Iteration and recursion

Due: February 2nd, 17:00. Push the function for Steffenson's iteration for 2(a) and a script for 2(b) to your assignmment repository. Make sure to use the input/output specified in the templates and check the result of the autograder. Typset your solution for question 1 in LaTeX and push botht the `.tex` and the `.pdf` file to your repository.
Make sure all files submitted include a comment line with your name and student number, e.g.

```
# Peter Peterson 100456789
```

and list the names and student numbers of class mates you collaborated with, if any.
Also, be a **good programmer** and include comments with a brief description of the functionality, input and output arguments and usage of each function or script. Also, add some comments that explain what steps are taken. Marks will be awarded or subtracted based on the readability and transparency of your code.
A discussion thread for this assignment is available on Slack. Pose your questions there before approaching the lecturer or TA.

Question 1 **50 marks**

You wish you use bisection for finding a root of the continuous functions below. For each function, use tools from calculus to find a domain (a, b) that contains precisely one root (so that you satisfy the sufficient condition for bisection to converge). if the function has more than one root, select the one closest the to origin.
Note that, while looking at a graph is a good starting point, it is not sufficient.

(a) $f(x) = (x-1)\cos(x) + 1/2$

(b) $f(x) = \sqrt{x^2+1} - x^3$

(c) $f(x) = x\ln(x) - 2x + x^2 + 1$

(d) $f(x) = 1/(x+1)^2 + 4/(x+2)^2 - 1$

(e) $f(x) = \exp(-x^3 + x - 2) - 1/10$

Question 2 **50 marks**

(a) Write a function that implements the following pseudo-code:
Input: f, f', x, ϵ, N.
Output: x^*.

1. Repeat N times:
   (a) Set $y_1 = x$.
   (b) Take one Newton step, starting from y_1. Call the result y_2.
   (c) Take one Newton step, starting from y_2. Call the result y_3.
   (d) Set
   $$x = y_1 - \frac{(y_2 - y_1)^2}{y_3 - 2y_2 + y_1}$$
   (e) Display $|f(x)|$.

(f) If $|f(x)| < \epsilon$ print "converged!", break.

2. Output $x^* = x$.

This algorithm is called Steffensen's iteration.

(**b**) Test your routine on the problem

$$\exp(-x^2 + x) - \frac{1}{2}x = 1.0836 \quad \text{(with initial guess } x = 1\text{)}$$

Show that Newton iteration does not converge quadratically, but your new iterative algorithm does.