無線通訊積體電路 Homework 4

107501019 魏子翔

1.

Let

$$\mathbf{H} = \frac{1}{\sqrt{L}}\sum_{l=1}^{L} \alpha_l \, \boldsymbol{a}^{\boldsymbol{ULA,r}}(\theta_l^r)\boldsymbol{a}^{\boldsymbol{ULA,t}}(\theta_l^t)^H$$

and

$$\boldsymbol{a}^{\boldsymbol{ULA}}(\theta) = [1 \quad e^{jkdsin(\theta)} \quad \dots \quad e^{j(N-1)kdsin(\theta)}]^T.$$

Assume $kd = \pi$.

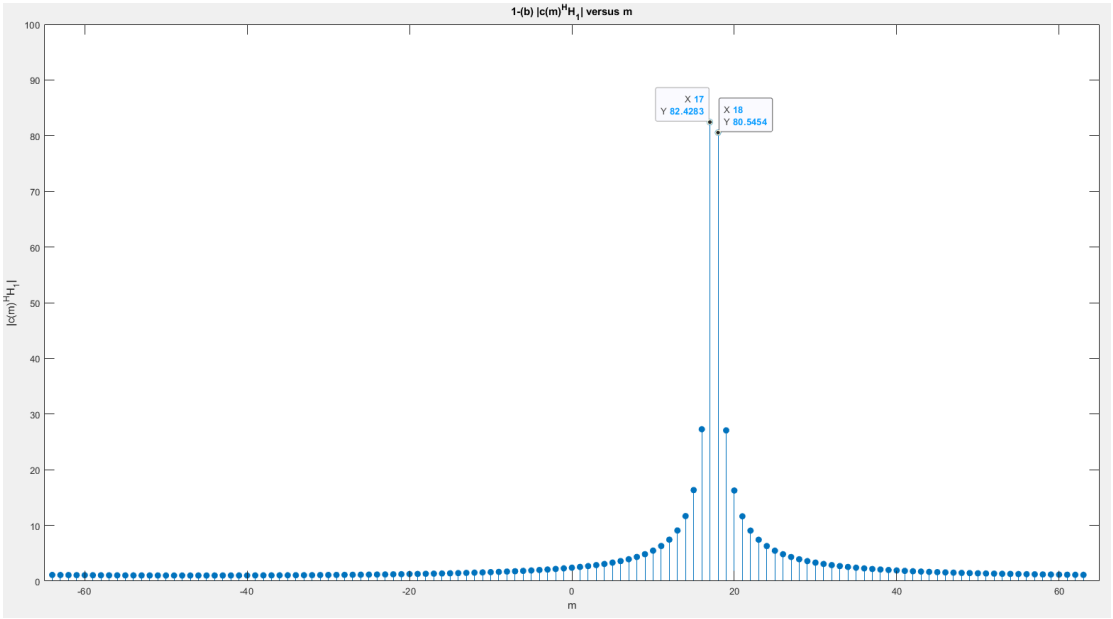
(a). Generate θ_1^r 、θ_1^t 、H_1

```
rl =

    0.2769


tl =

    1.2495
```

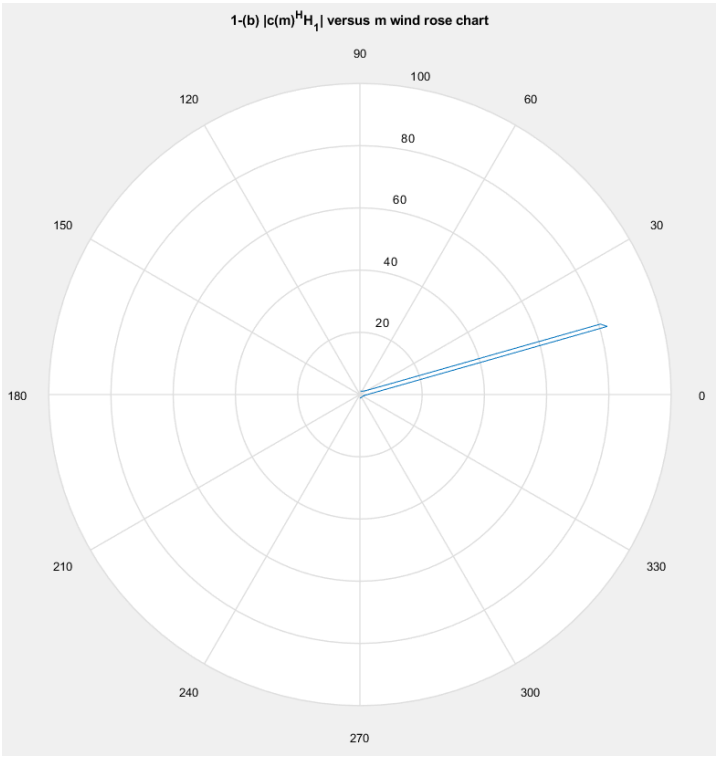
```
H1_reshape =

  Columns 1 through 7

   1.0000 + 0.0000i   0.6534 + 0.7570i  -0.1462 + 0.9893i  -0.8444 + 0.5357i  -0.9573 - 0.2892i  -0.4065 - 0.9136i   0.4260 - 0.9047i

  Columns 8 through 14

   0.9632 - 0.2686i   0.8327 + 0.5537i   0.1249 + 0.9922i  -0.6695 + 0.7429i  -0.9998 - 0.0214i  -0.6370 - 0.7708i   0.1673 - 0.9859i

  Columns 15 through 21

   0.8557 - 0.5175i   0.9509 + 0.3096i   0.3869 + 0.9221i  -0.4453 + 0.8954i  -0.9688 + 0.2479i  -0.8207 - 0.5714i  -0.1037 - 0.9946i

  Columns 22 through 28

   0.6852 - 0.7283i   0.9991 + 0.0428i   0.6204 + 0.7843i  -0.1884 + 0.9821i  -0.8666 + 0.4991i  -0.9440 - 0.3299i  -0.3670 - 0.9302i

  Columns 29 through 35

   0.4644 - 0.8856i   0.9739 - 0.2271i   0.8082 + 0.5888i   0.0823 + 0.9966i  -0.7007 + 0.7135i  -0.9979 - 0.0642i  -0.6034 - 0.7974i

  Columns 36 through 42

   0.2094 - 0.9778i   0.8771 - 0.4804i   0.9367 + 0.3501i   0.3470 + 0.9379i  -0.4832 + 0.8755i  -0.9785 + 0.2062i  -0.7954 - 0.6060i

  Columns 43 through 49

  -0.0610 - 0.9981i   0.7158 - 0.6983i   0.9963 + 0.0856i   0.5862 + 0.8102i  -0.2303 + 0.9731i  -0.8872 + 0.4615i  -0.9290 - 0.3701i

  Columns 50 through 56

  -0.3268 - 0.9451i   0.5019 - 0.8649i   0.9827 - 0.1852i   0.7823 + 0.6229i   0.0396 + 0.9992i  -0.7306 + 0.6828i  -0.9943 - 0.1069i

  Columns 57 through 63

  -0.5687 - 0.8225i   0.2511 - 0.9680i   0.8968 - 0.4424i   0.9209 + 0.3899i   0.3065 + 0.9519i  -0.5203 + 0.8540i  -0.9864 + 0.1641i
```

```
Columns 64 through 70

-0.7688 - 0.6395i  -0.0181 - 0.9998i   0.7450 - 0.6670i   0.9917 + 0.1282i   0.5510 + 0.8345i  -0.2718 + 0.9624i  -0.9061 + 0.4231i

Columns 71 through 77

-0.9123 - 0.4095i  -0.2861 - 0.9582i   0.5385 - 0.8426i   0.9897 - 0.1429i   0.7549 + 0.6559i  -0.0033 + 1.0000i  -0.7592 + 0.6509i

Columns 78 through 84

-0.9888 - 0.1494i  -0.5329 - 0.8461i   0.2923 - 0.9563i   0.9150 - 0.4035i   0.9033 + 0.4290i   0.2655 + 0.9641i  -0.5564 + 0.8309i

Columns 85 through 91

-0.9926 + 0.1217i  -0.7407 - 0.6719i   0.0247 - 0.9997i   0.7729 - 0.6345i   0.9853 + 0.1706i   0.5147 + 0.8574i  -0.3128 + 0.9498i

Columns 92 through 98

-0.9234 + 0.3838i  -0.8939 - 0.4482i  -0.2448 - 0.9696i   0.5741 - 0.8188i   0.9949 - 0.1004i   0.7261 + 0.6876i  -0.0461 + 0.9989i

Columns 99 through 105

-0.7863 + 0.6178i  -0.9815 - 0.1916i  -0.4962 - 0.8682i   0.3330 - 0.9429i   0.9314 - 0.3640i   0.8841 + 0.4673i   0.2239 + 0.9746i

Columns 106 through 112

-0.5915 + 0.8063i  -0.9969 + 0.0791i  -0.7112 - 0.7030i   0.0675 - 0.9977i   0.7994 - 0.6008i   0.9771 + 0.2126i   0.4775 + 0.8786i

Columns 113 through 119

-0.3532 + 0.9356i  -0.9390 + 0.3439i  -0.8739 - 0.4861i  -0.2030 - 0.9792i   0.6086 - 0.7935i   0.9983 - 0.0577i   0.6960 + 0.7181i

Columns 120 through 126

-0.0889 + 0.9960i  -0.8121 + 0.5835i  -0.9724 - 0.2335i  -0.4586 - 0.8887i   0.3731 - 0.9278i   0.9461 - 0.3237i   0.8633 + 0.5047i

Columns 127 through 128

 0.1820 + 0.9833i  -0.6255 + 0.7802i
```

(b). 求$|c(m)^H H_1|$



圖一 $|c(m)^H H_1| - m$圖



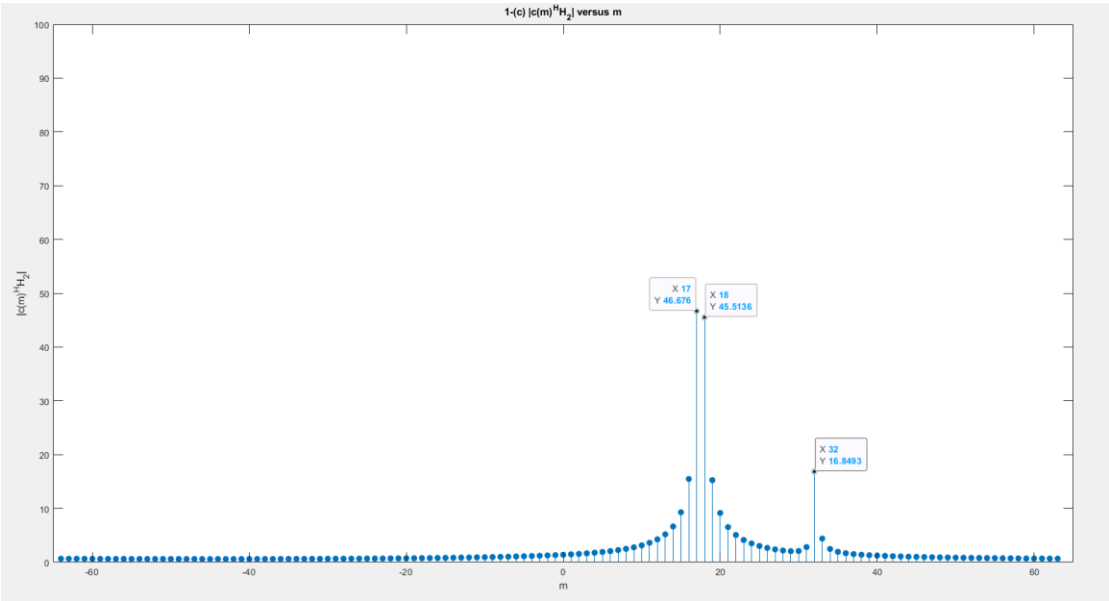
圖二 $|c(m)^H H_1| - m$風花圖

根據圖一，$|c(m)^H H_1|$的最大值發生在 m=17，大小為 82.4283，若將 m=17 換算成角度則為$\varphi = \sin^{-1}\left(\frac{2m}{N}\right) = \sin^{-1}(\frac{2\times17}{128}) = 15.404^o = 0.2688$，與 1-(a)的$\theta_1^r = 0.2679$相近。而圖二的風花圖的最大值也指在$15^o$附近。

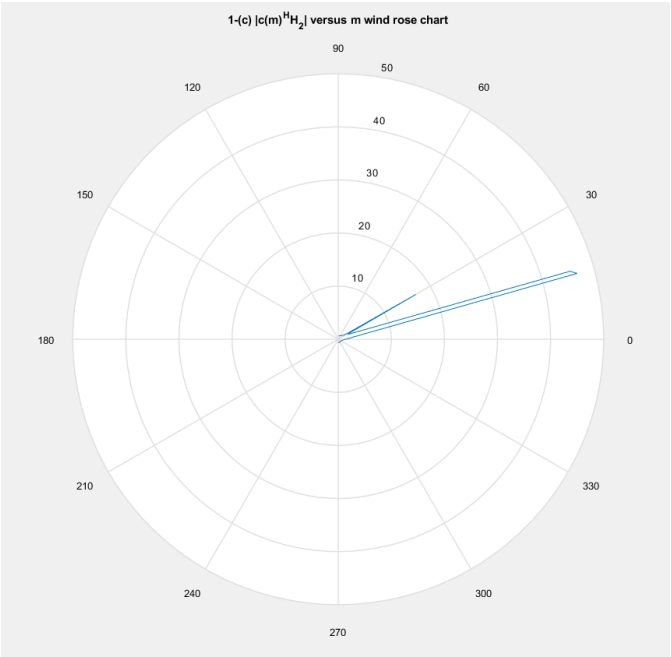
(c). 求H_2、$|c(m)^H H_2|$

```
H2_reshape =

  Columns 1 through 7

   0.7071 + 0.0000i   0.3684 + 0.5697i  -0.2241 + 0.5571i  -0.4739 + 0.1617i  -0.4002 - 0.1586i  -0.2363 - 0.3755i   0.0998 - 0.5193i

  Columns 8 through 14

   0.5537 - 0.2931i   0.6121 + 0.3233i   0.0594 + 0.7022i  -0.5196 + 0.4077i  -0.5517 - 0.1529i  -0.2197 - 0.4210i   0.0783 - 0.4172i

  Columns 15 through 21

   0.3437 - 0.3103i   0.5567 + 0.0350i   0.3588 + 0.5417i  -0.2732 + 0.6463i  -0.6876 + 0.1177i  -0.4405 - 0.4626i   0.0806 - 0.5376i

  Columns 22 through 28

   0.3614 - 0.2731i   0.4264 - 0.0033i   0.3797 + 0.3052i   0.0316 + 0.5855i  -0.5214 + 0.4202i  -0.6717 - 0.2191i  -0.1740 - 0.6636i

  Columns 29 through 35

   0.3998 - 0.4661i   0.5148 + 0.0083i   0.3208 + 0.2958i   0.0851 + 0.4277i  -0.2606 + 0.4433i  -0.6054 + 0.0990i  -0.4763 - 0.4932i

  Columns 36 through 42

   0.1618 - 0.6878i   0.6304 - 0.2272i   0.4842 + 0.3319i   0.0629 + 0.4836i  -0.2253 + 0.3623i  -0.4210 + 0.1659i  -0.5004 - 0.2107i

  Columns 43 through 49

  -0.1662 - 0.6162i   0.4577 - 0.5262i   0.6943 + 0.1024i   0.2765 + 0.5885i  -0.2600 + 0.4942i  -0.4444 + 0.1318i  -0.3968 - 0.1508i

  Columns 50 through 56

  -0.2446 - 0.4064i   0.1562 - 0.5501i   0.6179 - 0.2319i   0.5691 + 0.4155i  -0.0418 + 0.6912i  -0.5387 + 0.3209i  -0.4960 - 0.1853i

  Columns 57 through 63

  -0.1975 - 0.3977i   0.0734 - 0.4239i   0.3843 - 0.3200i   0.5918 + 0.0982i   0.2952 + 0.6104i  -0.3673 + 0.6042i  -0.6785 + 0.0187i
```

```
Columns 64 through 70

-0.3597 - 0.4816i   0.1089 - 0.4894i   0.3442 - 0.2589i   0.4433 - 0.0058i   0.3910 + 0.3550i  -0.0374 + 0.6248i  -0.5940 + 0.3549i

Columns 71 through 77

-0.6310 - 0.3141i  -0.0783 - 0.6562i   0.4180 - 0.3922i   0.4744 + 0.0318i   0.3151 + 0.2845i   0.0856 + 0.4546i  -0.3191 + 0.4567i

Columns 78 through 84

-0.6488 + 0.0250i  -0.4102 - 0.5691i   0.2568 - 0.6489i   0.6247 - 0.1359i   0.4177 + 0.3489i   0.0447 + 0.4511i  -0.2196 + 0.3654i

Columns 85 through 91

-0.4577 + 0.1649i  -0.5160 - 0.2772i  -0.0881 - 0.6634i   0.5361 - 0.4601i   0.6577 + 0.1962i   0.1905 + 0.5844i  -0.2754 + 0.4358i

Columns 92 through 98

-0.4200 + 0.1196i  -0.4090 - 0.1503i  -0.2425 - 0.4527i   0.2299 - 0.5681i   0.6686 - 0.1507i   0.5037 + 0.4956i  -0.1335 + 0.6571i

Columns 99 through 105

-0.5358 + 0.2412i  -0.4462 - 0.1984i  -0.1916 - 0.3813i   0.0778 - 0.4453i   0.4398 - 0.3173i   0.6123 + 0.1782i   0.2118 + 0.6643i

Columns 106 through 112

-0.4483 + 0.5402i  -0.6470 - 0.0697i  -0.2871 - 0.4797i   0.1192 - 0.4485i   0.3356 - 0.2599i   0.4738 + 0.0029i   0.3881 + 0.4191i

Columns 113 through 119

-0.1229 + 0.6479i  -0.6506 + 0.2704i  -0.5691 - 0.3950i   0.0059 - 0.6276i   0.4169 - 0.3275i   0.4427 + 0.0389i   0.3233 + 0.2836i

Columns 120 through 126

 0.0730 + 0.4941i  -0.3912 + 0.4540i  -0.6745 - 0.0650i  -0.3254 - 0.6278i   0.3367 - 0.5897i   0.5990 - 0.0569i   0.3616 + 0.3482i

Columns 127 through 128

 0.0414 + 0.4289i  -0.2259 + 0.3810i
```



圖三 $|c(m)^H H_2| - m$圖



圖四 $|c(m)^H H_2| - m$風花圖

(d). Comment the result in (b) & (c).

根據圖三，$|c(m)^H H_2|$的最大值發生在 m=17，大小為 46.676，第二大值發生在 m=32，大小為 16.8493，若將其換算成角度則為$\varphi_1 = \sin^{-1}(\frac{2\times17}{128}) = 15.404^o = 0.2688$、$\varphi_2 = \sin^{-1}(\frac{2\times32}{128}) = 30^o = 0.523$，分別與$\theta_1^r = 0.2679$和$\theta_2^r = \theta_1^r + 0.25 = 0.5179$都非常相近，風花圖的指向也指在$15^o$、$30^o$。(b)與(c)相比，(c)在接收端多了$\theta_2^r$，因此在兩張的作圖上可以發現$H_2$在會多出一個指向角度 m=32、$\varphi = 30^o$，讓$H_2$除了在$15.404^o$可以利用外，多出了$30^o$可以利用。

2.

$$y_1 = A\cos(2\pi f_c t + \theta_k + \phi)\cdot\cos(2\pi f_c t)$$
$$= \frac{A}{2}\left(\cos(4\pi f_c t + \theta_k + \phi) + \cos(\theta_k + \phi)\right)$$

$$y_2 = A\cos(2\pi f_c t + \theta_k + \phi)\cdot\cos(2\pi f_c t + \frac{\pi}{4})$$
$$= \frac{A}{2}\left(\cos(4\pi f_c t + \theta_k + \phi + \frac{\pi}{4}) + \cos(\theta_k + \phi - \frac{\pi}{4})\right)$$

$$y_3 = A\cos(2\pi f_c t + \theta_k + \phi)\cdot\cos(2\pi f_c t + \frac{\pi}{2})$$
$$= \frac{A}{2}\left(\cos(4\pi f_c t + \theta_k + \phi + \frac{\pi}{2}) + \cos(\theta_k + \phi - \frac{\pi}{2})\right)$$

$$y_4 = A\cos(2\pi f_c t + \theta_k + \phi)\cdot\cos(2\pi f_c t + \frac{3\pi}{4})$$
$$= \frac{A}{2}\left(\cos(4\pi f_c t + \theta_k + \phi + \frac{3\pi}{4}) + \cos(\theta_k + \phi - \frac{3\pi}{4})\right)$$

$$\Rightarrow z_1 = LPF\{y_1\} = \frac{A}{2}\cos(\theta_k + \phi)$$

$$z_2 = LPF\{y_2\} = \frac{A}{2}\cos(\theta_k + \phi - \frac{\pi}{4})$$

$$z_3 = LPF\{y_3\} = \frac{A}{2}\cos(\theta_k + \phi - \frac{\pi}{2})$$

$$z_4 = LPF\{y_4\} = \frac{A}{2}\cos(\theta_k + \phi - \frac{3\pi}{4})$$

$$\Rightarrow Z = z_1 z_2 z_3 z_4$$

$$= \frac{A^4}{16}\cos(\theta_k+\phi)\cos(\theta_k+\phi-\tfrac{\pi}{4})\cos(\theta_k+\phi-\tfrac{\pi}{2})\cos(\theta_k+\phi-\tfrac{3\pi}{4})$$

$$= \frac{A^4}{16}\tfrac{1}{2}\left[\cos(2\theta_k+2\phi-\tfrac{\pi}{2}) + \cos(\tfrac{\pi}{2})^{0}\right]$$

$$\times \tfrac{1}{2}\left[\cos(2\theta_k+2\phi-\pi) + \cos(\tfrac{\pi}{2})^{0}\right]$$

$$= \frac{A^4}{64}\cos(2\theta_k+2\phi-\tfrac{\pi}{2})\cos(2\theta_k+2\phi-\pi)$$

$$= \frac{A^4}{64}\cdot\tfrac{1}{2}\left(\cos(4\theta_k+4\phi-\tfrac{3\pi}{2}) + \cos\tfrac{\pi}{2}\right)$$

$$= \frac{A^4}{128}\cos(4\theta_k+4\phi-\tfrac{3\pi}{2}) = \frac{A^4}{128}\cos(4\theta_k+4\phi+\tfrac{\pi}{2})$$

$$= \frac{A^4}{128}\left(\cos(4\theta_k+4\phi)\cos\tfrac{\pi}{2} - \sin(4\theta_k+4\phi)\sin\tfrac{\pi}{2}\right)$$

$$= -\frac{A^4}{128}\sin(4\theta_k+4\phi)$$

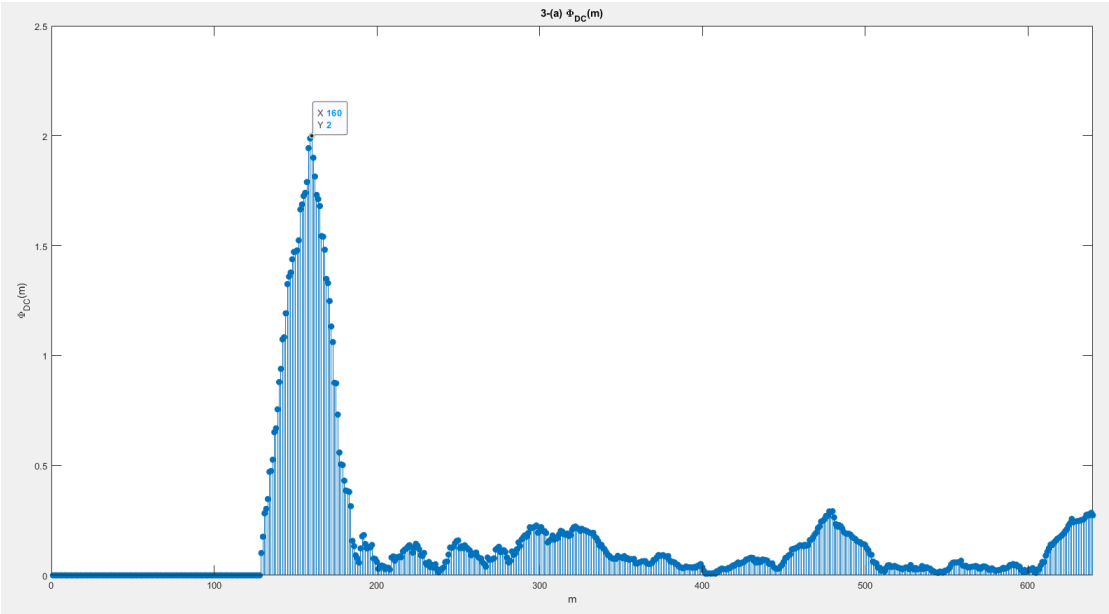
$$= -\frac{A^4}{128}\sin(k\pi+4\phi)\ ,\ k\in\{\pm1,\pm3\}$$

$$= -\frac{A^4}{128}\sin(4\phi+\pi)$$

$$= \frac{A^4}{128}\sin(4\phi)$$ #

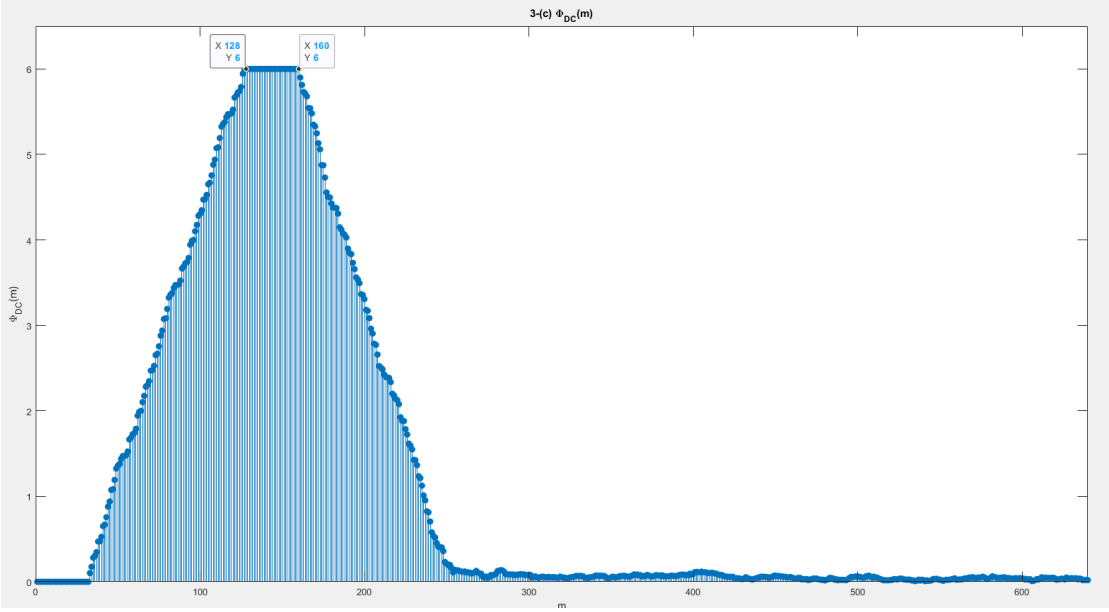
3.

(a). Draw the madnitude of delay and correlate $\Phi_{DC}(m)$ $R = 32$ $L = 128$



(b). $\hat{m}_{DC} = 160$

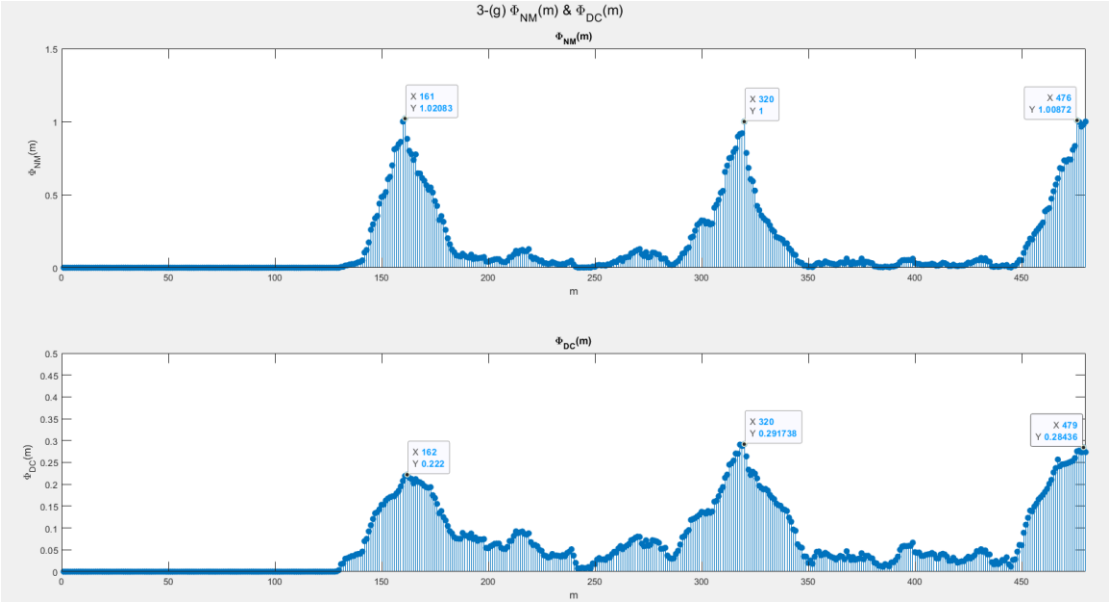
(c). Draw the madnitude of delay and correlate $\Phi_{DC}(m)$ $R = 96$ $L = 32$



(d). $\hat{m}_{DC} = 128\sim160$

(e). R 是取樣區間長度，L 是延遲訊號的延遲長度，R 越小容易造成原本訊號與延遲訊號重複性高導致平台的發生，但平台的發生也要考慮到 L 延遲的大小，若 L 越小 R 沒有隨之提高就越容易造成平台的發生，R 太長則容易有 ISI 取樣到下一個 symbol，因此在選擇 R、L 時，讓 R=N_g、L=N 是最好的選擇，讓Φ_{DC}不會取樣太超過，也不會有平台的產生。

(f). Draw $\Phi_{DC}(m)$ & $\Phi_{NM}(m)$



$\hat{m}_{DC} = 320$

$\hat{m}_{NM} = 161、320、476$

Normalized 後的Φ讓原本大小不一的三個峰值都 normalize 成接近 1 的值，理論上峰值都要是 1，而得出的三個峰值也非常接近 1。