

1). Prove by contradiction.

Assume for any given flow network, if the current flow is maximal then no edge in the residual graph has a weight of zero.

Then, we can say that the corresponding residual graph has all edges of non-zero weight. In this case, we are guaranteed to have at least one augmenting path added to the max flow.

Thus, the previous flow is smaller than the new max flow, which makes the previous flow not maximal. There is a contradiction.

It must be the case that for any flow network, if the current flow is maximal then at least one edge has a flow that matches its capacity.

2). Modify the given graph $G=(V,E)$ by setting all edges to weight K . Then run the max flow (Ford-Fulkerson) algorithm on the modified graph. Return the sequence of nodes passed by the flow from the start node to end node as the longest set of edge K -disjoint

3). Reducing the problem to max flow requires $O(|E|)$ running time

Applying the max flow algorithm requires $O(|E| \cdot k)$

Constructing the path requires $O(V+E)$ running time.

The running time of solving this problem is $O(|E| \cdot k)$