

Guidelines and Rules:

- 1) You can attempt this project in a group of three members. Please restrict your collaborations within your group. You are allowed to collaborate in problem formulation, approach and code development.
- 2) **Each group will turn in a report in a pdf format.** The report will contain problem formulation, solution methodology and discussion of results.
- 3) Each report should clearly indicate the contribution of each group member in percentage.
- 4) Each group member should sign the honor statement indicating that **they have neither given nor received assistance on this project.**

I. PROJECT STATEMENT: MANEUVER RECONSTRUCTION

Maneuver reconstruction is an important part of the astrodynamics suite of applications. Specially in sparse data environments, or in the event that the spacecraft is performing multiple maneuvers, identifying how much delta-v was expended is an important metric in characterizing the capabilities of a satellite. In this project, we will idealize a simplified maneuver reconstruction experiment.

Consider a spacecraft that is continuously being tracked. The ground station loses tracking when a maneuver is performed, but has regained its custody at a later time. (*Maneuver detection and tracking is another important and difficult problem in astrodynamics. For this project, let us assume that this problem is solved*). A schematic of one such maneuver reconstruction is shown as follows:

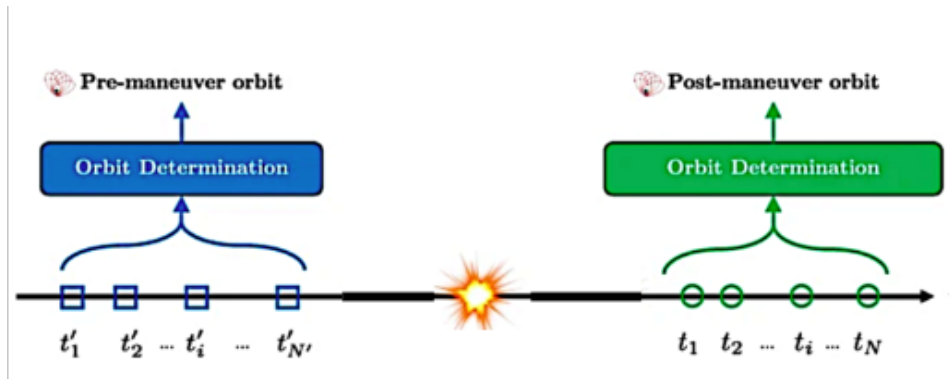


Fig. 1. Maneuver Reconstruction Schematic

We will utilize Gauss' initial orbit determination to identify the orbit that the spacecraft occupies before and after the maneuver and calculate how much delta-v was used to transfer between them.

II. ANALYTICAL PART: COMPUTING THE MANEUVER

Given the pre-maneuver and post-maneuver orbit parameters, compute the delta-v used to transfer from one to the other. Let us assume a coplanar case for this project, i.e. the inclination and RAAN of the pre and post-maneuver orbit are the same. However, the argument of perigee may be different.

In such a case, let us suppose that we adopt the perifocal reference frame for the first ellipse. In this reference frame, then, the argument of periapsis of the second ellipse is rotated counter-clockwise from the x-axis by an angle $\Delta\omega$ - where $\Delta\omega$ is the difference in the argument of periapsis of the two co-planar orbits.

So, the radial distance of a satellite on the first orbit can be thought of as:

$$r_1 = \frac{p_1}{1 + e_1 \cos f}$$

and that on the second orbit is

$$r_2 = \frac{p_2}{1 + e_2 \cos(f - \Delta\omega)}$$

If the two orbits intersect, the two radial distances must be equal:

$$r_1 = r_2 = r$$

Our aim in this analytical part of the project is to find the value of the radius where the two orbits intersect. This leads to a quadratic equation in the free variable r . Let's go about this step-by-step.

A. Derivation of radius of intersecting orbits

- 1) Using the first relation:

$$r_1 = \frac{p_1}{1 + e_1 \cos f}$$

rearrange to identify an expression for $\cos f$. Also, using trigonometric identities, identify an expression for $r^2 \sin^2 f$. We will keep these quantities aside, as we will be using them at a later step.

- 2) Using the relation

$$r_2 = \frac{p_2}{1 + e_2 \cos(f - \Delta\omega)}$$

rearrange to identify an expression for $\cos(f - \Delta\omega)$

- 3) Expand the above expression using the trigonometric identity for $\cos(A - B)$ and rearrange to find an expression for $r \sin f$. At this stage, you may also substitute the expression for $\cos f$ from step 1.

On performing this step, you should find the resulting expressions contain the terms:

$$\alpha = e_2 \cos \Delta\omega - e_1$$

$$\beta = e_1 p_2 - e_2 p_1 \cos \delta\omega$$

$$\gamma = e_1 e_2 \sin \Delta\omega$$

you may use these placeholder variables (α, β, γ) to simplify your math process.

- 4) Now, you may square the resulting expression on either side to obtain another expression for $r^2 \sin^2 f$ i.e., you will obtain an expression as:

$$r^2 \sin^2 f =$$

- 5) Plug in your relation for $r^2 \sin^2 f$ from step 1. At this stage, we have eliminated true anomaly from the equations.
- 6) You will notice that you have a quadratic expression in r .

$$ar^2 + br + c = 0$$

Collect the terms that multiply r^2 , r , and the constant term, i.e. a, b, c . These are the coefficients you will use to compute the roots of a quadratic equation:

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

As the final step, you should obtain something of the form:

$$a = \frac{e_1^2 - 1}{e_1^2} - \frac{\alpha^2}{\gamma^2}$$

$$b = \frac{2p_1}{e_1^2} - \frac{2\alpha\beta}{\gamma^2}$$

$$c = -\left(\frac{p_1^2}{e_1^2} + \frac{\beta^2}{\gamma^2}\right)$$

B. Testing the algorithm

Now, consider two sets of orbital elements of two spacecraft:

	Orbit 1	Orbit 2
semimajor axis (km)	13000	7226.58
eccentricity	0.3	0.444819
inclination (deg)	20	20.0
RAAN (deg)	30	30.0
AOP (deg)	50	301.901

- 1) Identify the radius at the intersecting point.
- 2) Compute the position and velocity vectors at these points on the two orbits. Note, that position vectors will be the same, but the velocity vectors may be different.
- 3) Compute the delta-v at the intersection point to transfer from one orbit to another.