Quantum Information Theory

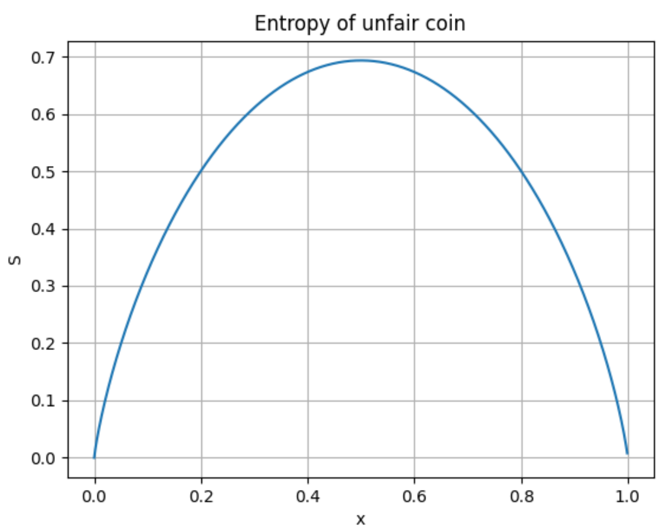
Adrian Perez Keilty

HT 6: Classical and quantum entropy

Problem 1

Given an unfair coin with $p_1 = x$ and $p_2 = 1 - x$ for $x \in [0, 1]$ we graph the classical entropy defined by Shannon:

$$\mathcal{S}(p) = -p_1 \ln p_1 - p_2 \ln p_2$$



Problem 2

Consider the state represented by the density operator

$$\rho = \begin{pmatrix} \frac{1}{2} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{2} \end{pmatrix}$$

with eigenvalues $p_1 = 0.625$ and $p_2 = 0.375$. If we take as basis the orthonormal eigenvectors e_1 and e_2 of ρ then the Von Neumann entropy of this state is given by

$$\mathcal{S}(\rho) = -\mathrm{Tr}(\rho)\ln(\rho) = -p_1 \ln p_1 - p_2 \ln p_2 \approx 0.662$$

Problem 3

Now, let $\alpha, \beta \in \mathbb{C}$ such that $|\alpha|^2 + |\beta|^2 = 1$ and consider the state represented by the density operator

$$\rho = \begin{pmatrix} |\alpha|^2 & \alpha\overline{\beta} \\ \overline{\alpha}\beta & |\beta|^2 \end{pmatrix}$$

From the identity $z \cdot \overline{z} = |z|^2$ it is easy to verify that $\rho^2 = \rho$, so in particular, ρ is a pure quantum state. By the first property of the Von Neumann entropy we have that $\mathcal{S}(\rho)$=0.