# *Wireless Communication IC*

## *Homework #5*

(Due on 05/23)

Total points: 100 points

1. Assume that a MIMO equation is described by

$$\mathbf{y} = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ y^{(3)} \end{bmatrix} = \begin{bmatrix} H^{(1,1)} & H^{(1,2)} & H^{(1,3)} \\ H^{(2,1)} & H^{(2,2)} & H^{(2,3)} \\ H^{(3,1)} & H^{(3,2)} & H^{(3,3)} \end{bmatrix} \begin{bmatrix} x^{(1)} \\ x^{(2)} \\ x^{(3)} \end{bmatrix} + \begin{bmatrix} v^{(1)} \\ v^{(2)} \\ v^{(3)} \end{bmatrix} = \mathbf{Hx} + \mathbf{v},$$

where $y^{(n)}$ is the received signal at $n$th antenna; $x^{(m)}$ is the transmitted signal at $m$th antenna; $v^{(n)}$ is the noise; $H^{(n,m)}$ is the channel response from antenna $m$ to antenna $n$.

(a) Please download HW5-1a.mat, which contains two variables $\mathbf{H}$(Hmatrix) and $\mathbf{y}$. Given that $v^{(n)} = 0$, please find the ZF detection matrix $\mathbf{G}_{\text{ZF}}$ so that we can detect $\mathbf{x}$ by $\mathbf{G}_{\text{ZF}}\mathbf{y}$. (5%) If the constellation of 16QAM, which contains symbols of $\{\pm 1 \pm 1j, \pm 1 \pm 3j, \pm 3 \pm 1j, \pm 3 \pm 3j\}$ is used as the transmitted signal, what is $\hat{\mathbf{x}}_1$, the output after ZF detection. (10%)

(b) Following (a), if the received signal $\mathbf{y}$ contains noise and become $\mathbf{y}'$ in HW5-1b.mat, what is the output after you apply $\mathbf{G}_{\text{ZF}}\mathbf{y}'$? (5%), what is the detection output $\hat{\mathbf{x}}_2$? (10%) Note that $\hat{\mathbf{x}}_2$ must belong to $\{\pm 1 \pm 1j, \pm 1 \pm 3j, \pm 3 \pm 1j, \pm 3 \pm 3j\}$.

2. Given that noisy $\mathbf{y}' = \mathbf{Hx} + \mathbf{v} = \begin{bmatrix} H^{(1,1)} & H^{(1,2)} & H^{(1,3)} \\ H^{(2,1)} & H^{(2,2)} & H^{(2,3)} \\ H^{(3,1)} & H^{(3,2)} & H^{(3,3)} \end{bmatrix} \begin{bmatrix} x^{(1)} \\ x^{(2)} \\ x^{(3)} \end{bmatrix} + \mathbf{v}$, now we want to use OSIC to detect transmitted signals from 16-QAM constellation $\{\pm 1 \pm 1j, \pm 1 \pm 3j, \pm 3 \pm 1j, \pm 3 \pm 3j\}$. Please download HW5-2.mat, which contains two variables, $\mathbf{H}$(Hmatrix) and $\mathbf{y}'$(yprime).

(a) Which signal should be detected first? $x^{(1)}$, $x^{(2)}$ or $x^{(3)}$? (5%)

(b) From (a), please write down your first detected output $\hat{x}^{(\alpha_1)}$ after decision. (5%)

(c) Please write down the equation $\mathbf{y}(1) = \mathbf{H}(1)\mathbf{x}(1)$ after you apply interference cancellation so that the channel matrix $\mathbf{H}(1)$ in the second iteration becomes a 3×2 matrix, $\mathbf{y}(1)$ is a 3×1 vector after interference removal and $\mathbf{x}(1)$ is a 2×1 vector containing two unknown to be detected. (5%)

(d) From (c), please write down the second detected output $\hat{x}^{(\alpha_2)}$, where $\alpha_2 \neq \alpha_1$ in

Q2(b) . (5%)

(e) Repeat (c), Please write down the equation $\mathbf{y}(2) = \mathbf{H}(2)\mathbf{x}(2)$ after you apply interference cancellation so that the channel matrix $\mathbf{H}(2)$ in the third iteration becomes a 3×1 matrix, $\mathbf{y}(2)$ is a 3×1 vector after interference removal and $\mathbf{x}(2)$ is a 1×1 vector containing one unknown to be detected. (5%)

(f) Please write down the second detected output $\hat{x}^{(\alpha_3)}$. (5%)

3. Assume that the MIMO configuration is 4×4. The constellation of BPSK is used at the transmitter, which means the element $x^{(n)}$ of the 4×1 transmitted vector $\mathbf{x}$ belongs to $\{+1, -1\}$. Now, please download HW5-3.mat. It contains the 4×4 channel matrix, **H** (Hmatrix) and the 4×1 received signal, **y(=Hx+n)**. Note that

(a) Define the cost function as $\Gamma(\bar{\mathbf{x}}) = ||\mathbf{y} - \mathbf{H}\bar{\mathbf{x}}||^2$ and $\bar{\mathbf{x}} = [\bar{x}^{(1)} \quad \bar{x}^{(2)} \quad \dots \quad \bar{x}^{(4)}]^{\mathrm{T}}$. First, compute 16 values of $\Gamma(\bar{\mathbf{x}})$ corresponding to 16 possibilities of $\bar{\mathbf{x}}$. Please draw $\Gamma(\bar{\mathbf{x}})$ versus the index of possible $\bar{\mathbf{x}}$ vectors and then determine $\hat{\mathbf{x}}_{\mathrm{ML}} = \arg\min_{\bar{\mathbf{x}}} \Gamma(\bar{\mathbf{x}})$. (15%)

(b) If $\mathbf{H} = \mathbf{QR}$ (you can use command “qr” in Matlab) and $\mathbf{z} = \mathbf{Q}^{\mathrm{H}}\mathbf{y}$, write down $\mathbf{z} = [z^{(1)} \quad z^{(2)} \quad \dots \quad z^{(4)}]^{\mathrm{T}}$. (5%)

(c) From (b), denote the $(i, j)$-th element in the upper triangular matrix **R** as $r_{i,j}$. If $\Phi(\bar{\mathbf{x}}) = ||\mathbf{z} - \mathbf{R}\bar{\mathbf{x}}||^2 = T(1) + T(2) + T(3) + T(4)$ and $T(i) = \left|z^{(i)} - \sum_{j=i}^{4} r_{i,j}\bar{x}^{(j)}\right|^2$. Use the 6-best algorithm to find out the detection output $\hat{\mathbf{x}}_{6\mathrm{B}}$ that has minimum $\Phi(\bar{\mathbf{x}})$ at the bottom layer of the visiting nodes. Mark the partial Euclidean distance (PED) of (2+4+8+12) nodes that you visited. Note that at layer $m$, PED equals to $\sum_{l=4}^{m} T(l)$ (25%)

(d) Does the $\hat{\mathbf{x}}_{6\mathrm{B}}$ same as $\hat{\mathbf{x}}_{\mathrm{ML}}$. If yes, why? If no, why? Please comment. (5%)