

Applied Statistical Analysis

EDUC 6050

Week 4

Finding clarity using data

Today

1. Z-scores (for individuals and samples)
2. Intro to Hypothesis Testing
3. Estimation & Confidence Intervals

Hypothesis Testing

Null Hypothesis

No effect

Alternative Hypothesis

Effect exists

Essentially, analyze data and see if null hypothesis seems plausible

- If not plausible, we believe the alternative
- If plausible, we assume there is no effect

Hypothesis Testing

“P-Values”

- The probability of observing an effect that large or larger, given the null hypothesis is true.
- It is trying to tell us if an effect exists in the population

“less than”

Usually a p-value $<$.05 is considered
“statistically significant”

Hypothesis Testing

P-Values

- Researchers rely on them too much (Cumming, 2014)
- **Effect sizes** should be used with them
 - We need to highlight that effect sizes are **uncertain**
 - A “significant” finding may not be meaningful or reproducible

Z-Scores

Important Point:

- There are **distributions of single scores**
- There are **distributions of statistics**
 - This is generally in reference to the sample mean

Chapter 4 is about single scores

Z-Scores for an Individual Point

$$Z = \frac{X - \mu}{\sigma}$$

Tells us:

- If the score is above or below the mean
- How large (the magnitude) the deviation from the mean is to other data points

Z-Score Examples

$$z = \frac{X - \mu}{\sigma}$$

1. $M = 20$, Score = 10, SD = 10, $z = ?$
2. $M = 5$, Score = 5, SD = 1, $z = ?$
3. $M = 5$, Score = 6, SD = 1, $z = ?$
4. $Z = 1$, Mean = 1, SD = 1, $M = ?$
5. $Z = -1$, Mean = 0, SD = 0.5, $M = ?$

Z-Score Interpretations

- If the score is $+$ then above the mean
- If the score is $-$ then below the mean
- If score is more than ± 1 then score is considered “atypical”
- If score is less than ± 1 then score is considered “typical”

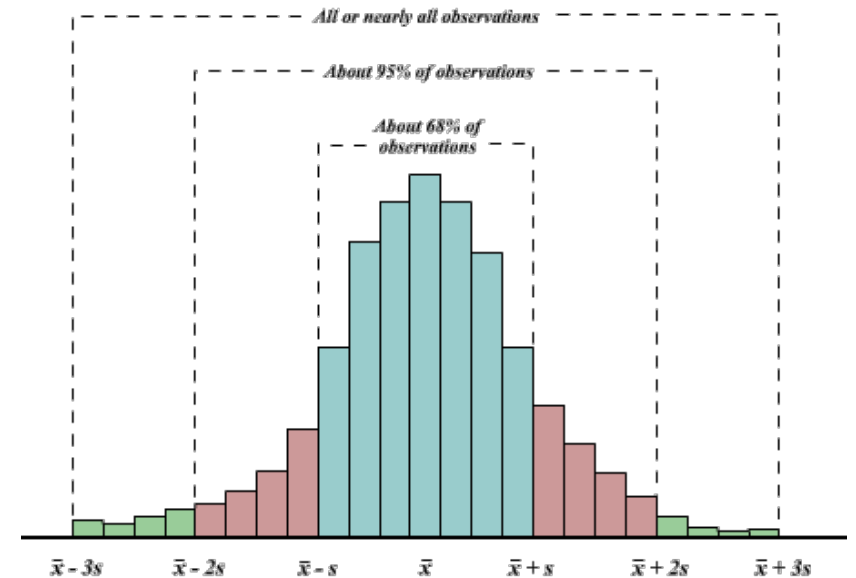
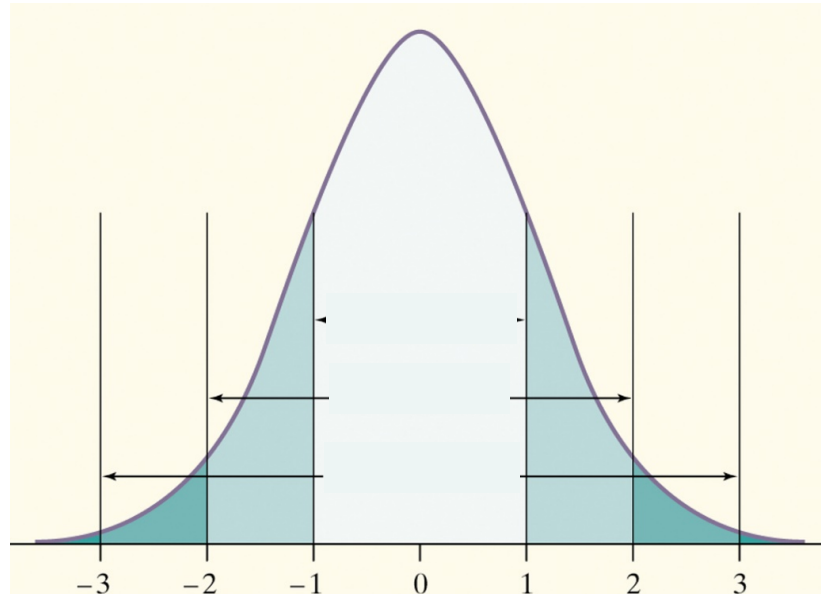
The z tells us more information than just a score. Why?

Z-Score and the Standard Normal Curve

The 68-95-99.7 Rule

In the Normal distribution with mean μ and standard deviation σ :

- Approximately **68%** of the observations fall within σ of μ .
- Approximately **95%** of the observations fall within 2σ of μ .
- Approximately **99.7%** of the observations fall within 3σ of μ .



Z-Score and the Standard Normal Curve

So...

- We can use the same idea to estimate the probability of scoring higher or lower than a certain level

Example: If the scores on an exam have a mean of 70, an SD of 10, we know the distribution is normal, what is the probability of scoring 90 or higher.

Distribution of Sample Means

!!! Important Point !!!

- There are **distributions of single scores**
- There are **distributions of statistics**
 - This is generally in reference to the sample mean

Chapter 5 is about distributions of statistics

Distribution of Sample Means

Inferential statistics
is all about using the
sample to infer
population parameters

But the sample is almost
certainly going to differ
from the population (at
least a little)

So what if we took 5 different samples
(or 10, or 50, etc.). Will each sample
have the same mean?

Standard Error of the Mean

“SEM” or “SE”

- Depends on **sample size** (bigger sample, smaller SEM)
- Tells us, *if we were to collect many samples*, how much the sample means would vary

$$SEM = \frac{\sigma}{\sqrt{N}}$$

Since we don't want to take lots of samples...

We use statistical theory! (or “the magic of math”)

- **Central Limit Theorem**
 - Tells us the shape (normal), center (μ) and spread (SEM) of the distribution of sampling means
- **Law of Large Numbers**
 - As N increases, the sample statistic is better and better at estimating the population parameter

The Z for a Sample Mean

$$Z_{Mean} = \frac{Mean - \mu}{SEM}$$

This is important because of what we will talk about in Chapter 6

- Hypothesis Testing with Z Scores

The Z for a Sample Mean

$$Z_{Mean} = \frac{Mean - \mu}{SEM}$$

1. $N = 100$, $Mean = 10$, $\mu = 5$, $\sigma = 5$,
 $Z_{Mean} = ?$
2. $N = 100$, $Mean = 2$, $\mu = 0$, $\sigma = 10$,
 $Z_{Mean} = ?$
3. What is the probability of having a mean greater than 10 for the first example?

Hypothesis Testing with Z Scores

Hypothesis Testing uses Inferential Statistics

- Is there evidence that this sample (maybe because of an intervention) is different than the population?

Hypothesis Testing with Z Scores

We'll use a 6-step approach

We'll use this throughout the class so get familiar with it

1. Examine Variables to Assess Statistical Assumptions
2. State the Null and Research Hypotheses (symbolically and verbally)
3. Define Critical Regions
4. Compute the Test Statistic
5. Compute an Effect Size and Describe it
6. Interpreting the results

Hypothesis Testing with Z Scores

Because assessing z-scores and t-tests are so similar, we will talk about both next week

Read Chapter 7

Questions?

Please post them to the
discussion board before
class starts

End of Pre-Recorded Lecture Slides

In-class discussion slides



Review of Z-Scores (Chapter 4)

1. What does a z-score about an individual point tell us?
2. Is it possible to make a specific probability statement about a z-score if the distribution is normal?
3. What proportion of scores are between z-scores of 0 and 1? (hint: use shading and the appendix)

Review of Sample Mean Distributions (Chapter 5 and Intro to 6)

1. Why is understanding the distribution of sample means important?
2. What does the standard error of the mean tell us?
3. How would we get a smaller SEM?
4. What are the steps in the 6-step approach?

Distribution of Sample Means

Inferential statistics
is all about using the
sample to infer
population parameters

But the sample is almost
certainly going to differ
from the population (at
least a little)

So, how can we visualize the sampling distribution of sample means?
(or, more generally, the sampling distribution of any statistic?)

http://shiny.stat.calpoly.edu/Sampling_Distribution/

Application

Example Using the Class Data &
The Office/Parks and Rec Data Set

Z-scores and Intro to Hypothesis Tests