# One-Way ANOVA

## Cohen Chapter 12

EDUC/PSY 6600

“It is easy to lie with statistics.
It is hard to tell the truth
without statistics.”

*-Andrejs Dunkels*

# Motivating examples

- Dr. Vito randomly assigns 30 individuals to 1 of 3 study groups to evaluate whether one of **2 new approaches** to therapy for adjustment disorders with mixed anxiety and depressed mood are more effective than the **standard approach**. Participants are matched on current levels of anxiety and depressed mood at baseline. Scores from the BAI and BDI are collected after 2 months of therapy.

- Dr. Creft wishes to assess differences in oral word fluency **among three groups of participants**: Right hemisphere stroke, left hemisphere stroke, and healthy controls. Scores on the COWAT are collected from 20 participants per group and the means of each group are compared.

# Research Design Vocab

- ***Experimental design***
  - Participants are randomly **assigned** to levels and at least one factor is **manipulated**
  - Participants are randomly selected from multiple **preexisting (observed)** populations
- ***Fixed or random effects***
  - **Fixed** effects design: Levels of each factor systematically chosen by researcher
  - **Random** factors design: Levels of each factor are chosen randomly from a larger subset (rarer)
- ***Independent** (Between-Subjects) or **Repeated** (Within-Subjects) factors*
  - **Independent**: Participants randomly allocated to each level of a factor
  - **Repeated** measures design: Participants are paired or a dependency exists (multiple observations)

# Research Design Vocab

- ***Experimental design***
  - Participants are randomly **assigned** to levels and at least one factor is **manipulated**
  - Participants are randomly selected from multiple **preexisting (observed)** populations

If the levels of the grouping variable are **highly ordinal or continuous** in nature, **regression** or a rank type test will be more powerful than ANOVA

- ANOVA is appropriate in cases where the groups are more nominal in nature.

*Some variables can be construed as both!!! (e.g. Grade level)*

- *probably want to analyze both ways*

# Analysis of Variance (ANOVA)

- ANOVA designs can be used for…
  - Experimental research
  - Quasi-experimental studies
  - Field/observational research

- *Other names for 1-way ANOVA…*
  - Single factor ANOVA
  - *Univariate ANOVA*
  - Simple ANOVA
  - Independent-ANOVA
  - Between-subjects ANOVA

**Omnibus test for group (MEAN) differences**

**ONE Dependent Variable (DV)**
"outcome"

Continuous (interval/ratio)
&
normally distributed

**ONE Independent Variable (IV)**
"predictor"

Categorical (nominal)
$\geqslant$ 3 independent samples or groups
*Factor* with *k levels*

# Example: noise & words memorized

| Group A | B | C |
|---|---|---|
| 8 | 7 | 4 |
| 10 | 8 | 8 |
| 9 | 5 | 7 |
| 10 | 8 | 5 |
| 9 | 5 | 7 |

- Study to determine if noise inhibits learning ($N$ = 15)
- Students **randomized** to 1 of 3 groups (k = 3 & $n$ = 5)
  - IV = grouping factor with 3 levels
    - Group A: **No** noise (no music, quiet room)
    - Group B: **Moderate** noise (classical music)
    - Group C: **Extreme** noise (rock music)
- Participants are given 1 minutes to memorize list of 15 nonsense words
  - DV = # of correct nonsense words recalled

# Steps of a Hypothesis test

1) State the **Hypotheses** (Null & Alternative)
2) Select the **Statistical Test** & Significance Level
   - *Examples include: $z$, $t$, $F$, $\chi^2$*
   - *$\alpha$ level (commonly use .05)*
   - *One vs. Two tails (usually prefer 2)*
3) Select random **samples** and **collect** data
4) Find the region of **Rejection**
   - *Based on $\alpha$ & # of tails*
5) Calculate the **Test Statistic**
   - *Select the appropriate formula*
   - *May need to find degrees of freedom*
6) Make the Statistical **Decision**

# Hypotheses of ANOVA

- Means: $\mu_1, \mu_2, \mu_3, \ldots, \mu_k$
- Variances: $\sigma_1^2, \sigma_2^2, \sigma_3^2, \ldots, \sigma_k^2$

$H_0: \mu_1 = \mu_2 = \mu_3 = \ldots = \mu_k$
$H_1$: Not $H_0$

Many ways to reject $H_0$

**NOT $H_1: \mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_k$**

**Example: Noise & Words Memorized**

**Null Hypothesis:**

The number of words recalled is the same regardless of the music/noise.

$H_0: \mu_{none} = \mu_{moderate} = \mu_{extreme}$

**Alternative Hypothesis:**

**At least one** music/noise level results in a **different** number of words recalled.

$H_1$: Not $H_0$

# Example: noise & words memorized

| Group | | |
|---|---|---|
| A | B | C |
| 8 | 7 | 4 |
| 10 | 8 | 8 |
| 9 | 5 | 7 |
| 10 | 8 | 5 |
| 9 | 5 | 7 |

## 1. Enter data into R

```
# A tibble: 15 x 2
   outcome group
     <dbl> <fct>
1       8.  A
2      10.  A
3       9.  A
4      10.  A
5       9.  A
6       7.  B
7       8.  B
8       5.  B
9       8.  B
10      5.  B
11      4.  C
12      8.  C
13      7.  C
14      5.  C
15      7.  C
```

## 2. Calculate the Group Means

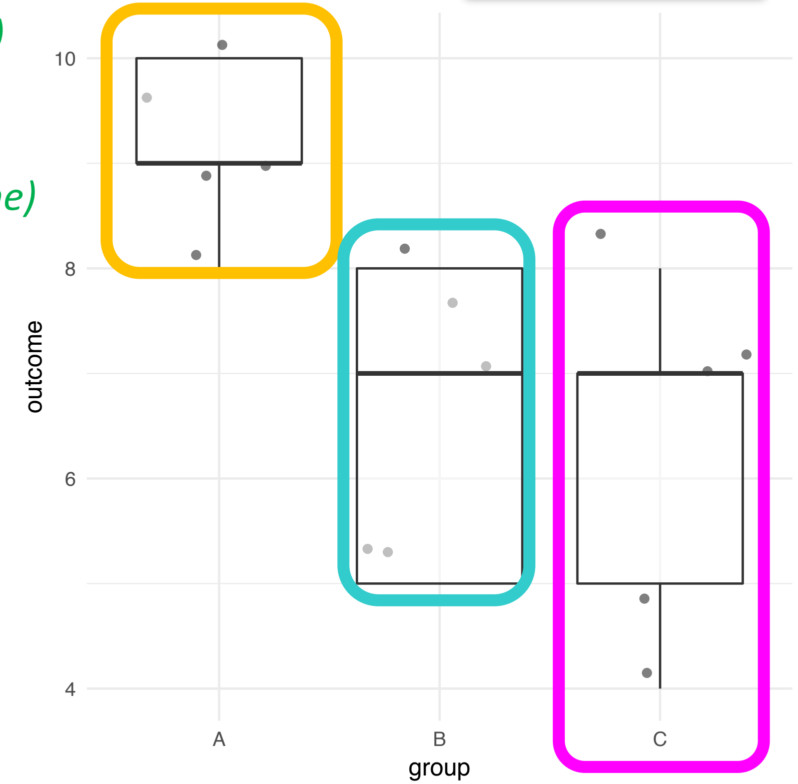
```
df %>%
  group_by(group) %>%
  furniture::table1(outcome)
```

*IV (groups)*

*DV (outcome)*

```
                         group
           A             B             C
           n = 5         n = 5         n = 5
outcome
           9.2 (0.8)     6.6 (1.5)     6.2 (1.6)
```

## 3. Visualize the data



# Link: Independent sample "t-test" & ANOVA

## *Same principle underlies many statistical tests*

$$F = \frac{MS_B}{MS_W} = \frac{\text{Measure of effect (or treatment) assessed by examining variance (or differences) between groups}}{\text{Measure of random variation (or error) assessed by examining variance (or differences) within groups}}$$

$$\text{Stats} = \frac{\text{Stuff we can explain with our variables}}{\text{Stuff we CANNOT explain with our variables (random error)}}$$

**Numerator**

$MS_B$ : Compute variance <u>between</u> (among) sample means, multiply by $n_j$

**Denominator**

$MS_W$ : Compute average of sample variances

# Link: Independent sample "t-test" & ANOVA

- Same question as before…
  - **Do group means significantly differ?**
  - Or…Do mean differences on DV 'between' groups **EXCEED** differences 'within' groups?
    - **Between**-groups differences
      - Differences in DV **due to IV (group)**
    - **Within**-groups differences
      - Differences in DV **due to pooled random error or variation**

- Same analysis approach as before…

# Link: Independent sample "t-test" & ANOVA

- Same question as before…
  - **Do group means significantly differ?**
  - Or…Do mean differences on DV 'between' groups **EXCEED** differences 'within' groups?
    - **Between**-groups differences
      - Differences in DV **due to IV (group)**
    - **Within**-groups differences
      - Differences in DV **due to pooled random error or variation**

$$t = \frac{(\bar{X}_1 - \bar{X}_2)}{s_{\bar{X}_1 - \bar{X}_2}} = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}} = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

- Same analysis approach as before…

$$\boxed{F = t^2}$$

# Link: Independent sample “t-test” & ANOVA

- Same question as before…
  - **Do group means significantly differ?**
  - Or…Do mean differences on DV ‘between’ groups **EXCEED** differences ‘within’ groups?
    - **Between**-groups differences
      - Differences in DV **due to IV (group)**
    - **Within**-groups differences
      - Differences in DV **due to pooled random error or variation**
- Same analysis approach as before…

$$\boxed{F = t^2}$$

$$t = \frac{\left(\bar{X}_1 - \bar{X}_2\right)}{s_{\bar{X}_1 - \bar{X}_2}} = \frac{\left(\bar{X}_1 - \bar{X}_2\right)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}} = \frac{\left(\bar{X}_1 - \bar{X}_2\right)}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Could rewrite as… $t = \dfrac{\left(\bar{X}_1 - \bar{X}_2\right)}{\sqrt{s_p^2\left(\frac{2}{n_j}\right)}}$,

Where $n_j$ = sample size for any group j. Then....

# Link: Independent sample "t-test" & ANOVA

- Same question as before…
  - **Do group means significantly differ?**
  - Or…Do mean differences on DV 'between' groups **EXCEED** differences 'within' groups?
    - **Between**-groups differences
      - Differences in DV **due to IV (group)**
    - **Within**-groups differences
      - Differences in DV **due to pooled random error or variation**
- Same analysis approach as before…

$$\boxed{F = t^2}$$

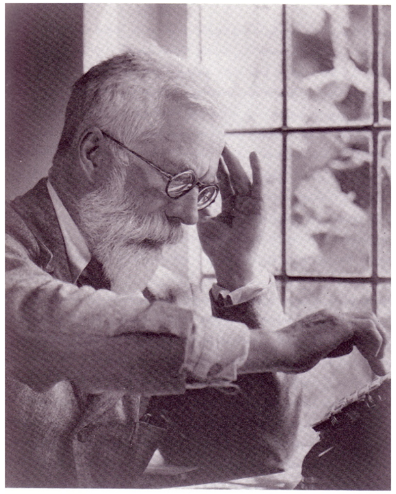
$$t = \frac{(\bar{X}_1 - \bar{X}_2)}{s_{\bar{X}_1 - \bar{X}_2}} = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}} = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Could rewrite as… $t = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{s_p^2\left(\frac{2}{n_j}\right)}}$,

Where $n_j$ = sample size for any group j. Then….

$$t^2 = \frac{(\bar{X}_1 - \bar{X}_2)^2}{\frac{2s_p^2}{n_j}} = \frac{n_j(\bar{X}_1 - \bar{X}_2)^2}{2s_p^2} = \boxed{\frac{n_j\frac{(\bar{X}_1 - \bar{X}_2)^2}{2}}{s_p^2} = F}$$

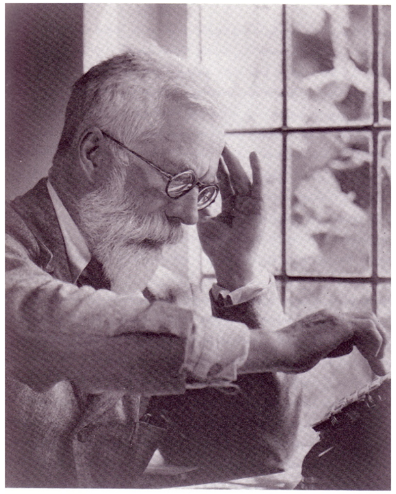
# F-distribution


Fisher at his desk calculator at Whittinghame Lodge, 1952

Sir **Ronald A. Fisher** (1920-40's) & **agricultural** experiments…

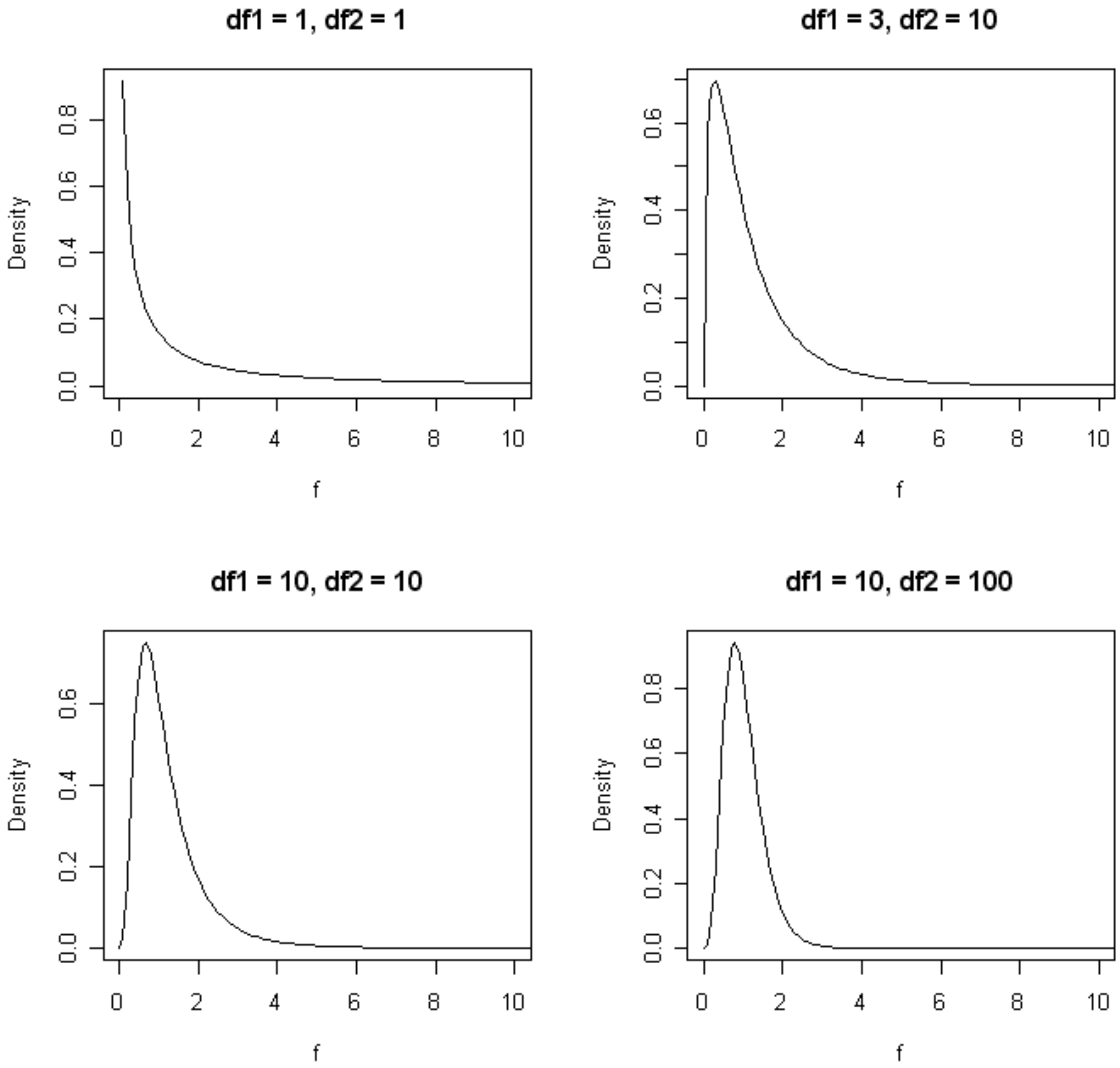
- *F*-distribution
  - Continuous theoretical probability distribution
  - Probability of **ratios** (fraction) of variance **between** groups to variance **within** groups
- Positively skewed
  - Range: 0 to ∞
  - one-tailed
  - More "normal" as $N\uparrow$
  - Mean ≈1… $M = \frac{df_W}{df_{W-2}}$
- Family of distributions
  - Need **2 *df*** and $\alpha$ to determine $F_{crit}$
    - $df_{Within}$ and $df_{Between}$ (more later…)

$$\boldsymbol{F = t^2}$$



Fisher at his desk calculator at Whittingehame Lodge, 1952

Sir **Ronald A. Fisher** (1920-40’s) & **agricultural** experiments…



# Link: Independent sample “t-test” & ANOVA

**Specific situation:** **2 groups, when $n_1 = n_2$**

**Numerator:** Variation **between** (among) group **means**

‘Variance’ of 2 means multiplied by $n_i$

Mean Square Between ($MS_B$) or Mean Square Treatment ($MS_T$)

$$t^2 = \frac{n_j \dfrac{\left(\bar{X}_1 - \bar{X}_2\right)^2}{2}}{s_p^2} = F$$

**Denominator**: Pooled variation **within groups**

Pooled variance ($s_p^2$) = average of 2 variances when *ns* are equal

Mean Square Within ($MS_W$) or Mean Square Error ($MS_E$)

# Link: Independent sample "t-test" & ANOVA

***'Mean Square'*** or ***MS***

is another term for the **variance**

'Square': Refers to the sum of SQUARED (*SS*) deviations from the mean

Mean: AVERAGE of the *SS* deviations

SS is divided by *N* or *N* - 1 to yield variance

So, Mean of the sum of SQUARED deviations = Variance

**All we want to know is whether variation among group means exceeds that variation within groups**

Will create a **ratio of the *MS*s**, the *F*-statistic, to see if this ratio is significantly **different from 1**

# Prior example

- Applying data from independent-samples *t*-test example
- (drug *v.* placebo and depression)
  - Recall, $t = 1.96$, $p = .085$

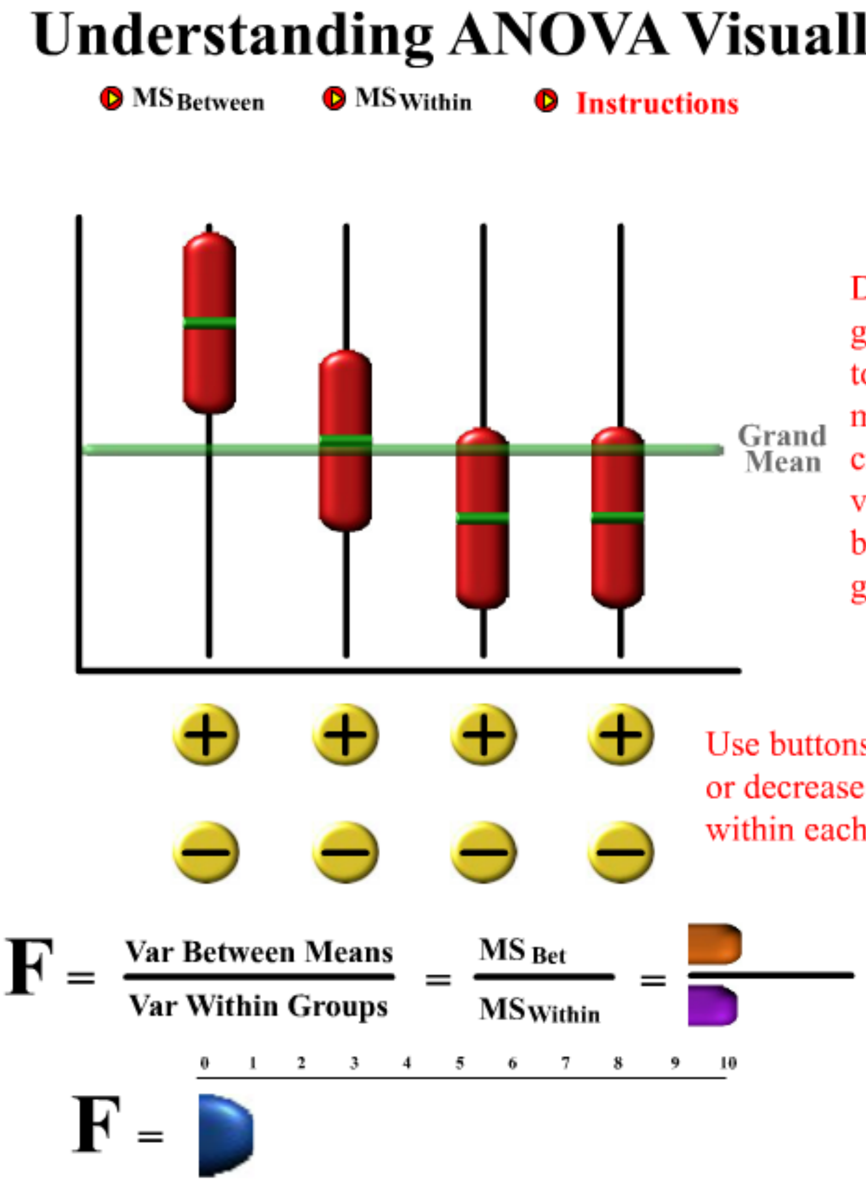
**$1.96^2 = 3.84$**

$$\boxed{t^2 = F}$$

| Group 1 - Drug | Group 2 - Placebo |
|---|---|
| 11 | 11 |
| 1 | 11 |
| 0 | 5 |
| 2 | 8 |
| 0 | 4 |

$$t^2 = \frac{n_j \frac{\left(\bar{X}_1 - \bar{X}_2\right)^2}{2}}{s_p^2}$$

# Interactive Applet




- http://www.psych.utah.edu/stat/introstats/anovaflash.html

# Assumptions

**Large or multiple violations will GREATLY increase risk of inaccurate *p*-values**
**Increased probability of Type I or II error**

**Independent, Random Sampling (for the IV)** ← ensure by planning ahead!

- For **preexisting** (observed) populations: randomly select a sample from each population
- For **experimental** (assigned) conditions: randomly divide your sample *(of convenience)* for assignment to groups
- Ensure no connection between subjects in the different groups (no matching!) ← MUST!!!

**Normally distributed (DV)**

- Robust requirement…if samples are large, this isn't as important
- If not normal (or small samples)
  - alternatives: use the Krukal-Wallis H test

**HOV: homogeneity of Variance (DV)**

- Since an average variance is computed for denominator of *F*-statistic, variance should be similar for all groups: $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \ldots = \sigma_k^2$
- $\sigma_e^2$, pooled (averaged) variance, must be representative of each group so that $MS_W$ is accurate
- Testing: Levene's Test
- All test for HOV are underpowered if samples are small, so you have to use judgement ;)
- If NOT HOV
  - alternatives: Welch, Brown-Forsythe, etc.

Recall from CLT, relationship between variance of population ($\sigma^2$) & variance of *SDM* ($SE^2 = \sigma_{\bar{X}}^2$)

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n_j}} \rightarrow \sigma_{\bar{X}}^2 = \frac{\sigma^2}{n_j} \rightarrow \sigma_{\bar{X}}^2 \cdot n_j = \sigma_e{}^2 = MS_B$$

# F-statistic: numerator = $MS_B$

## One estimate of population variance ($\sigma_e{}^2$)

- Cannot compute population variance of all possible means as we only have a sample
  - Estimate population **variance** with sample means and **multiply** by sample size:

**Equal Sample Sizes**

$$MS_B = n \cdot s_{\bar{X}}^2$$

**UN-equal Sample Sizes**

$$MS_B = \frac{\sum n_j(\bar{X}_j - \bar{X}_G)^2}{k - 1}$$

## If $H_0$ true, $MS_B = \sigma_e{}^2$

Have drawn *k* independent samples
From the SAME population
(i.e. group differences = 0)

## If $H_0$ false, $MS_B \neq \sigma_e{}^2$

$MS_B$ reflects BOTH
population variance
AND
group differences

# Example: noise & words memorized

1. *Find grand mean:*

$$\overline{X_G} = \frac{9.2+6.6+6.2}{3} = \frac{22}{3} = 7.33$$

| | group | | |
|---|---|---|---|
| | A | B | C |
| | n = 5 | n = 5 | n = 5 |
| outcome | | | |
| | 9.2 (0.8) | 6.6 (1.5) | 6.2 (1.6) |

2. *Find the SD of the means:*

$$s_{\bar{X}}^2 = \frac{(9.2-7.33)^2 + (6.6-7.33)^2 + (6.2-7.33)^2}{3-1} = \frac{5.3067}{2} = 2.65$$

3. *Multiply by n*

$$MS_B = 5 \cdot 2.65 = \mathbf{13.267}$$

**Equal**
**Sample Sizes**

$$MS_B = n \cdot s_{\bar{X}}^2$$

# F-STATISTIC: DENOMINATOR = $MS_W$

**Second estimate of population variance ($\sigma_e^2$)**

- **Pooling** sample variances yields best estimate
  - $\sigma_1^2 = s_1^2$; $\sigma_2^2 = s_2^2$; … ; $\sigma_j^2 = s_j^2$
- Average subgroup (*j*) variance: $\sigma_e^2 = s_e^2$

**Equal**
**Sample Sizes**

$$MS_W = \sigma_e^2 = \frac{\sum s_j^2}{k}$$

**UN-equal**
**Sample Sizes**

$$MS_W = \sigma_e^2 = \frac{\sum (n_j - 1) s_j^2}{n_T - k}$$

Goal should be to obtain equal *ns*
BUT…
1 group > 50% larger other group: too much

k = # subgroups
*j* denotes the *j-th* subgroup

**Regardless of whether $H_0$ true:**

$$MS_W = \sigma_e^2$$

Not affected by group MEANS

# Example: noise & words memorized

- *1. Average the* ***<u>VARIANCES's</u>*****:**

$$MS_W = \frac{0.8^2 + 1.5^2 + 1.6^2}{3} = \frac{5.5}{3} = \mathbf{1.9}$$

| | group | | |
|---|---|---|---|
| | A | B | C |
| | n = 5 | n = 5 | n = 5 |
| outcome | | | |
| | 9.2 $\boxed{(0.8)}$ | 6.6 $\boxed{(1.5)}$ | 6.2 $\boxed{(1.6)}$ |

**<u>Equal</u>**
**Sample Sizes**

$$MS_W = \sigma_e{}^2 = \frac{\sum s_j{}^2}{k}$$

# Logic of “anova”

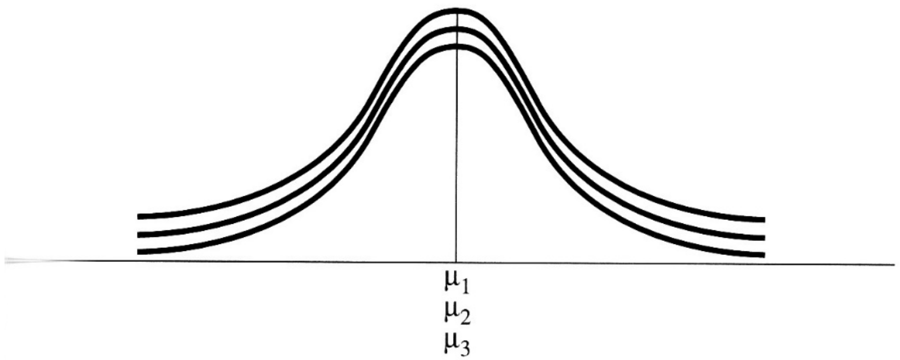
*IF all samples are the **same sizes…***

$$\left.\begin{aligned} MS_B &= \sigma_e{}^2 = n_j \cdot s^2{}_{\bar{X}} \\ MS_W &= \sigma_e{}^2 = \frac{\sum s_j{}^2}{k} \end{aligned}\right\} \xrightarrow{ratio} F = \frac{MS_B}{MS_W}$$

When estimates of $\sigma_e{}^2$ (variances) are…

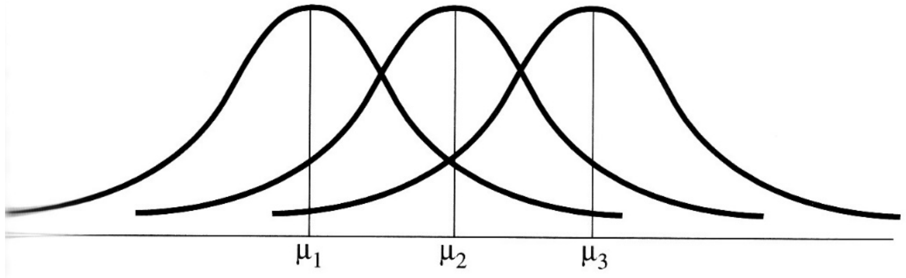
**Equal: Fail to reject $H_0$**

- All means come from same population
- Both are estimates of the same population variance $\sigma_e{}^2$
- $F$-ratio ≈ 1




**Unequal: Reject $H_0$**

- **Unlikely** that all means come from same population
- **Effect of IV surpasses random error/variation within groups**
- $F$-ratio significantly > 1, $MS_B$ > $MS_W$



# CALCULATIONS:

2 Approaches

SUMMARY STATS KNOWN
(shown on previous few slides)

SUM OF SQUARES (SS) APPROACH
(alternate formulas here)

$$SS = \sum_{i=1}^{n} (X_i - \bar{X})^2$$

Can '**partition**' total variation in DV due to group effects (IV) and error

$$SS_{Total} = SS_{Between} + SS_{Within}$$

## Total

How different are ALL individuals from the "GRAND MEAN"

**Inner Sum:** individuals in each subgroup

**Outter Sum:** subgroups in the whole

$$SS_{Total} = \sum_{j=1}^{k}\sum_{i=1}^{n}(X_{ij} - \bar{X}_{GM})^2$$

$$\boxed{df_T = n_T - 1}$$

## Between

How different are "GROUP MEANS" from the "GRAND MEAN"

$$SS_{Between} = n_j \sum_{j=1}^{k}(\bar{X}_j - \bar{X}_{GM})^2$$

$$\boxed{df_B = k - 1}$$

$$MS_{Between} = \frac{SS_B}{df_B} = \frac{n_j \sum_{j=1}^{k}(\bar{X}_j - \bar{X}_{GM})^2}{k-1}$$

## Within

How different are individuals from their "GROUP's MEAN"

**Inner Sum:** individuals in each subgroup

**Outter Sum:** subgroups in the whole

$$SS_{Within} = \sum_{j=1}^{k}\sum_{i=1}^{n}(X_{ij} - \bar{X}_j)^2$$

$$\boxed{df_w = n_T - k}$$

$$MS_{Within} = \frac{SS_W}{df_W} = \frac{\sum_{j=1}^{k}\sum_{i=1}^{n}(X_{ij} - \bar{X}_j)^2}{N-k}$$

$$F_{Ratio} \text{ or } F_{Statistic} = \frac{MS_B}{MS_W}$$

Can '**partition**' total variation in DV due to group effects (IV) and error

$$SS_{Total} = SS_{Between} + SS_{Within}$$

# F-statistic

$$F_{Ratio} \text{ or } F_{Statistic} = \frac{MS_B}{MS_W}$$

- $F_{crit}$ → $F$-distribution table
  - (different table per $\alpha$)
    - *Across the top: find $df_B$*
    - *Down the side: find $df_W$*

- **<u>If $H_0$ is true, $MS_B = MS_W$</u>**

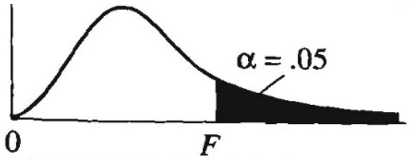
**$F$-statistic ≈ 1**

Both are estimates of variance of **same** population

- **<u>If $H_0$ is false, $MS_B > MS_W$</u>**

**$F$-statistic exceeds $F_{crit}$** by some amount

**At least one** mean significantly differs from another




| df Denominator | df NUMERATOR 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.74 | 8.70 |
| 4 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.91 | 5.86 |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.68 | 4.62 |
| 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 4.00 | 3.94 |
| 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.57 | 3.51 |
| 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.28 | 3.22 |
| 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.07 | 3.01 |
| 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.91 | 2.85 |
| 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.79 | 2.72 |
| 12 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.69 | 2.62 |
| 13 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.60 | 2.53 |
| 14 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.53 | 2.46 |
| 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.48 | 2.40 |
| 16 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.42 | 2.35 |
| 17 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.38 | 2.31 |
| 18 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.34 | 2.27 |
| 19 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.31 | 2.23 |
| 20 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.28 | 2.20 |
| 21 | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 | 2.32 | 2.25 | 2.18 |
| 22 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.23 | 2.15 |
| 23 | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 | 2.27 | 2.20 | 2.13 |
| 24 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.18 | 2.11 |
| 25 | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 | 2.24 | 2.16 | 2.09 |
| 26 | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 | 2.22 | 2.15 | 2.07 |
| 27 | 4.21 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |

# Example: noise & words memorized

*Test statistic: F-score observed*

$$df = (3-1, 15-3) = \mathbf{(2, 12)}$$

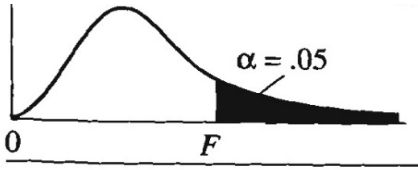
$$MS_B = \mathbf{13.267}$$

$$MS_W = \mathbf{1.9}$$

$$F(2,12) = \frac{13.267}{1.90} = \mathbf{6.98}$$

*Critical Value: F-crit for α=.05*

# Example: noise & words memorized




| df Denominator | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | df NUMERATOR | | | | | | | |
| 3 | 10.1 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.74 | 8.70 |
| 4 | 7.7 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.91 | 5.86 |
| 5 | 6.6 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.68 | 4.62 |
| 6 | 5.9 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 4.00 | 3.94 |
| 7 | 5.5 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.57 | 3.51 |
| 8 | 5.3 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.28 | 3.22 |
| 9 | 5.1 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.07 | 3.01 |
| 10 | 4.9 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.91 | 2.85 |
| 11 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 12 | 4.7 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.69 | 2.62 |
| 13 | 4.6 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.60 | 2.53 |
| 14 | 4.6 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.53 | 2.46 |
| 15 | 4.5 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.48 | 2.40 |
| 16 | 4.4 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.42 | 2.35 |
| 17 | 4.4 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.38 | 2.31 |
| 18 | 4.4 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.34 | 2.27 |
| 19 | 4.3 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.31 | 2.23 |
| 20 | 4.3 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.28 | 2.20 |
| 21 | 4.3 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 | 2.32 | 2.25 | 2.18 |
| 22 | 4.3 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.23 | 2.15 |
| 23 | 4.2 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 | 2.27 | 2.20 | 2.13 |
| 24 | 4.2 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.18 | 2.11 |
| 25 | 4.2 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 | 2.24 | 2.16 | 2.09 |
| 26 | 4.2 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 | 2.22 | 2.15 | 2.07 |
| 27 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |

*<u>Critical Value: F-crit for α=.05</u>*

$$F_{crit}(2, 12) = \mathbf{3.89}$$

$$F(2, 12) = \frac{13.267}{1.90} = \mathbf{6.98}$$

**Conclusion:**

- AT LEAST ONE noise/music levels has a different mean # of words memorized.
- In fact it is the no noise/music condition that has the most words memorized.
- What type of music is playing doesn't seem to make as much of a difference.

# R Code: ANOVA

```
df %>%
  group_by(group) %>%
  furniture::table1(outcome)
```

| | group | | |
|---|---|---|---|
| | A | B | C |
| | n = 5 | n = 5 | n = 5 |
| outcome | | | |
| | 9.2 (0.8) | 6.6 (1.5) | 6.2 (1.6) |

Same as with t-tests

```
df %>%
  car::leveneTest(outcome ~ group,
                  data = .,
                  center = mean)
```

```
Levene's Test for Homogeneity of Variance (center = mean)
       Df F value  Pr(>F)
group  2  2.8213   0.09902 .
      12
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

What does this tell us?

# R Code: ANOVA

```
df %>%
  group_by(group) %>%
  furniture::table1(outcome)
```

| | group | | |
|---|---|---|---|
| | A | B | C |
| | n = 5 | n = 5 | n = 5 |
| outcome | | | |
| | 9.2 (0.8) | 6.6 (1.5) | 6.2 (1.6) |

```
df %>%
  car::leveneTest(outcome ~ group,
                  data = .,
                  center = mean)
```

```
Levene's Test for Homogeneity of Variance (center = mean)
      Df F value  Pr(>F)
group  2  2.8213  0.09902 .
      12
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

aov() function performs ANOVA

```
df %>%
  aov(outcome ~ group,
      data = .) %>%
  summary()
```

```
            Df  Sum Sq  Mean Sq  F value  Pr(>F)
group        2  26.53   13.27    6.982    0.00974**
Residuals   12  22.80   1.90
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

summary() gives us the ANOVA table (similar to when we did regression)

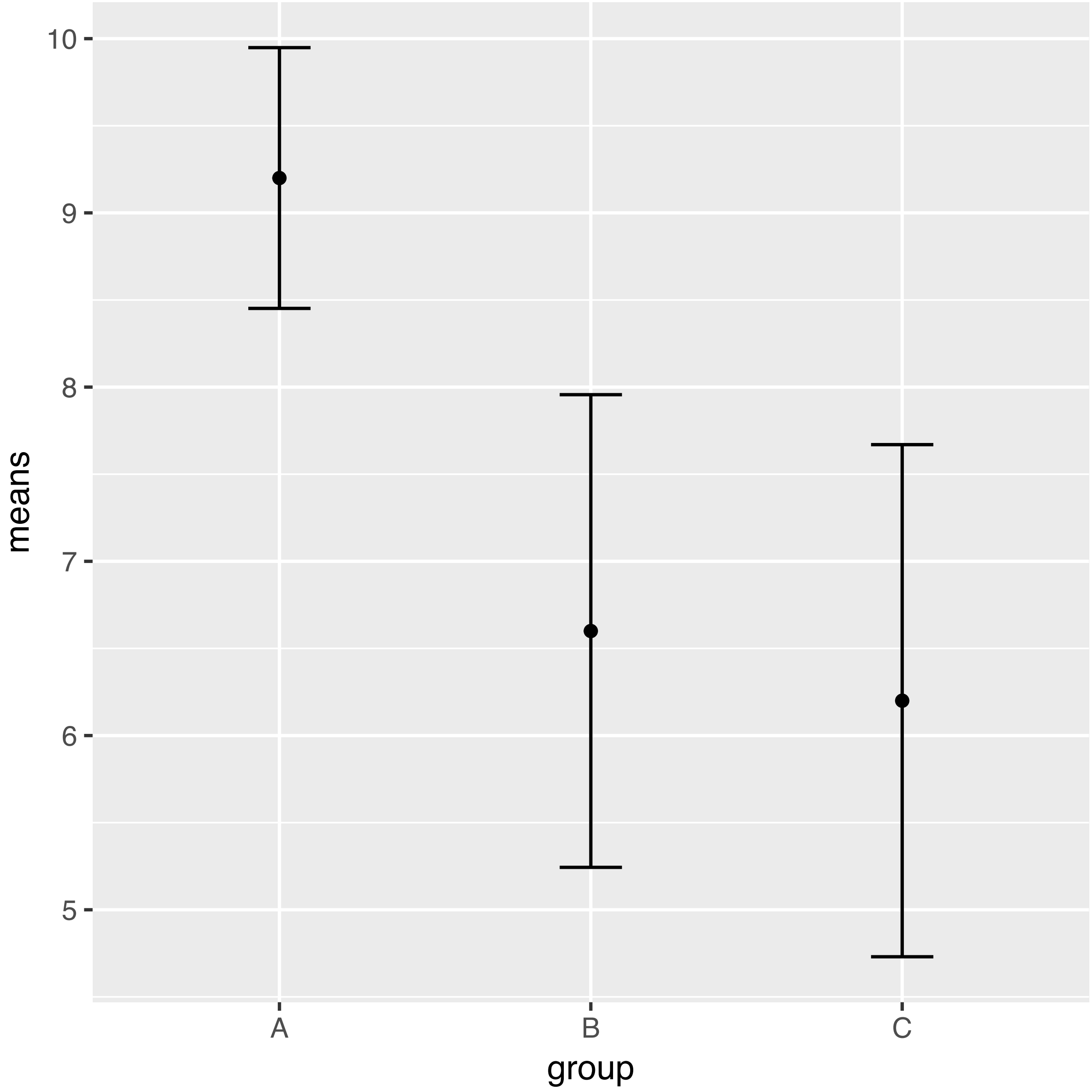
# R Code: ANOVA

Grabs the means and SE for each

```
df %>%
  dplyr::group_by(group) %>%
  dplyr::summarize(
    means = mean(outcome),
    ses   = sd(outcome)/sqrt(n())) %>%

  ggplot(aes(group, means)) +
    geom_point() +
    geom_errorbar(aes(ymin = means - 2*ses,
                      ymax = means + 2*ses),
                  width = .2)
```

Provides error bars that are approximately confidence intervals



# Measures of Association

- **Term preferred over "Effect size" for ANOVA**
  - Amount or % of variation in DV explained/accounted for by knowledge of group membership (IV)
  - Correlation between grouping variable (IV) and outcome variable (DV)

- **4 measures:**
  - Eta-squared ($\eta^2$)
  - Omega-squared ($\omega^2$)
  - Cohen's $f$
  - Intra-class Correlation Coefficients ($\rho$)

$\omega^2$ is least biased, but unfamiliarity and 'difficulty' of computation have limited use

$\eta^2$ probably sufficient in many cases

# Measures of Association: eta-squared

**$\eta^2$ : Measure of % reduction in error IN THIS DATA (SAMPLE)**

- $SS_{Total}$ = Error in DV around grand mean
- $SS_{Within}$ = Error around group means
- By knowing group membership we reduce error by $SS_{Between}$ = $SS_{Total} - SS_{Within}$
- % reduction in error expressed as: $\eta^2 = \frac{SS_B}{SS_T} = \frac{df_B \cdot F}{df_B \cdot F + df_W}$
- $\eta^2$ can be biased with sample data

- Adjusted $\eta^2 = 1 - \frac{MS_W}{MS_T}$

- Compute using information from ANOVA summary table
  - $\eta^2 = SS_B / SS_T$
  - $\eta^2_{adj} = 1 - (MS_W / MS_T)$

**Range: 0 to 1**
Small: .01 to .06
Medium: .06 to .14
Large: > .14

# Example: noise & words memorized

$df = (3-1, 15-3) = (\mathbf{2, 12})$

$MS_W = \mathbf{1.90} \xrightarrow{\text{"}SS = MS/df} SS_B = \mathbf{1.9 * (12) = 22.8}$

$F(2,12) = \dfrac{13.267}{1.90} = \mathbf{6.98}$

$MS_B = \mathbf{13.267} \xrightarrow{\text{"}SS = MS/df} SS_B = \mathbf{13.267 * (2) = 26.534}$

$$\eta^2 = \frac{SS_B}{SS_T} = \frac{df_B \cdot F}{df_B \cdot F + df_W}$$

## Using SS

$$\eta^2 = \frac{26.534}{26.534 + 22.8} = \mathbf{0.5378}$$

## Using F & df's

$$\eta^2 = \frac{2 \cdot 6.98}{2 \cdot 6.98 + 12} = \mathbf{0.5378}$$

# Example: noise & words memorized

$df = (3-1, 15-3) = (\mathbf{2}, \mathbf{12})$

$F(2,12) = \frac{13.267}{1.90} = \mathbf{6.98}$

$MS_W = \mathbf{1.90} \xrightarrow{"SS = MS/df} SS_B = \mathbf{1.9} * (\mathbf{12}) = \mathbf{22.8}$

$MS_B = \mathbf{13.267} \xrightarrow{"SS = MS/df} SS_B = \mathbf{13.267} * (\mathbf{2}) = \mathbf{26.534}$

$$\eta^2 = \frac{SS_B}{SS_T} = \frac{df_B \cdot F}{df_B \cdot F + df_W}$$

## *Using SS*

$$\eta^2 = \frac{26.534}{26.534 + 22.8} = \mathbf{0.5378}$$

## *Using F & df's*

$$\eta^2 = \frac{2 \cdot 6.98}{2 \cdot 6.98 + 12} = \mathbf{0.5378}$$

## Conclusion

The type of noise/music in the room accounts for 54% of the variation in the number of words each person was able to memorize

# Measures of Association: OMEGA-squared

- ***$\omega^2$* : Measure of % reduction in error IN THIS POPULATION (ESTIMATE TRUTH)**
- Alternative for "fixed-effects" ANOVA
  - More conservative than $\eta^2$ (and less biased)
  - Range: 0 to 1 (can be negative when $F < 1$)
    - Same interpretation as $\eta^2$
  - Compute using information from ANOVA summary table
    - Equation for fixed effects ANOVA only

$$\omega^2 = \frac{SS_B - (k-1)MS_W}{SS_T + MS_W} = \frac{(k-1)(F-1)}{(k-1)(F-1) + n_j \cdot k}$$

**Range: 0 to 1**
Small: .01 to .06
Medium: .06 to .14
Large: > .14

# Measures of association: Cohen's *f*

- **Traditional effect size index**
  - *Not a measure of association*
  - Generalization of Cohen's *d* to ANOVA
  - Compute using ANOVA summary information

$$f = \sqrt{\frac{\omega^2}{1-\omega^2}} = \sqrt{\frac{\frac{k-1}{n_j \cdot k}\left(MS_B - MS_W\right)}{MS_W}}$$

- Converting from $f$ to $\omega^2 \rightarrow$

$$\omega^2 = \frac{f^2}{1+f^2}$$

# Measures of Association: Intra-class correlation coefficient (ICC)

- Measure of association for random-effects ANOVA

- At least 6 *ICCs* available
  - Type selected depends on data structure
- Range: 0 to 1
  - Commonly used measure of agreement for continuous data

- Basic form: $$\rho_{\text{intraclass}} = \frac{MS_B - MS_W}{MS_B + (n_j - 1)MS_W}$$

  - Measures extent to which observations within a treatment are similar to one another relative to observations in different treatments

# APA Results

***Methods***

- Describe statistical and sample size analyses
- Describe factor and its levels
- Results of data screening

***Results***

- Reporting *F*-test:
  - $F(df_B, df_W)$ = *F*-statistic, $p$ = / <, measure of association and effect/effect size, power (optional)
- Don't need to include *MSE* (or $MS_W$) as Cohen suggests
- Discuss any follow-up tests, if any (next lecture)

**Method**

"A one-way ANOVA was used to test the hypothesis that the means of the three groups (Control, Moderate Noise, and Extreme Noise) were different following the experiment. A sample size analysis conducted prior to beginning the study indicated that five participants per group would be sufficient to reject the null hypothesis with at least 80% power if the effect size were moderate (Cohen's $f = .95$)."

**Results**

"Results indicated a significant difference among the group means, $F(2, 12) = 6.98$, $p < .01$, $\omega^2 = .44$"

# ANOVA vs. multiple t-tests

- Why not run series of independent-samples *t*-tests?

- Could, and will usually get same results, but this approach becomes more difficult under 2 conditions:
  - Large *k*
    - $k(k-1) / 2$ different *t*-tests!
  - Factorial designs

- Danger of increased risk of Type I error when conducting multiple *t*-tests on same data set
  - *In next lecture we explain ways to potentially limit this risk*

# Power: use G*Power

# Logic of "anova"

- In ANOVA, 2 independent estimates of same population (error) variance are computed: $\sigma^2$, now called $\sigma_e^2$
  - $MS_B$ : Variance between group means corrected by sample sizes ($n_j$)
  - $MS_W$ : Average variance within groups
- Ratio of *2* estimates of population variance
- Hence the term *Analysis of Variance,* instead of something related to means comparisons (even though that is what we are interested in doing)
- Increased **variance among means** indicates means are **spread out** & likely differ from one another or come from different populations
- Large *F*-ratio indicates differences among means is NOT likely due to chance

$$F_{Ratio} \text{ or } F_{Statistic} = \frac{MS_B}{MS_W}$$

**ANOVA is**

$$\frac{\textbf{Between-Group Measure of Variation Due to Estimate of Random Variation (Error)} + \textbf{Effect of IV (Group)}}{\textbf{Within-Group Estimate of Random Variation (Error)}}$$