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**Homework 7**

**17)**

**b)** {aibjcidj | i,j >= 0}

Proof: Suppose L is context-free. Let k be the number specified in PL2.

Let’s take z = akbkckdk. Note that z is a string from L and length(z) >= k. According to Pl2, z contains two pumpable substrings, v and x such that their parallel pumping produces strings of L. Note that at least one of the pumpable substrings, v or x, is not empty and that the two substrings, v and x, are located within k letters in z.

There are the following possible contents for substrings v and x in our z:

1) both v and x are strings over the same alphabet letter

2) v and x are strings over one – but different in each – alphabet letter

3) one of the two substrings, v or x, is a string over more than one alphabet letter

Let’s discuss each of these cases separately:

*Case 1*: both v and x are strings over the same alphabet letter. Namely:

a) v = an, x = am

b) v = bn, x = bm

c) v = cn, x = cm

d) v = dn, x = dm

n,m >= 0; n + m > 0

Let’s discuss subcase a) – other subcases will be handled the same way:

Let’s take z’ = uv2wx2y = ak+m+nbkckdk

z’ is not in L, because the ratio between a’s and c’s is wrong since n+m > 0

*Case 2*: v and x are strings over one – but different in each – alphabet letter. Namely:

a) v = an, x = bm

b) v = bn, x = cm

c) v = cn, x = dm

n,m >= 0; n + m > 0

Let’s discuss subcase a) – other subcases will be handled the same way:

Let’s take z’ = uv2wx2y = ak+nbk+mckdk

z’ is not in L, because the ratio between a’s and c’s, as well with b’s and d’s, is wrong since n+m > 0

*Case 3*: one of the two substrings, v or x, is a string over more than one alphabet letter. Namely:

a) v = aL, x = anbm

b) v = bL, x = bncm

c) v = cL, x = cndm

d) v = anbm, x = bL

e) v = bncm, x = cL

f) v = cndm, x = dL

L >= 0; n, m > 0

Let’s discuss subcase a) – other subcases will be handled the same way:

Let’s take z’ = uv2wx2y = ak+Lbmanbkckdk

z’ is not in L, because it doesn’t have the required pattern since n,m > 0 and thus there is an a after b

*Conclusion*: We showed that for every possible content of v and x substrings in z, their pumping creates a result (z’) that is not in L. This contradicts to PL2 and proves that our assumption of L being context-free is false

∴ L is not context-free.

**c)** {aib2iai| i >= 0}

Proof: Suppose L is context-free. Let k be the number specified in PL2.

Let’s take z = akb2kak. Note that z is a string from L and length(z) >= k. According to Pl2, z contains two pumpable substrings, v and x such that their parallel pumping produces strings of L. Note that at least one of the pumpable substrings, v or x, is not empty and that the two substrings, v and x, are located within k letters in z.

There are the following possible contents for substrings v and x in our z:

1) both v and x are strings over the same alphabet letter

2) v and x are strings over one – but different in each – alphabet letter

Let’s discuss each of these cases separately:

*Case 1*: both v and x are strings over the same alphabet letter. Namely:

a) v = an, x = am

b) v = bn, x = bm

n,m >= 0; n + m > 0

Let’s discuss subcase a) – other subcases will be handled the same way:

Let’s take z’ = uv2wx2y = ak+n+mb2kak

z’ is not in L, because the ratio between a’s wrong since n+m > 0

*Case 2*: v and x are strings over one – but different in each – alphabet letter. Namely:

a) v = an, x = bm

n,m >= 0; n + m > 0

Let’s discuss subcase a) – other subcases will be handled the same way:

Let’s take z’ = uv2wx2y = ak+nb2k+mak

z’ is not in L, because the ratio between a’s is wrong since n+m > 0

*Conclusion*: We showed that for every possible content of v and x substrings in z, their pumping creates a result (z’) that is not in L. This contradicts to PL2 and proves that our assumption of L being context-free is false

∴ L is not context-free.

**d)** {wwRw | w  {a,b}\*}

Proof: Suppose L is context-free. Let k be the number specified in PL2.

Let’s take z = akbkakbk (so R is 0). Note that z is a string from L and length(z) >= k. According to Pl2, z contains two pumpable substrings, v and x such that their parallel pumping produces strings of L. Note that at least one of the pumpable substrings, v or x, is not empty and that the two substrings, v and x, are located within k letters in z.

There are the following possible contents for substrings v and x in our z:

1) both v and x are strings over the same alphabet letter

2) v and x are strings over one – but different in each – alphabet letter

3) one of the two substrings, v or x, is a string over more than one alphabet letter

Let’s discuss each of these cases separately:

*Case 1*: both v and x are strings over the same alphabet letter. Namely:

a) v = an, x = am

b) v = bn, x = bm

where v,x in the first half

c) v = an, x = am

d) v = bn, x = bm

where v,x in the second half

n,m >= 0; n + m > 0

Let’s discuss subcase a) – other subcases will be handled the same way:

Let’s take z’ = uv2wx2y = ak+n+mbkakbk

z’ is not in L, because it can’t be split into two identical halves.

*Case 2*: v and x are strings over one – but different in each – alphabet letter. Namely:

a) v = an, x = bm  v,x are in first half

b) v = bn, x = am  v,x are in middle

c) v = an, x = bm v,x are in 2nd half

n,m >= 0; n + m > 0

Let’s discuss subcase a) – other subcases will be handled the same way:

Let’s take z’ = uv2wx2y = ak+nbk+makbk

z’ is not in L, because it can’t be split into 2 identical halves since either n or m is > 0

*Case 3*: one of the two substrings, v or x, is a string over more than one alphabet letter. Namely:

a) v = aL, x = anbm v,x are in first half

b) v = bL, x = bnam v,x are in middle

c) v = aL, x = anbm v,x are in second half

d) v = anbm, x = bL v,x are in first half

e) v = bnam, x = aL v,x are in middle

f) v = anbm, x = bL v,x are in second half

L >= 0; n, m > 0

Let’s discuss subcase a) – other subcases will be handled the same way:

Let’s take z’ = uv2wx2y = ak+L+nbk+makbk

z’ is not in L, because it can’t be split into two identical halves, since n,m > 0

*Conclusion*: We showed that for every possible content of v and x substrings in z, their pumping creates a result (z’) that is not in L. This contradicts to PL2 and proves that our assumption of L being context-free is false

∴ L is not context-free.

**18)**

**a)**

S → aSbb | A

A → cA | λ

**b)**

S → aS | A

A → bbAc | λ

**c)**

Resulting Language: L = L1 ∩ L2 = {aib2ici | i >= 0}

Similar to 17c

**19)**

**a)**

S → aSb | A

A → cAd | λ

**b)**

S → aS | A

A → bAc | B

B → dB | λ

**c)**

Resulting Language: L = L1 ∩ L2 = {aibicidi| i >= 0}

Similar to 17b