Project 2: Maneuver Reconstruction

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I. PROJECT STATEMENT: MANEUVER RECONSTRUCTION

Maneuver reconstruction is an important part of the astrodynamics suite of applications. Specially in sparse data environments, or in the event that teh spacecraft is performing multiple maneuvers, identifying how much delta-v was expended is an important metric in characterizing the capabilities of a satelltie. In this project, we will idealize a simplified maneuver reconstruction experiment.

Consider a spacecraft that is continuously being tracked. The ground station looses tracking when a maneuver is performed, but has regained its custody at a later time. (Maneuver detection and tracking is another important and difficult problem in astrodynamic. For this project, let us assume that this problem is solved). A schematic of one such maneuver reconstruction is shown as follows:

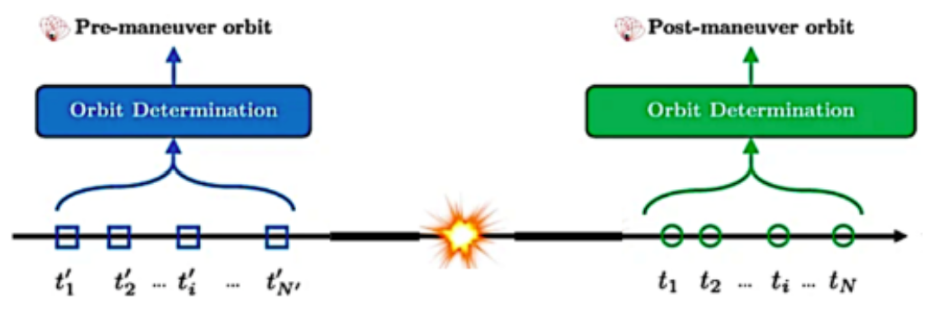

Figure 1. Maneuver Reconstruction Schematic

We will utilize Gauss' initial orbit determination to identify the orbit that the spacecraft occupies before and after the maneuver and calculate how much delta-v was used to transfer between them.

II. ANALYTICAL PART: COMPUTING THE MANEUVER

Given the pre-maneuver and post-maneuver orbit parameters, compute the delta-v used to transfer from one to the other. Let us assume a coplanar case for this project, i.e. the inclination and RAAN of the pre and post-maneuver orbit are the same. However, the argument of perigee may be different.

In such a case, let us suppose that we adopt the perifocal reference frame for the first ellipse. In this reference frame, then, the argument of periapsis of the second ellipse is rotated counter-clockwise from the x-axis by an angle $\Delta\omega$ - where $\Delta\omega$ is the difference in the argument of periapsis of the two co-planar orbits.

So, the radial distance of a satellite on the first orbit can be thought of as:

$$r_1 = \frac{p_1}{1 + e_1 cos(f)} \tag{1}$$

and that on the second orbit is:

$$r_2 = \frac{p_2}{1 + e_2 cos(f - \Delta\omega)} \tag{2}$$

If the two orbits intersect, the two radial distances must be equal:

$$r_1 = r_2 = r$$

Our aim in this analytical part of the project is to find the value of the radius where the two orbits intersect. This leads to a quadratic equation in the free variable r. Let's go about this step-by-step.

A. Derivation of radius of intersecting orbits

1. Using the first relation:

$$r_1 = \frac{p_1}{1 + e_1 cos(f)}$$

rearrange to identify an expression for $cos(f)$. Also, using trigonometric identities, identify an expression for $r^2 sin^2(f)$. We will keep these quantities aside, as we will be using them at a later step.

SOLUTION:

Using $r_1 = r$ and solving for $cos(f)$:

$$cos(f) = \frac{1}{e_1}\left(\frac{p_1}{r} - 1\right) \tag{3}$$

Solving for $r^2 sin^2(f)$ using Pythagoras' theorem ($sin^2(f) = 1 - cos^2(f)$) and squaring the previous, rearranged expression:

$$r^2 sin^2(f) = r^2(1 - cos^2(f))$$

$$r^2 sin^2(f) = r^2 \left(1 - \left(\frac{1}{e_1}\left(\frac{p_1}{r} - 1\right)\right)^2\right)$$

Expanding this expression (this will be helpful for step 5):

$$r^2 sin^2(f) = \left(\frac{e_1^2 - 1}{e_1^2}\right) r^2 + \left(\frac{2p_1}{e_1^2}\right) r + \left(-\frac{p_1^2}{e_1^2}\right) \tag{4}$$

2. Using the relation:

$$r_2 = \frac{p_2}{1 + e_2 cos(f - \Delta\omega)}$$

rearrange to identify an expression for $cos(f - \Delta\omega)$

SOLUTION:

Repeat the first part of step 1 but replacing subscripts and substituting $cos(f - \Delta\omega)$ for $cos(f)$:

$$cos(f - \Delta\omega) = \frac{1}{e_2}\left(\frac{p_2}{r} - 1\right) \tag{5}$$

3. Expand the above expression using the trigonometric identity for $cos(A - B)$ and rearrange to find an expression for $rsin(f)$. After this step, you may also substitute the expression for $cos(f)$ from step 1.
On performing this step, you should find the resulting expressions contain the terms:

$$\alpha = e_2 cos(\Delta\omega) - e_1$$

$$\beta = e_1 p_2 - e_2 p_1 cos(\Delta\omega)$$

$$\gamma = e_1 e_2 sin(\Delta\omega)$$

you may use these placeholder variables (α, β, γ) to simplify your math process.

SOLUTION:

Expanding the resulting expression from Equation (5) and solving for $sin(f)$:

$$cos(f - \Delta\omega) = cos(f)cos(\Delta\omega) - sin(f)sin(\Delta\omega)$$

$$sin(f) = \frac{cos(f - \Delta\omega) - cos(f)cos(\Delta\omega)}{sin(\Delta\omega)}$$

Multiplying r to both sides:

$$rsin(f) = r\left(\frac{cos(f - \Delta\omega) - cos(f)cos(\Delta\omega)}{sin(\Delta\omega)}\right)$$

Substituting the results from steps 1 and 2:

$$rsin(f) = r\left(\frac{\frac{1}{e_2}\left(\frac{p_2}{r} - 1\right) - \frac{1}{e_1}\left(\frac{p_1}{r} - 1\right)cos(\Delta\omega)}{sin(\Delta\omega)}\right)$$

Simplifying by factoring out $\frac{1}{r}$ in the numberator and multiplying the numberator and denominator by e_1 and e_2:

$$rsin(f) = \left(\frac{e_1(p_2 - r) - e_2(p_1 - r)cos(\Delta\omega)}{e_1 e_2 sin(\Delta\omega)}\right)$$

$$rsin(f) = \left(\frac{(e_2 cos(\Delta\omega) - e_1)r + e_1 p_2 - e_2 p_1 cos(\Delta\omega)}{e_1 e_2 sin(\Delta\omega)}\right)$$

Substituting for the placeholder variables mentioned earlier in this step:

$$rsin(f) = \frac{\alpha r + \beta}{\gamma} \tag{6}$$

4. Now, you may square the resulting expression on either side to obtain another expression for $r^2 sin^2(f)$ i.e., you will obtain an expression as:

$$r^2 sin^2(f) =$$

SOLUTION:

Squaring both sides of the resulting expression from Equation (6):

$$r^2 sin^2(f) = \frac{(\alpha r + \beta)^2}{\gamma^2}$$

Expanding this expression (this will be helpful for the next step):

$$r^2 sin^2(f) = \frac{\alpha^2}{\gamma^2} r^2 + 2\frac{\alpha\beta}{\gamma^2} r + \frac{\beta^2}{\gamma^2} \tag{7}$$

5. Plug in your relation for $r^2 sin^2(f)$ from step 1. At this stage, we have eliminated true anomaly from the equations.

SOLUTION:

Equating the results for $r^2 sin^2(f)$ from Equation (4) and Equation (7):

$$\left(r^2 sin^2(f)\right)_{step1} = \left(r^2 sin^2(f)\right)_{step4}$$

$$\left(\frac{e_1^2 - 1}{e_1^2}\right) r^2 + \left(\frac{2p_1}{e_1^2}\right) r + \left(-\frac{p_1^2}{e_1^2}\right) = \left(\frac{\alpha^2}{\gamma^2}\right) r^2 + \left(\frac{2\alpha\beta}{\gamma^2}\right) r + \left(\frac{\beta^2}{\gamma^2}\right) \tag{8}$$

6. You will notice that you have a quadratic expression in r.

$$ar^2 + br + c = 0$$

Collect the terms that multiply r^2, r, and the constant term, i.e. a, b, c. These are the coefficients you will use to compute the roots of a quadratic equation:

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

As the final step, you should obtain something of the form:

$$a = \frac{e_1^2 - 1}{e_1^2} - \frac{\alpha^2}{\gamma^2}$$

$$b = \frac{2p_1}{e_1^2} - \frac{2\alpha\beta}{\gamma^2}$$

$$c = -\left(\frac{p_1^2}{e_1^2} + \frac{\beta^2}{\gamma^2}\right)$$

SOLUTION:

Collecting terms from Equation (8):

$$\left(\frac{e_1^2 - 1}{e_1^2}\right) r^2 + \left(\frac{2p_1}{e_1^2}\right) r + \left(-\frac{p_1^2}{e_1^2}\right) = \left(\frac{\alpha^2}{\gamma^2}\right) r^2 + \left(\frac{2\alpha\beta}{\gamma^2}\right) r + \left(\frac{\beta^2}{\gamma^2}\right)$$

$$\left(\frac{e_1^2 - 1}{e_1^2} - \frac{\alpha^2}{\gamma^2}\right) r^2 + \left(\frac{2p_1}{e_1^2} - \frac{2\alpha\beta}{\gamma^2}\right) r + \left(-\left(\frac{p_1^2}{e_1^2} + \frac{\beta^2}{\gamma^2}\right)\right) = 0$$

Equating the previous result to a quadratic expression with constants a, b, and c ($a^2r + br + c = 0$):

$$a = \frac{e_1^2 - 1}{e_1^2} - \frac{\alpha^2}{\gamma^2}$$

$$b = \frac{2p_1}{e_1^2} - \frac{2\alpha\beta}{\gamma^2}$$

$$c = -\left(\frac{p_1^2}{e_1^2} + \frac{\beta^2}{\gamma^2}\right)$$

B. Testing the algorithm

Now, consider two sets of orbital elements of two spacecraft:

	Orbit 1	Orbit 2
semimajor axis (km)	13000	7226.58
eccentricity	0.3	0.444819
inclination (deg)	20	20
RAAN (deg)	30	30
AOP (deg)	50	301.901

1. Identify the radius at the intersecting point.

   SOLUTION:

   Calculating semi-latus rectum (in km) using given eccentricities and semimajor axes for orbits 1 and 2 (p_1 and p_2 respectively):

$$p_n = a_n\left(1 - e_n^2\right)$$

$$p_1 = 13000\left(1 - (0.3)^2\right) = 11830$$

$$p_2 = 7226.58\left(1 - (0.444819)^2\right) = 5796.7$$

   Calculating the difference in the argument of periapsis ($\Delta\omega$) between the two orbits:

$$\Delta\omega = \omega_2 - \omega_1 = 301.901^\circ - 50^\circ = 251.901^\circ$$

   Calculating the constants from part A, step 3 (α, β, γ):

$$\alpha = e_2 cos(\Delta\omega) - e_1 = -0.4382$$

$$\beta = e_1 p_2 - e_2 p_1 cos(\Delta\omega) = 3373.767$$

$$\gamma = e_1 e_2 sin(\Delta\omega) = -0.1268$$

   Calculating the constants from part A, step 6 (a, b, and c):

$$a = \frac{e_1^2 - 1}{e_1^2} - \frac{\alpha^2}{\gamma^2} = -22.045$$

$$b = \frac{2p_1}{e_1^2} - \frac{2\alpha\beta}{\gamma^2} = 446657.9$$

$$c = -\left(\frac{p_1^2}{e_1^2} + \frac{\beta^2}{\gamma^2}\right) = -2262440535.1$$

Calculating the radius using the relation obtained from part A, step 6:

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where the two possible solutions are $r = 10129.35$ km and $r = 10131.71$ km. Both values are valid solutions of where the orbits intersect, so we chose to continue using $r = 10129.35$ km.

2. Compute the position and velocity vectors at these points on the two orbits. Note, that position vectors will be the same, but the velocity vectors may be different.

SOLUTION:

Calculating $cos(f)$ using Equation (3):

$$cos(f) = \frac{1}{e_1}\left(\frac{p_1}{r} - 1\right) = 0.5596$$

Calculating $sin(f)$ using Equation (6):

$$sin(f) = \frac{\alpha r + \beta}{r\gamma} = 0.8287$$

Solving for f using a quadrant check between the two relations above:

$$f = cos^{-1}(0.5596) = 55.9689°/-55.9689°$$

$$f = sin^{-1}(0.8287) = 55.9689°/124.0311°$$

Using the value that is similar, $f = 55.9689°$. Finding the position vector in the orbital reference frame of the first orbit:

$$\vec{r}_{orbital} = [r \quad 0 \quad 0]^T = [10129.35 \quad 0 \quad 0]^T$$

Calculating the magnitude of angular momentum (h) of each orbit ($\mu_e = 398600.4418 \frac{km^3}{s^2}$):

$$h_n = \sqrt{\mu_e p_n}$$
$$h_1 = \sqrt{\mu_e p_1} = 68669.08$$
$$h_2 = \sqrt{\mu_e p_2} = 48068.36$$

There will be two velocity vectors at the intersection, one for each orbit. Calculating the radial and tangential components of each velocity (v_r and v_θ respectively) in the orbital reference frame of the first orbit (all in $\frac{km}{s}$):

$$v_{r,1} = \frac{h_1}{r} \frac{e_1 sin(f)}{1 + e_1 cos(f)} = 1.44315$$

$$v_{r,2} = \frac{h_2}{r}\frac{e_2 sin(f-\Delta\omega)}{1+e_2 cos(f-\Delta\omega)} = 1.01251$$

$$v_{\theta,1} = \frac{h_1}{r} = 6.77922$$

$$v_{\theta,2} = \frac{h_2}{r} = 4.74545$$

Constructing the velocity vectors in the orbital frame:

$$\vec{v}_{orbital,n} = [v_{r,n} \quad v_{\theta,n} \quad 0]^T$$

$$\vec{v}_{orbital,1} = [v_{r,1} \quad v_{\theta,1} \quad 0]^T = [1.44315 \quad 6.77922 \quad 0]^T$$
$$\vec{v}_{orbital,2} = [v_{r,2} \quad v_{\theta,2} \quad 0]^T = [1.01251 \quad 4.74545 \quad 0]^T$$

Converting the three vectors from orbital frame to inertial (ECI) frame by using a 3-1-3 rotation by angles $(\omega + f)$, I, Ω:

$$\vec{r}_{orbital} = \boldsymbol{C}_{ON}\vec{r}_{ECI}$$

$$\vec{r}_{ECI} = \boldsymbol{C}_{ON}^{-1}\vec{r}_{orbital} \tag{9}$$

where $\boldsymbol{C}_{ON}$ is:

$$\boldsymbol{C}_{ON} = \begin{bmatrix} \cos(\omega+f) & \sin(\omega+f) & 0 \\ -\sin(\omega+f) & \cos(\omega+f) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(I) & \sin(I) \\ 0 & -\sin(I) & \cos(I) \end{bmatrix} \begin{bmatrix} \cos(\Omega) & \sin(\Omega) & 0 \\ -\sin(\Omega) & \cos(\Omega) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

I is the inclination, where $I = 20°$ for both orbits. Ω is right ascension of the ascending node, where $\Omega = 30°$ for both orbits. f is the true anomaly relative to the first orbit, where $f = 55.9689°$ for both orbits. ω is the argument of perigee, where $\omega_1 = 50°$ for the first orbit and $\omega_2 = 301.901°$ for the second orbit.

Calculating $\boldsymbol{C}_{ON}^{-1}$ for both orbits:

$$\boldsymbol{C}_{ON,1}^{-1} = \begin{bmatrix} -0.6900 & -0.7033 & 0.1710 \\ 0.6448 & -0.7046 & -0.2962 \\ 0.3288 & -0.0941 & 0.9397 \end{bmatrix}$$

$$\boldsymbol{C}_{ON,2}^{-1} = \begin{bmatrix} 0.8829 & -0.4373 & 0.1710 \\ 0.4694 & 0.8318 & -0.2962 \\ -0.0127 & 0.3418 & 0.9397 \end{bmatrix}$$

Obtaining position and velocity vectors using equation (9):

$$\vec{r}_{ECI} = \boldsymbol{C}_{ON}^{-1}\vec{r}_{orbital} = [-6988.98 \quad 6531.78 \quad 3330.75]^T$$

$$\vec{v}_{ECI,1} = \boldsymbol{C}_{ON,1}^{-1}\vec{v}_{orbital,1} = [-5.76386 \quad -3.84599 \quad -0.16335]^T$$
$$\vec{v}_{ECI,2} = \boldsymbol{C}_{ON,2}^{-1}\vec{v}_{orbital,2} = [-4.03629 \quad -2.69071 \quad -0.11359]^T$$

3. Compute the delta-v at the intersection point to transfer from one orbit to another.

   SOLUTION:

   Calculating the delta-v (ΔV):

$$\Delta V = |\vec{v}_{ECI,2} - \vec{v}_{ECI,1}|$$

$$\Delta V = |[-1.72757 \quad -1.15529 \quad -0.04976]^T|$$

$$\Delta V = \sqrt{(-1.72757)^2 + (-1.15529)^2 + (-1.15529)^2}$$

$$\Delta V = 2.0789\,\frac{km}{s}$$

III. NUMERICAL PART

Now that you have performed the analytic part, the numerical part will have you implement an Initial Orbit Determination Algorithm. The following data has been provided to you:

DATE	AZIMUTH	ELEVATION	Rx	Ry	Rz
00:12:10:27 AM	48.699	-51.221	-1522.8	-5212.8	3386.4
00:12:10:31 AM	48.684	-51.054	-1521	-5213.3	3386.4
00:12:10:36 AM	48.67	-50.887	-1519.1	-5213.9	3386.4
00: 2:28:45 AM	64.042	11.984	1704.6	-5156.2	3386.4
00: 2:28:55 AM	64.116	12.166	1708.2	-5155	3386.4
00: 2:29:04 AM	64.19	12.347	1711.7	-5153.8	3386.4
00: 3:19:35 AM	112.87	49.16	2797	-4655	3386.4
00: 3:19:45 AM	113.13	49.189	2800.3	-4653	3386.4
00: 3:19:54 AM	113.38	49.217	2803.5	-4651.1	3386.4
00: 7:55:43 AM	100.67	66.224	5347.6	946.06	3386.4
00: 7:56:04 AM	100.9	66.283	5346.2	954.16	3386.4
00: 7:56:24 AM	101.12	66.342	5344.7	962.25	3386.4
00:10:15:36 AM	-161.62	47.097	3839.8	3840.3	3386.4
00:10:15:57 AM	-161.48	46.93	3834	3846.1	3386.4
00:10:16:17 AM	-161.35	46.762	3828.2	3851.9	3386.4
00: 2:24:49 PM	40.907	50.536	-1615.9	5184.7	3386.4
00: 2:25:21 PM	40.959	50.637	-1627.9	5180.9	3386.4
00: 2:25:53 PM	41.012	50.738	-1639.9	5177.2	3386.4
01: 7:47:29 AM	13.125	-64.381	5378.2	753.3	3386.4
01: 7:48:40 AM	13.344	-64.115	5374.2	781.21	3386.4
01: 7:49:51 AM	13.56	-63.848	5370.1	809.1	3386.4
01: 3:43:22 PM	46.265	56.699	-3264.3	4340.1	3386.4
01: 3:44:34 PM	46.427	56.971	-3286.8	4323.1	3386.4
01: 3:45:45 PM	46.591	57.244	-3309.2	4306	3386.4

Figure 2. Measurement Data

In this data,

1. The date and time of the observation is recorded

2. The Azimuth and Elevation values of the satellite expressed in a Topocentric Equatorial Frame are provided in degrees
3. The telescope's position (on the Earth's surface) is provided in the inertial frame in km. You may be interested to know that the telescope side is located at latitude = 31.9466 N and longitude = 108.8977 W, which is also the proposed location of the Penn State University Dynamical Observatory (PSUDO) lab (check it out on Google maps!)

You may also load this data in using the .mat file attached using the matlab command:

load('IODMeasurements.mat')

A. Your tasks are the following:

1. Taking 3 measurements at a time in sequence, perform the Gauss' method of Initial Orbit Determination to obtain the value of $\boldsymbol{r}_1$, $\boldsymbol{r}_2$, and $\boldsymbol{r}_3$ of the satellite.

   SOLUTION:

   Gauss' Method for Initial Orbit Determination was used to find the position vectors from the given dataset. The multiple solutions for data set 7 and data set 8 are explained in task 2. Below are three tables of the three position vectors obtained using Gauss' Method.

Dataset	$\boldsymbol{r}_{1,x}$	$\boldsymbol{r}_{1,y}$	$\boldsymbol{r}_{1,z}$
1	3457.9	456.5	-6006.4
2	7943.6	7659.6	6412.1
3	-884.5	4073.1	14345.4
4	3391.9	11323.5	27356.6
5	-10566.6	-947.7	19721.8
6	12872.5	17737.9	26671.5
7a	-54161.3	-13129.4	130877.6
7b	-16248.9	-4289.4	49696.2
7c	23032.2	4869.7	-34415.9
8a	121425.7	134662.5	277952.1
8b	11438.0	19706.5	35760.7
8c	-19200.7	-12316.2	-31705.4

Table 1. Obtaining $\boldsymbol{r}_1$ using Gauss' Method

Dataset	$\boldsymbol{r}_{2,x}$	$\boldsymbol{r}_{2,y}$	$\boldsymbol{r}_{2,z}$
1	3482.6	479.0	-5990.6
2	7928.3	7663.9	6458.1
3	-916.9	4050.9	14346.8
4	3354.0	11303.6	27377.0
5	-10587.2	-983.8	19655.6
6	12845.2	17744.2	26749.4
7a	-54646.4	-13456.3	130506.8
7b	-16428.0	-4390.5	49562.3
7c	23170.7	5002.7	-34305.6
8a	120153.2	134070.5	278853.6
8b	11266.5	19620.0	35863.3
8c	-19066.1	-12262.5	-31826.5

Table 2. Obtaining r_2 using Gauss' Method

Dataset	$\boldsymbol{r}_{3,x}$	$\boldsymbol{r}_{3,y}$	$\boldsymbol{r}_{3,z}$
1	3507.3	501.5	-5974.6
2	7912.8	7668.1	6504.0
3	-949.3	4028.6	14348.1
4	3316.2	11283.7	27397.1
5	-10607.5	-1019.8	19589.2
6	12817.7	17750.3	26827.1
7a	-55131.5	-13783.1	130135.9
7b	-16606.9	-4491.5	49427.7
7c	23308.5	5135.6	-34194.2
8a	118880.7	133478.4	279755.1
8b	11094.7	19532.9	35965.0
8c	-18930.8	-12208.3	-31946.5

Table 3. Obtaining r_3 using Gauss' Method

2. **Hint: Recall that Gauss' method requires you to find the roots of a 8th degree polynomial. You may use the roots() command in matlab to do so.** Also note that Of the 8 roots, 4 are imaginary, and you can reject them. But there will be 4 real roots. Negative roots can also be rejected. However, if you obtain three positive real roots, you have to now figure out which of the roots is a viable value for $|\boldsymbol{r}_2|$. You may need to identify all the orbits involved to make this decision.

   SOLUTION:

   In the dataset given, it can be seen that observatory measurements were recorded in sets of 3 measurements and 8 total sets were recorded. With the given data, Gauss' method of Initial Orbit Determination can be used to obtain three position vectors of the satellite: $\boldsymbol{r}_1$, $\boldsymbol{r}_2$, and $\boldsymbol{r}_3$. In this algorithm, an 8th degree polynomial involving the magnitude of the $\boldsymbol{r}_2$ vector needs to be solved which results in 8 roots. Immediately, complex roots and negative real roots can be rejected as those are not realistic values for a radius magnitude. For most of the sets of data in this project, this results in one remaining root. However, in two of the sets of data, this resulted in three valid roots. In Table 4, the valid roots of the 8th degree polynomial can be found.

Dataset	$\lvert\boldsymbol{r}_2\rvert$
1	6945.9
2	12778.9
3	14935.9
4	29808.0
5	22347.2
6	34574.4
7a	142124.3
7b	52398.3
7c	41698.7
8a	331920.2
8b	42403.5
8c	39074.4

Table 4. 8th degree polynomial roots

   From the valid roots above, the position vectors were calculated using Gauss' method. For the 7th and 8th dataset, a position vector was calculated for each possible root. These possibilities were later analyzed for their validity by analyzing the orbit. The position vectors from the observations can be seen in the tables from task 1.

3. Using the three position vectors you have obtained by employing the Gauss' method, perform Gibb's method to find the position and velocity of the middle measurement. *(Note that there are 24 measurements, so you will have to Perform Gauss method 8 times. It may be useful to write a function that does this.)*

   SOLUTION:

Next, using these positions, a velocity vector was calculated utilizing Gibb's method. This method requires the three position vectors previously calculated to calculate the corresponding velocity at position 2, $\boldsymbol{v}_2$. These values can be seen in the followng table.

Dataset	$\boldsymbol{v}_{2,x}$	$\boldsymbol{v}_{2,y}$	$\boldsymbol{v}_{2,z}$
1	5.078	4.624	3.266
2	-1.615	0.445	4.819
3	-3.399	-2.334	0.141
4	-1.817	-0.954	0.972
5	-0.982	-1.729	-3.183
6	-0.862	0.196	2.446
7a	-0.862	0.196	2.446
7b	0.000	0.000	0.000
7c	1.935	1.863	1.553
8a	-0.862	0.196	2.446
8b	-2.405	-1.216	1.431
8c	1.935	1.863	1.553

Table 5. Velocity Vector at Position 2 for Each Dataset

In this table it can immediately be seen that the 7b velocity of all zeros is not realistic and can be rejected. To determine whether or not the other solutions were valid for both the 7th and 8th datasets, the orbits were plotted. To plot the orbits, the two-body problem was solved using MATLAB's ode45 function with the initial conditions being the $\boldsymbol{r}_2$ and corresponding $\boldsymbol{v}_2$ previously calculated.

In the first plots analyzed, the first seven sets of data were used. It was shown that the 7a values resulted in a plot with imaginary numbers which is not realistic. With the 7b values already being rejected due having no velocity, the 7c values was chosen to be the correct orbit from the dataset given. In addition, the orbit from the 7c values were confirmed because it intersects the previous orbit.

The same process was used to evaluate the validity of the three possibilities for the 8th dataset. The 8a values resulted in an orbit with imaginary numbers which is not realistic. The 8c values resulted in an orbit that did not intersect the previous orbit meaning it would not be possible for there to be one delta V maneuver. It would be possible with two delta V maneuvers, but that is outside the scope of this project. Therefore, the 8b values were chosen. In addition, the orbit from the 8b values confirmed because there was no orbital maneuver from the previous orbit (7c).

4. Convert this position and velocity pair to orbital elements.

SOLUTION:

The orbital elements were then calculated for each of the datasets to determine where the orbital transfers occurred. These values can be seen in the following table.

Dataset	Semi-major axis, a (km)	Eccentricity, e	Inclination, I (rad)	RAAN (rad)	AOP (rad)
1	7000.0	0.010	1.309	0.611	-1.798
2	10965.0	0.368	1.309	0.611	-1.798
3	10965.0	0.368	1.309	0.611	-1.798
4	18464.8	0.625	1.309	0.611	-1.798
5	18465.1	0.625	1.309	0.611	-1.798
6	24465.0	0.717	1.309	0.611	-1.798
7	41999.7	0.010	1.309	0.611	-1.798
8	41999.9	0.010	1.309	0.611	-1.798

Table 6. Orbital Elements for Each Dataset

In this table it can be seen that the orbital maneuvers occurred between dataset 1 and 2, 3 and 4, 5 and 6, and 6 and 7. In total, there are four orbital maneuvers. The orbital elements of the unique orbits can be found in the table of task 5. Additionally, datasets 2/3, 4/5, and 7/8 have the same orbital elements; the table in task 5 was reduced to avoid redundancy.

5. After performing these three tasks above, how many unique orbits do you observe? Enter your answer in the following form:

   SOLUTION:

   In total, there were four orbital maneuvers and five unique orbits. The orbital elements of the five orbits can be found in the table below.

Orbit	Semi-major axis, a (km)	Eccentricity, e	Inclination, I (rad)	RAAN (rad)	AOP (rad)
1	7000.0	0.010	1.309	0.611	-1.798
2	10965.0	0.368	1.309	0.611	-1.798
3	18465.0	0.625	1.309	0.611	-1.798
4	24465.0	0.717	1.309	0.611	-1.798
5	41999.8	0.010	1.309	0.611	-1.798

Table 7. Orbital Elements for Each Unique Orbit

6. Plot all of these orbits in one plot. Just by observation can you identify the potential locations of the delta-v maneuvers? Show them on the plot. Also, mark with arrows the direction of motion of your spacecraft.

   SOLUTION:

   With the final selected values, the orbits reflected in the dataset given can be found in the figure below.

   In this plot the orbits can be seen in order of occurrence as well as the direction the spacecraft is traveling. It can be seen that the first three orbital transfers occur near the perigee point of the first four orbits, and the last orbital transfer occurs at apogee between Orbit 4 and 5.

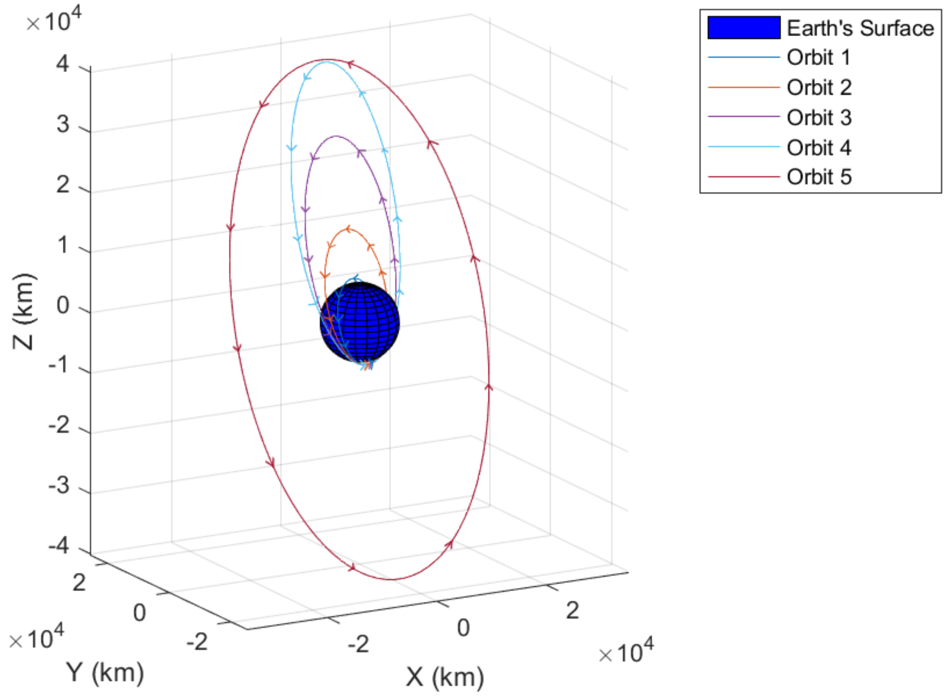

Figure 3. Plot of Orbits

7. Finally perform the task you were assigned in the analytic part to identify each delta-v that explains the measurements and the orbits you have got. Fill in your answers as follows:

SOLUTION:

Using these orbital elements, the delta Vs can be calculated between each orbit. This process was derived in the beginning of this project in the analytical section. The calculated values can be seen in Table 6.

Transfer	Delta V (km/s)
1	1.249
2	0.797
3	0.270
4	1.429
Total	3.745

Table 8. Delta V Values for the Orbital Maneuvers