Applied Statistical Analysis

EDUC 6050

Week 4

Finding clarity using data

Today

1. **Hypothesis Testing** with Z Scores (continued)
2. Chapters 6 and 7 in Book

Review

$$z = \frac{X - \mu}{\sigma} \qquad z = \frac{X - \mu}{\sigma/\sqrt{N}}$$

1. Which formula is for individual z scores and which is for z scores for sample means?
2. What are the six steps to hypothesis testing?
3. What are we trying to accomplish with hypothesis testing?

Z-Scores for an Individual Point

$$z = \frac{X - \mu}{\sigma}$$

Tells us:

- If the score is above or below the mean
- How large (the magnitude) the deviation from the mean is to other data points

The Z for a Sample Mean

$$Z_{Mean} = \frac{Mean - \mu}{SEM}$$

$$SEM = \frac{\sigma}{\sqrt{N}}$$

Depends on sample size (bigger sample, smaller SEM)

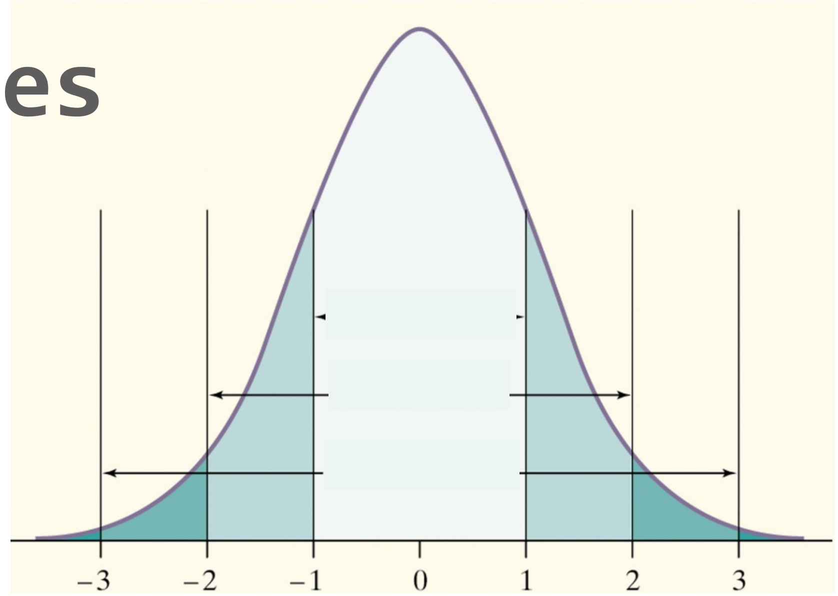
Z-Score and the Standard Normal Curve

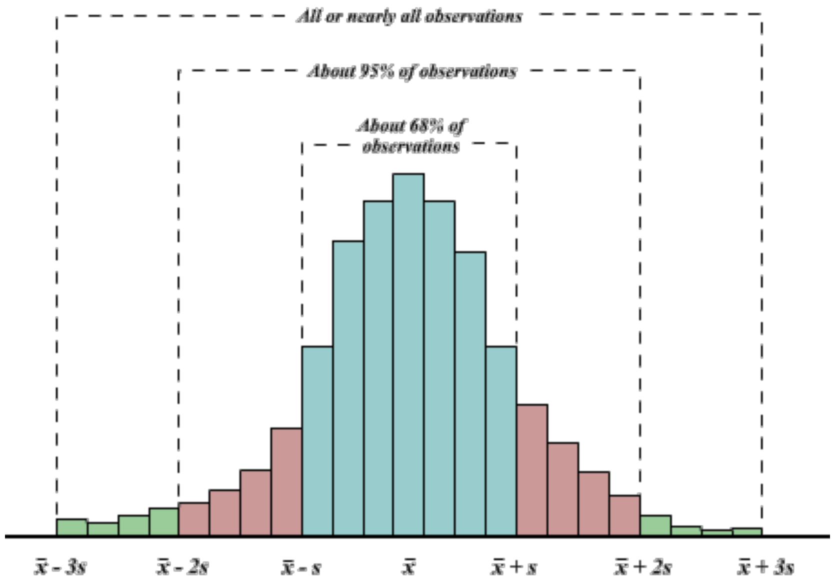
The 68-95-99.7 Rule

In the Normal distribution with mean μ and standard deviation σ:

- Approximately **68%** of the observations fall within σ of μ.
- Approximately **95%** of the observations fall within 2σ of μ.
- Approximately **99.7%** of the observations fall within 3σ of μ.

Appendix A shows more exact p-values





Hypothesis Testing with Z Scores

We’ll use a 6-step approach

We’ll use this throughout the class so get familiar with it

1. Examine Variables to Assess Statistical Assumptions
2. State the Null and Research Hypotheses (symbolically and verbally)
3. Define Critical Regions
4. Compute the Test Statistic
5. Compute an Effect Size and Describe it
6. Interpreting the results

1 Examine Variables to Assess Statistical Assumptions

Basic Assumptions

1. Independence of data
2. Appropriate measurement of variables for the analysis
3. Normality of distributions
4. Homogeneity of variance

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Basic Assumptions

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Individuals are independent of each other

- eg Jim and Pam could impact each other’s scores so they are NOT independent

1 Examine Variables to Assess Statistical Assumptions

Basic Assumptions

1. Independence of data
2. **Appropriate measurement** of variables for the analysis
3. Normality
4. Homo

Type of variable controls what analyses we can do

- eg nominal, ordinal, interval, ratio

1 Examine Variables to Assess Statistical Assumptions

Basic Assumptions

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   for the analysis
3. **Normality of distributions**
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Usually the outcome needs to be normal (for small samples)

1 Examine Variables to Assess Statistical Assumptions

Basic Assumptions

1. Independence of data
2. Appropriate measurement of variables for the
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4. Homogeneity of variance

Variances across groups should be approximately the same

1 Examine Variables to Assess Statistical Assumptions

Examining the Basic Assumptions

1. Independence: random sample
2. Appropriate measurement: know what methods work with that type of variable
3. Normality: Histograms, skew and kurtosis
4. Homogeneity: Histograms, compare SD's

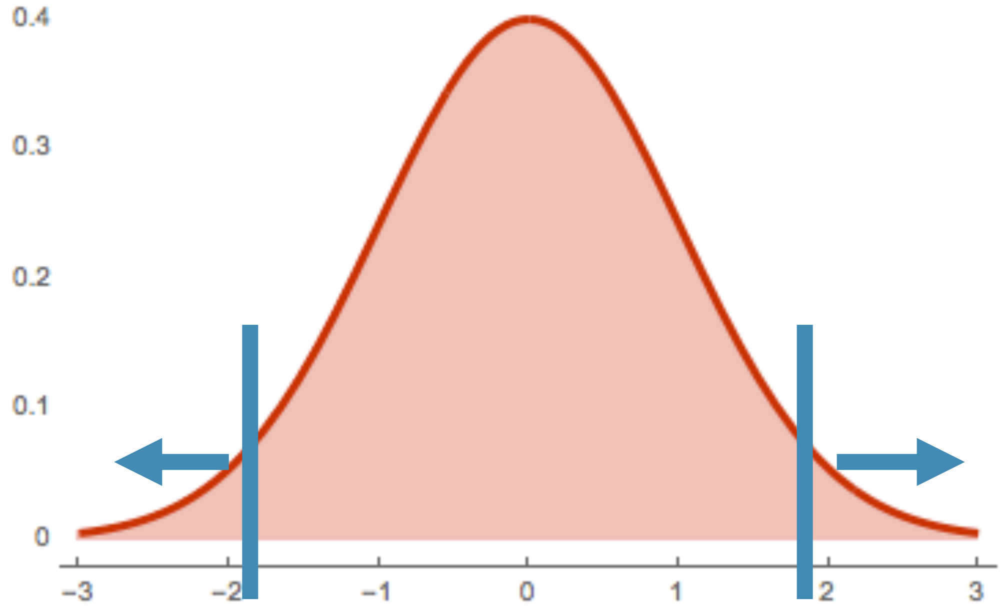
2 State the Null and Research Hypotheses (symbolically and verbally)

Hypothesis Type	Symbolic	Verbal	Difference between means created by:
Research Hypothesis	$\mu_{class} \neq \mu_{population}$	The class's mean is different than the population mean	True differences
Null Hypothesis	$\mu_{class} = \mu_{population}$	There is no *real* difference between the class and the population	Random chance (sampling error)

3 Define Critical Regions

How much evidence is enough to believe the null is not true?

Before analyzing the data, we define the critical regions (generally based on an alpha = .05)



3 Define Critical Regions

We decide on an alpha level first

↳ **Then calculate the critical value**

Look in the book for the z value
at alpha = .05 (two tails)

$$z_{critical} = 1.96$$

So our critical regions is defined as:

$$\alpha = .05$$

$$z_{critical} = 1.96$$

4 Compute the Test Statistic

$$z = \frac{M - \mu}{\sigma / \sqrt{N}}$$

The SEM, μ, and M will be given to you

Calculate it and compare to $z_{critical}$

Or

Calculate it, look up its p-value, and compare to our α level

5 Compute an Effect Size and Describe it

One of the main effect size estimates is **Cohen's d**

$$d = \frac{M - \mu}{\sigma}$$

d	Estimated Size of the Effect
Close to .2	Small
Close to .5	Moderate
Close to .8	Large

6 Interpreting the results

Put your results into words

Use the example on page 157 as a template

Break Time

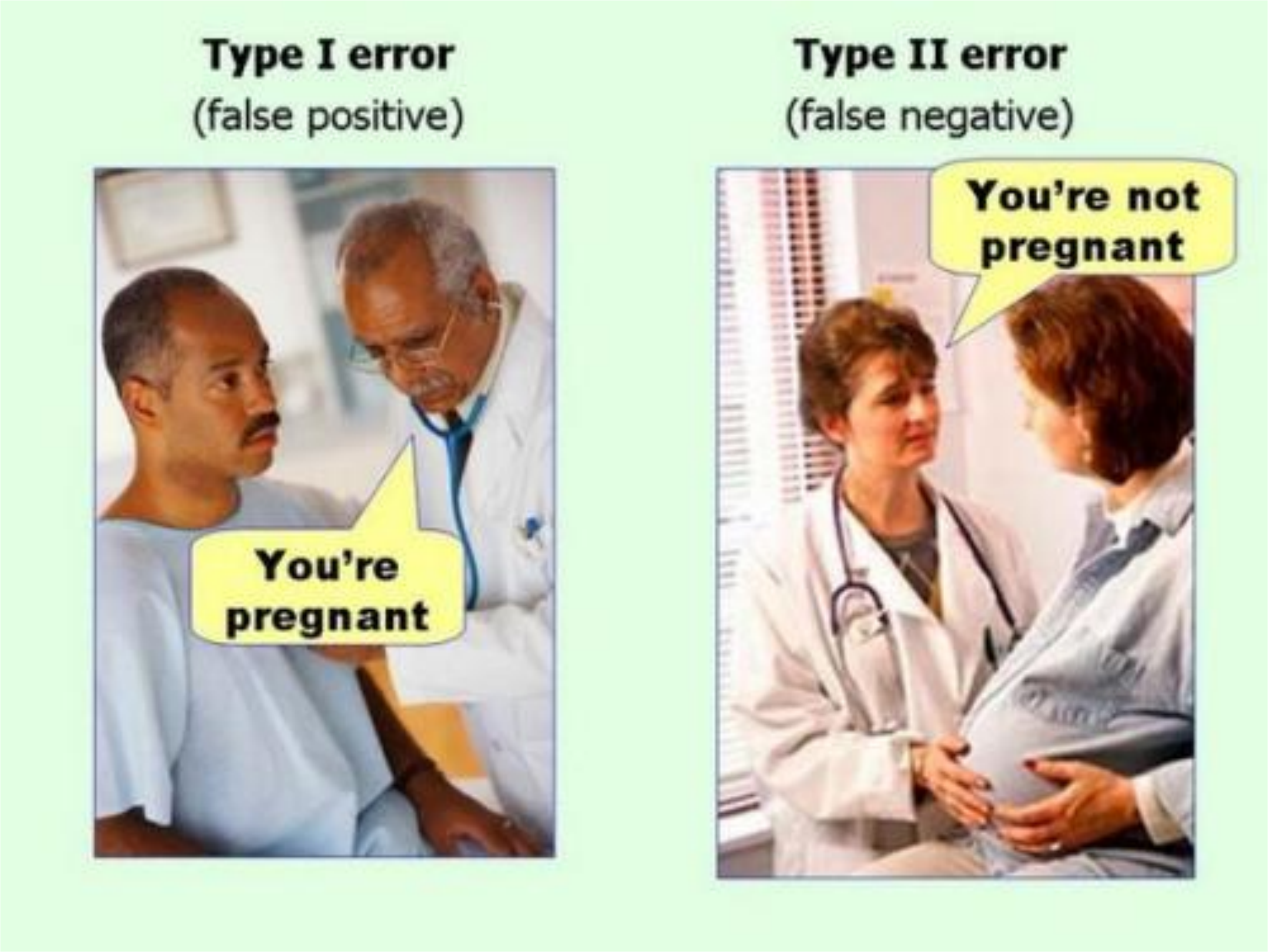
Hypothesis Testing with Z Scores

Let's practice!

- Study about IQ and our "Creation of SuperHumans" intervention
- $\mu_{pop} = 100$ and $\sigma = 15$ (https://en.wikipedia.org/wiki/IQ_classification)
- $M = 120$ and $SD = 13$ with a N = 100
- We think our intervention works so we want to test it

- Can we say that it does work?

Errors in Hypothesis Testing (any type)



Hypothesis Testing Rules

We make a handful of decisions along this process

- Alpha level
- Research and Null Hypotheses

These are related to Type I and Type II errors

What is a p-value?

The probability of getting an obtained value or a more extreme value assuming the null is true

Whether results are due to chance (sampling error)

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Reject that Null!

We don’t accept either the research or null hypotheses

Rather it is either evidence for or against the null

We do say that we “reject” or “fail to reject” the null

Evidence vs. Proof

Since we use p-values, there is always a chance that we made a Type I or Type II error

So we have evidence for or against it but we do NOT have PROOF of the research or null hypotheses

I like the discussion on Page 165 about this

Break Time

Single-Sample T-tests

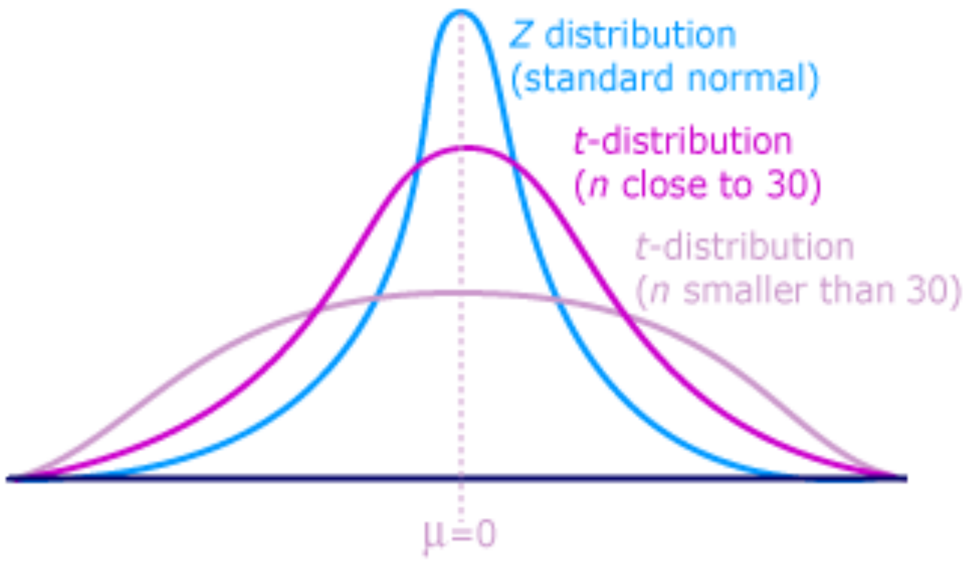
Situation	Test to Use	Formulas
Know population mean and standard deviation	Z-Test	$z = \frac{M - \mu}{\sigma / \sqrt{N}}$ $SEM = \sigma / \sqrt{N}$
Know population mean but not the standard deviation	One-Sample T-Test	$t = \frac{M - \mu}{SD / \sqrt{N}}$ $SEM = SD / \sqrt{N}$

Single-Sample T-tests

Requirements

1. Need a DV on an interval/ratio scale,
2. IV defines one sample, and
3. you do not know the population standard deviation

Slightly different distribution



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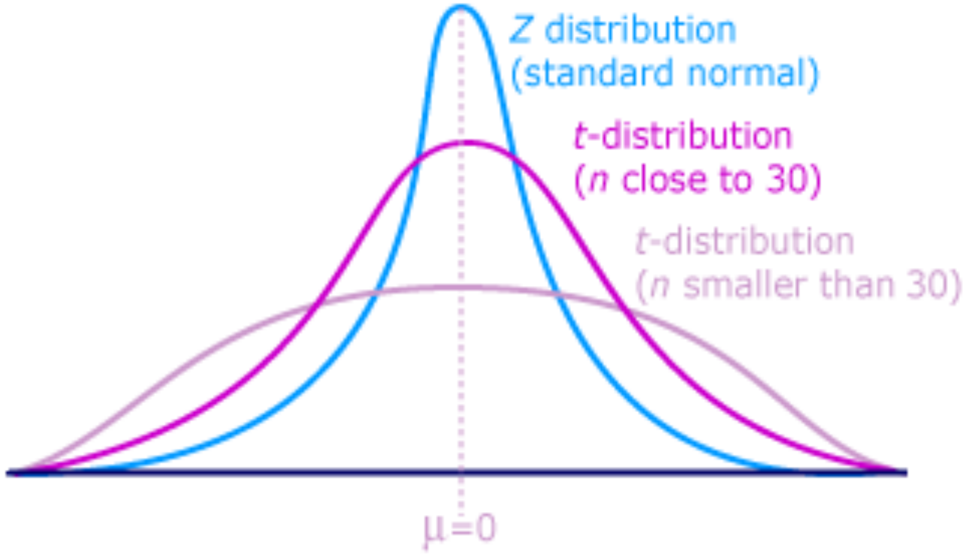

Hypothesis Testing with T-Tests

The same 6 step approach!

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2. State the Null and Research Hypotheses (symbolically and verbally)
3. Define Critical Regions
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Individuals are independent of each other (one person's scores does not affect another's)

1 Examine Variables to Assess Statistical Assumptions

Basic Assumptions

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Here we need interval/ratio DV and an IV that is for a single sample (group)

1 Examine Variables to Assess Statistical Assumptions

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The outcome needs to be normal (for small samples)

1 Examine Variables to Assess Statistical Assumptions

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The variance of our sample is supposed to match the population variance (but do we know it?)

1 Examine Variables to Assess Statistical Assumptions

Examining the Basic Assumptions

1. Independence: random sample
2. Appropriate measurement: know what your variables are
3. Normality: Histograms, skew and kurtosis
4. Homogeneity: Hard to assess

2 State the Null and Research Hypotheses (symbolically and verbally)

Hypothesis Type	Symbolic	Verbal	Difference between means created by:
Research Hypothesis	$\mu_{class} \neq \mu_{population}$	The class's mean is different than the population mean	True differences
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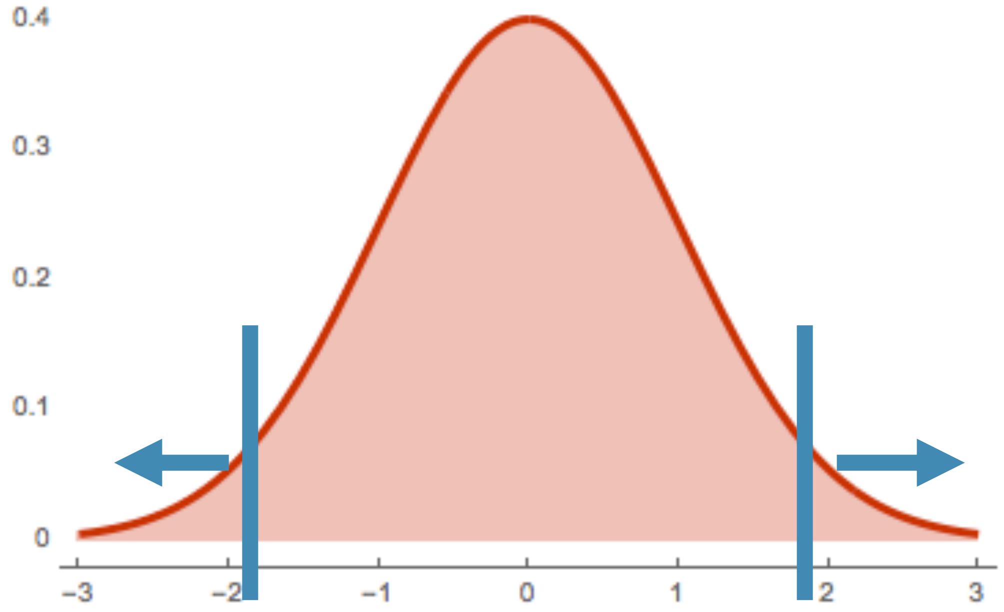
3 Define Critical Regions

How much evidence is enough to believe the null is not true?

Before analyzing the data, we define the critical regions (generally based on an alpha = .05)



3 Define Critical Regions

We decide on an alpha level first

↳ Then calculate the critical value (based on sample size)

3 Define Critical

We decide on an alpha le

Then calculate the c

(based on sample si

I'll provide a table for you for the t values

- Base on alpha and a specific df

$$df = N - 1$$

df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	43178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
z	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905
CI	——	——	80%	90%	95%	98%	99%	99.9%

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3 Define Critical Regions

We decide on an alpha level first

↳ Then calculate the critical value (based on sample size)

$$t_{critical,\,29} = 2.05$$

So our critical regions is defined as:

$$\alpha = .05$$

$$t_{critical,\,29} = 2.05$$

4 Compute the Test Statistic

$$t = \frac{M - \mu}{SD / \sqrt{N}}$$

The SEM, μ, and M will be given to you

Calculate it and compare to $t_{critical}$

Or

Calculate it, look up its p-value, and compare to our α level

5 Compute an Effect Size and Describe it

One of the main effect size estimates is **Cohen's d**

$$d = \frac{M - \mu}{SD}$$

d	Estimated Size of the Effect
Close to .2	Small
Close to .5	Moderate
Close to .8	Large

6 Interpreting the results

Put your results into words

Use the example on page 216 as a template

Hypothesis Testing with T-tests

Let's practice!

- Study about height and our "Creation of Super-Tall Humans" intervention
- $\mu_{pop} = 63$ and $\sigma = ?$ (https://en.wikipedia.org/wiki/IQ_classification)
- $M = 70$ and $SD = 10$ with a N = 36
- We think our intervention works so we want to test it

- Can we say that it does work?

Review Hypothesis Testing

1. When can we say a result is "significant"?
2. How would we get a smaller SEM?
3. What is Cohen's d?
4. What is the difference between a Z-test and a T-test?

Another look at Jamovi

One-Sample T-test

Questions?

Next week:

1. **Hypothesis Testing** with T-tests and **Confidence Intervals**
2. Chapters 7 and 8 in Book
3. Keep updating your Statistical Organizer