$$



4th equation: $\dot{\phi} = p$ for small perturbations

5th equation: $\dot{\psi} = r$ for " "

$$\mathbb{A} = \begin{bmatrix} Y_v & Y_p & Y_r & Y_\phi & Y_\psi \\ L_v & L_p & L_r & L_\phi & L_\psi \\ N_v & N_p & N_r & N_\phi & N_\psi \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Δ values?

$$\mathbb{B} = \begin{bmatrix} Y_\xi & Y_\zeta \\ L_\xi & L_\zeta \\ N_\xi & N_\zeta \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

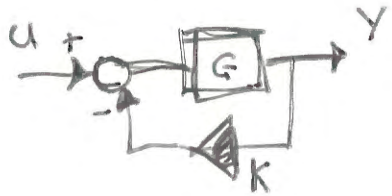
η: elevator
ε: throttle
ξ: aerilorons
ζ: rudder

u: peturbation in x
w: peturbation in z
q: pitching rate
θ: pitch angle
X: axial force
Z: normal force
M: pitching moment

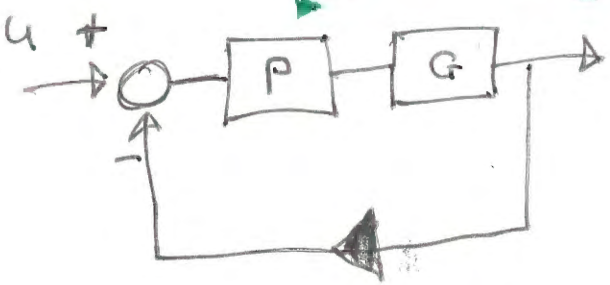
v: peturbation in y
p: roll rate
r: yaw rate
ϕ: roll angle
ψ: yaw angle
Y: sideslip force
L: rolling moment
N: yawing moment

Control: η elevator
output : θ pitch angle

Assume ~~thrust unaffected by~~ pitch?
(for simplicity) ~~i.e. τ = const?~~



PID: affordable & easy to tune



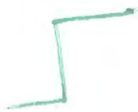
But not optimal.

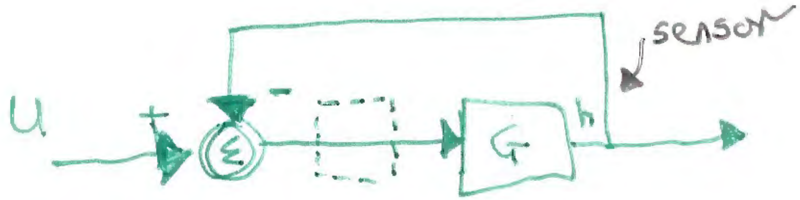
2 loops in previous project: will three loops work.

q → pitch rate responsiveness

θ → pitch angle smooth & dampen

h → regulate by external measurement?


altitude change.



State observer?
Non-linear system?