Quantum Information Theory

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HT 1: Hamiltonian Equations

(Note: Results were supported by the mathematical software Python.)

Consider the Hamiltonian functions

$$\mathcal{H}_1(q,p) = p^2 + q^2 \quad \text{and} \quad \mathcal{H}_2(q,p) = p^2 + q^4$$

yielding separately the Hamiltonian equations

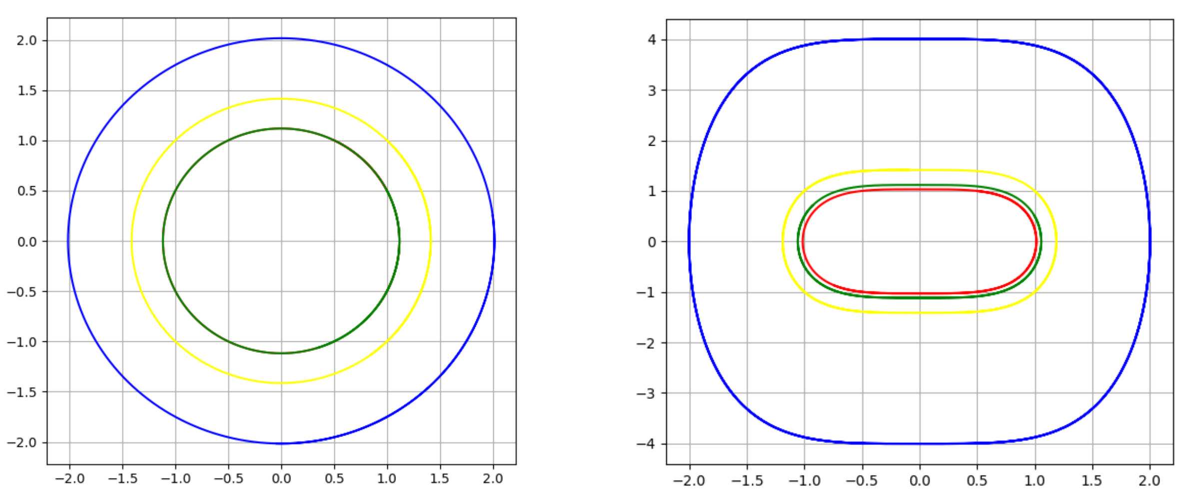
$$\begin{cases} \dot{q} = \frac{\partial \mathcal{H}_1}{\partial p} & = 2p \\ \dot{p} = -\frac{\partial \mathcal{H}_1}{\partial q} & = -2q \end{cases} \quad (1); \qquad \begin{cases} \dot{q} = \frac{\partial \mathcal{H}_2}{\partial p} & = 2p \\ \dot{p} = -\frac{\partial \mathcal{H}_2}{\partial q} & = -4q^3 \end{cases} \quad (2).$$

We observe that (1) is a linear system of ordinary differential equations and its general solution can be obtained analytically as

$$\begin{cases} q(t) = C_1 \sin(2t) + C_2 \cos(2t) \\ p(t) = C_1 \cos(2t) - C_2 \sin(2t) \end{cases}$$

(2) on the other hand is non linear so we compute its solution numerically. We graph both solutions with the following initial conditions:

- $(q(0), p(0)) = (1, 1) \Rightarrow (C_1, C_2) = (1, 1)$. See yellow curves.
- $(q(0), p(0)) = (0.5, 1) \Rightarrow (C_1, C_2) = (0.5, 1)$. See red curves.
- $(q(0), p(0)) = (1, 0.5) \Rightarrow (C_1, C_2) = (1, 0.5)$. See green curves.
- $(q(0), p(0)) = (2, 0.25) \Rightarrow (C_1, C_2) = (2, 0.25)$. See blue curves.



We note that for system (1), the initial values $(0.5, 1)$ and $(1, 0.5)$ yield solutions that cover the same trajectory.