Week 5 CS-312 Homework

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1 Problem 6.6

1.1 Question

Give CFG's for the following languages with alphabet $\{a, b\}$:

1.1.1 All strings of the form $a^m b^n$ with $m \leq n \leq 2m$.

$$\begin{array}{lcl} S & \rightarrow & \epsilon \mid aSB \\ B & \rightarrow & b \mid bb \end{array}$$

1.1.2 All strings such that the middle symbol is a.

$$S \rightarrow aSa \mid aSb \mid bSb \mid bSa \mid a$$

2 Problem 6.10

2.1 Question

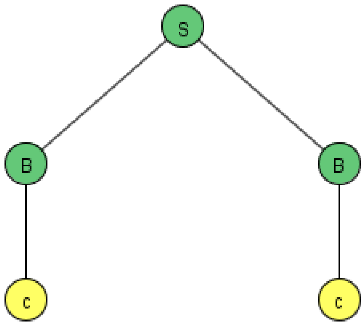
Consider the following CFG with start variable S:

$$\begin{array}{lcl} S & \rightarrow & BB \\ B & \rightarrow & SS \mid c \end{array}$$

2.1.1 What is the shortest string in the language of the grammar?

$$cc$$

2.1.2 Draw the derivation tree for the string from (2.1.1).



2.1.3 Is this grammar ambiguous? Explain.

Yes, because in the case of $ccccc$ (5 c's), the derivation can be achieved either by splitting the first B or the second B into SS.

2.1.4 Describe in English the language generated by the grammar.

Any string with 2mod3 number of c's.

2.1.5 Does this CFG generate a regular language? Explain.

Yes, because the equivalent regular expression is $cc(ccc)^*$

3 Problem 6.11

3.1 Question

Describe in English the language generated by the following grammar:

$$S \rightarrow 0S1 \mid 1S0 \mid \epsilon \mid SS$$

Sketch a proof that your answer is correct.

3.2 Answer

This language is any string with an equal number of 0's and 1's.

We can prove this by looking at every string's first and last symbol, solving it, removing it, and moving on to the inward set of symbols in the string.
For a string $0w1$ or $1w0$, the string can be broken into w using the rule $S \rightarrow 0S1$ or $S \rightarrow 1S0$. For a string $0w0$ or $1w1$, there must also be a $1s1$ or $0s0$ that is inside the w. This can be rewritten as $0u1j1u0$, which can be broken up into $0u1$, j, and $1u0$ by $S \rightarrow SS$. These individual parts are part of the grammar, and so any string with an equal number of 0's and 1's is accepted.

4 Problem 7.6

4.1 Question

Explain how, given a PDA for L_1 and a PDA for L_2, you can produce a PDA for the concatenation L_1L_2.

4.2 Answer

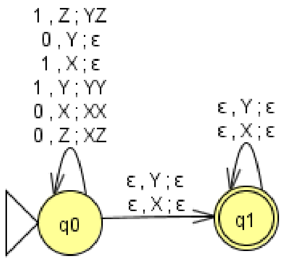
Create a Grammar for both L_1 and L_2, called G_1 and G_2. Concatenate the grammars, so that $G \to G_1G_2$. Convert G back into a PDA.

5 Problem 7.9

5.1 Question

Construct a PDA for the language of binary strings with an unequal number of 0's and 1's.

5.2 Answer



6 Problem 7.13

6.1 Question

Consider the language of strings where (at least) the first half is all the same symbol. To be specific, consider the set of all string $a^n x$ where $|x| \le n$ and $x \in \{a, b\}^*$. Give both a grammar and a PDA for this language.

6.2 Answer

$$\begin{array}{lcl} S & \to & aST \mid \epsilon \\ T & \to & a \mid b \mid \epsilon \end{array}$$