Q1.

s1=1 s2=9 s3=0 s4=5 s5=6 s6=6 s7=7

Q2.

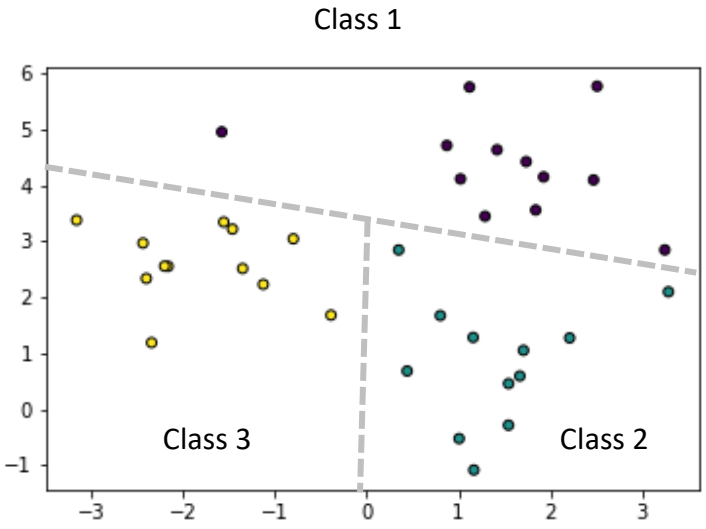
R_1=2

Use Binary decision tree

Q3.

Class 1		Class 2		Class 3	
X1	X2	X1	X2	X1	X2
-1.58	4.96	0.44	0.68	-2.16	2.56
0.87	4.71	3.29	2.1	-2.34	1.19
2.47	4.1	1.01	-0.52	-2.2	2.56
1.02	4.12	0.35	2.85	-2.43	2.97
3.25	2.85	1.16	1.28	-1.35	2.52
1.29	3.45	1.54	-0.28	-0.39	1.68
1.42	4.64	1.71	1.05	-1.46	3.22
1.93	4.15	2.21	1.28	-2.4	2.34
1.84	3.56	0.8	1.68	-0.8	3.05
2.51	5.77	1.55	0.46	-1.12	2.23
1.74	4.43	1.67	0.6	-1.56	3.35
1.12	5.76	1.17	-1.08	-3.16	3.38

Q4.



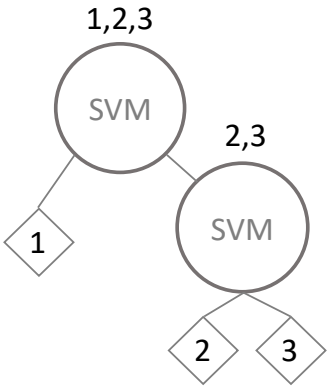
Q5.

Method: Binary decision tree

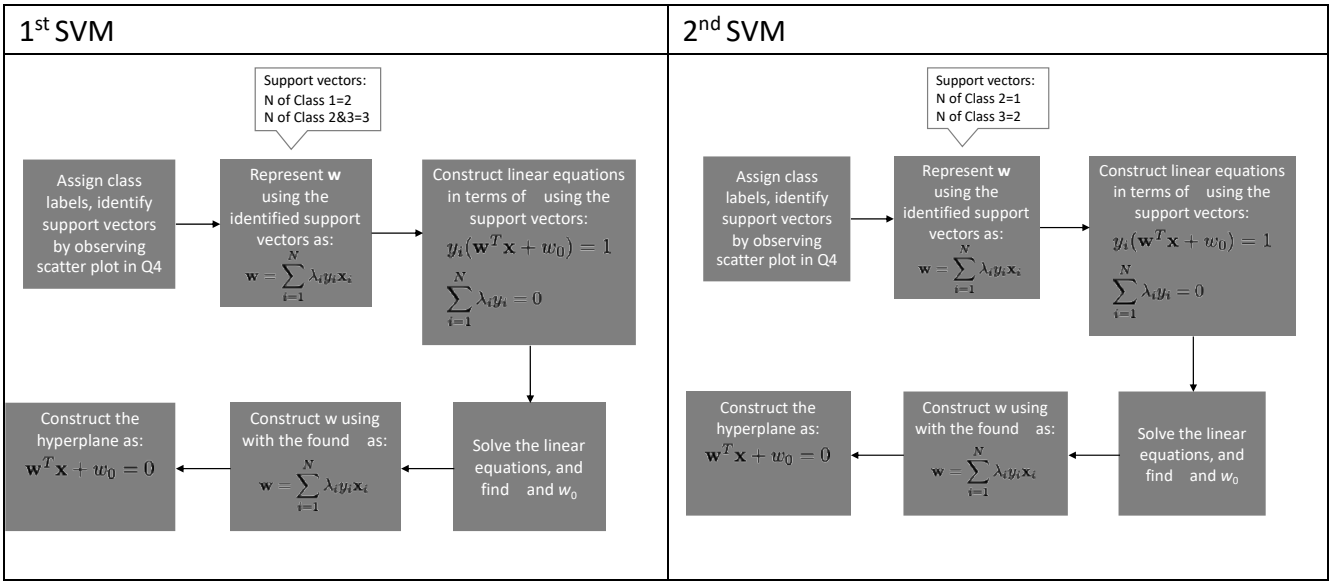
Class assignments:

SVM Number	Class assignment (+1|-1)
SVM 1	1|23
SVM 2	2|3

Steps of structuring the Binary decision tree classifier constructed by linear SVMs:



Steps of structing SVM:



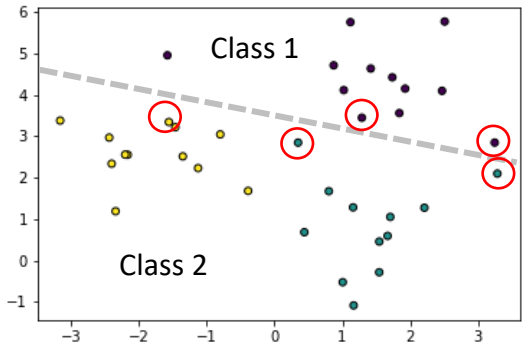
Summary:

	1st SVM	2nd SVM
number of inputs	Class 1: 2 (+1) Class 2: 3 (-1)	Class 1: 1 (+1) Class 2: 2 (-1)
number of outputs	λ: 5 & w_0	λ: 3 & w_0
number of SVMs used	2 (R-1)	
class label assignment	1|23	2|3

Q6.

SVM1

Step1:



Class 1	$x_1 = \begin{bmatrix}3.25\\2.85\end{bmatrix}$ $x_2 = \begin{bmatrix}1.29\\3.45\end{bmatrix}$ $, y = +1$
Class 2	$x_3 = \begin{bmatrix}3.29\\2.1\end{bmatrix}$ $x_4 = \begin{bmatrix}0.35\\2.85\end{bmatrix}$ $x_5 = \begin{bmatrix}-1.56\\3.35\end{bmatrix}$ $, y = -1$

Step2:

$$\omega = \sum_{i=1}^{5} \lambda_i y_i x_i = \lambda_1 x_1 + \lambda_2 x_2 - \lambda_3 x_3 - \lambda_4 x_4 - \lambda_5 x_5 = \lambda_1 \begin{bmatrix}3.25\\2.85\end{bmatrix} + \lambda_2 \begin{bmatrix}1.29\\3.45\end{bmatrix} - \lambda_3 \begin{bmatrix}3.29\\2.1\end{bmatrix} - \lambda_4 \begin{bmatrix}0.35\\2.85\end{bmatrix} - \lambda_5 \begin{bmatrix}-1.56\\3.35\end{bmatrix}$$

Step3:

$$y_1(\omega^T x_1 + \omega_0) = 1 \times \left(\left(\lambda_1 \begin{bmatrix}3.25\\2.85\end{bmatrix} + \lambda_2 \begin{bmatrix}1.29\\3.45\end{bmatrix} - \lambda_3 \begin{bmatrix}3.29\\2.1\end{bmatrix} - \lambda_4 \begin{bmatrix}0.35\\2.85\end{bmatrix} - \lambda_5 \begin{bmatrix}-1.56\\3.35\end{bmatrix}\right)^T \begin{bmatrix}3.25\\2.85\end{bmatrix} + \omega_0\right) = 1$$

$$\Rightarrow 18.685\lambda_1 + 14.025\lambda_2 - 16.678\lambda_3 - 9.26\lambda_4 - 4.478\lambda_5 + \omega_0 = 1$$

$$y_2(\omega^T x_2 + \omega_0) = 1 \times \left(\left(\lambda_1 \begin{bmatrix}3.25\\2.85\end{bmatrix} + \lambda_2 \begin{bmatrix}1.29\\3.45\end{bmatrix} - \lambda_3 \begin{bmatrix}3.29\\2.1\end{bmatrix} - \lambda_4 \begin{bmatrix}0.35\\2.85\end{bmatrix} - \lambda_5 \begin{bmatrix}-1.56\\3.35\end{bmatrix}\right)^T \begin{bmatrix}1.29\\3.45\end{bmatrix} + \omega_0\right) = 1$$

$$\Rightarrow 14.025\lambda_1 + 13.567\lambda_2 - 11.489\lambda_3 - 10.284\lambda_4 - 9.545\lambda_5 + \omega_0 = 1$$

$$y_3(\omega^T x_3 + \omega_0) = (-1) \times \left(\left(\lambda_1 \begin{bmatrix}3.25\\2.85\end{bmatrix} + \lambda_2 \begin{bmatrix}1.29\\3.45\end{bmatrix} - \lambda_3 \begin{bmatrix}3.29\\2.1\end{bmatrix} - \lambda_4 \begin{bmatrix}0.35\\2.85\end{bmatrix} - \lambda_5 \begin{bmatrix}-1.56\\3.35\end{bmatrix}\right)^T \begin{bmatrix}3.29\\2.1\end{bmatrix} + \omega_0\right) = 1$$

$$\Rightarrow 16.678\lambda_1 + 11.489\lambda_2 - 15.234\lambda_3 - 7.137\lambda_4 - 1.903\lambda_5 + \omega_0 = -1$$

$$y_4(\omega^T x_4 + \omega_0) = (-1) \times \left(\left(\lambda_1 \begin{bmatrix}3.25\\2.85\end{bmatrix} + \lambda_2 \begin{bmatrix}1.29\\3.45\end{bmatrix} - \lambda_3 \begin{bmatrix}3.29\\2.1\end{bmatrix} - \lambda_4 \begin{bmatrix}0.35\\2.85\end{bmatrix} - \lambda_5 \begin{bmatrix}-1.56\\3.35\end{bmatrix}\right)^T \begin{bmatrix}0.35\\2.85\end{bmatrix} + \omega_0\right) = 1$$

$$\Rightarrow 9.26\lambda_1 + 10.284\lambda_2 - 7.137\lambda_3 - 8.245\lambda_4 - 9.002\lambda_5 + \omega_0 = -1$$

$$y_5(\omega^T x_5 + \omega_0) = (-1) \times \left(\left(\lambda_1 \begin{bmatrix}3.25\\2.85\end{bmatrix} + \lambda_2 \begin{bmatrix}1.29\\3.45\end{bmatrix} - \lambda_3 \begin{bmatrix}3.29\\2.1\end{bmatrix} - \lambda_4 \begin{bmatrix}0.35\\2.85\end{bmatrix} - \lambda_5 \begin{bmatrix}-1.56\\3.35\end{bmatrix}\right)^T \begin{bmatrix}-1.56\\3.35\end{bmatrix} + \omega_0\right) = 1$$

$$\Rightarrow 4.478\lambda_1 + 9.545\lambda_2 - 1.903\lambda_3 - 9.002\lambda_4 - 13.656\lambda_5 + \omega_0 = -1$$

$$\sum_{i=1}^{5} \lambda_i y_i = 0 \Rightarrow \lambda_1 + \lambda_2 - \lambda_3 - \lambda_4 - \lambda_5 = 0$$

Step4:

$$\begin{bmatrix} 18.685 & 14.025 & -16.678 & -9.26 & -4.478 & 1 \\ 14.025 & 13.567 & -11.489 & -10.284 & -9.545 & 1 \\ 16.678 & 11.489 & -15.234 & -7.137 & -1.903 & 1 \\ 9.26 & 10.284 & -7.137 & -8.245 & -9.002 & 1 \\ 4.478 & 9.545 & -1.903 & -9.002 & -13.656 & 1 \\ 1 & 1 & -1 & -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \omega_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ 0 \end{bmatrix}$$

$$\Rightarrow \lambda_1 = 520.8103, \lambda_2 = -461.4698, \lambda_3 = 95.8143, \lambda_4 = 379.3294, \lambda_5 = -415.8033, \omega_0 = -9.6108$$

Step5:

$$\omega = \lambda_1 \begin{bmatrix} 3.25 \\ 2.85 \end{bmatrix} + \lambda_2 \begin{bmatrix} 1.29 \\ 3.45 \end{bmatrix} - \lambda_3 \begin{bmatrix} 3.29 \\ 2.1 \end{bmatrix} - \lambda_4 \begin{bmatrix} 0.35 \\ 2.85 \end{bmatrix} - \lambda_5 \begin{bmatrix} -1.56 \\ 3.35 \end{bmatrix}$$

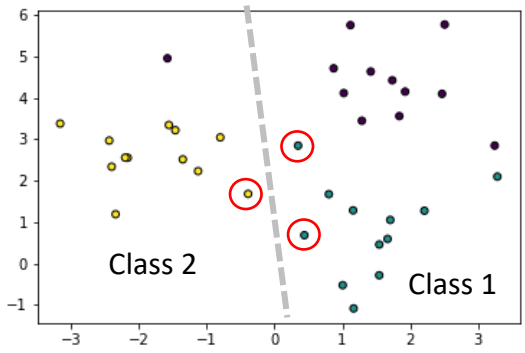
$$= 520.8103 \begin{bmatrix} 3.25 \\ 2.85 \end{bmatrix} - 461.4698 \begin{bmatrix} 1.29 \\ 3.45 \end{bmatrix} - 95.8143 \begin{bmatrix} 3.29 \\ 2.1 \end{bmatrix} - 379.3294 \begin{bmatrix} 0.35 \\ 2.85 \end{bmatrix} + 415.8033 \begin{bmatrix} -1.56 \\ 3.35 \end{bmatrix} = \begin{bmatrix} 0.69 \\ 2.88 \end{bmatrix}$$

Step6:

The hyperplane is $\omega^T x_i + \omega_0 = 0.69x_1 + 2.88x_2 - 9.6108$ $(SVM1)$

<u>SVM2</u>

Step1:



Class 1	$x_1 = \begin{bmatrix} 0.35 \\ 2.85 \end{bmatrix}$ $x_2 = \begin{bmatrix} 0.44 \\ 0.68 \end{bmatrix}$ $, y = +1$
Class 2	$x_3 = \begin{bmatrix} -0.39 \\ 1.68 \end{bmatrix}$ $, y = -1$

Step2:

$$\omega = \sum_{i=1}^{3} \lambda_i y_i x_i = \lambda_1 x_1 + \lambda_2 x_2 - \lambda_3 x_3 = \lambda_1 \begin{bmatrix} 0.35 \\ 2.85 \end{bmatrix} + \lambda_2 \begin{bmatrix} 0.44 \\ 0.68 \end{bmatrix} - \lambda_3 \begin{bmatrix} -0.39 \\ 1.68 \end{bmatrix}$$

Step3:

$$y_1(\omega^T x_1 + \omega_0) = 1 \times \left(\left(\lambda_1 \begin{bmatrix}0.35\\2.85\end{bmatrix} + \lambda_2 \begin{bmatrix}0.44\\0.68\end{bmatrix} - \lambda_3 \begin{bmatrix}-0.39\\1.68\end{bmatrix}\right)^T \begin{bmatrix}0.35\\2.85\end{bmatrix} + \omega_0\right) = 1$$

$$\Rightarrow 8.245\lambda_1 + 2.092\lambda_2 - 4.6515\lambda_3 + \omega_0 = 1$$

$$y_2(\omega^T x_2 + \omega_0) = 1 \times \left(\left(\lambda_1 \begin{bmatrix}0.35\\2.85\end{bmatrix} + \lambda_2 \begin{bmatrix}0.44\\0.68\end{bmatrix} - \lambda_3 \begin{bmatrix}-0.39\\1.68\end{bmatrix}\right)^T \begin{bmatrix}0.44\\0.68\end{bmatrix} + \omega_0\right) = 1$$

$$\Rightarrow 2.092\lambda_1 + 0.656\lambda_2 - 0.9708\lambda_3 + \omega_0 = 1$$

$$y_3(\omega^T x_3 + \omega_0) = (-1) \times \left(\left(\lambda_1 \begin{bmatrix}0.35\\2.85\end{bmatrix} + \lambda_2 \begin{bmatrix}0.44\\0.68\end{bmatrix} - \lambda_3 \begin{bmatrix}-0.39\\1.68\end{bmatrix}\right)^T \begin{bmatrix}-0.39\\1.68\end{bmatrix} + \omega_0\right) = 1$$

$$\Rightarrow 4.6515\lambda_1 + 0.9708\lambda_2 - 2.9745\lambda_3 + \omega_0 = -1$$

$$\sum_{i=1}^{3} \lambda_i y_i = 0 \Rightarrow \lambda_1 + \lambda_2 - \lambda_3 = 0$$

Step4:

$$\begin{bmatrix}8.245 & 2.092 & -4.6515 & 1\\ 2.092 & 0.656 & -0.9708 & 1\\ 4.6515 & 0.9708 & -2.9745 & 1\\ 1 & 1 & -1 & 0\end{bmatrix}\begin{bmatrix}\lambda_1\\\lambda_2\\\lambda_3\\\omega_0\end{bmatrix} = \begin{bmatrix}1\\1\\-1\\0\end{bmatrix} \Rightarrow \lambda_1 = 1.5333, \lambda_2 = 1.6888, \lambda_3 = 3.2221, \omega_0 = -0.1875$$

Step5:

$$\omega = \lambda_1 \begin{bmatrix}0.35\\2.85\end{bmatrix} + \lambda_2 \begin{bmatrix}0.44\\0.68\end{bmatrix} - \lambda_3 \begin{bmatrix}-0.39\\1.68\end{bmatrix} = 1.5333 \begin{bmatrix}0.35\\2.85\end{bmatrix} + 1.6888 \begin{bmatrix}0.44\\0.68\end{bmatrix} - 3.2221 \begin{bmatrix}-0.39\\1.68\end{bmatrix} = \begin{bmatrix}2.54\\0.11\end{bmatrix}$$

Step6:

The hyperplane is $\omega^T x_i + \omega_0 = 2.54x_1 + 0.11x_2 - 0.1875$ $(SVM2)$

Summary

SVM1	SVM2	Output
+1		Class 1
-1	+1	Class 2
	-1	Class 3

Q7. SVM1: $\omega^T \mathcal{X}_i + \omega_0 = 0.69x_1 + 2.88x_2 - 9.6108$ SVM2: $\omega^T \mathcal{X}_i + \omega_0 = 2.54x_1 + 0.11x_2 - 0.1875$

Sample		Output of SVM 1	Output of SVM 2	Classification
X1	X2			
-1.1	2.73	-2.5074	-2.6812	Class 3
0.61	2.67	-1.5003	1.6556	Class 2
0.43	2.05	-3.4101	1.1302	Class 2
-0.35	3.31	-0.3195	-0.7124	Class 3
1.02	2.22	-2.5134	2.6475	Class 2
0.81	1.62	-4.3863	2.0481	Class 2
0.56	2.97	-0.6708	1.5616	Class 2
0.58	2.59	-1.7514	1.5706	Class 2
0.61	2.76	-1.2411	1.6655	Class 2
0.98	2.82	-0.813	2.6119	Class 2
0.62	2.8	-1.119	1.6953	Class 2
-0.29	2.69	-2.0637	-0.6282	Class 3