Wireless Communication IC

Homework #2

(Due on 03/28)

Please upload your simulation codes to eeClass system.
Total points: 120 points

1. The properties of **constant amplitude zero auto-correlation** (CAZAC) Zadoff-Chu sequence will be examined. Define

$$S(n,u) = e^{\frac{j\pi u n(n+1)}{N}}$$

for $0 \le n \le N-1$.

(a) First, given $N = 29$, select u_1 so that $\gcd(u_1, N) = 1$. Write down your u_1.

(b) Please plot the real part and imaginary part of sequence $S(n,u_1)$ for $0 \le n \le N-1$. (10%)

(c) Now, examine its autocorrelation property

$$\Phi_s(k) = \tfrac{1}{N}\textstyle\sum_{n=0}^{N-1} S(n,u_1)S^*(n-k,u_1)$$

Plot $|\Phi_s(k)|$ for $-N \le k \le N$. (10%)

(d) Now, examine its cross-correlation property

$$\Omega_s(m) = \tfrac{1}{N}\textstyle\sum_{n=0}^{N-1} S(n,u_1)S^*(n-k,u_1+m)$$

Plot $|\Omega_s(m)|$ for $-N \le m \le N$ and $k = 3$. (10%)

(e) From (d), plot $|\Omega_s(m)|$ for $-N \le k \le N$ and $m = 3$. (10%)

(f) Assume that the received signal of length $N+6$ is given by

$$y(n) = [0.3S(N-2,u_1) \quad 0.3S(N-1,u_1) \quad 0.3S(0,u_1) \quad 0.3S(1,u_1) \ldots.$$
$$\ldots \quad 0.3S(N-1,u_1) \quad 0.3S(0,u_1) \quad 0.3S(1,u_1) \quad 0.3S(2,u_1)\ ];$$

The receiver then performs $p(m) = \sum_{n=0}^{N-1} y(n+m)S^*(n,u_1)$ for synchronization. Please show $|p(m)|$ for $1 \le m \le N$ and explain the result. According to the properties in (c) to (e) that you examine, which one is suitable for the explanation. (10%)

2. Given the Barker code list in Table 1, please check the periodic autocorrelation function defined as

$$\Phi_s(k) = \frac{1}{N}\sum_{n=0}^{N-1} S(n)S^*([n-k]_N)$$

for $N = 7$ and 13, where $[\cdot]_N$ is the modulo-N operation. (20%)

Table 1 Barker code

N	Barker code												
7	[−1	−1	−1	1	1	−1	1]						
11	[−1	−1	−1	1	1	1	−1	1	1	−1	1]		
13	[−1	−1	−1	−1	−1	1	1	−1	−1	1	−1	1	−1]

3. Walsh Hadamard code is also widely used for multiple access because of good cross-correlation. A Walsh Hadamard code matrix $\mathbf{W_N}$ of size $N \times N$ can be defined recursively as in the following.

$$\mathbf{W_1} = [\mathbf{1}] \quad \mathbf{W_{2N}} = \begin{bmatrix} \mathbf{W_N} & \mathbf{W_N} \\ \mathbf{W_N} & -\mathbf{W_N} \end{bmatrix} .$$

   (a) Denote your last digit of student ID as α. Please write a program to generate $\mathbf{W_{32}}$. Print out the $(\alpha + 1)$th and 24$^{\text{th}}$ columns of the matrix $\mathbf{W_{32}}$. (6%)

   Check for their cross correlation $\frac{1}{32}\sum_{j=1}^{32}(\mathbf{W_{32}}(j, \alpha+1)\mathbf{W_{32}}(j, 24))$. (4%)

   (b) Now, choose $\mathbf{c_1} = \mathbf{W_{32}}(:, \alpha+1)$ and $\mathbf{c_2} = \mathbf{W_{32}}(:, 14)$ as two codes for user 1 and user 2. Randomly generate 5 symbols (d_0~d_4) from the set {+1,-1}. Spread the data by code 1 as $\mathbf{y} =$ $[d_0c_{1,1} \quad d_0c_{2,1} \quad \ldots d_0c_{32,1} \quad d_1c_{1,1} \quad \ldots \quad d_1c_{32,1} \ldots d_4c_{1,1} \ldots d_4c_{32,1} \; 0 \; 0 \; 0 \; 0]$.
   Use "stem" to plot the signals before spreading and after spreading. (10%)

   (c) Assume in the Rx side, perfect synchronization is not achieved. Then, check the results if the receiver use code 1 for dispreading, which includes multiplying the spreading code again and then summing them, i.e.

   $$p(i) = \frac{1}{32}\sum_{j=1}^{32}(y_{32i+j+1})\, c_{j,1}, \text{ where } y_r \text{ is the } r\text{th element of vector } \mathbf{y}.$$

   Plot $p(i)$ using index i as the x-axis. (10%)

   (d) Assume in the Rx side, perfect synchronization is achieved. Then, check the results if the receiver uses code 2 for dispreading, which includes multiplying the spreading code again and then summing them, i.e.

   $$p(i) = \frac{1}{32}\sum_{j=1}^{32}(y_{32i+j})\, c_{j,2} = \frac{1}{32}\sum_{j=1}^{32}(d_i c_{j,1})\, c_{j,2}, \text{ where } y_r \text{ is the } r\text{th}$$

   element of vector $\mathbf{y}$. Plot $p(i)$ using index i as the x-axis. (10%)

(e) Please comment the results in (c) and (d). (10%)