# Chapter 3
Image Enhancement in the Spatial Domain-2

3.5 Basics of Spatial Filtering

- Some neighborhood operations work with the values of the image pixels in the neighborhood and the corresponding values of a sub-image that has the same dimensions as the neighborhood.

- The sub-image is called a filter, mask, kernel, template, or window.

- The values in a filter sub-image are referred to as coefficients, rather than pixels.

- The concept of filtering has its roots in the use of the Fourier transform for signal processing in the so-called frequency domain.

- We use the term spatial filtering to differentiate this type of process from the more traditional frequency domain filtering.

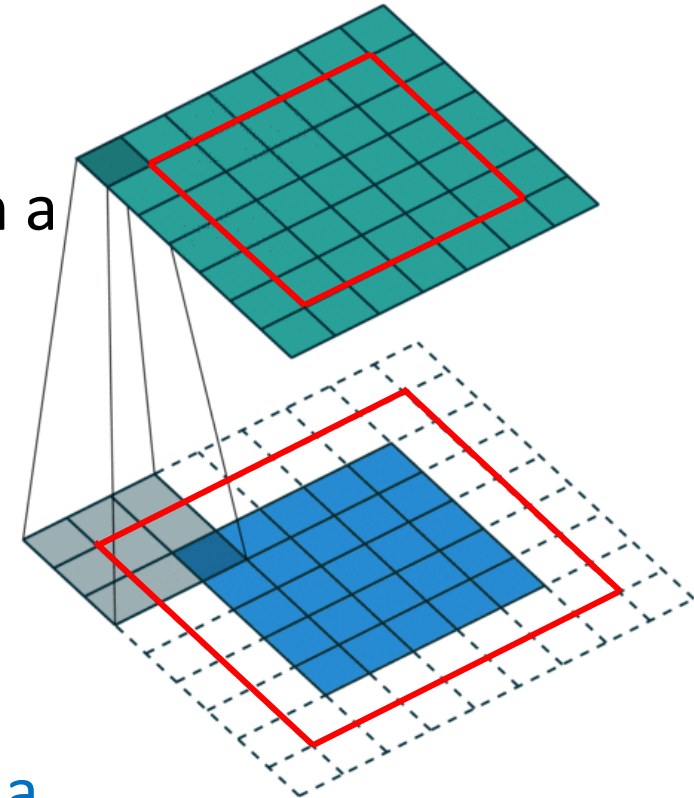# 3.5 Basics of Spatial Filtering

Filtered result.

Source Image.
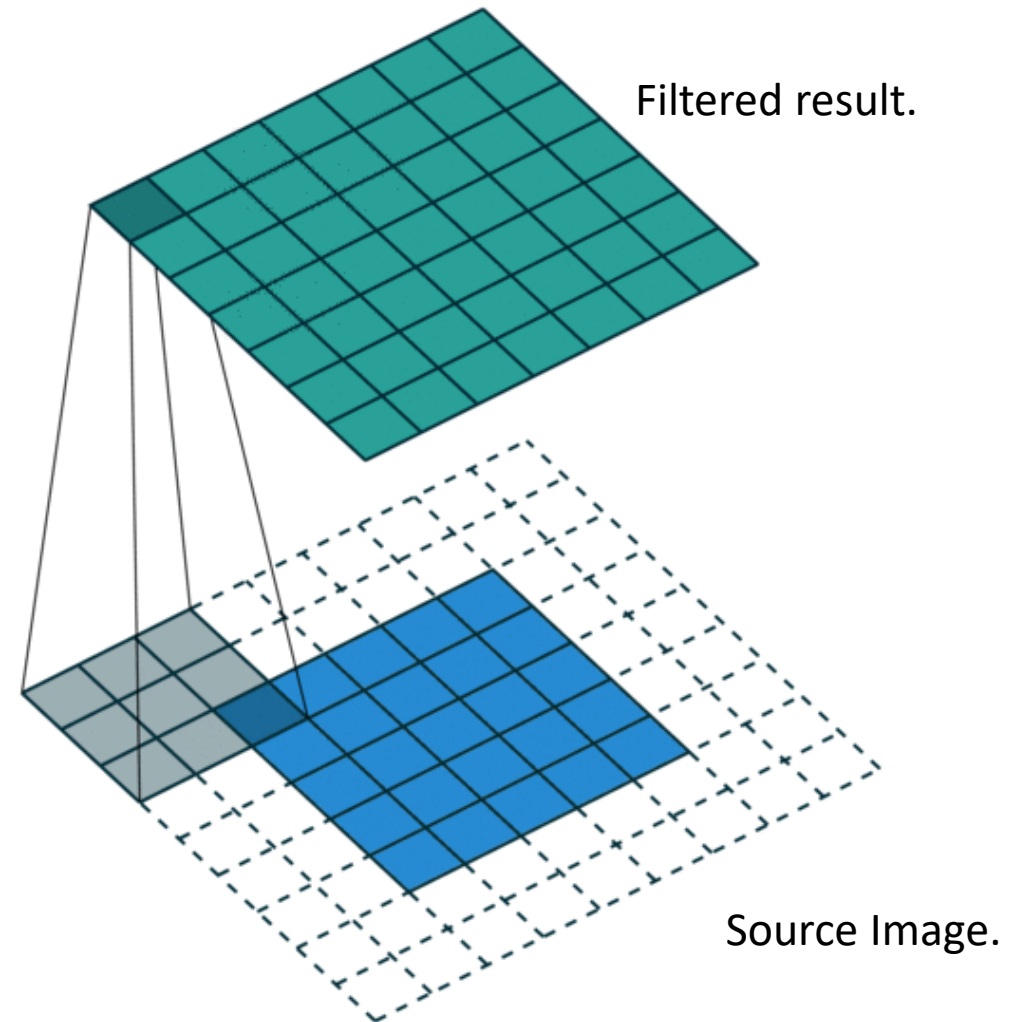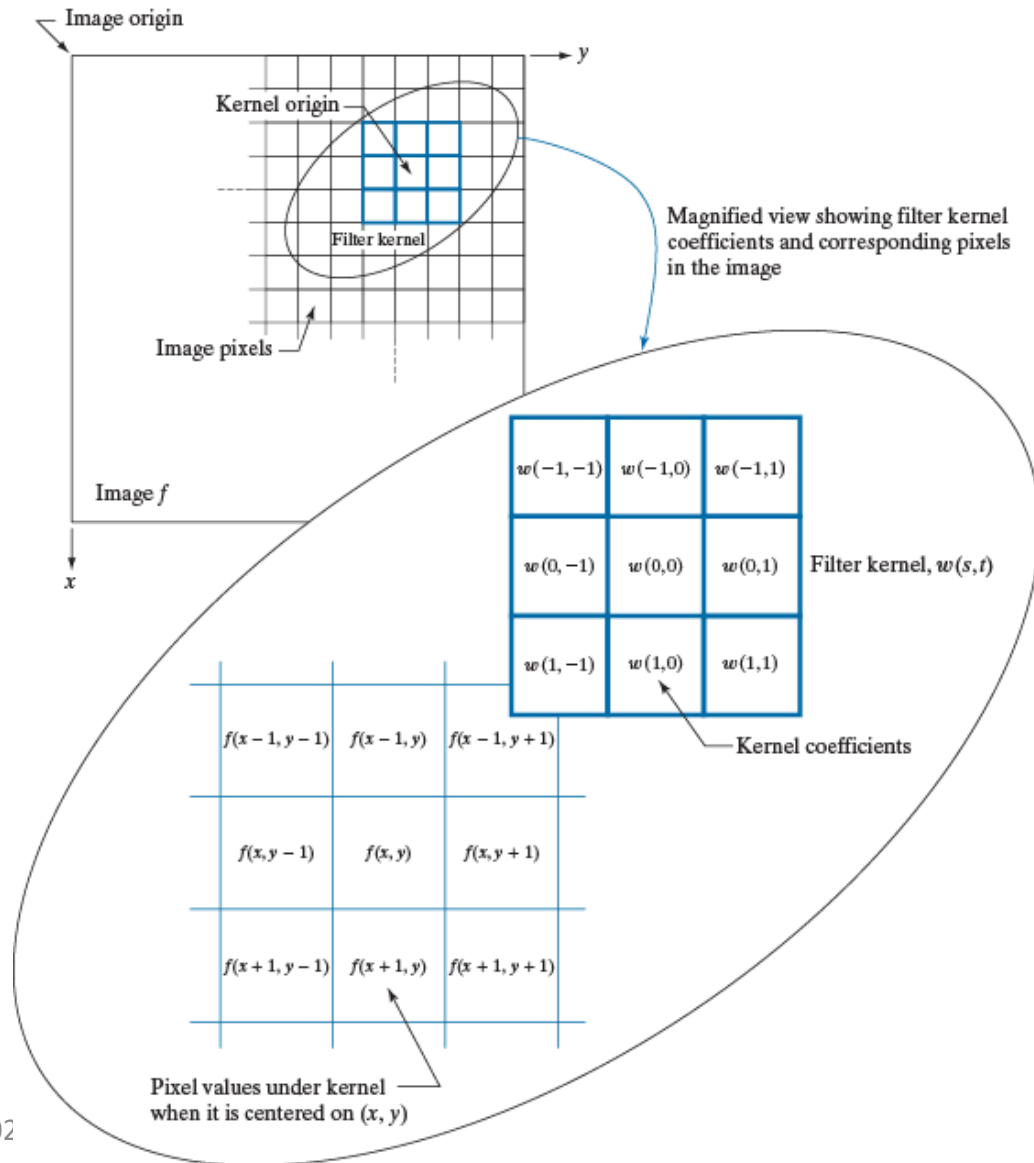
3.5 Basics of Spatial Filtering

- In general, linear filtering of an image f of size $M \times N$ with a filter mask of size $m \times n$ is given by the expression:

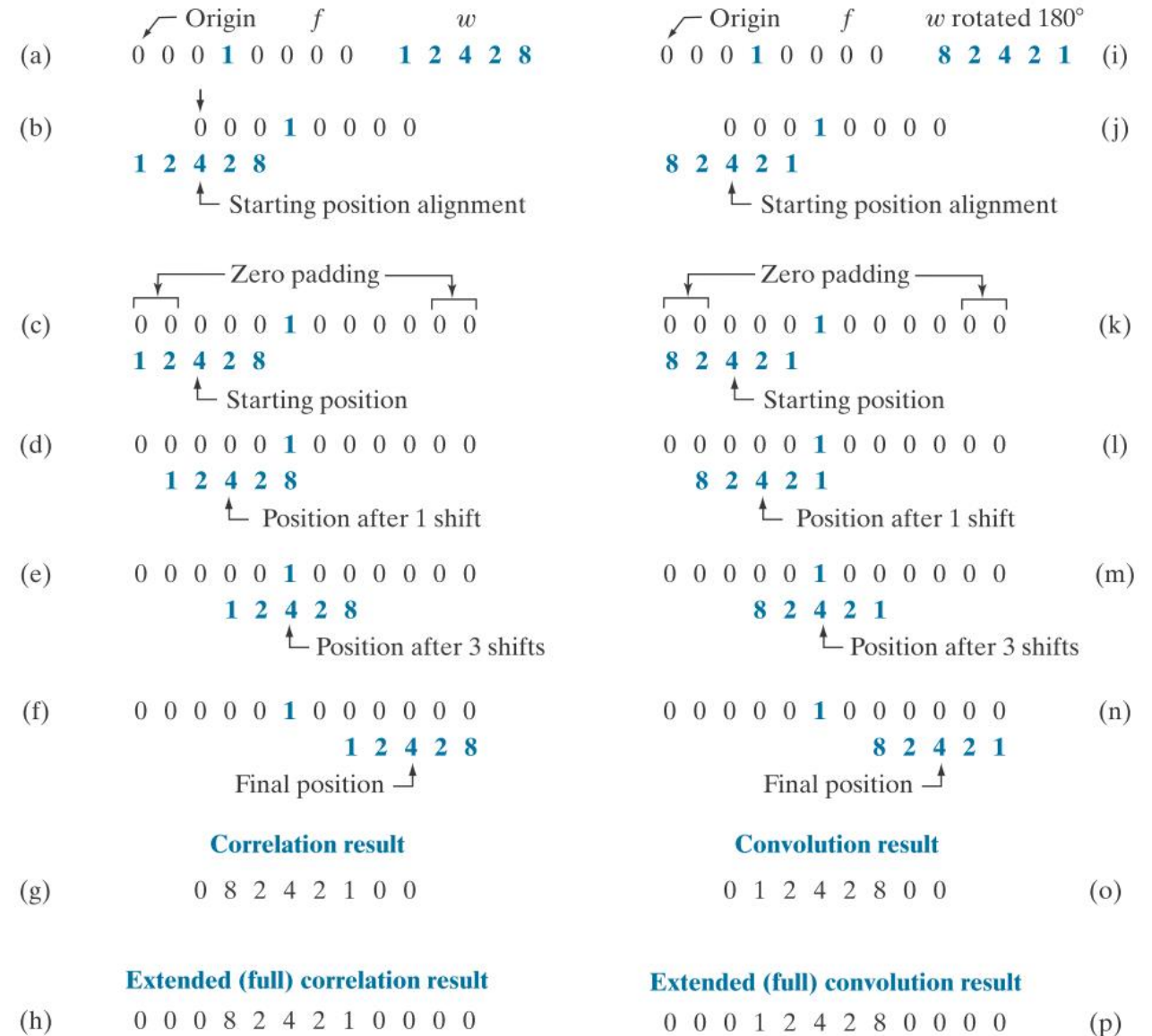$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) \quad .....(3.5\text{-}1)$$

  - where $a=(m\text{-}1)/2$ and $b=(n\text{-}1)/2$
  - To generate a complete filtered image, this equation must be applied for $x=0, 1, 2, \dots , M\text{-}1$ and $y=0, 1, 2, \dots , N\text{-}1$.

- Linear spatial filtering often is referred to as "convolving a mask with an image." Similarly, filter masks are sometimes called convolution masks. The term convolution kernel also is in common use.

- 訊號處理中的 convolution 定義為：$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) \boxed{f(x-s, y-t)}$

3.5 Basics of Spatial Filtering

- Correlation and convolution results of a 1-D function f, and a kernel w.

3.5 Basics of Spatial Filtering

- 訊號處理中的 Correlation:

$$g(x, y) = w \Leftrightarrow f(x, y) \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) \quad \ldots\ldots(3.5\text{-}2)$$

- 訊號處理中的 Convolution:

$$g(x, y) = w \star f(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t) \quad \ldots\ldots(3.5\text{-}3)$$

- Though both the operations are slightly different, yet it doesn't matter if the kernel is symmetric.

- Size of full correlation/convolution:
  - $S_v = m + M - 1$
  - $S_h = n + N - 1$

3.5 Basics of Spatial Filtering

TABLE 3.5
Some fundamental properties of convolution and correlation. A dash means that the property does not hold.

Property	Convolution	Correlation
Commutative	$f \star g = g \star f$	—
Associative	$f \star (g \star h) = (f \star g) \star h$	—
Distributive	$f \star (g + h) = (f \star g) + (f \star h)$	$f \star (g + h) = (f \star g) + (f \star h)$

- Suppose an image f is filtered with a kernel w_1, the result filtered with kernel w_2, that result filtered with a third kernel, and so on, for Q stages.

$$w \star f = \left(w_1 \star w_2 \star w_3 \star ... \star w_Q \right) \star f$$

  - The size of w is
    - $W_v = Q \times (m - 1) + m$
    - $W_h = Q \times (n - 1) + n$

3.5 Basics of Spatial Filtering

- Separatable filter kernels:
  - A spatial filter kernel is a matrix, and a separable kernel is a matrix that can be expressed as the outer product of two vectors.
  - For example:

  $$w = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} = cr^T$$

  - A separable kernel of size $m \times n$ can be expressed as the outer product of two vectors : $w = \mathbf{v}\mathbf{w}^T$
    - $\mathbf{v}$ and $\mathbf{w}$ are vectors of size $m \times 1$ and $n \times 1$.
  - If we have a kernel w that can be decomposed into two simpler kernels, such that $w = w_1 \star w_2$, then

  $$w \star f = (w_1 \star w_2) \star f = (w_2 \star w_1) \star f = w_2 \star (w_1 \star f) = (w_1 \star f) \star w_2 \quad \text{.....(3.5-4)}$$

  - Convolving a separable kernel with an image is the same as convolving w_1 with f first, and then convolving the result with w_2.

3.5 Basics of Spatial Filtering

- Separatable filter kernels:
  - For an image of size $M \times N$ and a kernel of size $m \times n$, implementation of Eq. (3.5-2) requires on the order of $MNmn$ multiplications and additions.
  - If the kernel is separable and we use Eq. (3.5-4), then the first convolution, $w_1 \star f$, requires on the order of MNm multiplications and additions. The result is of size $M \times N$, so the convolution of w_2 with the result requires MNn such operations, for a total of $MN(m+n)$ multiplication and addition operations.
  - Thus, the computational advantage of performing convolution with a separable:

$$C = \frac{MNmn}{MN(m+n)} = \frac{mn}{m+n}$$

  - If $m \times n = 11 \times 11$, C=5.2 times faster with separatable kernels.

3.5 Basics of Spatial Filtering

- The response, *R*, of an *m×n* mask at any point (*x, y*):

$$R = w_1 z_1 + w_2 z_2 + \ldots + w_{mn} z_{mn} = \sum_{i=1}^{mn} w_i z_i \quad \ldots\ldots(3.5\text{-}2)$$

  - where the *w*'s are mask coefficients, the *z*'s are the values of the image gray levels corresponding to those coefficients, and *mn* is the total number of coefficients in the mask.

- For the 3*3 general mask shown in Fig. 3.33 the response at any point (*x, y*) in the image is given by

$$R = w_1 z_1 + w_2 z_2 + \ldots + w_9 z_9 = \sum_{i=1}^{9} w_i z_i \quad \ldots\ldots(3.5\text{-}3)$$

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

3.5 Basics of Spatial Filtering

- How to handle border issue?
  - Limit the excursions of the center of the mask to be at a distance no less than (n-1)/2 pixels from the border.
    - The resulting filtered image will be smaller than the original one.
  - Partial filter.
    - There will be bands of pixels near the border that will have been processed with a partial filter mask.
  - Padding
    - Pad zeros.
    - Padding by replicating rows or columns.
    - Padding by reflecting (mirroring) rows or columns.
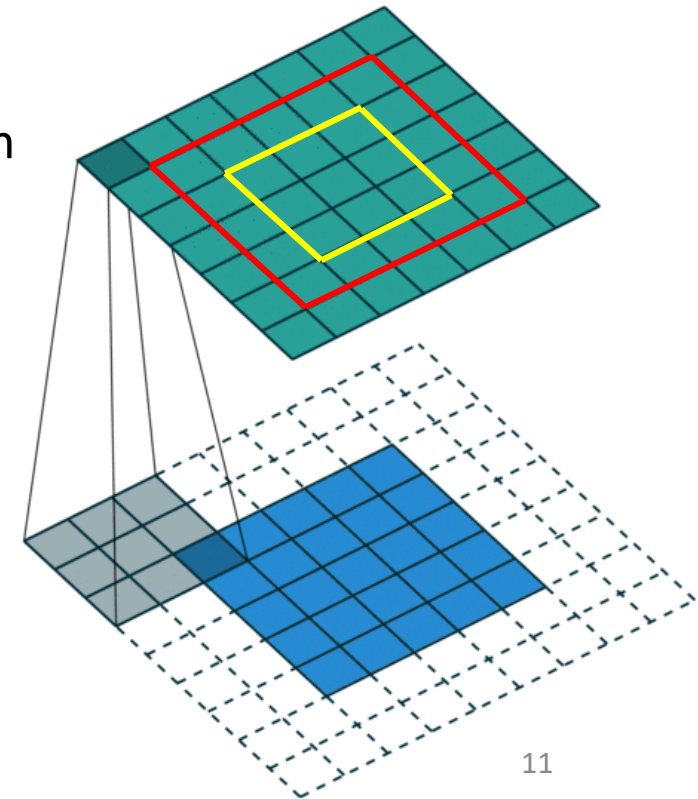
3.6 Smoothing Spatial Filters

- Smoothing filters are used for blurring and for noise reduction.
  - Blurring is used in preprocessing steps, such as removal of small details from an image prior to (large) object extraction, and bridging of small gaps in lines or curves.
  - Noise reduction can be accomplished by blurring with a linear filter and also by non-linear filtering.

3.6 Smoothing Spatial Filters

- Smoothing Linear Filters:
  - The output (response) of a smoothing, linear spatial filter is simply the average of the pixels contained in the neighborhood of the filter mask. These filters sometimes are called averaging filters (lowpass filters).

1	1	1
1	1	1
1	1	1

$\frac{1}{9} \times$

$$g(x, y) = \frac{1}{\sum\limits_{s=-a}^{a}\sum\limits_{t=-b}^{b} w(s,t)} \sum\limits_{s=-a}^{a}\sum\limits_{t=-b}^{b} w(s,t)f(x+s, y+t)$$

A spatial averaging filter in which all coefficients are equal is sometimes called a box filter.

3.6 Smoothing Spatial Filters

- Smoothing Linear Filters:
  - Gaussian filter:

$$w(s,t) = Ke^{-\frac{s^2+t^2}{2\sigma^2}}$$

$\frac{1}{16} \times$

1	2	1
2	4	2
1	2	1

a b

FIGURE 3.35
(a) Sampling a Gaussian function to obtain a discrete Gaussian kernel. The values shown are for $K = 1$ and $\sigma = 1$. (b) Resulting 3×3 kernel [this is the same as Fig. 3.31(b)].
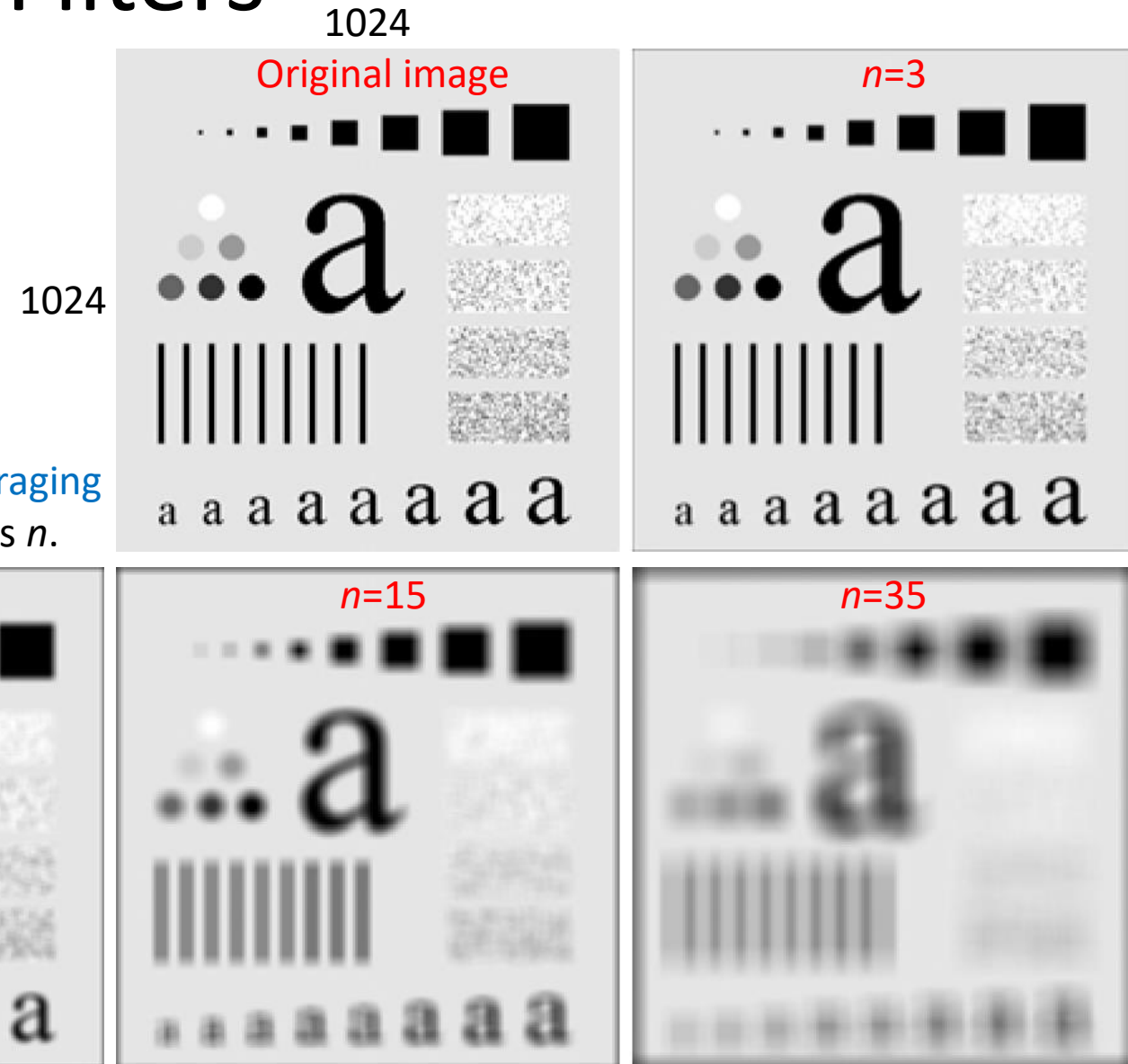
$G(s,t)$
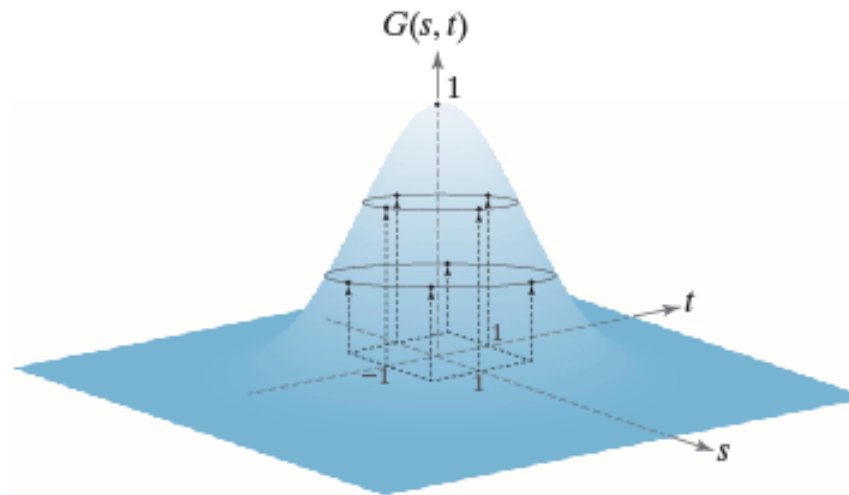
$\frac{1}{4.8976} \times$

0.3679	0.6065	0.3679
0.6065	1.0000	0.6065
0.3679	0.6065	0.3679

3.6 Smoothing Spatial Filters

- Smoothing Linear Filters:

Smoothing with square averaging filter masks of different sizes n.

3.6 Smoothing Spatial Filters

- Smoothing Linear Filters – Gaussian filter
  - The values of a Gaussian function at a distance larger than 3σ from the mean are small enough that they can be ignored.
  - We select the size of a Gaussian kernel to be $\lceil 6\sigma \rceil \times \lceil 6\sigma \rceil$ (celling of 6σ)

a b c
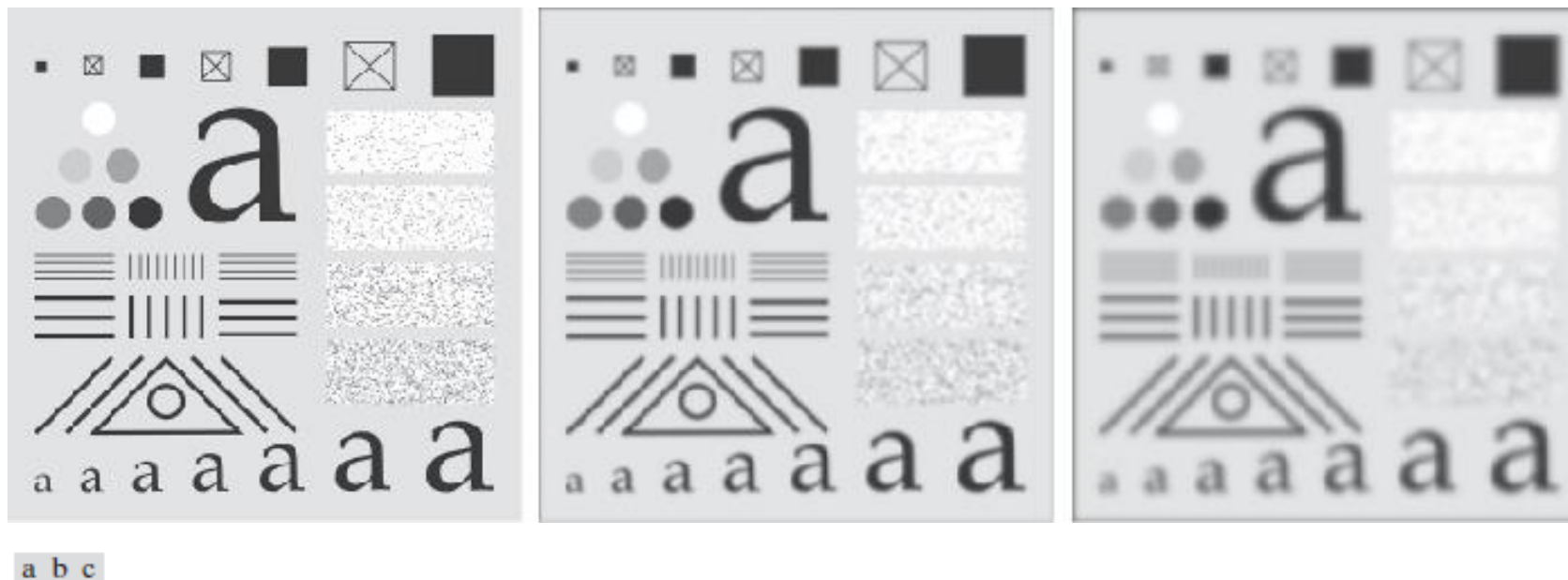
FIGURE 3.36 (a)A test pattern of size 1024×1024. (b) Result of lowpass filtering the pattern with a Gaussian kernel of size 21×21, with standard deviations $\sigma = 3.5$. (c) Result of using a kernel of size 43×43, with $\sigma = 7$. This result is comparable to Fig. 3.33(d). We used $K = 1$ in all cases.

3.6 Smoothing Spatial Filters

- Smoothing Linear Filters – Gaussian filter

TABLE 3.6 Mean and standard deviation of the product ($\times$) and convolution ($\star$) of two 1-D Gaussian functions, f and g. These results generalize directly to the product and convolution of more than two 1-D Gaussian functions (see Problem 3.25).

	f	g	$f \times g$	$f \star g$
Mean	m_f	m_g	$m_{f \times g} = \dfrac{m_f \sigma_g^2 + m_g \sigma_f^2}{\sigma_f^2 + \sigma_g^2}$	$m_{f \star g} = m_f + m_g$
Standard deviation	σ_f	σ_g	$\sigma_{f \times g} = \sqrt{\dfrac{\sigma_f^2 \sigma_g^2}{\sigma_f^2 + \sigma_g^2}}$	$\sigma_{f \star g} = \sqrt{\sigma_f^2 + \sigma_g^2}$

3.6 Smoothing Spatial Filters

- Smoothing Linear Filters:

a b c

FIGURE 3.37 (a) Result of filtering Fig. 3.36(a) using a Gaussian kernels of size 43×43, with $\sigma = 7$. (b) Result of using a kernel of 85×85, with the same value of σ. (c) Difference image.

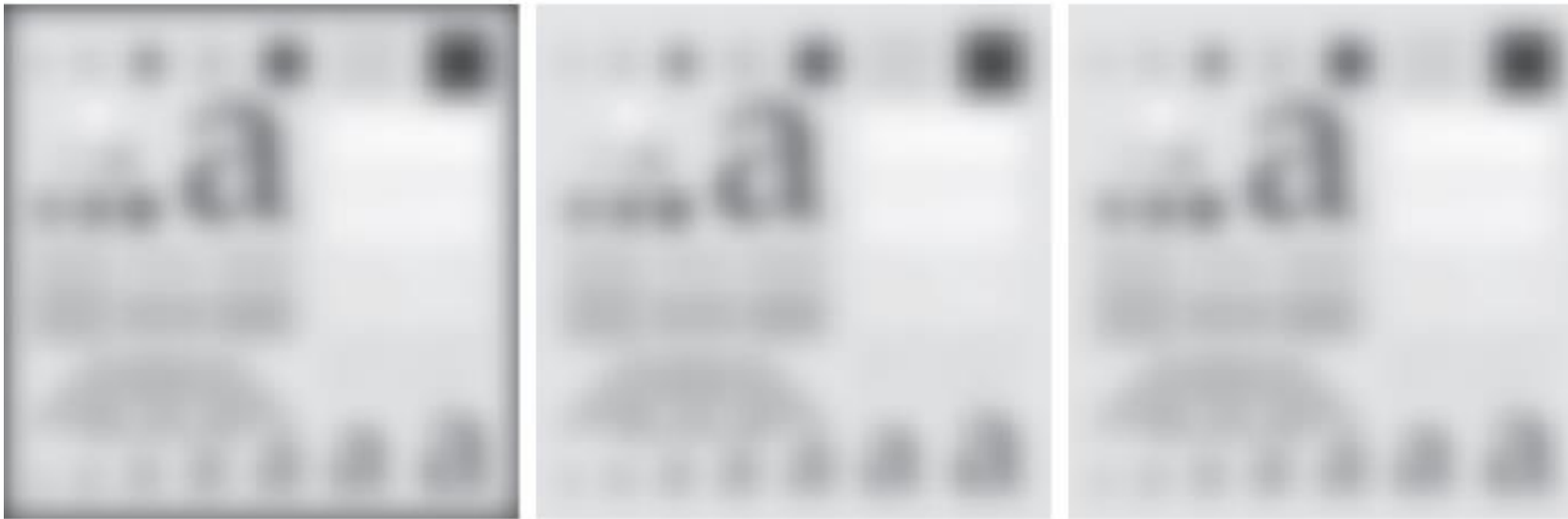# 3.6 Smoothing Spatial Filters

- Smoothing Linear Filters:
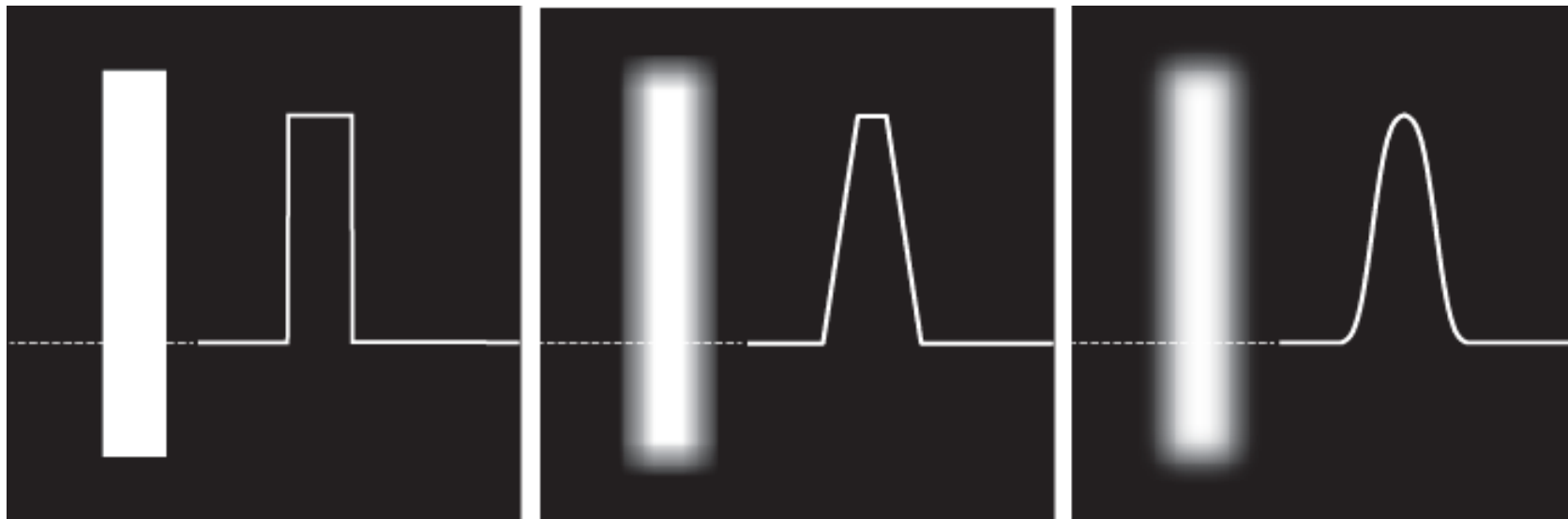
a b c

FIGURE 3.38 (a) Image of a white rectangle on a black background, and a horizontal intensity profile along the scan line shown dotted. (b) Result of smoothing this image with a box kernel of size 71×71, and corresponding intensity profile. (c) Result of smoothing the image using a Gaussian kernel of size 151×151, with $K = 1$ and $\sigma = 25$. Note the smoothness of the profile in (c) compared to (b). The image and rectangle are of sizes 1024×1024 and 768×128 pixels, respectively.

3.6 Smoothing Spatial Filters

- Smoothing Linear Filters:

a b c

FIGURE 3.39 Result of filtering the test pattern in Fig. 3.36(a) using (a) zero padding, (b) mirror padding, and (c) replicate padding. A Gaussian kernel of size 187×187, with $K = 1$ and $\sigma = 31$ was used in all three cases.

3.6 Smoothing Spatial Filters

- Smoothing Linear Filters:

a b c
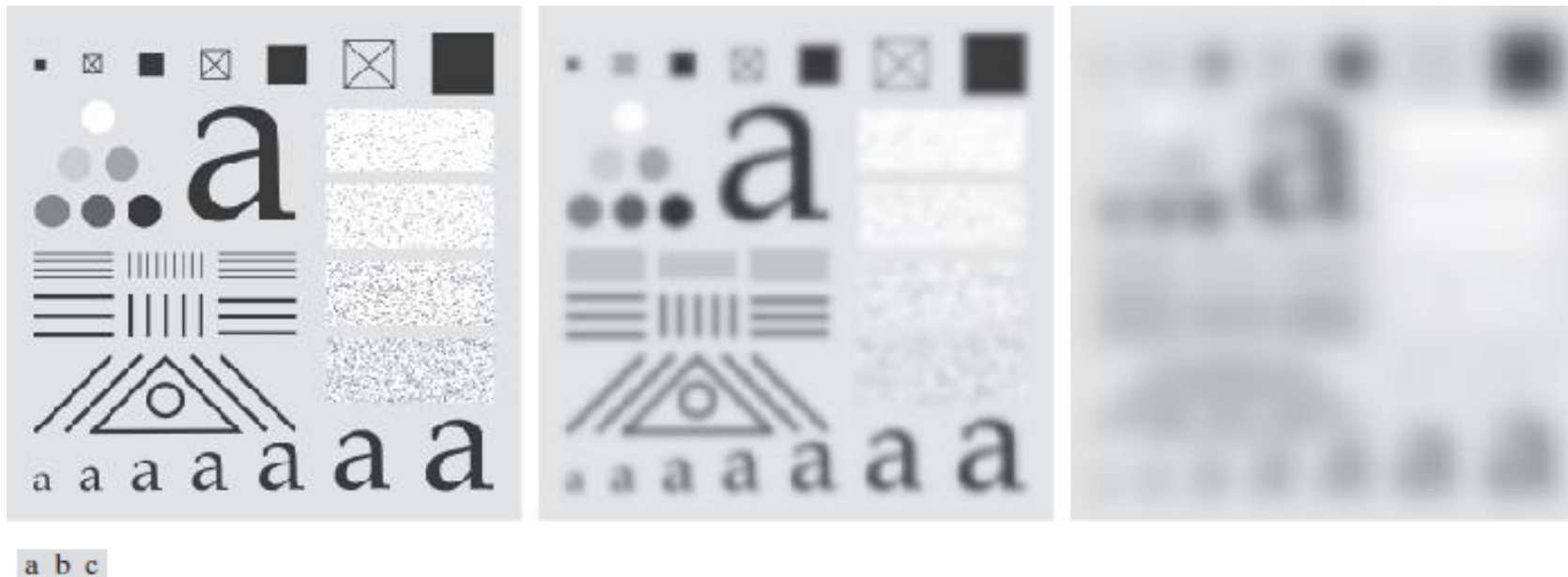
FIGURE 3.40 (a) Test pattern of size 4096 × 4096 pixels. (b) Result of filtering the test pattern with the same Gaussian kernel used in Fig. 3.39. (c) Result of filtering the pattern using a Gaussian kernel of size 745 × 745 elements, with $K = 1$ and $\sigma = 124$. Mirror padding was used throughout.

3.6 Smoothing Spatial Filters

- Lowpass + thresholding = region extraction
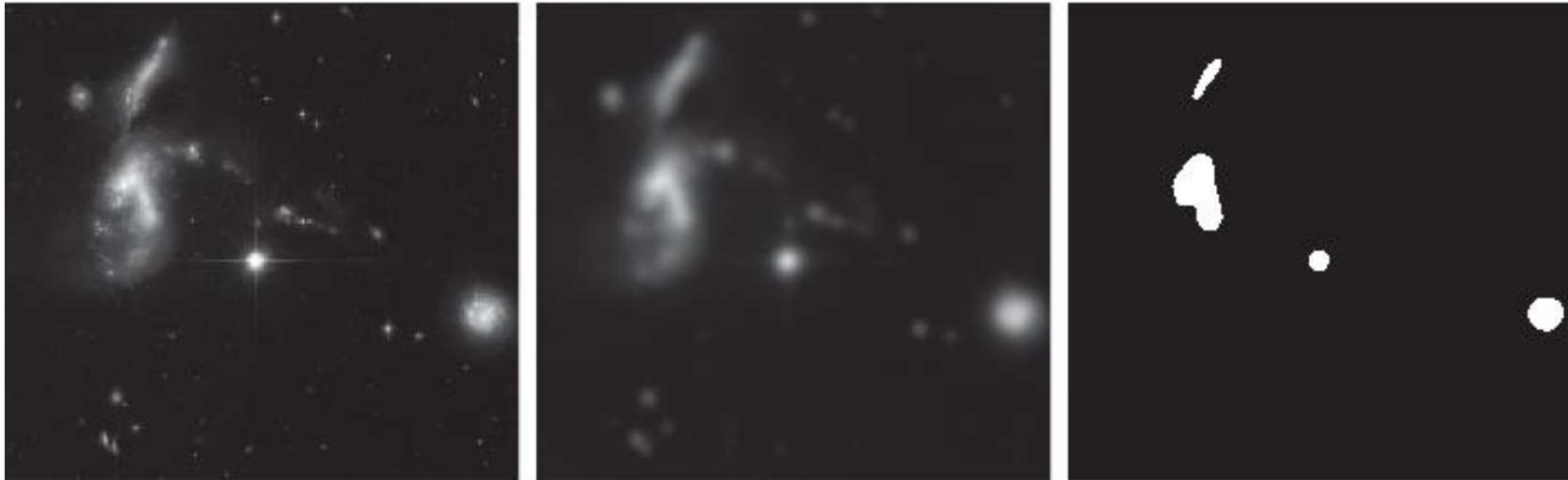
a b c

FIGURE 3.41 (a) A 2566 × 2758 Hubble Telescope image of the *Hickson Compact Group*. (b) Result of lowpass filtering with a Gaussian kernel. (c) Result of thresholding the filtered image (intensities were scaled to the range [0, 1]). The Hickson Compact Group contains dwarf galaxies that have come together, setting off thousands of new star clusters. (Original image courtesy of NASA.)

3.6 Smoothing Spatial Filters

- Shading correction:
  - Lowpass filtering the image with a 512×512 Gaussian kernel (four times the size of the squares), $K = 1$, and $\sigma = 128$ (equal to the size of the squares)

a b c

FIGURE 3.42 (a) Image shaded by a shading pattern oriented in the −45° direction. (b) Estimate of the shading patterns obtained using lowpass filtering. (c) Result of dividing (a) by (b). (See Section 9.8 for a morphological approach to shading correction).

3.6 Smoothing Spatial Filters

- Order-Statistics Filters:
  - Order-statistics filters are nonlinear spatial filters whose response is based on ordering (ranking) the pixels contained in the image area encompassed by the filter, and then replacing the value of the center pixel with the value determined by the ranking result.
  - The best-known example in this category is the median filter.
    - For certain types of random noise, they provide excellent noise-reduction capabilities, with considerably less blurring than linear smoothing filters of similar size.
    - Median filters are particularly effective in the presence of impulse noise, also called salt-and-pepper noise.
  - Max. filter, Min. filter.

3.6 Smoothing Spatial Filters

- Order-Statistics Filters:
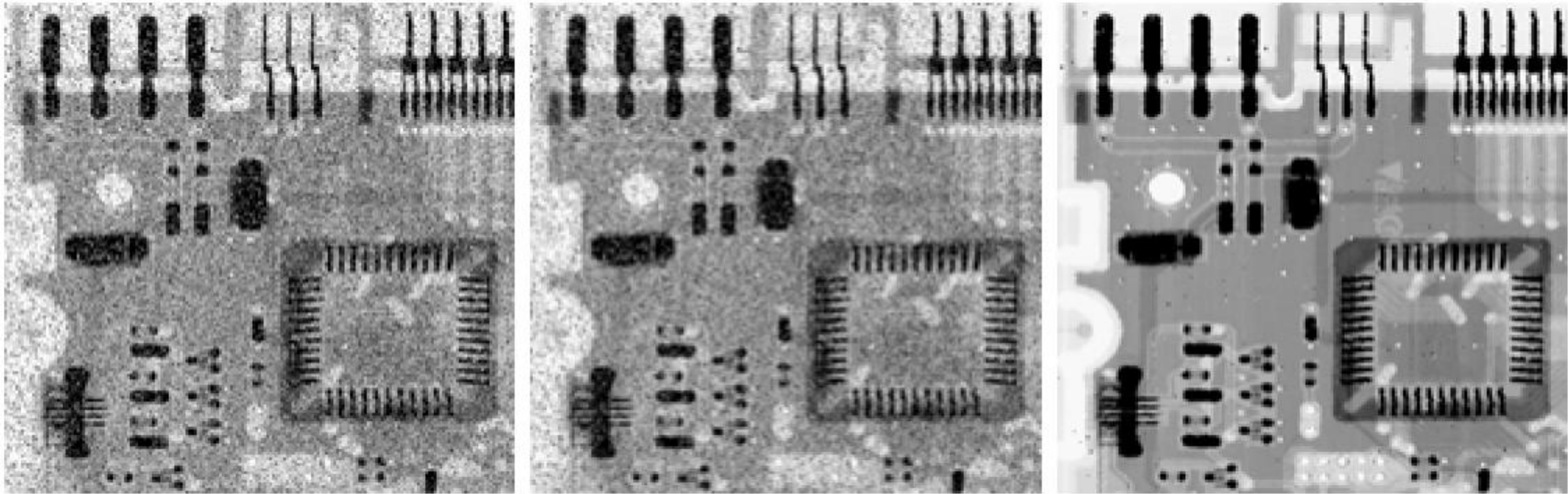
a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

3.7 Sharpening Spatial Filters

- The principal objective of sharpening is to highlight fine detail in an image or to enhance detail that has been blurred, either in error or as a natural effect of a particular method of image acquisition.
  - in some detail sharpening filters that are based on first- and second-order derivatives.

- For first derivative:
  1. Must be zero in areas of constant intensity;
  2. Must be nonzero at the onset of an intensity step or ramp;
  3. Must be nonzero along intensity ramps.

- For second derivative
  1. Must be zero in areas of constant intensity;
  2. Must be nonzero at the onset and end of a intensity step or ramp;
  3. Must be zero along ramps of constant slope.

3.7 Sharpening Spatial Filters

- A basic definition of the first-order derivative of a one-dimensional function $f(x)$ is the difference
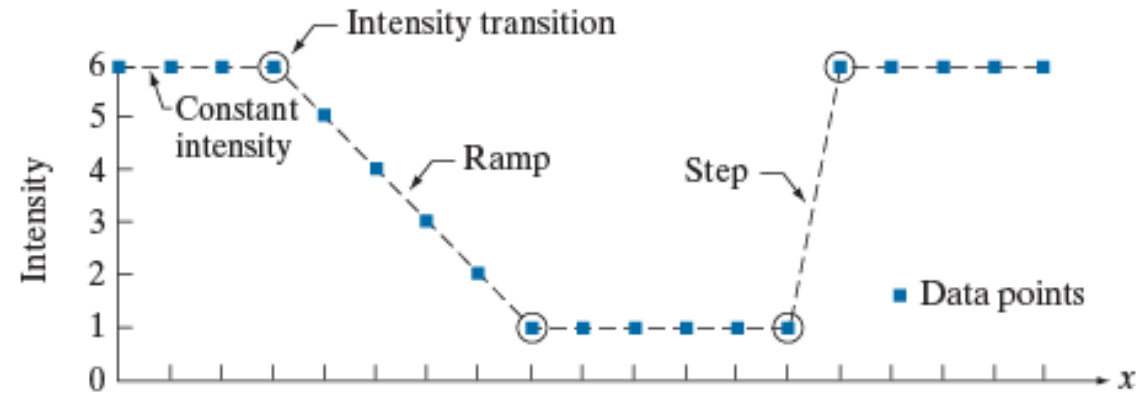
$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

- A second-order derivative as the difference

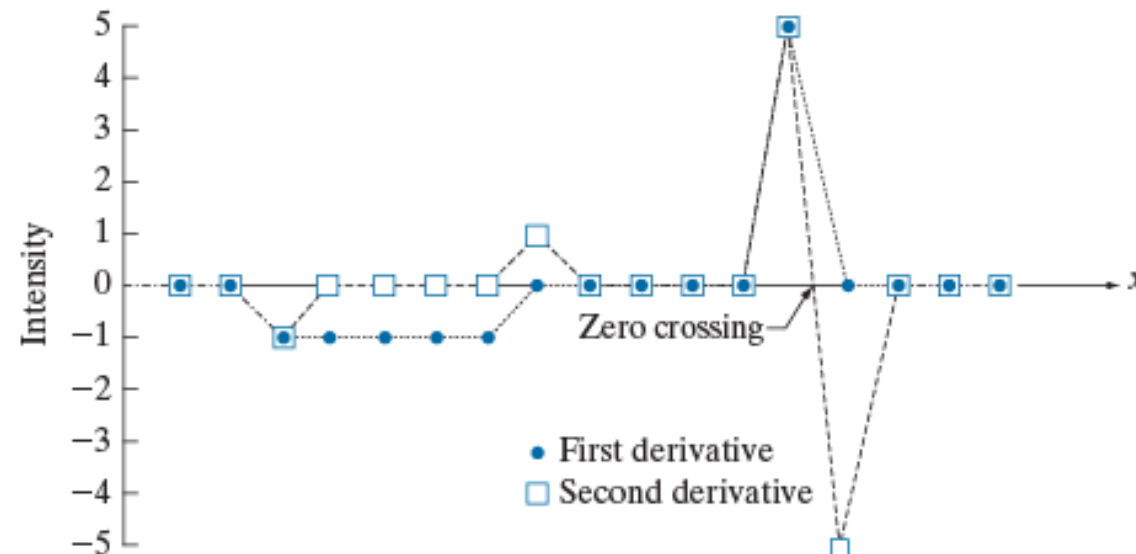$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

3.7 Sharpening Spatial Filters
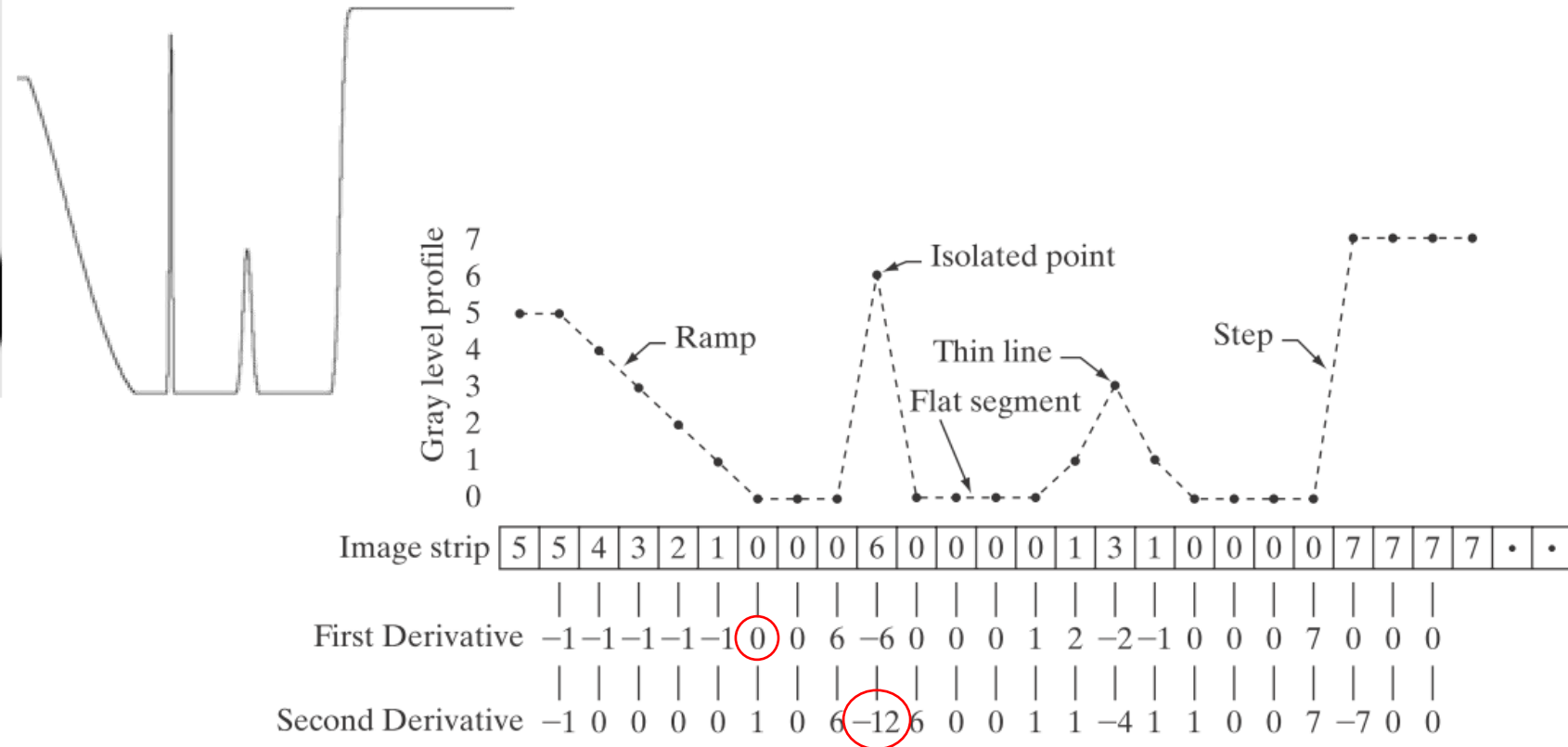
3.7 Sharpening Spatial Filters

3.7 Sharpening Spatial Filters

1. First-order derivatives generally produce thicker edges in an image.

2. Second-order derivatives have a stronger response to fine detail, such as thin lines, isolated points, and noise.

3. Second-order derivatives produce a double-edge response at ramp and step transitions in intensity.

4. The sign of the second derivative can be used to determine whether a transition into an edge is from light to dark or dark to light.

3.7 Sharpening Spatial Filters

- Use of Second Derivatives for Enhancement–The Laplacian:
  - The approach basically consists of defining a discrete formulation of the second-order derivative and then constructing a filter mask based on that formulation.

  isotropic filters are rotation invariant

- It can be shown that the simplest isotropic derivative operator is the Laplacian, which, for a function (image) $f(x, y)$ of two variables, is defined as

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \quad .....(3.7\text{-}1)$$

  - Because derivatives of any order are linear operations, the Laplacian is a linear operator.

3.7 Sharpening Spatial Filters

- Partial second-order derivative in the x-direction:

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x) \quad \text{.....(3.7-2)}$$

- Partial second-order derivative in the y-direction:

$$\frac{\partial^2 f}{\partial y^2} = f(y+1) + f(y-1) - 2f(y) \quad \text{.....(3.7-3)}$$

- Therefore:

$$\nabla^2 f = \left[f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) \right] - 4f(x, y) \quad \text{.....(3.7-4)}$$

  - Gives an isotropic result for rotations in increments of 90°
  - The diagonal directions can be incorporated in the definition of the digital Laplacian by adding two more terms to Eq. (3.7-1), one for each of the two diagonal directions.

3.7 Sharpening Spatial Filters

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

a b
c d

FIGURE 3.39
(a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4).
(b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.

isotropic results for increments of 90°.

isotropic results for increments of 45°.

3.7 Sharpening Spatial Filters

- The basic way in which we use the Laplacian for image enhancement is as follows:

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) & \text{if the center coefficient of the Laplacian mask is negative.} \\ f(x,y) + \nabla^2 f(x,y) & \text{if the center coefficient of the Laplacian mask is positive.} \end{cases} \quad \dots\text{(3.7-5)}$$

- Simplifications:

$$g(x,y) = f(x,y) - \left[f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) \right] + 4f(x,y)$$

$$= 5f(x,y) - \left[f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) \right]$$

0	−1	0
−1	5	−1
0	−1	0

−1	−1	−1
−1	9	−1
−1	−1	−1
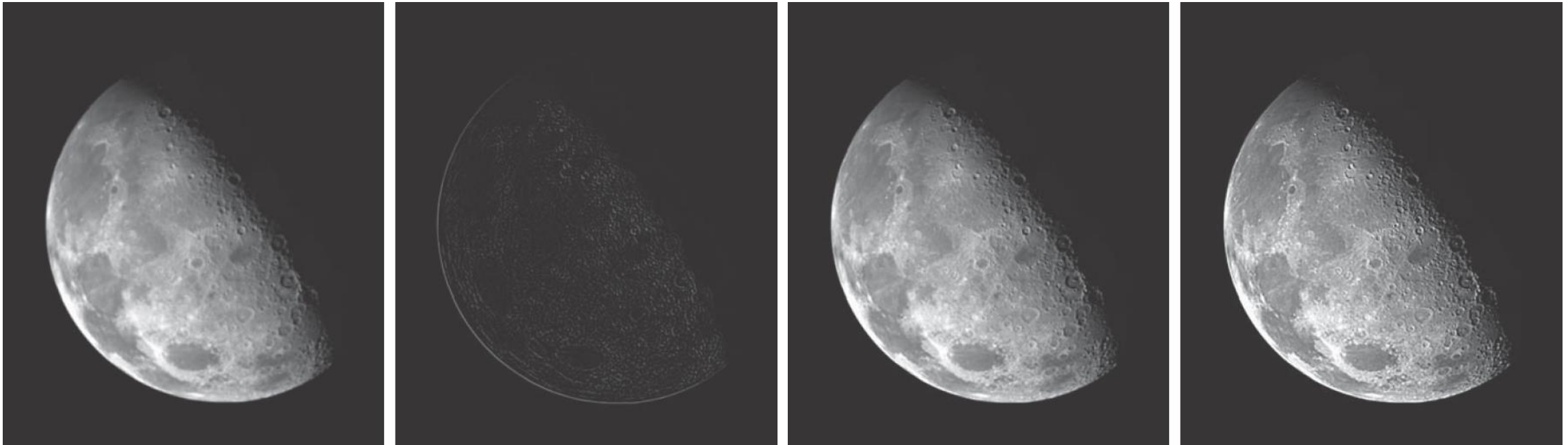
3.7 Sharpening Spatial Filters

0	−1	0
−1	5	−1
0	−1	0

−1	−1	−1
−1	9	−1
−1	−1	−1

3.7 Sharpening Spatial Filters

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)
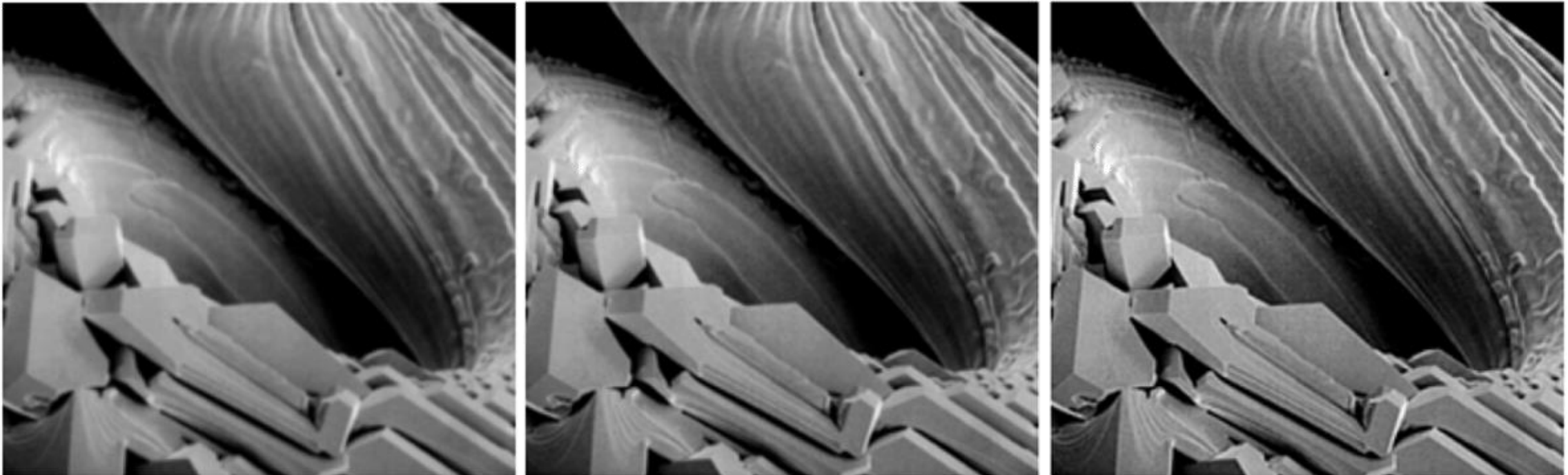
3.7 Sharpening Spatial Filters

- Unsharp masking and highboost filtering:
  - A process used for many years in the publishing industry to sharpen images consists of subtracting a blurred version of an image from the image itself. This process, called unsharp masking,

  $$g_{mask}(x, y) = f(x, y) - \overline{f}(x, y) \quad \text{.....(3.7-7)}$$

  $$g(x, y) = f(x, y) + kg_{mask}(x, y)$$

  where $g_{mask}(x, y)$ is the unsharp mask, and $\overline{f}(x, y)$ is a blurred version of $f(x, y)$. When k=1, we have unsharp masking, and k>1 is called highboost filtering.
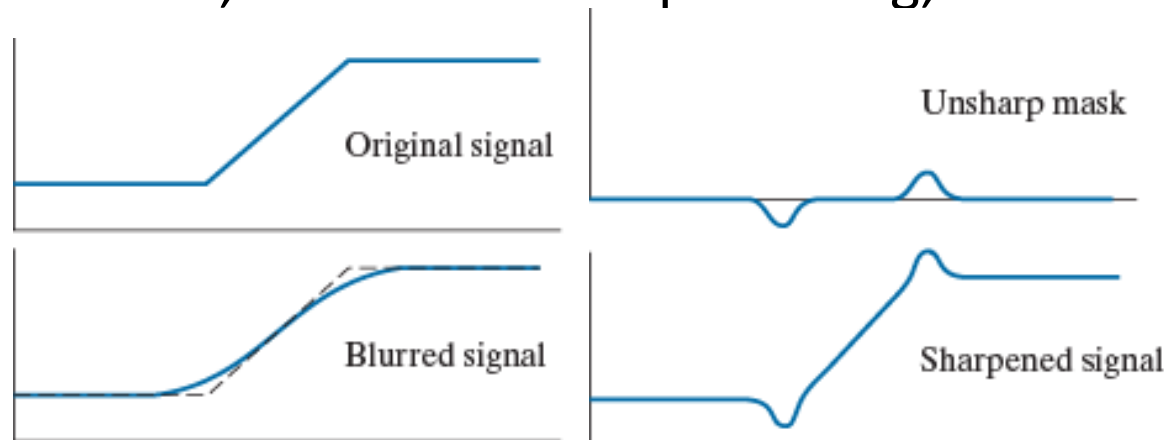
Original signal

Unsharp mask

Blurred signal

Sharpened signal

3.7 Sharpening Spatial Filters

- Unsharp masking and highboost filtering:

a b c
d e

FIGURE 3.49 (a) Original image of size 600×259 pixels. (b) Image blurred using a 31×31 Gaussian lowpass filter with $\sigma = 5$. (c) Mask. (d) Result of unsharp masking using Eq. (3-56) with $k = 1$. (e) Result of highboost filtering with $k = 4.5$.

3.7 Sharpening Spatial Filters

- Use of First Derivatives for Enhancement—The Gradient:
  - For a function f(x, y), the gradient of f at coordinates (x, y) is defined as the two-dimensional column vector

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix} \quad \text{.....(3.7-12)}$$

  - The magnitude of this vector is given by

$$\nabla f = mag\left(\nabla \mathbf{f}\right) = \left[G_x^2 + G_y^2 \right]^{1/2} = \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2} \quad \text{.....(3.7-13)}$$

    - the magnitude of the gradient vector often is referred to as the gradient.

3.7 Sharpening Spatial Filters

- Use of First Derivatives for Enhancement—The Gradient:
  - The computational burden of implementing Eq. (3.7-13) over an entire image is not trivial, and it is common practice to approximate the magnitude of the gradient by using absolute values instead of squares and square roots.

  $$\nabla f \approx \left| G_x \right| + \left| G_y \right| \quad \text{.....(3.7-14)}$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

  - The simplest approximations to a first-order derivative that satisfy the conditions stated in that section are $Gx = (z_8 - z_5)$ and $Gy = (z_6 - z_5)$.
  - Another definitions proposed by Roberts (Roberts cross-gradient operators) [1965] in the early development of digital image processing use cross differences $Gx = (z_9 - z_5)$ and $Gy = (z_8 - z_6)$

-1	0
0	1

0	-1
1	0

3.7 Sharpening Spatial Filters

- Use of First Derivatives for Enhancement—The Gradient:
 - Masks of even size are awkward to implement. The smallest filter mask in which we are interested is of size 3×3.
  - An approximation using absolute values, still at point z_5, but using a 3×3 mask, is

$$\nabla f \approx \left| \left(z_7 + 2z_8 + z_9 \right) - \left(z_1 + 2z_2 + z_3 \right) \right| + \left| \left(z_3 + 2z_6 + z_9 \right) - \left(z_1 + 2z_4 + z_7 \right) \right| \quad \ldots\ldots(3.7\text{-}18)$$

Sobel operators:

−1	−2	−1
0	0	0
1	2	1

Sobel Y

−1	0	1
−2	0	2
−1	0	1

Sobel X

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

3.7 Sharpening Spatial Filters

- Use of First Derivatives for Enhancement—The Gradient:
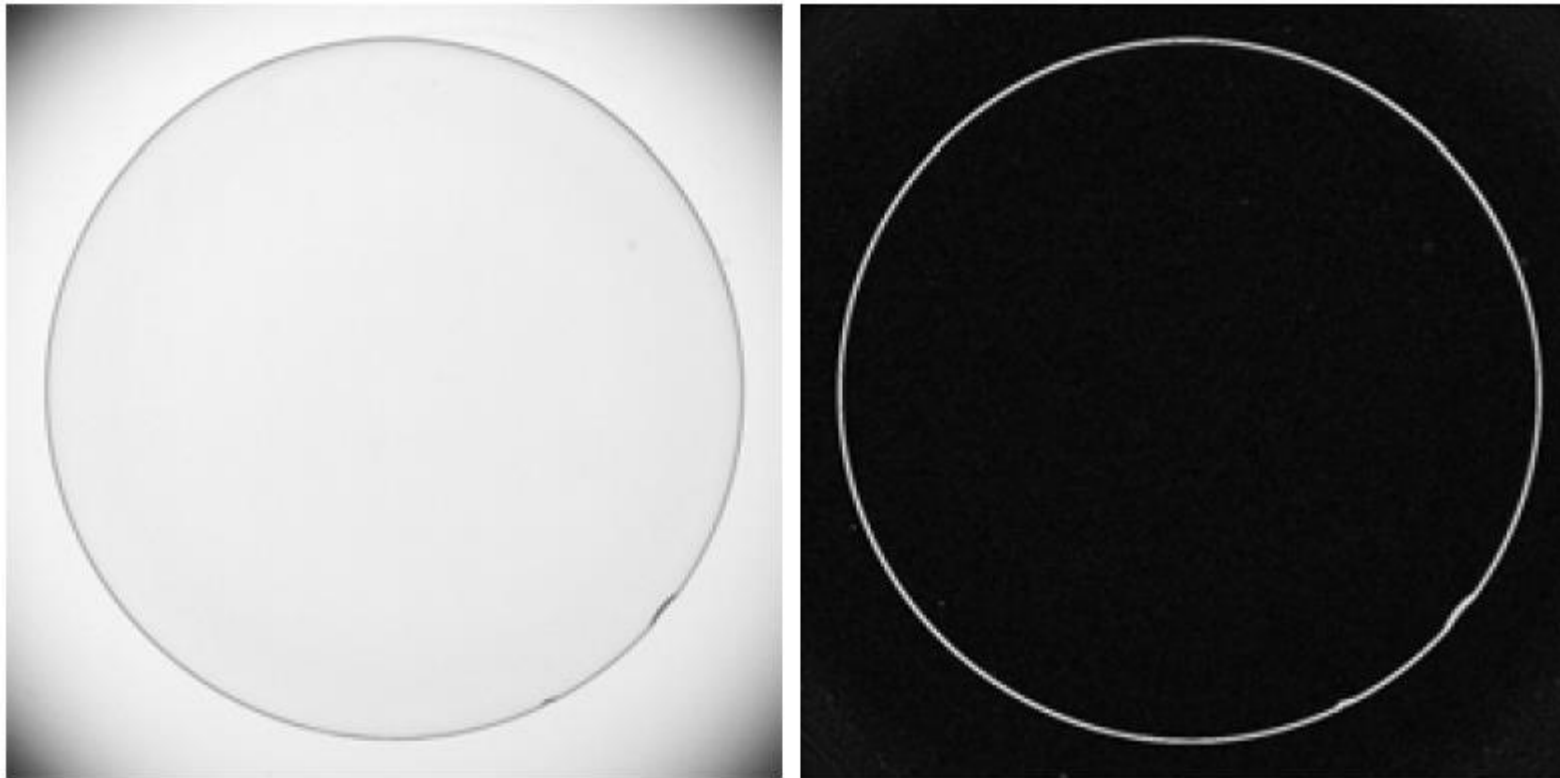
a b

FIGURE 3.45
Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)

3.8 Highpass, band reject, and bandpass filters from lowpass filters
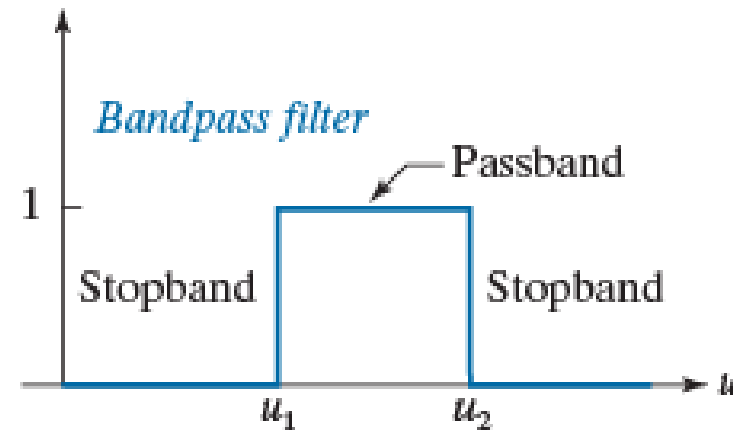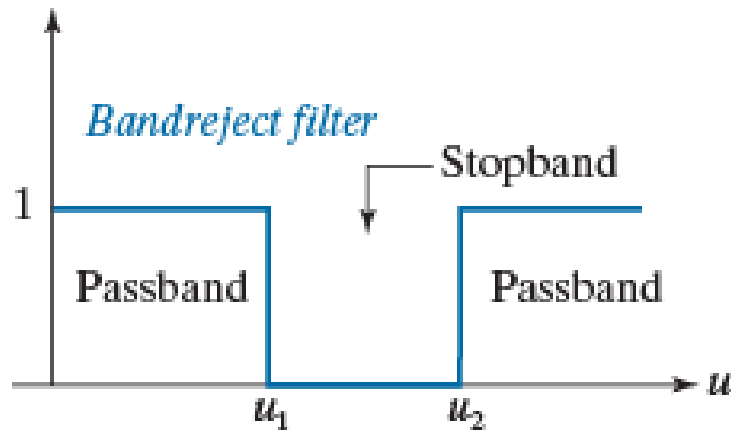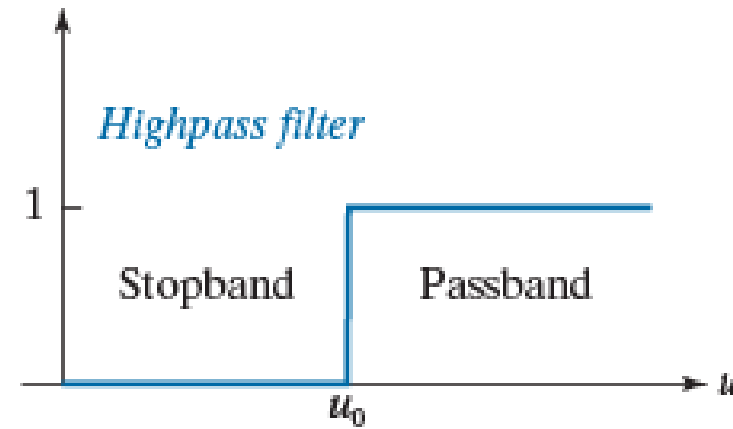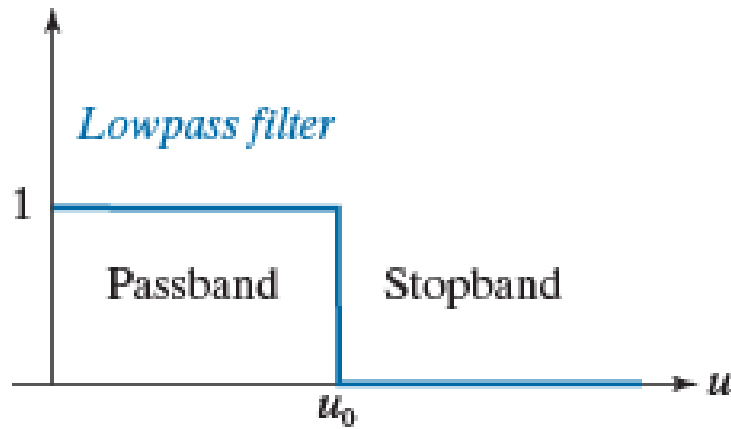
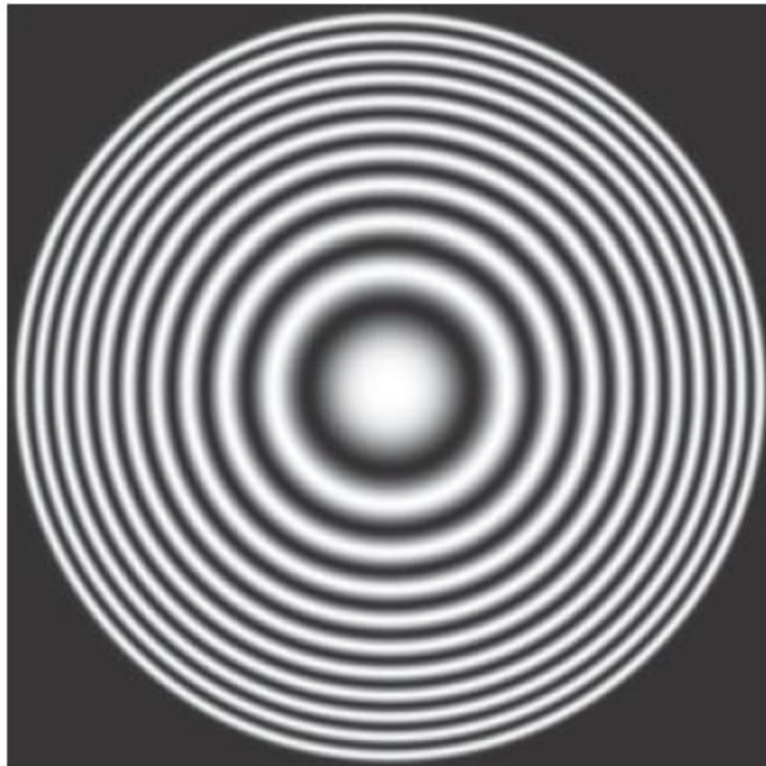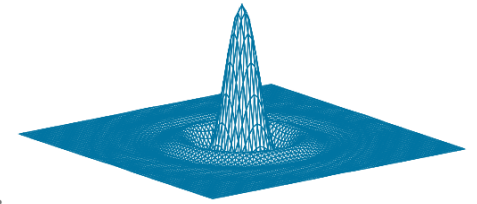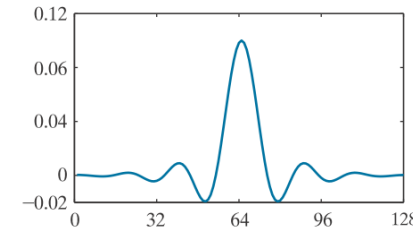# 3.8 Highpass, band reject, and bandpass filters from lowpass filters

TABLE 3.7
Summary of the four principal spatial filter types expressed in terms of low-pass filters. The centers of the unit impulse and the filter kernels coincide.

Filter type	Spatial kernel in terms of lowpass kernel, lp
Lowpass	$lp(x,y)$
Highpass	$hp(x,y) = \delta(x,y) - lp(x,y)$
Bandreject	$br(x,y) = lp_1(x,y) + hp_2(x,y)$
	$= lp_1(x,y) + \left[\delta(x,y) - lp_2(x,y)\right]$
Bandpass	$bp(x,y) = \delta(x,y) - br(x,y)$
	$= \delta(x,y) - \left[lp_1(x,y) + \left[\delta(x,y) - lp_2(x,y)\right]\right]$

3.8 Highpass, band reject, and bandpass filters from lowpass filters
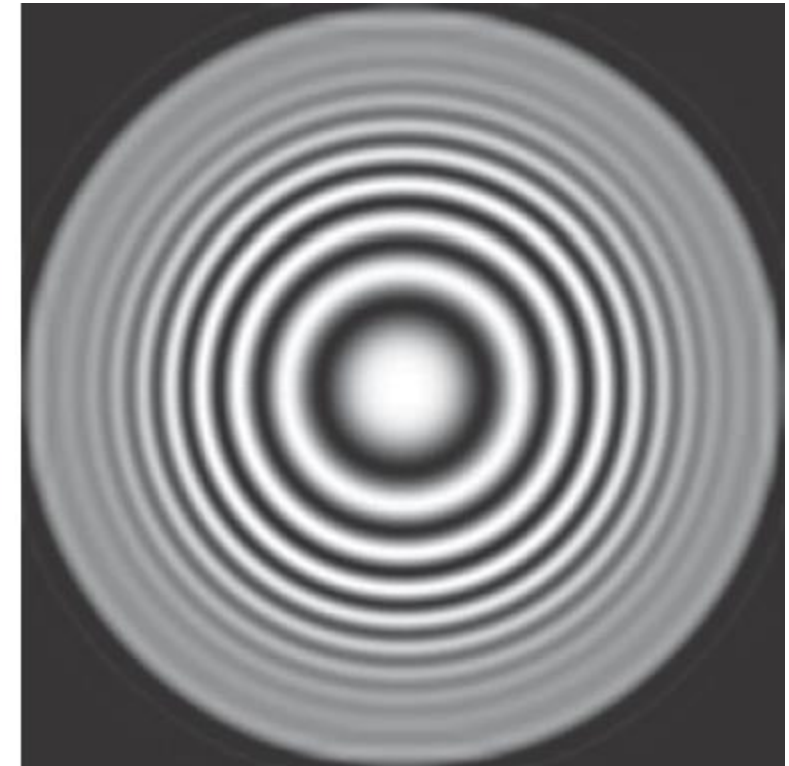
FIGURE 3.54
(a) A 1-D spatial lowpass filter function. (b) 2-D kernel obtained by rotating the 1-D profile about its center.

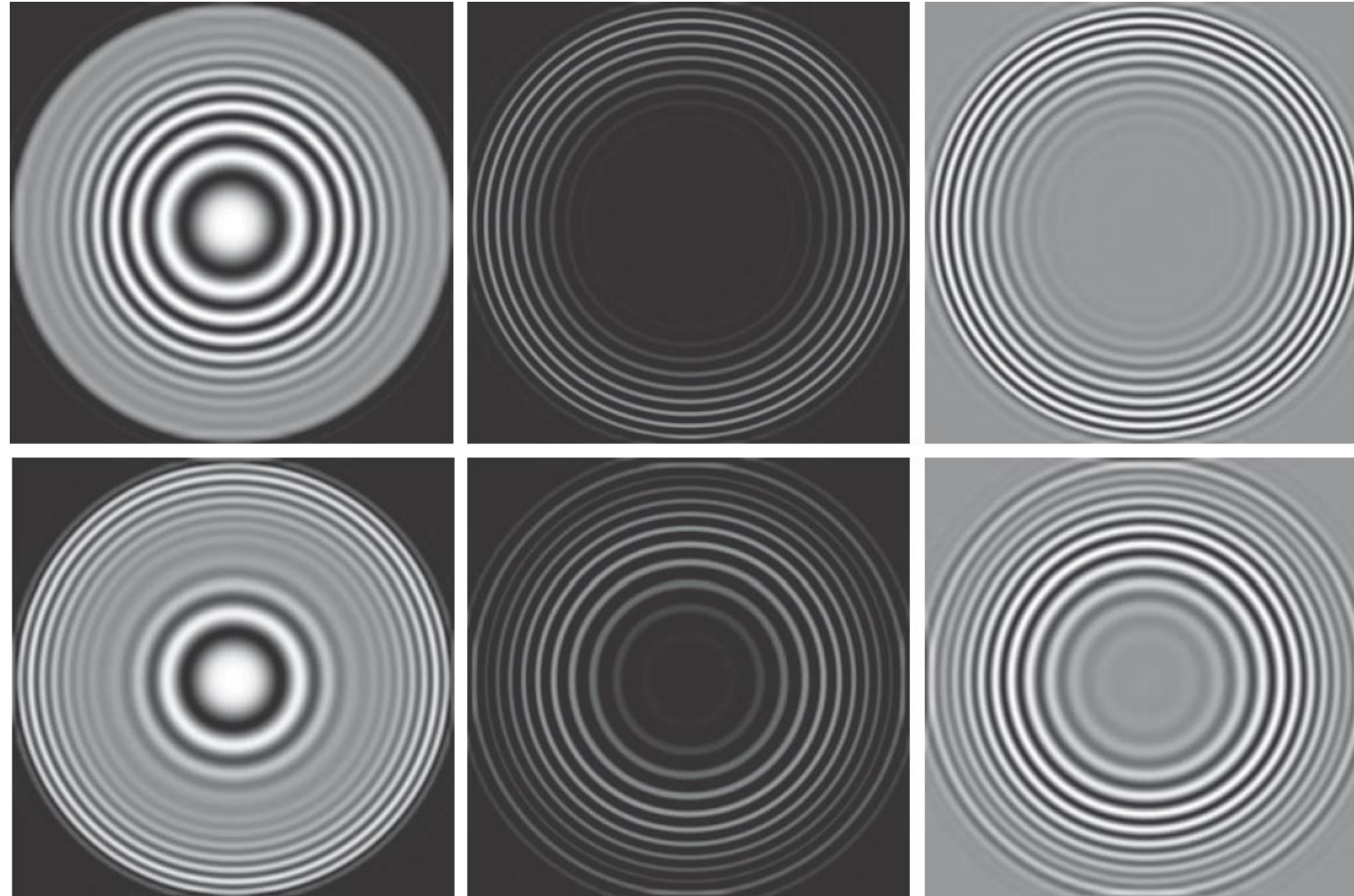$$z(x, y) = \frac{1}{2}\left[1 + \cos\left(x^2 + y^2\right)\right]$$
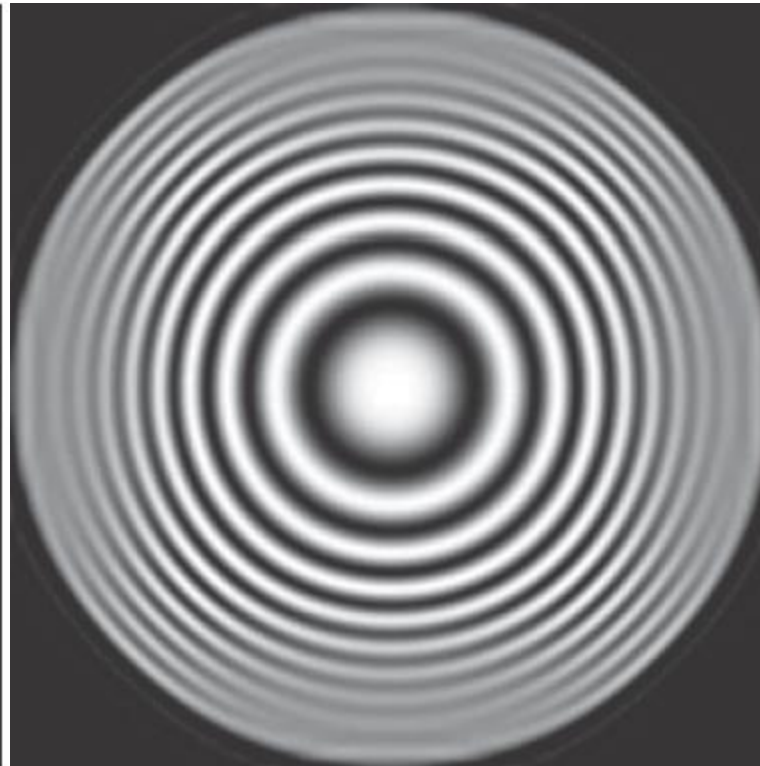
filtered with a separable lowpass kernel.

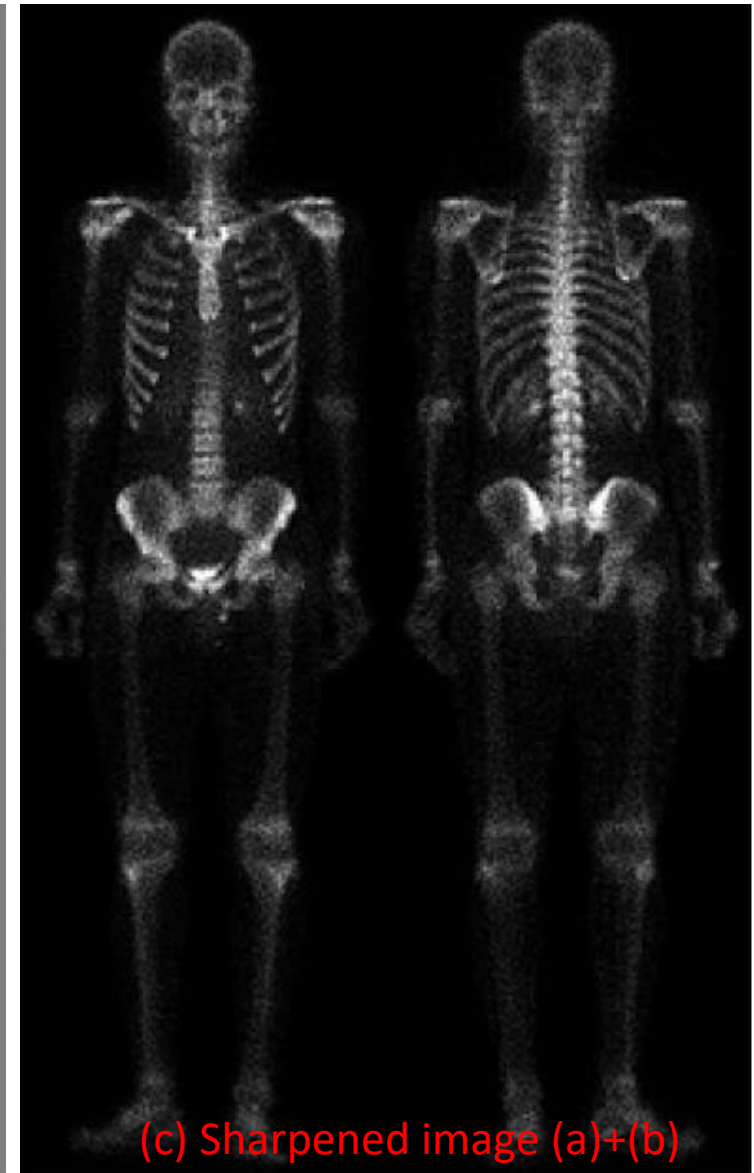filtered with the isotropic lowpass kernel.
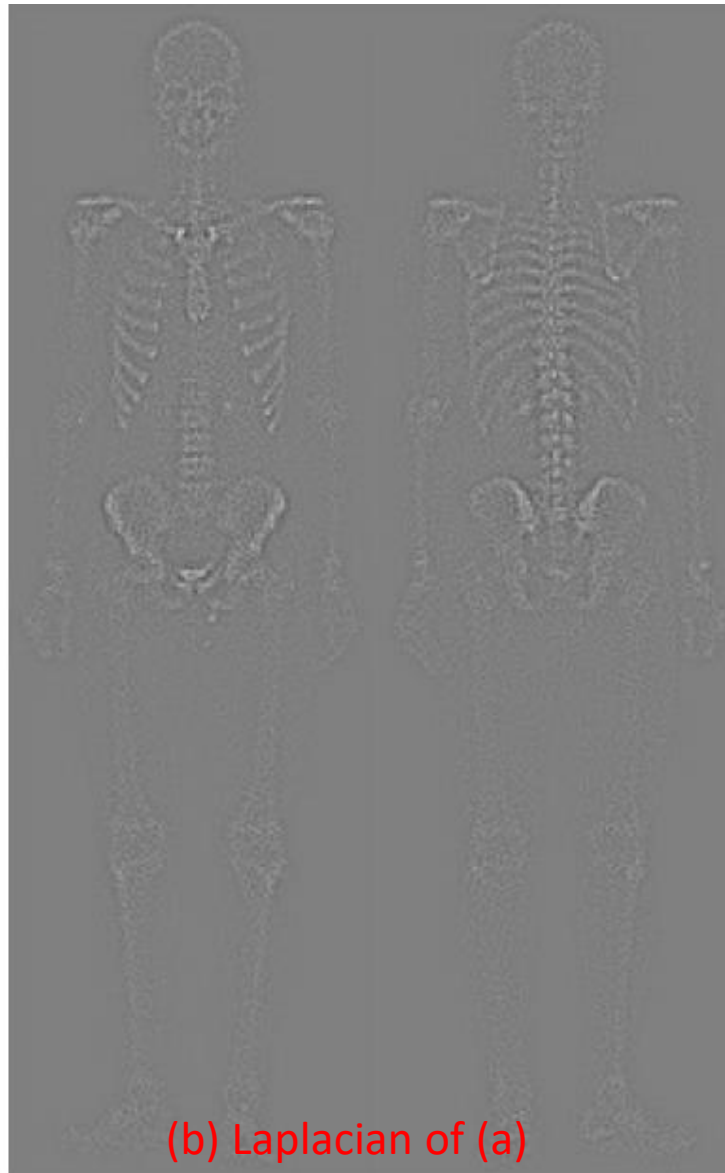
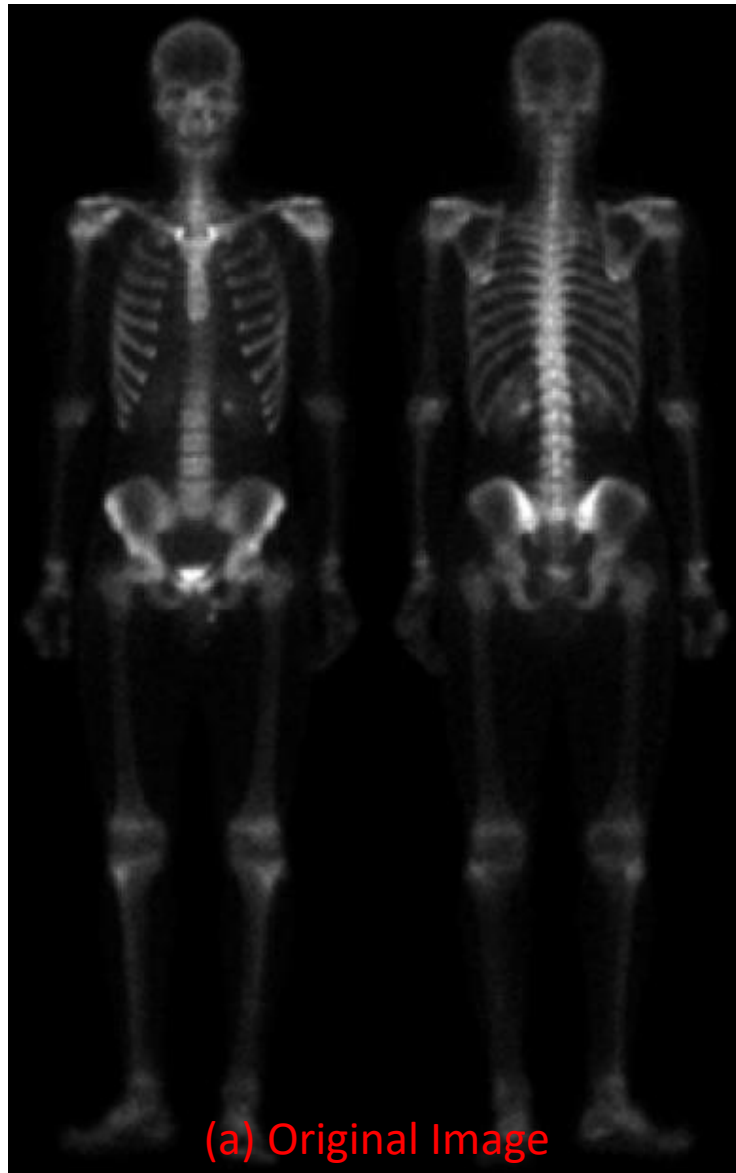# 3.8 Highpass, band reject, and bandpass filters from lowpass filters
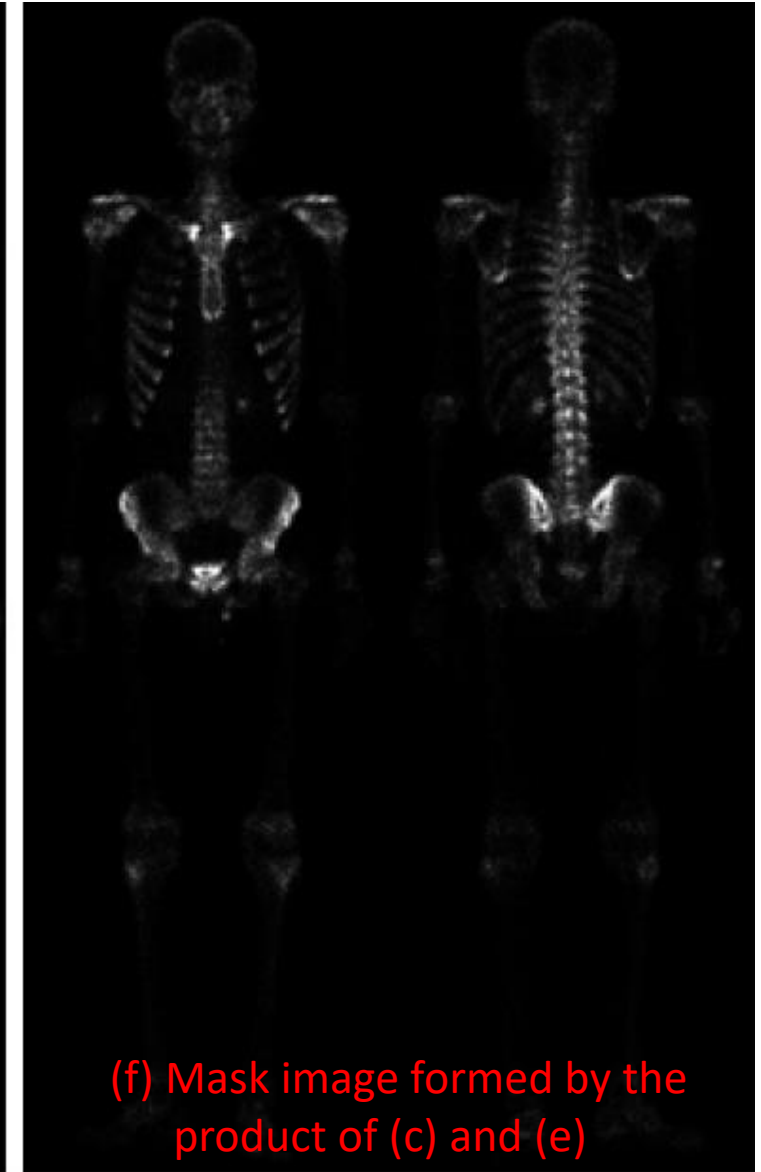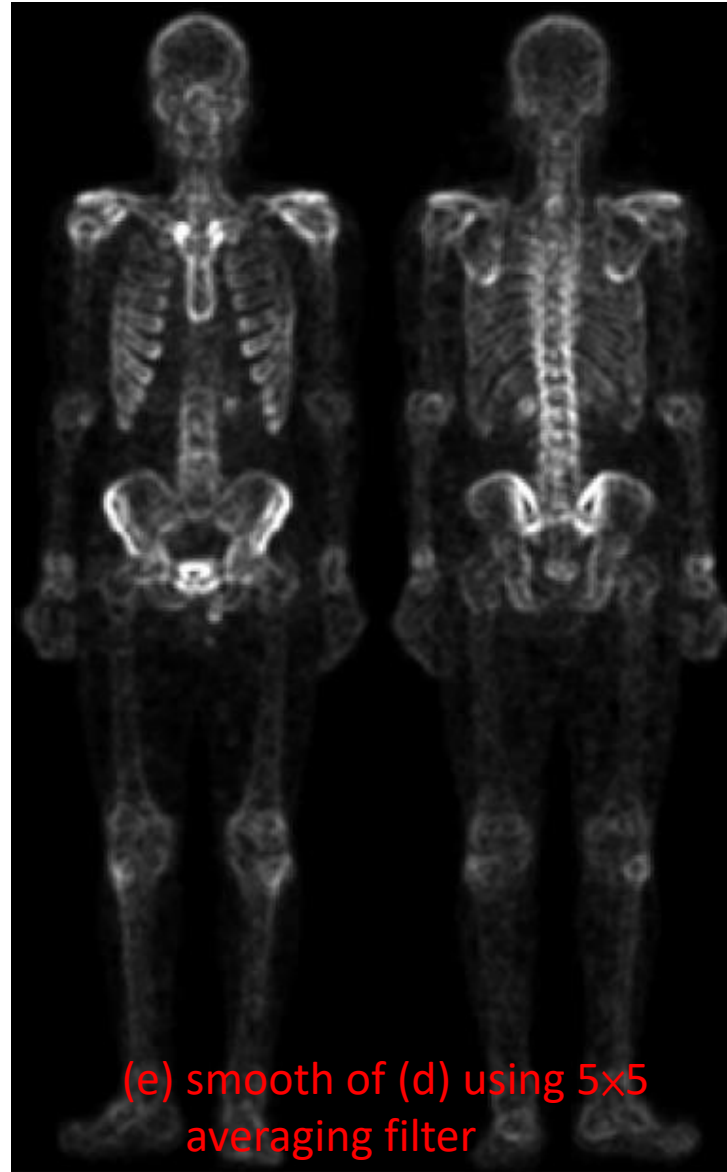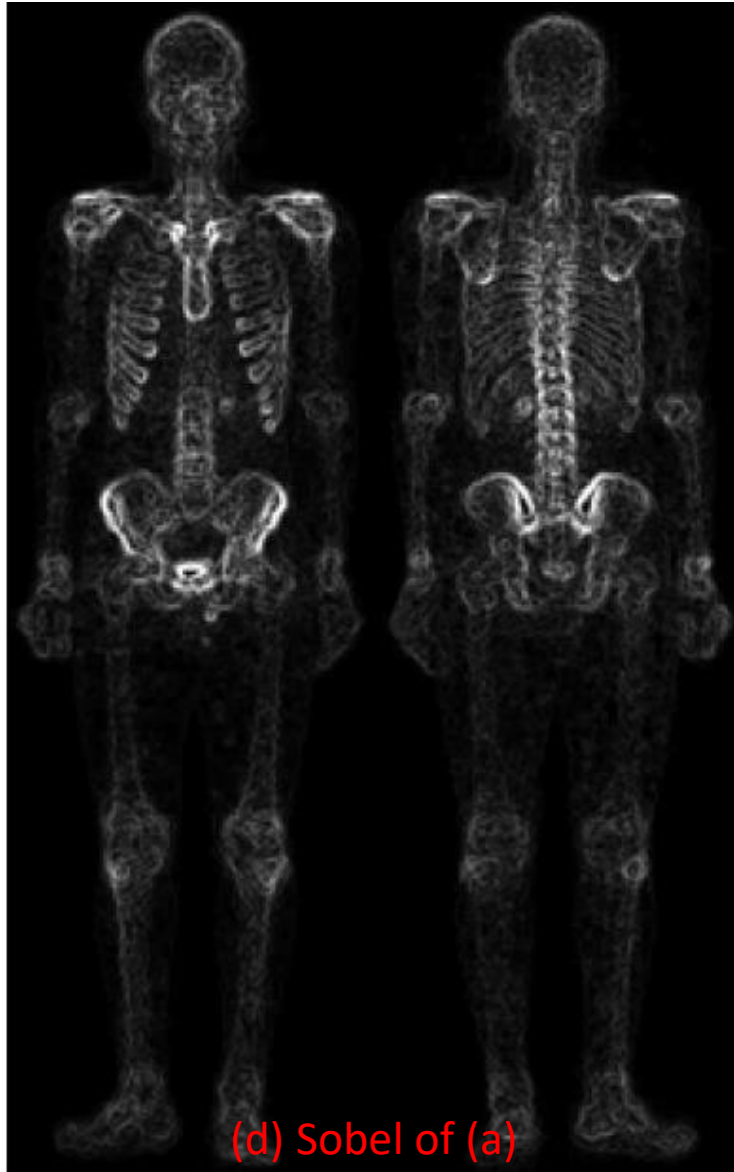
a b c
d e f

FIGURE 3.56
Spatial filtering of the zone plate image. (a) Lowpass result; this is the same as Fig. 3.55(b). (b) Highpass result. (c) Image (b) with intensities scaled. (d) Bandreject result. (e) Bandpass result. (f) Image (e) with intensities scaled.

3.9 Combining Spatial Enhancement Methods

(a) Original Image

(b) Laplacian of (a)

(c) Sharpened image (a)+(b)

3.9 Combining Spatial Enhancement Methods

(d) Sobel of (a)

(e) smooth of (d) using 5×5 averaging filter

(f) Mask image formed by the product of (c) and (e)

3.9 Combining Spatial Enhancement Methods

(g) Sharpened image obtained by (a) + (f)

(h) Final result obtained by applying a power-law transformation to (g)

(a) Original Image