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Brief Paper

On controllability and trajectory tracking of a kinematic vehicle model[☆]

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Abstract

This paper presents some further results concerning the issues of controllability and trajectory tracking regarding a front-wheel drive vehicle kinematic model. A simple procedure for computing an open-loop control strategy that transfers the system between given initial and final states, is presented. In particular, the input function is computed by means of a set of linear algebraic equations. The resulting motion planning procedure allows us to present a control scheme for solving the trajectory (a time-parameterized reference signal) tracking problem. Various applications of the approach in forward and backward motions are considered, and simulation results are presented.

Keywords: Nonholonomic constraints; Kinematic model; Controllability; Lyapunov stability; Trajectory tracking

1. Introduction

Recent years have witnessed a significant volume of activity in the studies of nonholonomic systems. Canudas de Wit and Sørdalen (1992) presented a piecewise smooth controller that ensures an exponential convergence of the mobile robot to the origin. Samson (1995) has established a global time-varying stabilizing feed-back control for a general class of nonholonomic chained form systems. Application of homogeneous time-varying stabilizing control laws for controllable systems with no drift was developed by Morin, Pomet, and Samson (1999). A control algorithm based on a neural network back-stepping approach for mobile robots for tracking and stabilizing a desired posture was established by Fierro and Lewis (1998).

The vehicle tracking problems consist of two main control objectives: *geometric path tracking* (e.g., Altafini, 1999), and tracking a *time parameterized reference*. Liu (1997) proposes an algorithm for synthesizing a sequence of trajectories of the systems that converges to a given trajectory of a reference model. The construction is complicated and the resulting controllers oscillate in high frequencies. An exponentially stabilizing control law for systems with nonholonomic constraints was proposed by Walsh, Tilbury, Sastry, Murray, and Laumond (1994), where the controller depends on an elaborate procedure, which requires the evaluation of a transition matrix of a time-varying system due to linearization about a nominal trajectory. Time-varying feedback based on the back-stepping technique for mobile robots is considered by Jiang and Nijmeijer (1997). Egerstedt, Hu, and Stotsky (2001) propose algorithms for controlling mobile platforms by utilizing the virtual vehicle approach. The controllability problem in systems with nonholonomic constraints was the subject of several studies, e.g., Laumond (1993), and the references therein. Methods for steering a system with nonholonomic constraints by means of sinusoids was presented by Murray and Sastry (1993).

This work deals with the controllability and trajectory following problems of the *kinematic car-like model* shown

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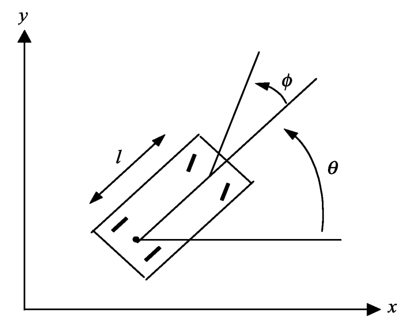

Fig. 1. A front-wheel drive kinematic model.

in Fig. 1, with a nonholonomic constraint that restricts the wheels to roll with no slip. The set $\{x, y, \theta, \phi\}$ denotes the configuration of the car, where $\{x, y\}$ is the location of the midpoint of the rear axle, θ is the angle between the x-axis and a reference line on the car frame and ϕ is the steering angle. We may take $l = 1$. Assuming that u_1 and u_2 are, respectively, the *driving* and the *steering* velocities, the resulting system is given by Murray and Sastry (1993):

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \\ \tan\phi \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_2; \quad t \geqslant 0. \tag{1}$$

This study was motivated by the earlier works of Ailon, Baratchart, Grimm, and Langholz (1986), Ailon and Langholz (1986) and Aeyels (1987), which show that in linear time-invariant systems controllability is equivalent to polynomial controllability. That is, it is always possible to transfer a controllable linear system from a given state to a desired one along a polynomial trajectory, by means of a polynomial input. The polynomial coefficients are computed simply by solving a linear algebraic equation of the form $Ha = c$ where H is a constant matrix, c is a vector associated with the initial and final states, and a is the vector of the unknown polynomial trajectory coefficients. The concept of a flat system (Fliess, Levine, Martin, & Rouchon, 1995) is useful in solving nonholonomic motion planning problems. However, as far as linear systems are concerned, the concept of polynomial controllability is an older but relevant notion to that of flatness. Here we go further and present a sort of a closed-form solution for motion planning in the nonlinear system (1), which is similar to the one obtained for the linear system case in the sense that the polynomial coefficients of the relevant variables are computed simply by solving a linear algebraic equation. Having established tools for motion planning, a control scheme is presented for the trajectory-tracking task. This note is organized as follows. Section 2 is devoted to the controllability and motion-planning problems. Section 3 deals with the trajectory-following problem. Simulation results are presented in Section 4.

2. Tools relating to controllability and motion planning

The restrictive conditions that we impose now, will be removed subsequently.

Let $0 \leqslant x_0 = x(0) < x_f = x(t_f)$, $0 < t_f$. Consider the functions $f : [0, t_f] \to \Re$ and $g : [x_0, x_f] \to \Re$ where $f \in C^2[0, t_f]$, $0 < \lambda_1 \leqslant \mathrm{d}f/\mathrm{d}t$ (λ_1 is an arbitrary constant), and $g \in C^3[x_0, x_f]$. Let $x(t) = f(t)$ and $y(t) = (g \circ f)(t) = g(x(t))$. Define

$$\alpha(t) \doteq \left(\frac{\mathrm{d}g}{\mathrm{d}x}\right)(t). \tag{2}$$

Recalling (1), $u_1(t) = \sqrt{\dot{x}^2(t) + \dot{y}^2(t)} > 0$. Therefore

$$u_1(t) = \dot{f}(t)\sqrt{1 + \alpha^2(t)} \geqslant \lambda_1 > 0. \tag{3}$$

Since $\theta(t) = \tan^{-1}\alpha(t)$, (3) and (1) yield

$$\dot{\theta}(t) = \dot{\alpha}(t)/(1 + \alpha^2(t))$$
$$\Rightarrow \tan\phi(t) = \dot{\alpha}(t)/[(1 + \alpha^2(t))^{3/2}\dot{f}(t)], \tag{4}$$

where $\dot{\alpha}(t) = (\mathrm{d}^2 g/\mathrm{d}x^2)\dot{f}(t)$. Recalling (1)

$$u_2(t) = \dot{\phi}(t) = \dot{\beta}(t)/(1 + \beta^2(t));$$
$$\beta(t) \doteq \dot{\theta}(t)/u_1(t) = \tan\phi(t). \tag{5}$$

Application of (2)–(5) yields the following result.

Theorem 2.1. *Let* $\varphi_0 = [x_0, y_0, \theta_0, \phi_0]^{\mathrm{T}}$ *and* $\varphi_f = [x_f, y_f, \theta_f, \phi_f]^{\mathrm{T}}$ *be state vectors with* $-\pi/2 < \theta_0, \theta_f < \pi/2$, $-\pi/2 < \phi_0, \phi_f < \pi/2$, *and* $0 \leqslant x_0 < x_f$. *Suppose that a pair* $\{f(t), g(x)\}$ *can be selected such that*

$$f(0) = x_0; \quad f(t_f) = x_f; \quad g(x_0) = y_0; \quad g(x_f) = y_f;$$
$$\alpha(0) = \tan\theta_0; \quad \alpha(t_f) = \tan\theta_f;$$
$$\dot{\alpha}(0) = \tan\phi_0[(1 + \alpha^2(0))^{3/2}\dot{f}(0)];$$
$$\dot{\alpha}(t_f) = \tan\phi_f[(1 + \alpha^2(t_f))^{3/2}\dot{f}(t_f)]. \tag{6}$$

Then the input $u = [u_1, u_2]^{\mathrm{T}}$ *in* (3) *and* (5) *transfers system* (1) *from* φ_0 *to* φ_f.

The proof of Theorem 2.1 is given in Ailon, Berman, and Arogeti (2001). Recall that $x = f(t)$. Let

$$y = g(x) = \sum_{i=0}^{n} a_i \exp(-\mathrm{i}\lambda x), \tag{7}$$

where $\lambda > 0$ is an arbitrary constant. The constants a_i and the integer n are yet to be determined. (Other set of independent functions can be used in (7).) We have

$$\alpha(t) = -\sum_{i=0}^{n} a_i i \lambda \exp(-\mathrm{i}\lambda x(t))$$
$$\Rightarrow \dot{\alpha}(t) = \dot{f}(t) \sum_{i=0}^{n} a_i i^2 \lambda \exp(-\mathrm{i}\lambda x(t)). \tag{8}$$

The boundary conditions in Theorem 2.1 require

$$\sum_{i=0}^{n} a_i \exp(-\mathrm{i}\lambda x_0) = c_{0,0}; \quad \sum_{i=0}^{n} a_i i\lambda \exp(-\mathrm{i}\lambda x_0) = c_{1,0};$$
$$\sum_{i=0}^{n} a_i i^2\lambda^2 \exp(-\mathrm{i}\lambda x_0) = c_{2,0},$$
$$\sum_{i=0}^{n} a_i \exp(-\mathrm{i}\lambda x_f) = c_{0,f}; \quad \sum_{i=0}^{n} a_i i\lambda \exp(-\mathrm{i}\lambda x_f) = c_{1,f};$$
$$\sum_{i=0}^{n} a_i i^2\lambda^2 \exp(-\mathrm{i}\lambda x_f) = c_{2,f}, \tag{9}$$

where $c_{i,0} = c_i(0)$, $c_{i,f} = c_i(t_f)$, $i = 0, 1, 2$, and $c_i(t)$ are

$$c_0(t) \doteq y(t); \quad c_1(t) \doteq -\tan\theta(t);$$
$$c_2(t) \doteq (1 + \tan^2\theta(t))^{3/2} \tan\phi(t). \tag{10}$$

We may choose $n = 5$. Define $\tau_0 \doteq \mathrm{e}^{-\lambda x_0}$ and $\tau_f \doteq \mathrm{e}^{-\lambda x_f}$. Then, (9) becomes

$$\begin{bmatrix} 1 & \tau_0 & \tau_0^2 & \tau_0^3 & \tau_0^4 & \tau_0^5 \\ 0 & \tau_0 & 2\tau_0^2 & 3\tau_0^3 & 4\tau_0^4 & 5\tau_0^5 \\ 0 & \tau_0 & 4\tau_0^2 & 9\tau_0^3 & 16\tau_0^4 & 25\tau_0^5 \\ 1 & \tau_f & \tau_f^2 & \tau_f^3 & \tau_f^4 & \tau_f^5 \\ 0 & \tau_f & 2\tau_f^2 & 3\tau_f^3 & 4\tau_f^4 & 5\tau_f^5 \\ 0 & \tau_f & 4\tau_f^2 & 9\tau_f^3 & 16\tau_f^4 & 25\tau_f^5 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} c_{0,0} \\ c_{1,0} \\ c_{2,0} \\ c_{0,f} \\ c_{1,f} \\ c_{2,f} \end{bmatrix}. \tag{11}$$

Proposition 2.1. *Select arbitrarily x_0 and x_f with $x_0 < x_f$. Then* (11) *has a unique solution $a = [a_0, a_1, a_2, a_3, a_4, a_5]^{\mathrm{T}}$.*

Proof. Let $A(\tau_0, \tau_f)$ denote the matrix in (11) with $0 \neq \tau_0 \neq \tau_f \neq 0$ and apply the following sequence of elementary row operations. (i) Add (-1) times the second row to the third row and the fifth row to the sixth row. (ii) Multiply the second and the fifth rows, respectively, by $1/\tau_0$ and $1/\tau_f$. (iii) Multiply the third and the sixth rows, respectively, by $1/\tau_0^2$ and $1/\tau_f^2$. The resulting equivalent matrix is denoted by $\tilde{A}(\tau_0, \tau_f) \approx A(\tau_0, \tau_f)$. If the matrix $\tilde{A}(\tau_0, \tau_f)$ is singular, there exists $d = [d_0, \ldots, d_5]^{\mathrm{T}} \neq 0$ such that $\tilde{A}(\tau_0, \tau_f)d = 0$. Then

$$\sum_{i=0}^{5} d_i \tau_0^i = \sum_{i=1}^{5} d_i i \tau_0^{i-1} = \sum_{i=2}^{5} d_i i(i-1)\tau_0^{i-2} = 0,$$
$$\sum_{i=0}^{5} d_i \tau_f^i = \sum_{i=1}^{5} d_i i \tau_f^{i-1} = \sum_{i=2}^{5} d_i i(i-1)\tau_f^{i-2} = 0. \tag{12}$$

Eqs. (12) imply that $p(\tau) = \sum_{i=0}^{5} d_i \tau^i$ and its first and second derivatives have two distinct roots: τ_0 and τ_f. Since $p(\tau_0) = p(\tau_f) = 0$, by the *mean value theorem* there is a $\tau_1 \in (\tau_0, \tau_f)$ at which $\dot{p}(\tau_1) = \mathrm{d}p(\tau)/\mathrm{d}\tau|_{\tau=\tau_1} = 0$. Hence, by (12), the polynomial $\dot{p}(\tau)$ has three different roots: τ_0, τ_1, and τ_f. By the mean value theorem there are $\tau_2 \in (\tau_0, \tau_1)$ and $\tau_3 \in (\tau_1, \tau_f)$ such that $\ddot{p}(\tau_2) = \ddot{p}(\tau_3) = 0$. Hence $\ddot{p}(\tau)$ has four different roots: τ_0, τ_2, τ_3, τ_f, which is impossible since $\ddot{p}(\tau)$ is a polynomial of degree three, and thus $A(\tau_0, \tau_f) \approx \tilde{A}(\tau_0, \tau_f)$ is nonsingular. □

Recalling Theorem 2.1, $u = [u_1, u_2]^{\mathrm{T}}$ defined by (3) and (5) drives the vehicle from φ_0 to φ_f.

Remark 2.1. (i) The integer $n = 5$ is required for satisfying the boundary conditions. Obviously one can select $n > 5$ while satisfying additional conditions (e.g., sub-minimizing a certain index of performance). (ii) As indicated previously, the proposed approach is based on the *linear algebraic equation* (11), similar to the one established in the linear system framework.

The open-loop controls of the backward and forward motions, are associated as follows.

Proposition 2.2. *Let $u = [u_1, u_2]^{\mathrm{T}}$ with $u_1(t) > 0$ for all $t \in [0, t_f]$ be the control signal that transfers the system* (1) *from φ_0 to φ_f with $-\pi/2 < \theta_0, \theta_f, \phi_0, \phi_f < \pi/2$, $0 \leqslant x_0 < x_f$ along the trajectory $\varphi(t)$. Then $u^*(t) = -u(t_f - t)$ takes the system from φ_f to φ_0 along the trajectory φ^* with $\varphi^*(t) = \varphi(t_f - t)$, $t \in [0, t_f]$.*

Proof. Let $u^*(t) = -u(t_f - t)$. From the last equation of (1) we have for any fixed $t_i \in [0, t_f]$,

$$\phi^*(t_i) = \phi_f + \int_0^{t_i} u_2^*(\tau)\,\mathrm{d}\tau = \phi_f - \int_0^{t_i} u_2(t_f - \tau)\,\mathrm{d}\tau$$
$$= \phi_f - \int_{t_f - t_i}^{t_f} u_2(\rho)\,\mathrm{d}\rho, \tag{13}$$

where $\rho = t_f - \tau$. But since $\phi_f = \phi_0 + \int_0^{t_f} u_2(\rho)\,\mathrm{d}\rho$

$$\phi^*(t_i) = \phi_0 + \int_0^{t_f} u_2(\rho)\,\mathrm{d}\rho - \int_{t_f - t_i}^{t_f} u_2(\rho)\,\mathrm{d}\rho$$
$$= \phi_0 + \int_0^{t_f - t_i} u_2(\rho)\,\mathrm{d}\rho = \phi(t_f - t_i). \tag{14}$$

Observing the third equation in (1), we have $\dot{\theta}^*(t) = \tan\phi^* u_1^* = \tan\phi(t_f - t)(-u_1(t_f - t)) = -\mu(t_f - t)$, where $\mu(t) \doteq \tan\phi(t)u_1(t)$. Therefore $\theta^*(t_i) = \theta(t_f) + \int_0^{t_i} \mu^*(\tau)\,\mathrm{d}\tau$, where $\mu^*(t) = -\mu(t_f - t)$, and we obtain, similar to (13) and (14) $\theta^*(t_i) = \theta(t_f - t_i)$. But then, from the first two equation of (1) $\dot{y}^*(t) = \sin\theta^*(t)u_1^*(t) = \sin\theta(t_f - t)(-u_1(t_f - t))$ and $\dot{x}^*(t) = \cos\theta^*(t)u_1^*(t) = \cos\theta(t_f - t)(-u_1(t_f - t))$. Hence, from (13) and (14), we have $\varphi^*(t) = \varphi(t_f - t)$, as claimed. □

In further applications, the definition domains of $f(\cdot)$ and $g(\cdot)$ are extended, respectively, to $[0, \infty)$ and $[x_0, \infty)$, we assume that $0 < \lambda_1 \leqslant \mathrm{d}f/\mathrm{d}t \leqslant \lambda_2$, $|\mathrm{d}^2 f/\mathrm{d}t^2| \leqslant \chi$, and $|\mathrm{d}^i g/\mathrm{d}x^i| \leqslant \kappa_i$, $i = 1, 2, 3$, where $\chi, \kappa_i \geqslant 0$ are any constants. A pair $\{f, g\}$ possessing these properties will be designated by $\{f^*, g^*\}$. The previous analysis shows that

$\{f^*, g^*\}$ generates $\varphi^* = [x^*, y^*, \theta^*, \phi^*]^{\mathrm{T}}$ with $x^*(t) = f^*(t)$, $y^*(t) = g^*(f^*(t))$, $\theta^*(t) = \tan^{-1}[(\mathrm{d}g^*/\mathrm{d}x^*)(t)]$, $\phi^*(t) = \tan^{-1}[\dot{\theta}^*(t)/u_1^*(t)]$, and $u^* = [u_1^*, u_2^*]^{\mathrm{T}}$ with forward velocity $u_1^*(t) = \dot{f}^*(t)\sqrt{1 + [(\mathrm{d}g^*/\mathrm{d}x^*)(t)]^2}$ and steering velocity $u_2^*(t) = \dot{\phi}^*(t)$, and in particular, $\{\varphi^*, u^*\}$ satisfies (1). The properties of f^* and g^* ensure that (see (2)–(5)) the steering velocity $u_2^*(\cdot)$ is continuous and uniformly bounded and the functions $\theta^*(\cdot)$, $\phi^*(\cdot)$, and $u_1^*(\cdot)$, are sufficiently smooth and there exist positive constants $\xi_\theta, \xi_\phi, \xi_1, \xi_2$ such that

$$\begin{aligned} &-\pi/2 + \xi_\theta \leqslant \theta^*(t) \leqslant \pi/2 - \xi_\theta; \\ &-\pi/2 + \xi_\phi \leqslant \phi^*(t) \leqslant \pi/2 - \xi_\phi; \quad 0 < \xi_1 \leqslant u_1^*(t) \leqslant \xi_2. \end{aligned} \tag{15}$$

3. Trajectory tracking

A simple application of the above results concerns with the straight line tracking problem. For the sake of illustration, we choose the line $y=0$. Let $y=g(x)=\sum_{i=1}^n a_i \exp(-\mathrm{i}\lambda x)$, $\lambda > 0$ (λ affects the system's rate of convergence) and select $x = f(t) \in C^2[0, \infty)$, $0 < \lambda_1 \leqslant \mathrm{d}f/\mathrm{d}t$. To comply with the system's initial condition set (using the notation in (10))

$$\begin{aligned} &\sum_{i=1}^n a_i \exp(-\mathrm{i}\lambda x_0) = c_0(0); \\ &\sum_{i=1}^n a_i i\lambda \exp(-\mathrm{i}\lambda x_0) = c_1(0); \\ &\sum_{i=1}^n a_i i^2\lambda^2 \exp(-\mathrm{i}\lambda x_0) = c_2(0). \end{aligned} \tag{16}$$

Proposition 3.1. *Take $n = 3$. Then, for any $x_0 > 0$ there is a unique solution $a = [a_1, a_2, a_3]^{\mathrm{T}}$ to* (16).

The proof of the proposition is similar to the proof of Proposition 2.1. Since $x = f(t) > 0$ increases monotonically in t, $\lim_{t\to\infty} y(t) = \lim_{t\to\infty} \sum_{i=1}^n a_i \exp(-\mathrm{i}\lambda f(t)) = 0$. Thus, $u = [u_1, u_2]^{\mathrm{T}}$ defined by (3) and (5) ensures tracking of the line $y = 0$ ($x > 0$).

We consider now the trajectory-tracking problem. Recall that a pair $\{f^*, g^*\}$ generates a pair (of the *reference trajectory* and *input*) $\{\varphi^*, u^*\}$ that satisfies (1). Let $u = [u_1, u_2]^{\mathrm{T}}$ be an input generating the *actual trajectory* $\varphi = [x, y, \theta, \phi]^{\mathrm{T}}$ in (1). Letting $e = [e_1, e_2, e_3, e_4]^{\mathrm{T}} = [x - x^*, y - y^*, \theta - \theta^*, \phi - \phi^*]^{\mathrm{T}}$, (1) gives

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \end{bmatrix} = \begin{bmatrix} u_1 \cos\theta - u_1^* \cos\theta^* \\ u_1 \sin\theta - u_1^* \sin\theta^* \\ u_1 \tan\phi - u_1^* \tan\phi^* \\ u_2 - u_2^* \end{bmatrix}. \tag{17}$$

Let the selected feedback for u_1 be

$$u_1 = [u_1^* \cos\theta^* - \gamma e_1]/\cos(e_3 + \theta^*). \tag{18}$$

Regarding u_2, recalling that $\dot{\phi} = u_2$ in (1) and assuming that $\phi(0) \in (-\pi/2, \pi/2)$, the steering velocity will be determined such that $\phi(t)$ satisfies

$$u_1 \tan\phi - u_1^* \tan\phi^* = -\alpha e_2 - \beta e_3 - p\exp(-qt), \tag{19}$$

where the constants $\alpha, \beta > 0$ are yet to be determined, $q > 0$ is an arbitrary constant, and p is selected such that (19) holds at $t = 0$, i.e.,

$$\begin{aligned} -p = {} & u_1(0)\tan\phi(0) - u_1^*(0)\tan\phi^*(0) \\ & + \alpha e_2(0) + \beta e_3(0). \end{aligned} \tag{20}$$

Solving (19) for ϕ, we have (so long as $\phi(t) \in (-\pi/2, \pi/2)$ and $u_1(t) > 0$ in (18))

$$\phi = \tan^{-1}[(-\alpha e_2 - \beta e_3 - w + u_1^* \tan\phi^*)/u_1]. \tag{21}$$

where $w = p\exp(-qt)$. Using (21), we determine the steering signal by $u_2 = \dot{\phi}$, i.e.,

$$\begin{aligned} &u_2 = \dot{\phi} = \dot{\rho}/(1 + \rho^2); \\ &\rho \doteq (-\alpha e_2 - \beta e_3 - w + u_1^* \tan\phi^*)/u_1. \end{aligned} \tag{22}$$

Remark 3.1. (i) u_2 in (22) can be expressed in terms of the state variables (rather than their derivatives). In fact from (17), $\dot{e}_2 = u_1 \sin\theta - u_1^* \sin\theta^*$ and $\dot{e}_3 = u_1 \tan\phi - u_1^* \tan\phi^*$. Similarly, using (18) and (1), $\dot{u}_1$ can be expressed in terms of the state variables. (ii) If $\bar{e} \doteq [e_1, e_2, e_3, w]^{\mathrm{T}} = 0$, then $u_1 = u_1^*$ and $u_2 = \dot{\phi}^* = u_2^*$. (iii) Recalling (20), the right-hand side of (21) at $t = 0$ is $\phi(0)$, and therefore if we show that $u_2(\cdot)$ in (22) is well-defined (smooth and uniformly bounded), the unique solution of $\dot{\phi}(t) = u_2(t)$, $t \geqslant 0$, is $\phi(t)$ in (21).

Theorem 3.1. *Consider the error equation* (17) *subject to the action of the input signals u_1* (*with the gain γ*) *in* (18) *and u_2* (*with the gains α and β*) *in* (22). *Fix a $\gamma > 0$. Then $\alpha, \beta > 0$ can be selected such that $e = 0$ is an asymptotically stable equilibrium point.*

Proof. The proof will be divided into three parts.

Part 1: Consider first the error dynamics of the *unicycle model* ($\dot{x} = v_1\cos\theta$, $\dot{y} = v_1 \sin\theta$, $\dot{\theta} = v_2$), namely,

$$\begin{bmatrix} \dot{\varepsilon}_1 \\ \dot{\varepsilon}_2 \\ \dot{\varepsilon}_3 \end{bmatrix} = \begin{bmatrix} v_1 \cos\theta - v_1^* \cos\theta^* \\ v_1 \sin\theta - v_1^* \sin\theta^* \\ v_2 - v_2^* \end{bmatrix}, \tag{23}$$

$\varepsilon = [\varepsilon_1, \varepsilon_2, \varepsilon_3]^{\mathrm{T}}$ and θ^* and $v_1^* = u_1^*$ satisfy (15). Let

$$\begin{aligned} &v_1 = [v_1^* \cos\theta^* - \gamma\varepsilon_1]/\cos(\varepsilon_3 + \theta^*), \\ &v_2 = v_2^* - \alpha\varepsilon_2 - \beta\varepsilon_3. \end{aligned} \tag{24}$$

Since $\tan\theta - \tan\theta^* = \sin\varepsilon_3/(\cos(\varepsilon_3 + \theta^*)\cos\theta^*)$, by applying (24) in (23), we have

$$\begin{bmatrix} \dot{\varepsilon}_1 \\ \dot{\varepsilon}_2 \\ \dot{\varepsilon}_3 \end{bmatrix} = \begin{bmatrix} -\gamma\varepsilon_1 \\ -\gamma\varepsilon_1 \tan(\varepsilon_3 + \theta^*) + v_1^* \sin\varepsilon_3/\cos(\varepsilon_3 + \theta^*) \\ -\alpha\varepsilon_2 - \beta\varepsilon_3 \end{bmatrix}. \tag{25}$$

Fix $\gamma > 0$ and define $\sigma(\varepsilon, t) \doteq \gamma \tan(\varepsilon_3 + \theta^*(t))$ and $\varphi_1(\varepsilon, t) \doteq v_1^*(t)/\cos(\varepsilon_3 + \theta^*(t))$. Let

$$V(\varepsilon) = \tfrac{1}{2}\varepsilon^{\mathrm{T}} \begin{bmatrix} \delta & 0 & 0 \\ 0 & \delta_1 & \mu \\ 0 & \mu & 1 \end{bmatrix} \varepsilon \doteq \tfrac{1}{2}\varepsilon^{\mathrm{T}} P_1 \varepsilon, \tag{26}$$

where $\delta, \delta_1 > 0$, be a *Lyapunov candidate* function. Then, along the solution of (25) $\mathrm{d}V/\mathrm{d}t$ is given by

$$\begin{aligned}\dot V &= \delta\varepsilon_1\dot\varepsilon_1 + \delta_1\varepsilon_2\dot\varepsilon_2 + \varepsilon_3\dot\varepsilon_3 + \mu\varepsilon_3\dot\varepsilon_2 + \mu\varepsilon_2\dot\varepsilon_3 \\ &= -\delta\gamma\varepsilon_1^2 - \delta_1\sigma\varepsilon_1\varepsilon_2 - \mu\sigma\varepsilon_1\varepsilon_3 + \delta_1\varphi_1\varepsilon_2\sin\varepsilon_3 \\ &\quad + \mu\varphi_1\varepsilon_3\sin\varepsilon_3 - \alpha\mu\varepsilon_2^2 - \beta\varepsilon_3^2 - \alpha\varepsilon_2\varepsilon_3 - \beta\mu\varepsilon_2\varepsilon_3. \end{aligned}\tag{27}$$

Clearly for $\varepsilon = 0$ (an equilibrium point) we have in (24) $v_1 = v_1^*$. Thus, noting that v_1 is *independent of* α and β, it follows by continuity that one can determine $\rho > 0$ and $\rho_1, \rho_2 > 0$ such that

$$\|\varepsilon\| \leqslant \rho \Rightarrow 0 < \rho_1 \leqslant v_1(\varepsilon, \theta^*, v_1^*) \leqslant \rho_2, \tag{28}$$

for any θ^* and $v_1^* = u_1^*$ satisfying (15). Furthermore, ρ can be determined such that

$$\begin{aligned}&\|\varepsilon\| \leqslant \rho \Rightarrow -\pi/2 + \xi \leqslant \varepsilon_3 + \theta^* \leqslant \pi/2 - \xi; \\ &\eta \leqslant \varphi_1(\varepsilon, \tau) \leqslant \vartheta; \quad |\sigma(\varepsilon, \tau)| \leqslant \psi,\end{aligned}\tag{29}$$

where $\|\cdot\|$ denotes, say, the *Euclidean norm*, $0 < \xi \leqslant \xi_\theta$, and η, ψ, ϑ are some positive constants. By the mean value theorem, $\varepsilon_3 \in [-\pi/2, \pi/2]$ satisfies

$$|\varepsilon_3/(\pi/2)| \leqslant |\sin\varepsilon_3| \leqslant |\varepsilon_3|; \quad \mathrm{sgn}\{\sin\varepsilon_3\} = \mathrm{sgn}\{\varepsilon_3\}. \tag{30}$$

Let $\|\varepsilon(\tau)\| \leqslant \rho$. Consider the two cases: (i) if $\varepsilon_2(\tau)\varepsilon_3(\tau) > 0$ set $\dot V_1(\tau) = \dot V(\tau)$ and $h_1 \doteq \delta_1\vartheta$, and (ii) if $\varepsilon_2(\tau)\varepsilon_3(\tau) \leqslant 0$ set $\dot V_2(\tau) = \dot V(\tau)$ and $h_2 \doteq \delta_1\eta/(\pi/2)$. Then, in view of (27)–(30), we have

$$\begin{aligned}\dot V_i &\leqslant -\delta\gamma\varepsilon_1^2 - \delta_1\sigma(\varepsilon,\tau)\varepsilon_1\varepsilon_2 - \mu\sigma(\varepsilon,\tau)\varepsilon_1\varepsilon_3 - \alpha\mu\varepsilon_2^2 \\ &\quad - (\beta - \mu\vartheta)\varepsilon_3^2 - (\alpha + \beta\mu - h_i)\varepsilon_2\varepsilon_3 \\ &= -\varepsilon^{\mathrm{T}} \begin{bmatrix} \delta\gamma & \frac{1}{2}\delta_1\sigma(\varepsilon,\tau) & \frac{1}{2}\mu\sigma(\varepsilon,\tau) \\ \frac{1}{2}\delta_1\sigma(\varepsilon,\tau) & \alpha\mu & \frac{1}{2}(\alpha+\beta\mu-h_i) \\ \frac{1}{2}\mu\sigma(\varepsilon,\tau) & \frac{1}{2}(\alpha+\beta\mu-h_i) & (\beta-\mu\vartheta) \end{bmatrix} \varepsilon \\ &\doteq -\varepsilon^{\mathrm{T}} Q_i(\varepsilon,\tau)\varepsilon, \quad i = 1, 2.\end{aligned}\tag{31}$$

We may take without loss of generality $\mu=1$ in (26). Assume δ_1 in (26) is given by

$$\delta_1 \doteq 2(\alpha+\beta)/[\vartheta + \eta/(\pi/2)], \tag{32}$$

where ϑ and η are given in (29) and obviously $0 < \eta/(\pi/2) < \vartheta$. Using (32) we have

$$\begin{aligned}&-((\alpha+\beta) - \delta_1\vartheta) = [(\vartheta - \eta/(\pi/2))/(\vartheta \\ &\quad + \eta/(\pi/2))](\alpha+\beta) \doteq \zeta(\alpha+\beta) < (\alpha+\beta).\end{aligned}\tag{33}$$

Denote the 2×2 matrix in the bottom right-hand corner of Q_i in (31) by q_i. Since $\mu = 1$, $h_1 = \delta_1\vartheta$, and $h_2 = \delta_1\eta/(\pi/2)$, (31)–(33) imply $\det q_1 = \det q_2 = \alpha(\beta - \vartheta) - [\zeta(\alpha+\beta)/2]^2$. Hence, α and β can be selected such that $\det q_i > 0$ for $i = 1, 2$. Indeed, since $\zeta < 1$ in (33) (see (29)), for a sufficiently large $\alpha = \beta$, we have $\det q_i = (1-\zeta^2)\alpha^2 - \alpha\vartheta > 0$.

Since (see (29)) $\sigma(\varepsilon, \tau)$ is bounded and δ can be selected arbitrarily large (and hence δ det q_i becomes large) we see from (31) that one can select positive constants δ, α, and β such that $\beta - \mu\vartheta > 0$, $\det q_i > 0$ and $\det Q_i > 0$. From (32) and (26) for sufficiently large α and β the matrix P_1 is positive definite. Hence, the independent coefficients α, β, and δ can be selected such that $P_1, Q_1, Q_2 > 0$, and *V is a Lyapunov function*. Under these conditions there exist constants $r_1, a_1, b_1, c_1 > 0$ where (see (29)) $r_1 < \rho$, such that $a_1\|\varepsilon\|^2 \leqslant V(\varepsilon) \leqslant b_1\|\varepsilon\|^2$ and $\dot V \leqslant -c_1\|\varepsilon\|^2$ for $\|\varepsilon\| < r_1$, and $\varepsilon = 0$ in (23) is exponentially stable.

Part 2: Consider the system

$$\begin{bmatrix} \dot e_1 \\ \dot e_2 \\ \dot e_3 \\ \dot w \\ \dot e_4 \end{bmatrix} = \begin{bmatrix} u_1\cos\theta - u_1^*\cos\theta^* \\ u_1\sin\theta - u_1^*\sin\theta^* \\ -\alpha e_2 - \beta e_3 - w \\ -qw \\ u_2 - u_2^* \end{bmatrix}, \tag{34}$$

where the starred variables satisfy (15), $\theta = e_3 + \theta^*$ and u_1 is given in (18). Comparing u_1 in (18) to v_1 in (24), the first three equations of (34) are similar to those that appear in (23) with v_2 given by (24), except the extra variable w. Consider

$$\bar V(\bar e) = \frac{1}{2}\bar e^{\mathrm{T}} \begin{bmatrix} P_1 & 0 \\ 0 & \delta_w \end{bmatrix} \bar e \doteq \tfrac{1}{2}\bar e^{\mathrm{T}}\bar P\bar e, \tag{35}$$

where the block P_1 is given in (26), the zero blocks have appropriate dimensions, $\delta_w > 0$ is a scalar that can be selected arbitrarily large, $\bar e = [e_1, e_2, e_3, w]^{\mathrm{T}}$ is the state vector of the *first four equations* of (34), which are *independent* of e_4. Repeating Part 1 of the proof, for a fixed $\gamma > 0$ in (18) the gains α and β in (34) can be selected such that $\bar V(\bar e)$ is a Lyapunov function for the first four equations of (34). Thus, there exist constants $r_2, a_2, b_2, c_2 > 0$ with $r_2 \leqslant r_1$ (r_1 in Part 1) such that $a_2\|\bar e\|^2 \leqslant \bar V \leqslant b_2\|\bar e\|^2$ and $\mathrm{d}\bar V/\mathrm{d}t \leqslant -c_2\|\bar e\|^2$ for all $\|\bar e\| < r_2$. Let $\varrho_2 = r_2(a_2/b_2)^{1/2}$. Then

$$\begin{aligned}&\|\bar e(0)\| < \varrho_2 \Rightarrow \|\bar e(t)\| \\ &\quad \leqslant (b_2/a_2)^{1/2}\|\bar e(0)\| \exp[(-c_2/2b_2)t] < r_2; \quad \forall t \geqslant 0.\end{aligned}\tag{36}$$

Part 3: Consider (17) where u_1 is given by (18) and u_2 by (22) and the computed gains γ, α, and β are given in Part 2. Observing (34), $w = p\exp(-qt)$ satisfies $\dot w = -qw$ with $w(0) = p$. We emphasize now two facts.

Fact (i) By (20), (18) and (15) $e(0) = 0 \Rightarrow p = 0$ and by continuity for each $\zeta > 0$ and a sufficiently small $\|e(0)\|$, $|p| < \zeta$.

Fact (ii) From (21) and (18), $\bar e = [e_1, e_2, e_3, w]^{\mathrm{T}} = 0 \Rightarrow u_1 - u_1^* = 0$ and $e_4 = \phi - \phi^* = 0$ and by continuity for each $\varpi > 0$, a $\psi > 0$ can be selected such that $\|\bar e\| < \psi \Rightarrow |e_4| < \varpi$ for the starred variables in (15).

Fix $\varpi > 0$ such that $|e_4| < \varpi \Rightarrow \phi = e_4 + \phi^* \in (-\pi/2, \pi/2)$ for all $\phi^* \in [-\pi/2 + \xi_\phi, \pi/2 - \xi_\phi]$ (see (15)). Let $\psi > 0$ be selected such that $\|\bar e\| < \psi \Rightarrow |e_4| < \varpi$

(fact (ii)). Using fact (i) let $\xi > 0$ be taken such that

$$\|e(0)\| < \xi \leqslant \varpi \\ \Rightarrow \|\bar{e}(0)\| < \varrho_3 \doteq \min\{\varrho_2, \psi(a_2/b_2)^{1/2}\}, \quad (37)$$

where the constants with the subscript 2 are given in Part 2. Then for any $e(0)$ with $\|e(0)\| < \xi$ system (17) with $u = [u_1(t), u_2(t)]^{\mathrm{T}}$ given by (18) and (22) is well defined at least along an interval $(0, \tau]$ with a sufficiently small $\tau > 0$. In particular, $u_1(t) \in [\rho_1, \rho_2]$ for some positive constants ρ_i (see (28) and recall that $u_1(t)$ is defined as $v_1(t)$), and $\theta(t), \phi(t) \in (-\pi/2, \pi/2)$ for all $t \in [0, \tau]$, and thus $u_2(t)$ is smooth and bounded in $(0, \tau]$. But then u_2 in (22) generates ϕ in (21) and (19) holds. Applying (19) in the third equation of (17) and adding the dummy scalar equation $\dot{w} = -qw$, the resulting system is precisely (34). But then (36) states that $\|\bar{e}(t)\|$ is bounded by a monotonically decreasing function and by (37) $\|\bar{e}(t)\| < \psi$ and therefore $|e_4(t)| < \varpi$ for all $t \in [0, \tau]$. This implies that u_1 and u_2 are well defined (smooth and uniformly bounded) in $(0, \infty)$. In fact if this is not the case, there must be a point $t_i > \tau$ such that either $\|\bar{e}(t_i)\| = \psi$ and/or $|e_4(t_i)| = \varpi$, which is impossible due to (36) and fact (ii). Therefore $\|\bar{e}(t)\| \to 0$ exponentially. The error $e_4 = \phi - \phi^*$ depends on $\bar{e}$ and the reference trajectory (see (21) and (18)), and we write $e_4 = g(\bar{e}, \bar{\eta}^*)$, where $\bar{\eta}^*$ is the starred variables vector and $g(\cdot,\cdot)$ is continuous in any set of pairs $\{\bar{e}, \bar{\eta}^*\}$ with a sufficiently small $\|\bar{e}\|$ and $\bar{\eta}^*$ satisfying (15). Since $\|\bar{e}(t)\| \to 0$ exponentially, one can define a closed bounded set E such that $\bar{e}(t), \bar{\eta}^*(t) \in E$ for each $t \geqslant 0$. Since, by fact (ii), $e_4 = g(0, \bar{\eta}^*) = 0$ for any fixed $\bar{\eta}^*$ that satisfies (15) and E is compact (and thus the restriction of $g(\cdot,\cdot)$ on E is uniformly continuous), $\|\bar{e}(t)\| \to 0 \Rightarrow |e_4(t)| \to 0$. By similar reasoning $\|\bar{e}(t)\| \to 0 \Rightarrow |u_1(t) - u_1^*(t)| \to 0$ and (see Remark 3.1 and recall that $u_2^*(t) = \dot{\phi}^*(t)$ is uniformly bounded and $0 < \rho_1 \leqslant u_1(t) \leqslant \rho_2$) $u_2(t)$ is uniformly bounded with $|u_2(t) - u_2^*(t)| \to 0$. □

Remark 3.2. The controller parameters specification and an estimate for the resulting region of attraction could be obtained through the following steps. Let an *admissible nominal trajectory* be given. Then the starred variables satisfy (15) for a set of positive constants and u_2^* is uniformly bounded. Fix gains $\gamma > 0$ and select $\alpha, \beta > 0$ according to Part 1. Then, using (35) in Part 2 the coefficients in (36) could be determined. Applying facts (i) and (ii) in Part 3, the coefficients in (37) could be determined and a domain (at least a conservative one) of the admissible initial error $e(0)$, is obtained.

Finally, we remove some constraints imposed on the controller design. Let x and y be the coordinates of a given vector in the original frame, say **F**, in Fig. 1, and x_1 and y_1 be the coordinates of the same point in a frame $\mathbf{F}_1$. If p and p_1 are the representation of the same point in **F** and $\mathbf{F}_1$, respectively, we have $p = R_\eta p_1 + d$, where η is the angle of rotation between the two frames, R_η is the rotation (orthogonal) matrix, and $d = [d_x, d_y]^{\mathrm{T}}$ is the vector from the

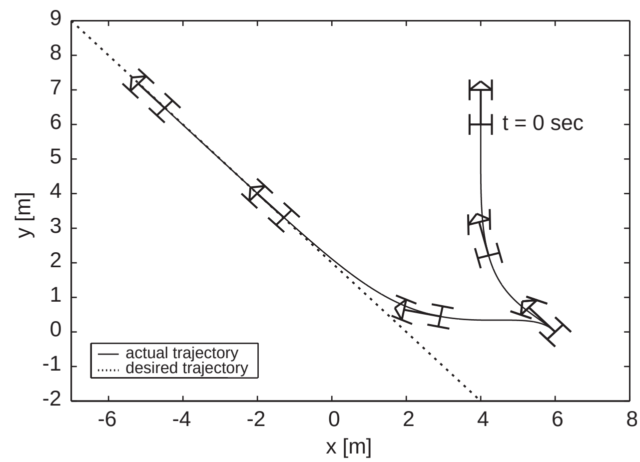

Fig. 2. State-to-state control for backward motion followed by a straight line tracking.

origin of **F** to the origin of $\mathbf{F}_1$, expressed in the coordinate system **F**. The variables $\{x, y, \theta, \phi\}$ in **F** are replaced by $\{x_1, y_1, \theta_1 = \theta - \eta, \phi_1 = \phi\}$ in $\mathbf{F}_1$. This allows us to extend the approach to the entire *xy* plane (see the examples).

To enhance the controller capabilities a *two-mode control strategy* is suggested. If, initially, the system is 'far' from the desired trajectory, by the motion planning procedure one can compute a control strategy that drives the system towards a *neighborhood* of a selected point of the desired trajectory. The computed target belongs to the estimated region of attraction with respect to the selected point. Applying the closed-loop controller, the vehicle reaches the selected neighborhood via the computed path. Finally, the closed-loop controller is implemented to ensure the tracking of the required trajectory.

4. Numerical examples

Example 4.1. *State-to-state control in a backward motion followed by a straight line tracking*. The vehicle state vector is $\varphi = [x, y, \theta, \phi]^{\mathrm{T}}$. The initial and final conditions of the backward motion are $\varphi_0 = \{4, 6, 90°, 0°\}$ and $\varphi_f = \{6, 0, 135°, 25°\}$. The reference line is $y(x) = -x + 2$, with $\dot{x}(t) < 0$. Using Proposition 2.2, the backward motion is computed by reversing the driving signals that produce a fictitious forward motion. We apply a new coordinate system $\mathbf{F}_1$ originating at the final location of the backward motion $\{x, y\} = \{6, 0\}$, with a rotation angle $\eta = \theta_f = 135°$. In reference to the frame $\mathbf{F}_1$, $f_1(t) = t$ and $\lambda = 0.001$. Fig. 2 describes the resulting vehicle motion. The arrow in the figures indicates the vehicle front part.

Example 4.2. *Tracking a circle with time scheduling at selected points.* The initial state is $\varphi_0 = \{0, 4, 0, 0\}$. The geometric path is the circle $x^2 + y^2 = 9$. The *a-priori* selected

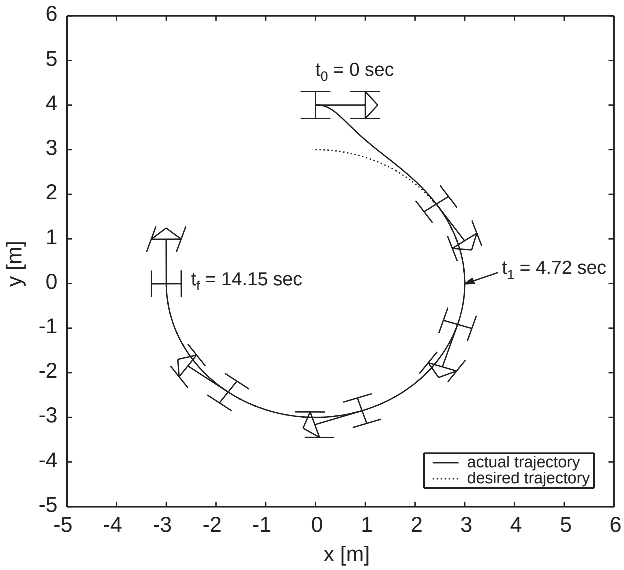

Fig. 3. Tracking a circle with time scheduling of arrivals at selected points.

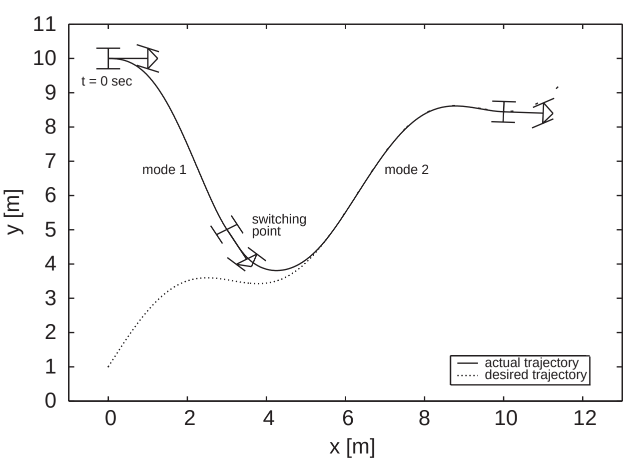

Fig. 4. Trajectory following achieved by the two-mode controller.

arrival times to $\{x_1, y_1\} = \{3, 0\}$ and $\{x_f, y_f\} = \{-3, 0\}$ are, respectively, $t_1^* = 4.71$ and $t_f^* = 14.14$ s. The results are given in Fig. 3. Actually the vehicle reaches a small neighborhood of each point $\{x_1, y_1\}$ and $\{x_f, y_f\}$ at times $t_1 = 4.72$ and $t_f = 14.15$ s, respectively.

Example 4.3. *A two-mode controller*. (i) *First mode*. The initial and final states are $\varphi_0 = \{0, 10, 0, -20°\}$ and $\varphi_f = \{3, 5, -60°, 20°\}$. Let $f(t) = t$ and $\lambda = 0.001$. (ii) *Second mode*. The controller gains in (18) and (22) are: $\gamma = 5$, $\alpha = \beta = 10$, and $q = 4$ (recall that w in (22) satisfies $\dot{w} = -qw$). The results are presented in Fig. 4.

5. Concluding remarks

In this paper, we have established tools for solving the motion planning and the trajectory tracking control problems for autonomous vehicles. By solving a set of algebraic equations an open-loop control for state-to-state is determined. Combining the motion planning procedure with the action of a closed-loop controller, a two-mode control strategy has been synthesized for achieving trajectory tracking.

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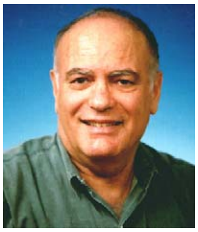

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