無線通訊積體電路 Homework 2

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1. The properties of constant amplitude zero auto-correlation (CAZAC) Zadoff-Chu sequence.

(a). We set u_1 as 9.

(b). Plot the real part and imaginary part of sequence $S(n, u_1)$.

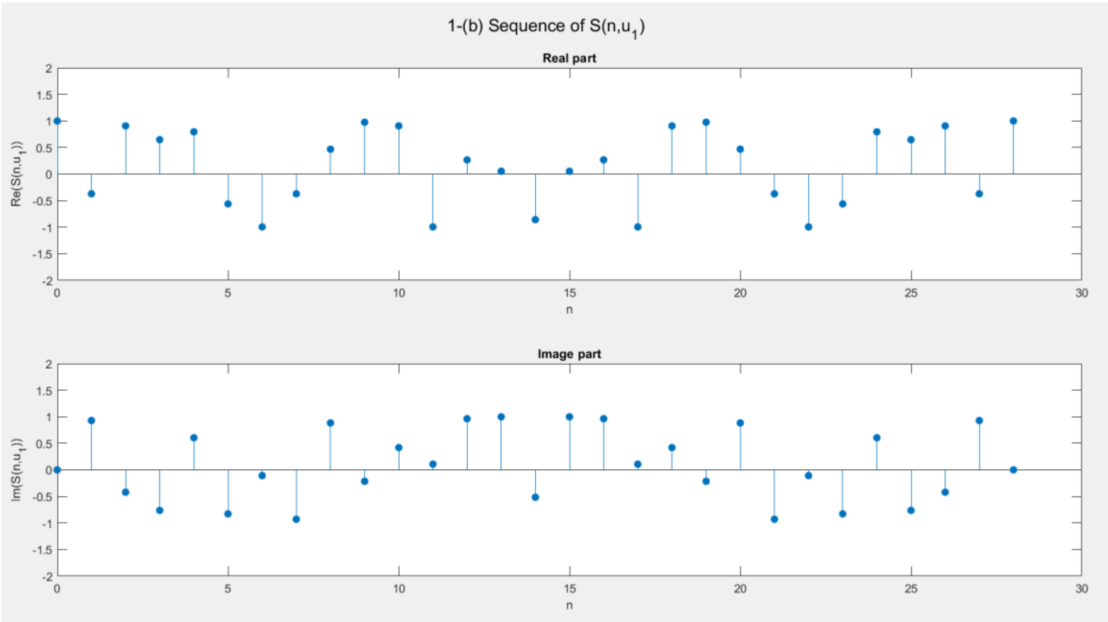

Figure 1: the real part and imaginary part of sequence $S(n, u_1)$

(c). Plot $|\Phi_s(k)|$, $\Phi_s(k) = \frac{1}{N}\sum_{n=0}^{N-1} S(n, u_1)S^*(n-k, u_1)$

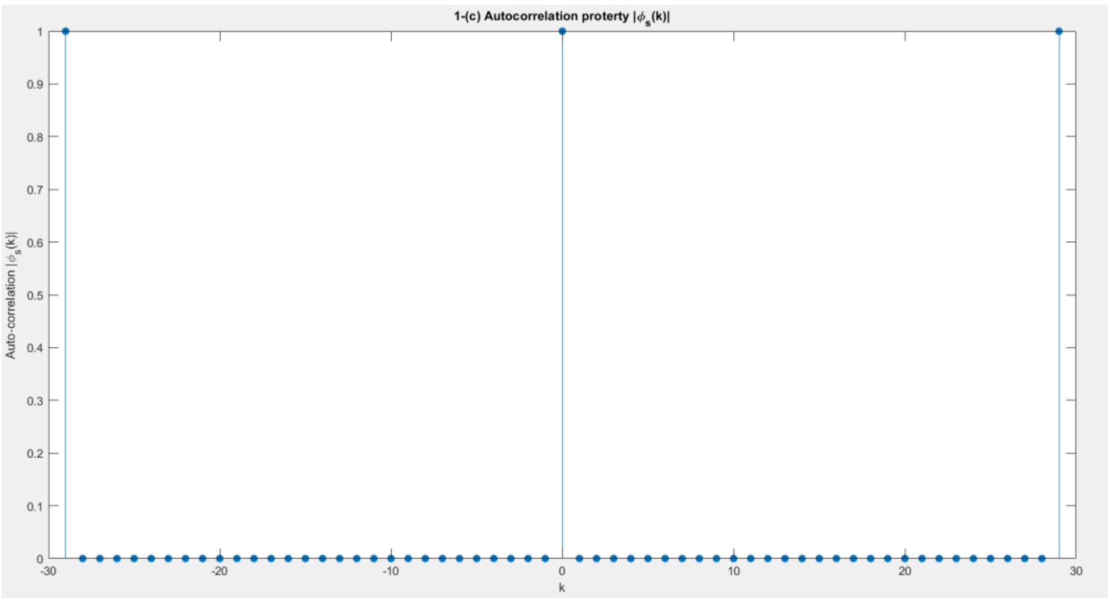

Figure 2: auto-correlation $|\Phi_s(k)|$

(d). Plot $|\Omega_s(m)|$, $\Omega_s(m) = \frac{1}{N}\sum_{n=0}^{N-1} S(n, u_1)S^*(n-k, u_1+m)$, k=3

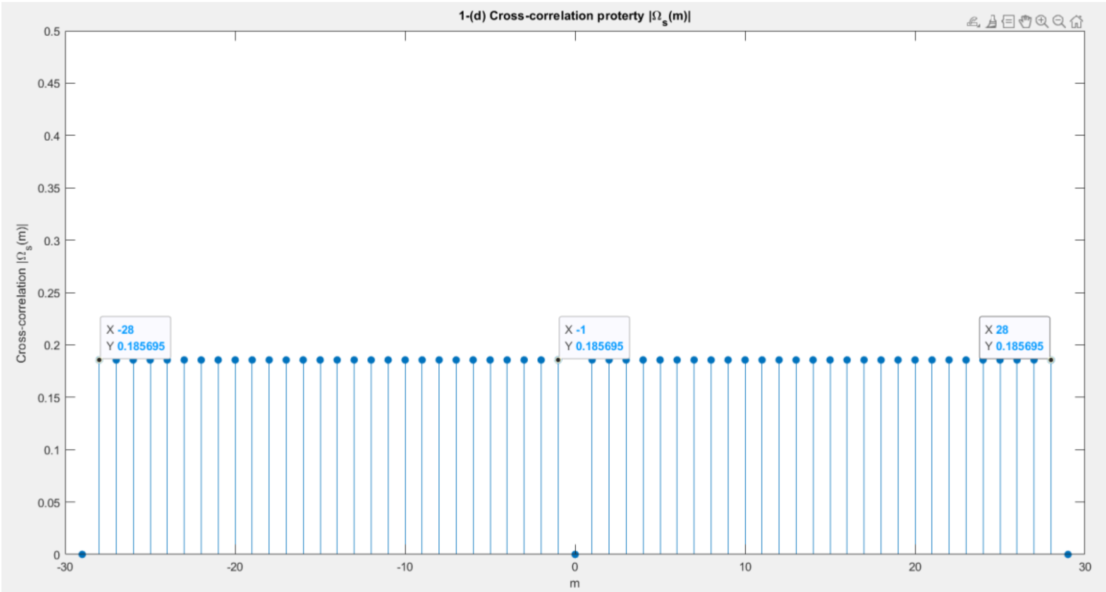

Figure 3: cross-correlation $|\Omega_s(m)|$

(e). Plot $|\Omega_s(k)|$, $\Omega_s(k) = \frac{1}{N}\sum_{n=0}^{N-1} S(n, u_1)S^*(n-k, u_1+m)$, m=3

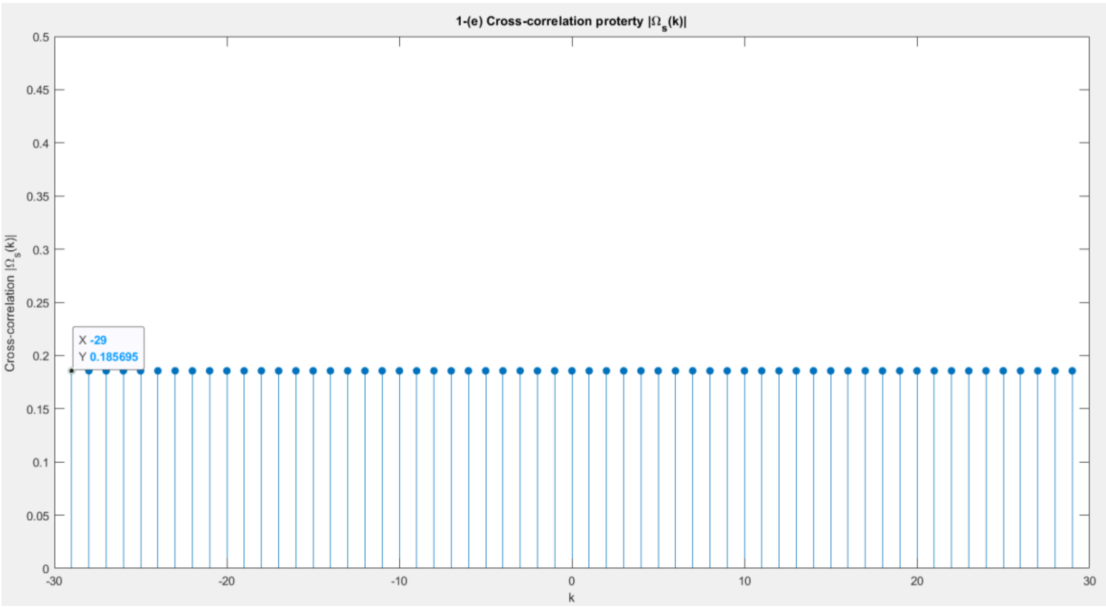

Figure 4: cross-correlation $|\Omega_s(k)|$

(f). Show $|\mathrm{p}(m)|$, $\mathrm{p}(m) = \sum_{n=0}^{N-1} \mathrm{y}(n+m)\mathrm{S}^*(n,u_1)$

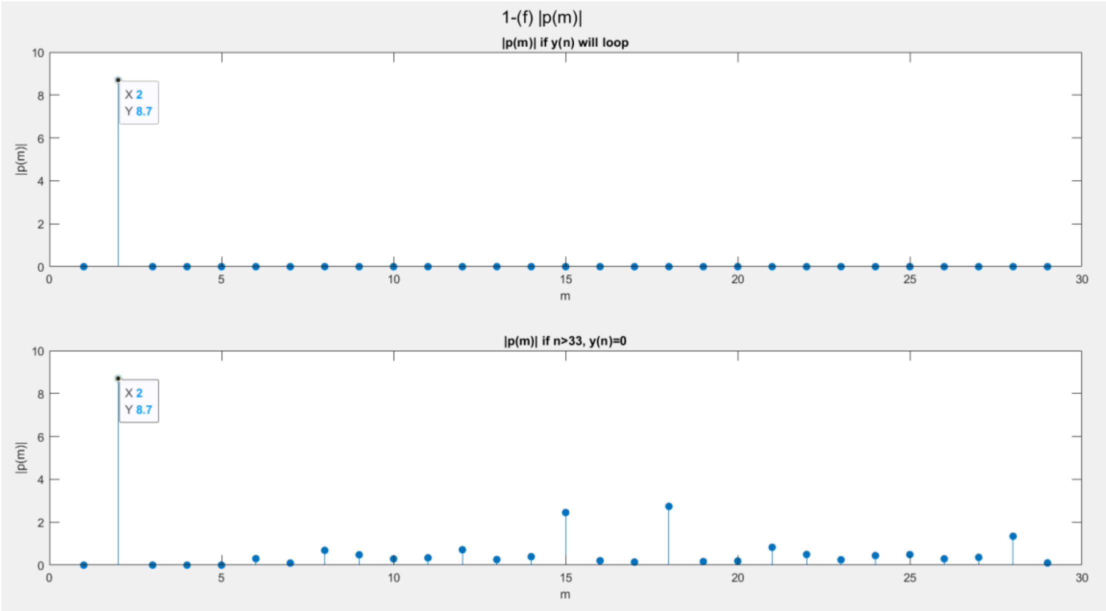

Figure 5: $|\mathrm{p}(m)|$

Initially, we define $\mathrm{y}(n)$ as $\mathrm{y}(n) = [0.3\mathrm{S}(N-2,u_1)\ 0.3\mathrm{S}(N-1,u_1)\ 0.3\mathrm{S}(0,u_1)\ 0.3\mathrm{S}(1,u_1) \ldots \ldots 0.3\mathrm{S}(N-1,u_1)\ 0.3\mathrm{S}(0,u_1)\ 0.3\mathrm{S}(1,u_1)\ 0.3\mathrm{S}(2,u_1)]$ .This indicates that $\mathrm{y}(n)$ is a right-shifted version of $\mathrm{S}(n,u_1)$ by 2 samples, and its amplitude is scaled by a factor of 0.3. The function $\mathrm{p}(m) = \sum_{n=0}^{N-1} \mathrm{y}(n+m)\mathrm{S}^*(n,u_1)$ represents the auto-correlation between $\mathrm{y}(n)$ and $\mathrm{S}^*(n,u_1)$. Since the parameter u remains unchanged for both $\mathrm{y}(n)$ and $\mathrm{S}^*(n,u_1)$, and only the index n in $\mathrm{y}(n)$ varies, option (c) is the most appropriate explanation among (c) to (e).

Because $\mathrm{y}(n)$ is a right-shifted version of $\mathrm{S}^*(n,u_1)$ by 2 samples, the pulse in Figure 5 also shifts right by 2 units. The observed peak value is 8.7, which, when divided by N=29, results in approximately 0.3—matching the scaling factor applied to $\mathrm{y}(n)$ in relation to $\mathrm{S}^*(n,u_1)$.

2. Given the Barker code list, please check the periodic autocorrelation function defined as $\Phi_s(k) = \frac{1}{N}\sum_{n=0}^{N-1} S(n)S^*([n-k]_N)$

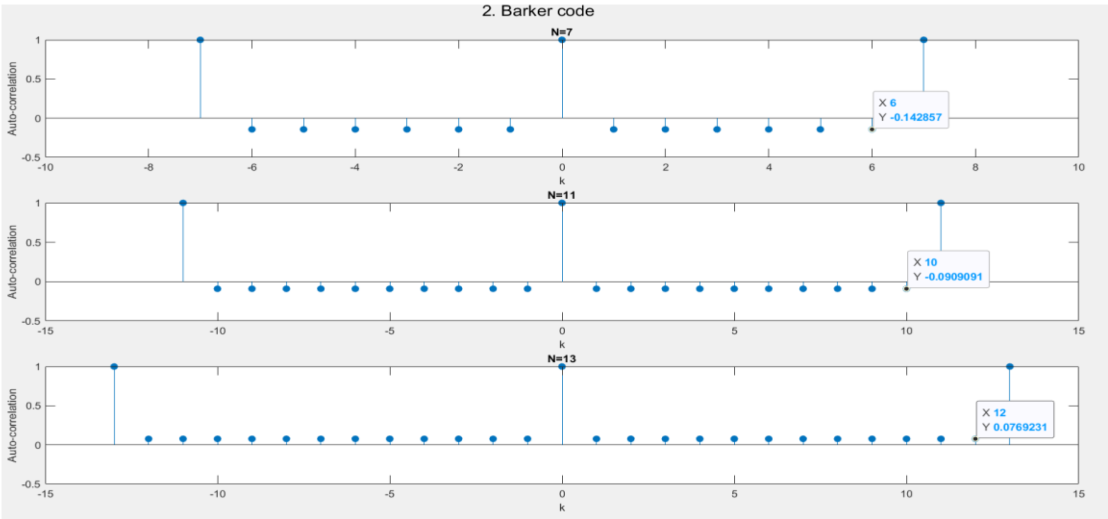

Figure 6: Barker code 的 auto-correlation

3. Walsh Hadamard code is also widely used for multiple access because of good cross-correlation.

(a). Print out the $(\alpha+1)$th and 24^{th} columns of the matrix W_{32}. $(\alpha = 9)$

```
ans =

  Columns 1 through 19

    1    -1     1    -1     1    -1     1    -1    -1     1    -1     1    -1     1    -1     1     1    -1     1

  Columns 20 through 32

   -1     1    -1     1    -1    -1     1    -1     1    -1     1    -1     1
```

Figure 7: 10^{th} columns of the matrix W_{32}

```
ans =

  Columns 1 through 19

    1    -1    -1     1    -1     1     1    -1     1    -1    -1     1    -1     1     1    -1    -1     1     1

  Columns 20 through 32

   -1     1    -1    -1     1    -1     1     1    -1     1    -1    -1     1
```

Figure 8: 24^{th} columns of the matrix W_{32}

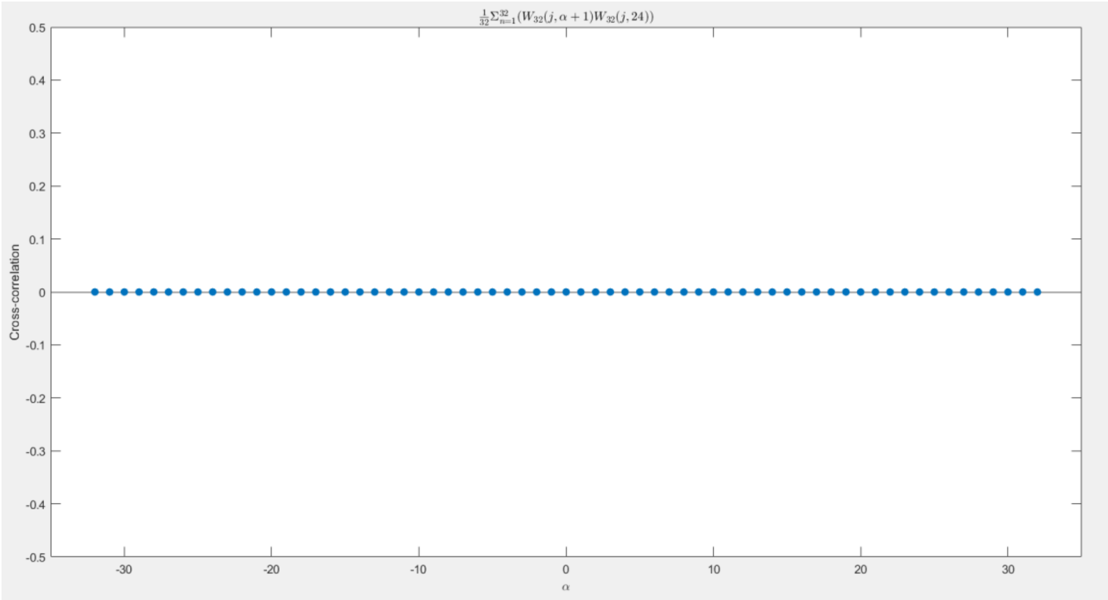

Figure 9: cross correlation $\frac{1}{32}\sum_{j=1}^{32}(W_{32}(j,\alpha+1)W_{32}(j,24))$

(b). Choose $c_1 = W_{32}(:,\ \alpha+1)$ and $c_2 = W_{32}(:,\ 14)$ as two codes for user 1 and user 2. Randomly generate 5 symbols $(d_0 \sim d_4)$ from the set {+1, -1}. Spread the data by code 1 as $y =$ $[d_0c_{1,1}\ d_0c_{2,1}\ \ldots d_0c_{32,1}\ d_1c_{1,1}\ \ldots d_1c_{32,1} \ldots d_4c_{1,1} \ldots d_4c_{32,1}\ 0\ 0\ 0\ 0]$.

We genetate $(d_0 \sim d_4)$ as $(+1, -1, +1, +1, +1)$

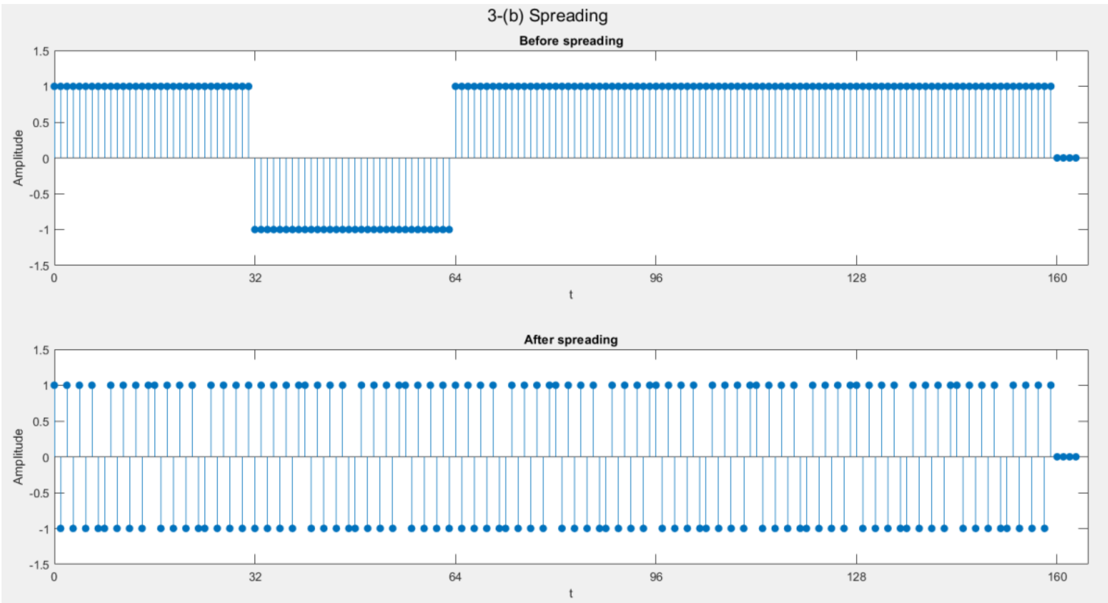

Figure 10: 5 symbols $(d_0 \sim d_4)$ before spreading and after spreading

(c). Plot p(i) using index I as the x-axis, $p(i) = \frac{1}{32}\sum_{j=1}^{32}(y_{32i+j+1})c_{j,1}$.

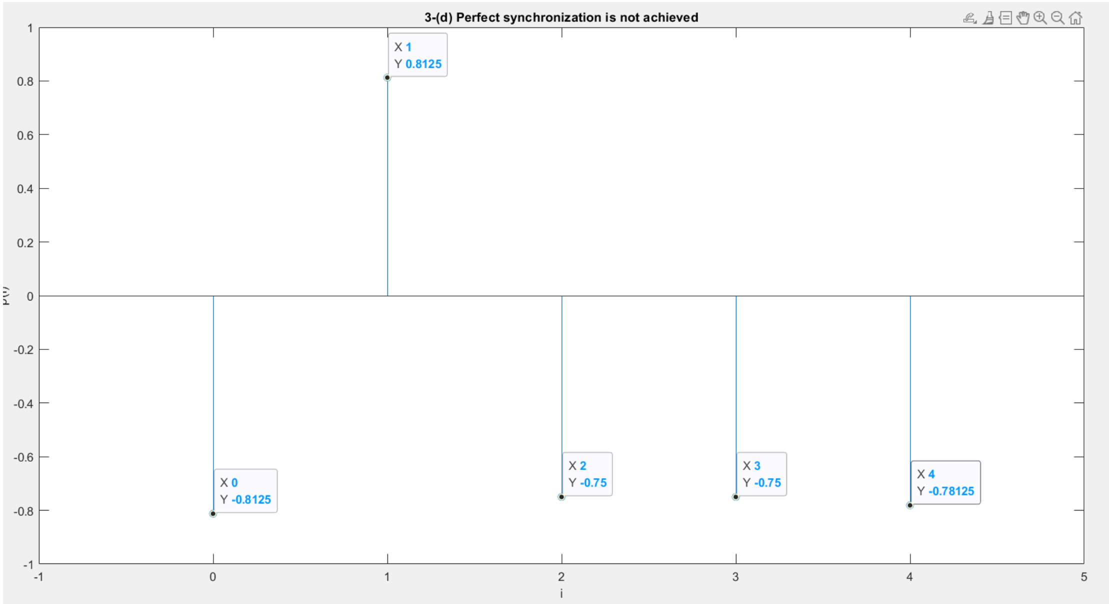

Figure 11: p(i) perfect synchronization is not achieved

(d). Plot p(i) using index I as the x-axis, $p(i) = \frac{1}{32}\sum_{j=1}^{32}(y_{32i+j})c_{j,2}$.

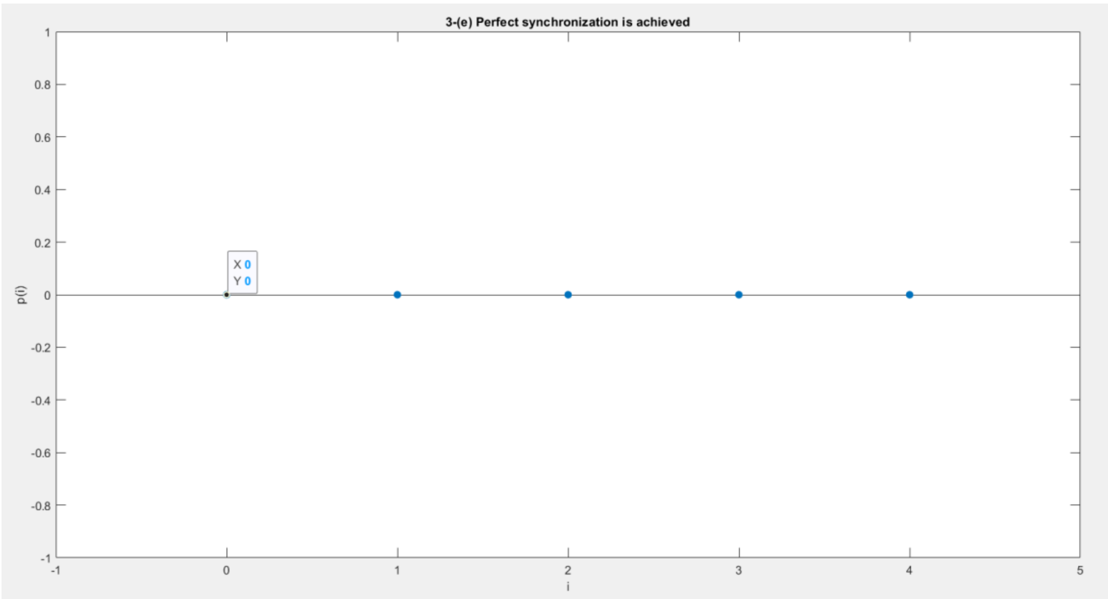

Figure 12: p(i) perfect synchronization is achieved

(e). Please comment the results in (c) and (d).

For sub-question (c), this case represents non-perfect synchronized despreading. When performing despreading on y, the signal has already been advanced by one sample. As a result, the despread signal becomes {-1, +1, -1, -1, -1}, which is exactly the inverse of the original signal {+1, -1, +1, +1, +1}. Moreover, the despread values are not strictly +1 or -1—they are significantly smaller in magnitude.

Through experimentation by adjusting the value of α\alphaα, we observe the following behavior:

- When α is **odd**, the despread result is the inverse of the original signal.
- When α is **even**, the despread result matches the original signal.

This shows how synchronization offset affects the correlation result.

For sub-question (d), this is a case of **perfectly synchronized despreading**. However, the despreading code used (Code 2) is completely different from the original y signal. Since the cross-correlation between the two sequences is zero, the result of the despreading operation is a sequence of all zeros.