無線通訊積體電路 Homework 5

107501019 魏子翔

1.

Assume that a MIMO equation is described by

$$\mathbf{y} = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ y^{(3)} \end{bmatrix} = \begin{bmatrix} H^{(1,1)} & H^{(1,2)} & H^{(1,3)} \\ H^{(2,1)} & H^{(2,2)} & H^{(2,3)} \\ H^{(3,1)} & H^{(3,2)} & H^{(3,3)} \end{bmatrix} \begin{bmatrix} x^{(1)} \\ x^{(2)} \\ x^{(3)} \end{bmatrix} + \begin{bmatrix} v^{(1)} \\ v^{(2)} \\ v^{(3)} \end{bmatrix} = \mathbf{Hx} + \mathbf{v},$$

where $y^{(n)}$ is the received signal at $n$th antenna; $x^{(m)}$ is the transmitted signal at $m$th antenna; $v^{(n)}$ is the noise; $H^{(n,m)}$ is the channel response from antenna $m$ to antenna $n$.

(a). Find the ZF detection matrix $\mathbf{G}_{ZF}$ so that we can detect $\mathbf{x}$ by $\mathbf{G}_{ZF}\mathbf{y}$ ,what is $\hat{\mathbf{x}}_1$, the output after ZF detection.

```
GZF =

   6.6453 + 5.0592i   0.8865 - 7.2630i   3.5066 - 5.2088i
   1.5506 + 3.0498i   1.4751 - 2.6577i   2.9094 - 0.8717i
  -2.9113 + 4.3892i   3.3007 - 0.0263i   2.4426 + 2.0726i


x1_bar =

  -1.0000 - 1.0000i
   1.0000 - 1.0000i
  -1.0000 + 1.0000i
```

(b). What is the output after you apply $\mathbf{G}_{ZF}\mathbf{y}'$? (5%), what is the detection output $\hat{\mathbf{x}}_2$?

```
x2prime =

  -0.0534 - 6.5251i
   2.3414 - 2.7152i
   2.0314 + 1.3313i


noise =

  -0.2290 - 0.3530i
   0.2131 - 0.1128i
   0.1660 + 0.1403i


x2_bar =

  -1.0000 - 1.0000i
   1.0000 - 1.0000i
  -1.0000 + 1.0000i
```

$x_2' = G_{ZF}y'$，我們這邊將$\hat{x}_2$進行兩種分析，首先就是直接將$x_2'$量化，可得

$$[-1-3\mathrm{i}\ \ 3-3\mathrm{i}\ \ 3+1\mathrm{i}]'$$

第二種，為了要求$\hat{x}_2$，首先要找出$nosie = y' - y$，所以$\hat{x}_2 = x_2' - G_{ZF} * noise$，得到的結果如上方 x2_bar，其值完全與(a)小題一模一樣。

2.

Given that noisy $\mathbf{y}' = \mathbf{Hx} + \mathbf{v} = \begin{bmatrix} H^{(1,1)} & H^{(1,2)} & H^{(1,3)} \\ H^{(2,1)} & H^{(2,2)} & H^{(2,3)} \\ H^{(3,1)} & H^{(3,2)} & H^{(3,3)} \end{bmatrix} \begin{bmatrix} x^{(1)} \\ x^{(2)} \\ x^{(3)} \end{bmatrix} + \mathbf{v}$, now we want to use OSIC to detect transmitted signals from 16-QAM constellation $\{\pm1\pm1j, \pm1\pm3j, \pm3\pm1j, \pm3\pm3j\}$. Please download HW5-2.mat, which contains two variables, $\mathbf{H}$(Hmatrix) and $\mathbf{y}'$(yprime).

(a). Which signal should be detected first? $x^{(1)}$, $x^{(2)}$ or $x^{(3)}$ ?

```
ROW1 =

    1.0351


ROW2 =

    1.5042


ROW3 =

    3.1402
```

我們藉由計算$G_0$每一行的平方和，得到上方的結果，可以看出第一航所得到的值最小，因此$x^{(1)}$先。

(b). Please write down your first detected output $\hat{x}^{(\alpha 1)}$ after decision.

```
xa1 =

  -1.1507 + 3.2353i
```

$\hat{x}^{(\alpha 1)}$計算得到上方的結果，我們再將其量化成**-1+3i** (c). Please write down the equation $\mathbf{y}(1) = \mathbf{H}(1)\mathbf{x}(1)$

```
H1 =

  -0.3297 - 0.8380i   0.1445 + 0.2389i
   0.4435 + 0.5897i  -0.5525 + 0.1579i
   0.1050 + 0.0530i  -0.7337 + 0.4045i


y1 =

   2.8785 + 2.7469i
  -3.1989 + 1.1005i
  -1.4017 + 3.6054i


x1 =

  -3.0633 + 0.7937i
   3.3269 - 3.3349i
```

我們將 H(0)的第$\alpha 1 = 1$列刪除後就能得到 H(1)，而$y(1) = y(0) - H(0)_{col,\alpha 1} * \hat{x}^{(\alpha 1)}$，$x(1) = H^{-1}(1)y(1)$

(d). Please write down the second detected output $\hat{x}^{(\alpha 2)}$.

```
ROW1 =

    1.0646


ROW2 =

    1.3132


xa2 =

  -3.0633 + 0.7937i
```

計算$G(1)$每一行的平方和，得到第一行的值最小，再將$G(1)_{row,\alpha 2} * y(1)$就能得到$\hat{x}^{(\alpha 2)}$，將其量化後得到**-3+1i** (e). Please write down the equation **y**(2) = **H**(2)**x**(2).

```
H2 =

   0.1445 + 0.2389i
  -0.5525 + 0.1579i
  -0.7337 + 0.4045i


y2 =

   1.0512 + 0.5626i
  -1.2787 + 2.4262i
  -1.0337 + 3.6595i


x2 =

   3.2563 - 3.2206i
```

我們將 H(1)的第$\alpha 2 = 1$列刪除後就能得到 H(2)，而$y(2) = y(1) - H(1)_{col,\alpha 2} * \hat{x}^{(\alpha 2)}$，$x(2) = H^{-1}(2)y(2)$

(f). Please write down the second detected output $\hat{x}^{(\alpha 3)}$.

```
xa3 =

   3.2563 - 3.2206i
```

最後直接將 G(2)*y(2)就能得到$\hat{x}^{(\alpha 3)}$，將其量化後得到 **3-3i**

3.

Assume that the MIMO configuration is 4×4. The constellation of BPSK is used at the transmitter, which means the element $x^{(n)}$ of the 4×1 transmitted vector **x** belongs to $\{+1,-1\}$. Now, please download HW5-3.mat. It contains the 4×4 channel matrix, **H** (Hmatrix) and the 4×1 received signal, **y(=Hx+n)**. Note that

(a). Please draw $\Gamma(\bar{x})$ versus the index of possible $\bar{x}$ vectors and then determine $\hat{x}_{ML} = \arg\min_{\bar{x}} \Gamma(\bar{x})$.

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 67.8616 |
| 1 | 1 | 1 | -1 | 58.2691 |
| 1 | 1 | -1 | 1 | 98.5133 |
| 1 | 1 | -1 | -1 | 90.2492 |
| 1 | -1 | 1 | 1 | 60.0647 |
| 1 | -1 | 1 | -1 | 53.8053 |
| 1 | -1 | -1 | 1 | 83.4545 |
| 1 | -1 | -1 | -1 | 78.5236 |
| -1 | 1 | 1 | 1 | 16.0189 |
| -1 | 1 | 1 | -1 | 1.6859 |
| -1 | 1 | -1 | 1 | 17.5500 |
| -1 | 1 | -1 | -1 | 4.5454 |
| -1 | -1 | 1 | 1 | 15.7033 |
| -1 | -1 | 1 | -1 | 4.7033 |
| -1 | -1 | -1 | 1 | 9.9726 |
| -1 | -1 | -1 | -1 | 0.3010 |

經過 matlab 模擬後得到上方的表格(map)，可以得到當$\bar{x} = [-1 -1 -1 -1]$時，可以得到$\min_{\bar{x}} \Gamma(\bar{x}) = 0.3010$，因此$\hat{x}_{ML} = [-1 -1 -1 -1]$。

(b). Write down $\mathbf{z} = [z^{(1)}\ z^{(2)}\ \ldots\ z^{(4)}]^T$ .

```
Q =

   -0.9283    0.0481    0.3591   -0.0841
   -0.3713   -0.1223   -0.9076    0.1532
   -0.0069    0.9497   -0.0738    0.3044
   -0.0204   -0.2843    0.2047    0.9364


R =

    4.2225    0.2215   -0.8621   -0.1403
         0   -1.1403    0.6286   -0.3926
         0         0   -0.3820   -0.7641
         0         0         0    1.3334


z =

   -3.8503
    0.5722
    1.0411
   -1.4460
```

(c). Use the 6-best algorithm to find out the detection output $\hat{\mathbf{x}}_{6B}$ that has minimum $\Phi(\overline{\mathbf{x}})$ at the bottom layer of the visiting nodes. Mark the partial Euclidean distance (PED) of (2+4+8+12) nodes that you visited.

```
best =

    -1    -1    -1    -1


ED =

    0.3010
```

| | | | | |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 7.7254 |
| 0 | 0 | 0 | -1 | 0.0127 |
| 0 | 0 | -1 | -1 | 0.0237 |
| 0 | 0 | 1 | -1 | 0.4470 |
| 0 | 0 | -1 | 1 | 9.7508 |
| 0 | 0 | 1 | 1 | 12.5093 |
| 0 | -1 | -1 | -1 | 0.1340 |
| 0 | 1 | 1 | -1 | 0.9247 |
| 0 | -1 | 1 | -1 | 2.9730 |
| 0 | 1 | -1 | -1 | 3.8201 |
| 0 | -1 | -1 | 1 | 9.9562 |
| 0 | -1 | 1 | 1 | 13.1559 |
| **-1** | **-1** | **-1** | **-1** | **0.3010** |
| -1 | 1 | 1 | -1 | 1.6859 |
| -1 | 1 | -1 | -1 | 4.5454 |
| -1 | -1 | 1 | -1 | 4.7033 |
| -1 | -1 | -1 | 1 | 9.9726 |
| -1 | -1 | 1 | 1 | 15.7033 |

上方表格為 6-best 演算法中所被選擇的點，而黃底的數據由上到下分別 PED of (2+4+8+12) nodes that you visited，

| | | |
|---|---|---|
| NODE 2 | 0 0 0 -1 | 0.0127 |
| NODE 4 | 0 0 1 -1 | 0.4470 |
| NODE 8 | 0 1 1 -1 | 0.9247 |
| NODE 12 | 0 -1 1 1 | 13.1559 |

（我的演算法在算每一層的 PED 時，除了 Level 4(Node 1、2)外，會將算到的結果進行排序，然後選擇當中的前 6 個，因此都會根據 PED 從小到大去拜訪 nodes）

最後得到當$\hat{x}_{6B} = [-1 - 1 - 1 - 1]$時，會得到 minimum $\Phi(\bar{x}) = 0.3010$

(d). Does the $\hat{\mathbf{x}}_{6B}$ same as $\hat{\mathbf{x}}_{ML}$. If yes, why? If no, why? Please comment.

$\hat{x}_{6B}$與$\hat{x}_{ML}$相同，這是因為當 K-best algorithm 的 K 值越大時所能得到的 ED 越有機會接近窮舉所得到的最佳答案，這次我們的 x vector 也夠小(4 個元素)，所以取 6 best 時，每一層最多都可以算到 12 個節點的 PED，而這題的 tree 的最底層的 leaf 只有 16 個，因此才有很高的機率得到最佳解；但是根據上方表格，在每一層中可以發現最小 PED 都發生在[~～～-1]、[~～-1 -1]、[~ -1 -1 -1]上，因此取 2-best 也可以得到[-1 -1 -1 -1]有最佳解。