無線通訊積體電路 Homework 3

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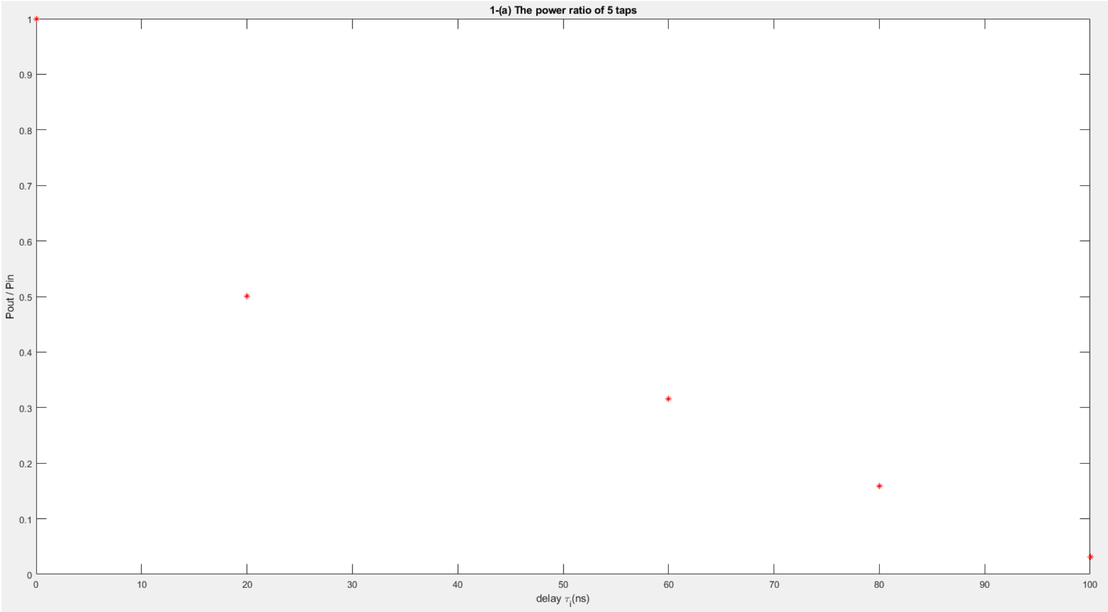
1. Consider one channel power delay profile.

(a). Please list the power ratio of 5 taps in linear scale.



圖一 Power ratio of 5 taps

表一 Power ratio of 5 taps

Tap	Power ratio(Pout/Pin)	Delay(ns)
0	1	0
1	0.5012	20
2	0.3162	60
3	0.1585	80
4	0.0316	100

(b). Please find the maximum excess delay.

因為經過 channel 最大的延遲訊號為 100 ns，所以 maximum excess delay=100 ns

(c). Please calculate the mean excess delay.

因為 channel power delay profile 並非連續的，因此

$$\bar{\tau} = \frac{\sum_{i=0}^{4} \tau_i P(\tau_i)}{\sum_{i=0}^{4} P(\tau_i)} = 22.3354\, ns.$$

(d). Please calculate the RMS excess delay.

$$\tau_{rms} = \sqrt{\int_{-\infty}^{\infty} (\tau - \bar{\tau})^2 P_{norm}(\tau) d\tau} = \sqrt{\sum_{i=0}^{4} (\tau_i - \bar{\tau})^2 P_{norm}(\tau_i)}$$

$$= \sqrt{\sum_{i=0}^{4} {\tau_i}^2 P_{norm}(\tau_i) - 2\bar{\tau} \sum_{i=0}^{4} \tau_i P_{norm}(\tau_i) + (\bar{\tau})^2 \sum_{i=0}^{4} P_{norm}(\tau_i)}$$

$$= \sqrt{\overline{\tau^2} - 2\bar{\tau}\bar{\tau} + (\bar{\tau})^2} = \sqrt{\overline{\tau^2} - (\bar{\tau})^2} = 28.8245\ ns$$

$$\overline{\tau^2} = \sum_{i=0}^{4} {\tau_i}^2 P_{norm}(\tau_i) = \frac{\sum_{i=0}^{4} {\tau_i}^2 P(\tau_i)}{\sum_{i=0}^{4} P(\tau_i)}$$

```
Pout =

    1.0000    0.5012    0.3162    0.1585    0.0316


D =

     0    20    60    80   100


Tmean =

   22.3354


Trms =

   28.8245
```

圖二 Q1 的 MATLAB 執行結果

2. Please generate complex Gaussian random variable with zero mean and unit variance to obtain γ_i，$i = 0, 1, 2, 3, 4.$

γ_i為以下數值，並且平均值趨近於 0，同時γ_i的變異數為 1、γ_i_X的變異數為 0.5、γ_i_Y的變異數為 0.5

```
r =

  -0.1926 + 0.7899i   0.6128 - 0.3335i   0.5856 + 0.0122i   0.1048 + 0.5257i  -1.1106 - 0.9942i


rmean =

  -4.4409e-17 + 2.2204e-17i


rvar =

     1


rxvar =

    0.5000


ryvar =

    0.5000
```

(a). $g_i = \alpha_i * \gamma_i$, now normalize g_i so that $h_i = Kg_i$ and $\sum_{i=0}^{4}|h_i|^2 = 1$

因此 K=1/sqrt(sum(abs(g).^2))

```
g =

  -0.1926 + 0.7899i   0.3071 - 0.1671i   0.1852 + 0.0038i   0.0166 + 0.0833i  -0.0351 - 0.0314i


k =

    1.0997


h =

  -0.2118 + 0.8686i   0.3377 - 0.1838i   0.2036 + 0.0042i   0.0183 + 0.0916i  -0.0386 - 0.0346i


hi2sum =

    1.0000
```

圖三 2-(a) MATLAB 執行結果

(b). Given that the channel impulse response is written as$\sum_{i=0}^{4} h_i \delta(t-\tau_i)$. Now, use Fourier transform to derive its frequency domain channel response. Write down your equation.

假設$x(t) = \sum_{i=0}^{4} h_i \delta(t-\tau_i)$

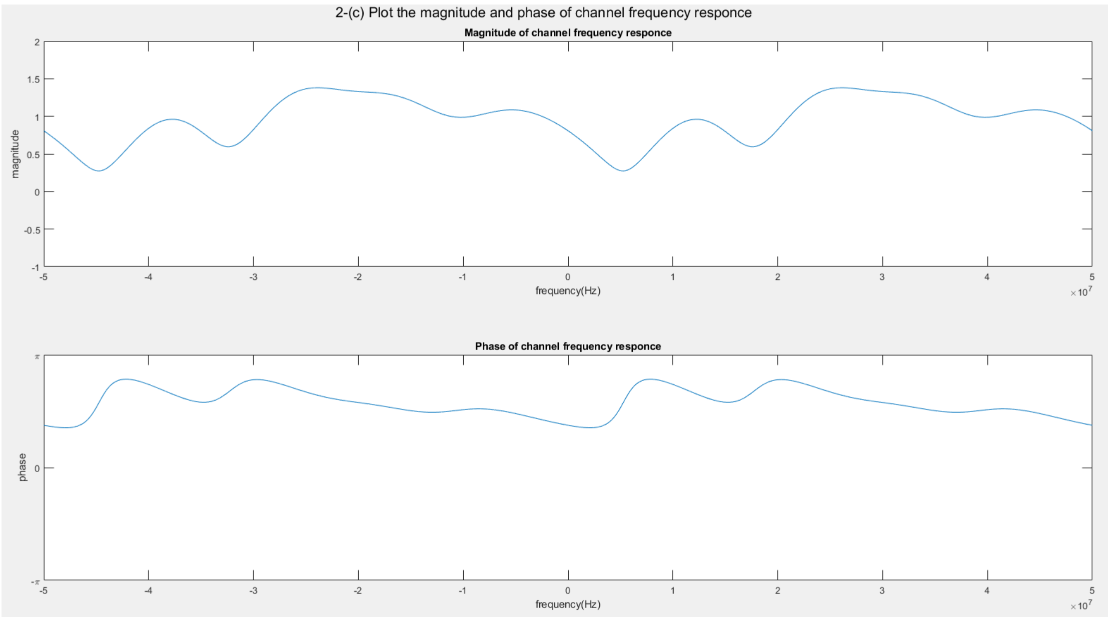
$$X(j2\pi f) = F\{x(t)\} = \int_{-\infty}^{\infty} x(t) * e^{-j2\pi f t} dt$$

$$= \int_{-\infty}^{\infty} \sum_{i=0}^{4} h_i \delta(t-\tau_i) * e^{-j2\pi f t} dt$$

$$= \int_{-\infty}^{\infty} [h_0\delta(t-\tau_0) + h_1\delta(t-\tau_1) + h_2\delta(t-\tau_2)$$
$$+ h_3\delta(t-\tau_3) + h_4\delta(t-\tau_4)] * e^{-j2\pi f t} dt$$

$$= \sum_{i=0}^{4} [h_0\delta(\tau_i-\tau_0) + h_1\delta(\tau_i-\tau_1) + h_2\delta(\tau_i-\tau_2)$$
$$+ h_3\delta(\tau_i-\tau_3) + h_4\delta(\tau_i-\tau_4)] * e^{-j2\pi f \tau_i}$$
$$= h_0 e^{-j2\pi f \tau_0} + h_1 e^{-j2\pi f \tau_1} + h_2 e^{-j2\pi f \tau_2} + h_3 e^{-j2\pi f \tau_3}$$
$$+ h_4 e^{-j2\pi f \tau_4}$$

(c). Plot the magnitude and phase of channel frequency response in the range of -50MHz and 50MHz.



圖四 The magnitude and phase of channel frequency response (d). If the coherence bandwidth is defined as $B_c = 1/(6\tau_{RMS})$. Please calculate the coherence bandwidth. How many subcarriers should be allocated in a 50 MHz bandwidth in flat-fading channel with OFDM systems.

$$B_c = \frac{1}{6\tau_{RMS}} = \frac{1}{6 \times 28.8245 \times 10^{-9}} = 5.782 \times 10^6 Hz = 5.782MHz$$

$$\because we\ want\ to\ produce\ a\ flat - flading\ channel$$

$$\therefore \frac{B}{N} < B_c$$

$$\Rightarrow \frac{B}{B_c} < N \Rightarrow N > \frac{50MHz}{5.782MHz} = 8.64$$

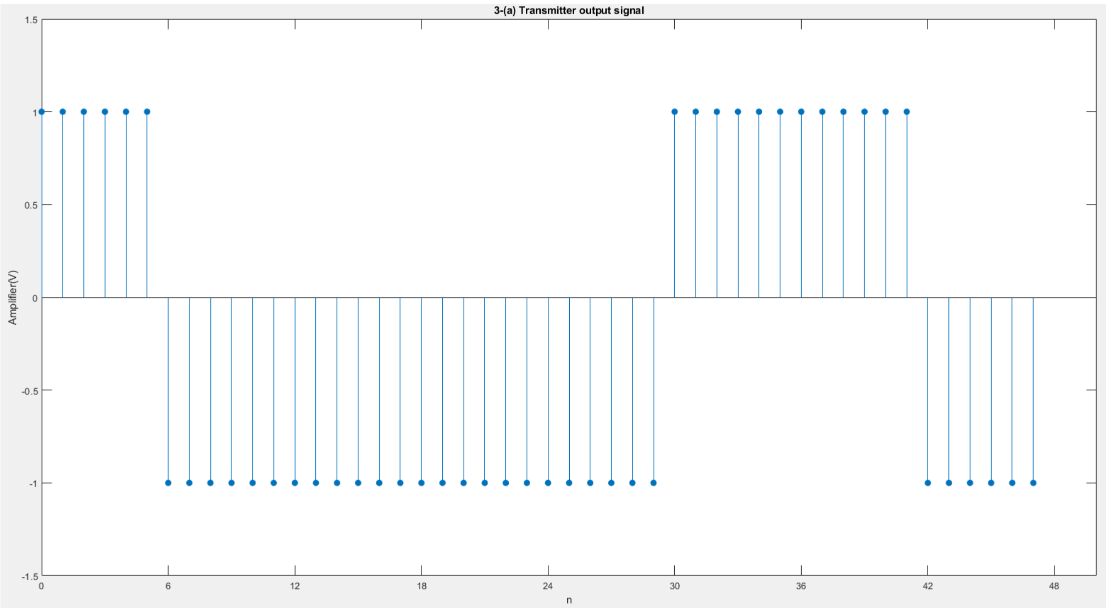
$At\ least\ 9\ subcarriers.$

```
Bc =

   5.7821e+06


N =

     9
```

3. Please randomly generate 8 binary data and write it down.

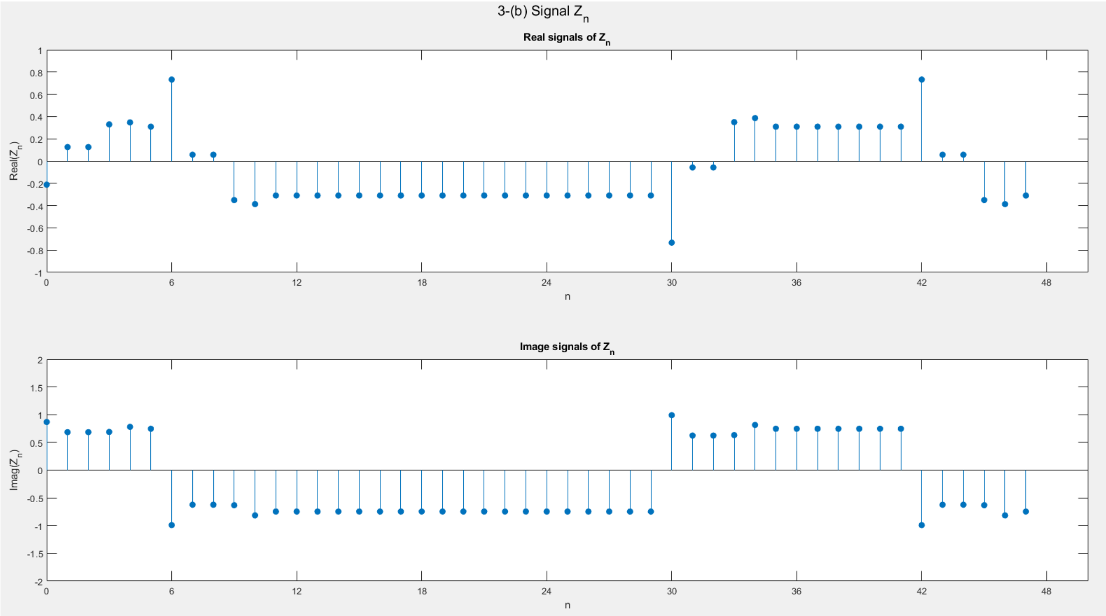
```
r8bits =

     0     1     1     1     1     0     0     1
```

(a). Please use command "stem" in MATLAB to draw the transmitter output signal, d_n for n=0, 1, …, 47.



圖五 Transmitter output signal

(b). Please draw the real part and imaginary part signals of the received signal z_n after multipath channel fading effect, namely $z_n = \sum_{i=0}^{4} h_i d_{n-\tau_i/T_s}$.



圖六 Signal of z_n

(c). 因為我的 transmitter signal 是 0 1 1 1 1 0 0 1，我們可以從上圖發現當 0 變 1、1 變 0 的這個區間非常容易發生 inter-symbol interference，如 n=6~8、30~31、42~44 這轉換區間，而當訊號保持不變時，inter-symbol interference 就不太明顯，如 1 1 1 1(n=6~29)、0 0(n=30~41)，訊號沒有甚麼明顯的失真表現。

4、

$$x(t) = (x_I(t) + j x_Q(t))(C(t) + jS(t))$$
$$= (x_I(t) + j x_Q(t))\left[(1-\tfrac{\varepsilon}{2})\cos(\omega_c t + \tfrac{\phi}{2}) + j(1+\tfrac{\varepsilon}{2})\sin(\omega_c t - \tfrac{\phi}{2})\right]$$
$$= \left[x_I(t)(1-\tfrac{\varepsilon}{2})\cos(\omega_c t + \tfrac{\phi}{2}) - x_Q(t)(1+\tfrac{\varepsilon}{2})\sin(\omega_c t - \tfrac{\phi}{2})\right]$$
$$+ j\left[x_I(1+\tfrac{\varepsilon}{2})\sin(\omega_c t - \tfrac{\phi}{2}) + x_Q(t)(1-\tfrac{\varepsilon}{2})\cos(\omega_c t + \tfrac{\phi}{2})\right]$$

$$\tilde{y}(t) = \mathrm{Re}\{x(t)\}$$
$$= x_I(t)(1-\tfrac{\varepsilon}{2})\cos(\omega_c t + \tfrac{\phi}{2}) - x_Q(t)(1+\tfrac{\varepsilon}{2})\sin(\omega_c t - \tfrac{\phi}{2})$$

$$\tilde{x}_I(t) = \mathrm{LPF}\{\tilde{y}(t)\cos(\omega_c t)\}$$
$$= \mathrm{LPF}\{x_I(t)(1-\tfrac{\varepsilon}{2})\cos(\omega_c t + \tfrac{\phi}{2})\cos(\omega_c t) - x_Q(t)(1+\tfrac{\varepsilon}{2})\sin(\omega_c t - \tfrac{\phi}{2})\cos(\omega_c t)\}$$
$$= \mathrm{LPF}\{x_I(t)(1-\tfrac{\varepsilon}{2})\tfrac{1}{2}\left[\cos(2\omega_c t + \tfrac{\phi}{2}) + \cos(\tfrac{\phi}{2})\right] - x_Q(t)(1+\tfrac{\varepsilon}{2})\tfrac{1}{2}\left[\sin(2\omega_c t - \tfrac{\phi}{2}) + \sin(-\tfrac{\phi}{2})\right]\}$$
$$= \tfrac{1}{2}(1-\tfrac{\varepsilon}{2})x_I(t)\cos(\tfrac{\phi}{2}) + \tfrac{1}{2}(1+\tfrac{\varepsilon}{2})x_Q(t)\sin(\tfrac{\phi}{2})$$

$$\tilde{x}_Q(t) = \mathrm{LPF}\{-\tilde{y}(t)\sin(\omega_c t)\}$$
$$= \mathrm{LPF}\{-x_I(t)(1-\tfrac{\varepsilon}{2})\cos(\omega_c t + \tfrac{\phi}{2})\sin(\omega_c t) + x_Q(t)(1+\tfrac{\varepsilon}{2})\sin(\omega_c t - \tfrac{\phi}{2})\sin(\omega_c t)\}$$
$$= \mathrm{LPF}\{-x_I(t)(1-\tfrac{\varepsilon}{2})\tfrac{1}{2}\left[\sin(2\omega_c t + \tfrac{\phi}{2}) - \sin(\tfrac{\phi}{2})\right] + x_Q(t)(1+\tfrac{\varepsilon}{2})\tfrac{1}{2}\left[\cos(-\tfrac{\phi}{2}) - \cos(2\omega_c t - \tfrac{\phi}{2})\right]$$
$$= \tfrac{1}{2}(1-\tfrac{\varepsilon}{2})x_I(t)\sin(\tfrac{\phi}{2}) + \tfrac{1}{2}(1+\tfrac{\varepsilon}{2})x_Q(t)\cos(\tfrac{\phi}{2})$$