Quantum Information Theory

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HT 3: Schrödinger's inequality

Consider the 2-dimensional complex Hilbert space along with the following state and Hermitian operators:

$$\psi = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}, \qquad \hat{A} = \begin{pmatrix} 1 & i \\ -i & 2 \end{pmatrix}, \qquad \hat{B} = \begin{pmatrix} 1 & 1-i \\ 1+i & 3 \end{pmatrix}$$

where

$$[\hat{A}, \hat{B}] = \begin{pmatrix} 2i & -1+3i \\ 1+3i & -2i \end{pmatrix}, \qquad \{\hat{A}, \hat{B}\} = \begin{pmatrix} 0 & 3+i \\ 3-i & 10 \end{pmatrix}$$

We find the averages

$$\langle \hat{A} \rangle_\psi = \frac{3}{2}, \qquad \langle \hat{B} \rangle_\psi = 3, \qquad \langle [\hat{A}, \hat{B}] \rangle_\psi = -3i, \qquad \langle \{\hat{A}, \hat{B}\} \rangle_\psi = 8$$

(Note that the commutator $[\hat{A}, \hat{B}]$ is anti-hermitian so Born's postulate shouldn't apply to compute its average)

and dispersions

$$\sigma^2_{\hat{A}}(\psi) = \frac{5}{4}, \qquad \sigma^2_{\hat{B}}(\psi) = 2$$

We can now check Schrödinger's inequality

$$\sigma^2_{\hat{A}} \sigma^2_{\hat{B}} = 2.5 > 2.3125 = \left| \frac{1}{2} \langle \{\hat{A}, \hat{B}\} \rangle - \langle \hat{A} \rangle \langle \hat{B} \rangle \right|^2 + \left| \frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle \right|^2$$