# Homework 4, 12/21/2021

## Due Jan/4/2022 9:00AM, to new E3

1. (40%) Find the shortest encoded bit string of the following source sequence:
   Source sequence: NCTUNCTUNYCUNCTUNYCU
   (a) (10%) Calculate the entropy of the source sequence.
   (b) (10%) Use Huffman coding to encode the string. You first calculate probability distribution, develop a binary tree for it, and perform the encoding.
   (c) (10%) Use extended Huffman coding to encode the string where k=2. You first calculate probability distribution, develop a binary tree for it, and perform the encoding.
   (d) (10%) Using arithmetic coding. For this case, you can ignore the termination issue in a decoder, and output the shortest bit string of the first 2 symbols only, while the probability distribution is based on the whole sequence.

2. (30%) For the 4x4 pixel values given below, calculate the first level of the 2D Haar wavelet transform and the second level of the 2D Haar wavelet transform on the LL band of the first-level transformed coefficients. Write the details of the procedures and the final 4x4 output.

| | | | |
|---|---|---|---|
| 120 | 20 | 20 | 15 |
| 40 | 30 | 08 | 12 |
| 30 | 20 | 84 | 126 |
| 20 | 20 | 120 | 110 |

3. (30%) For the final 4x4 output of Question 2, what are coded significance map and the outputs of the subordinate passes for the first round of EZW coding?

1. (40%) Find the shortest encoded bit string of the following source sequence:
   Source sequence: NCTUNCTUNYCUNCTUNYCU
   (a) (10%) Calculate the entropy of the source sequence.
   (b) (10%) Use Huffman coding to encode the string. You first calculate probability distribution, develop a binary tree for it, and perform the encoding.
   (c) (10%) Use extended Huffman coding to encode the string where k=2. You first calculate probability distribution, develop a binary tree for it, and perform the encoding.
   (d) (10%) Using arithmetic coding. For this case, you can ignore the termination issue in a decoder, and output the shortest bit string of the first 2 symbols only, while the probability distribution is based on the whole sequence.

1. (a)

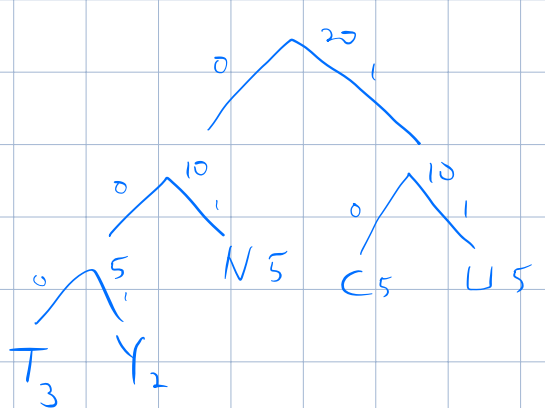
| | N | C | T | U | Y |
|---|---|---|---|---|---|
| 个数 | 5 | 5 | 3 | 5 | 2 |
| P | 0.25 | 0.25 | 0.15 | 0.25 | 0.1 |

$$\sum_{i=1}^{5} p_i \log_2 \frac{1}{p_i} = \left(\frac{1}{4}\cdot \log_2 4\right)\cdot 3 + \frac{3}{20}\log_2 \frac{20}{3} + \frac{1}{10}\log_2 10$$

$$= \frac{3}{2} + \frac{3}{20}\log_2 \frac{20}{3} + \frac{1}{10}\log_2 10 \quad \#$$

(b)

| | N | C | T | U | Y |
|---|---|---|---|---|---|
| 个数 | 5 | 5 | 3 | 5 | 2 |
| P | 0.25 | 0.25 | 0.15 | 0.25 | 0.1 |



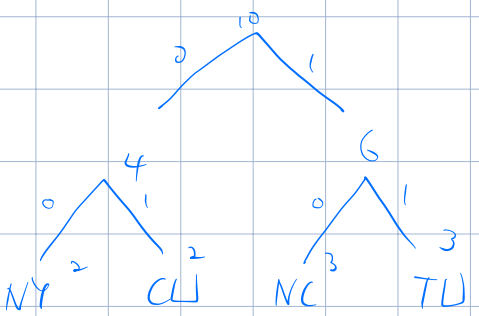

N : 01
C : 10
T : 000
U : 11
Y : 001

encode string:

01 10 000 11 01 10 000 11 01 001 10 11 01 10 000 11 01 001 10 11 #

(c)

| | NC | TU | NY | CU |
|---|---|---|---|---|
| 个数 | 3 | 3 | 2 | 2 |
| P | 0.3 | 0.3 | 0.2 | 0.2 |




NC: 10

TU: 11

NY: 00

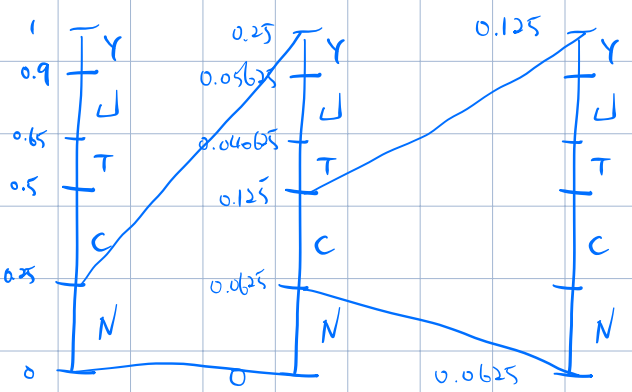
CU: 01

encode string:

10 11 10 11 00 01 10 11 00 01 #

(d)

| | N | C | T | U | Y |
|---|---|---|---|---|---|
| 个数 | 5 | 5 | 3 | 5 | 2 |
| P | 0.25 | 0.25 | 0.15 | 0.25 | 0.1 |




$0.1_{(2)} = 0.5 > 0.0625$

$\rightarrow 0.01_{(2)} = 0.25 > 0.0625$

$\rightarrow 0.001_{(2)} = 0.125 > 0.0625$

$\rightarrow 0.0001_{(2)} = 0.0625 \leq 0.0625$

$\Rightarrow$ 0.0001 #

2. (30%) For the 4x4 pixel values given below, calculate the first level of the 2D Haar wavelet transform and the second level of the 2D Haar wavelet transform on the LL band of the first-level transformed coefficients. Write the details of the procedures and the final 4x4 output.

| | | | |
|---|---|---|---|
| 120 | 20 | 20 | 15 |
| 40 | 30 | 08 | 12 |
| 30 | 20 | 84 | 126 |
| 20 | 20 | 120 | 110 |

2.

| | | | |
|---|---|---|---|
| 70 | 17.5 | 50 | 2.5 |
| 35 | 10 | 5 | -2 |
| 25 | 105 | 5 | -21 |
| 20 | 115 | 0 | 5 |

1st row 运算

| | | | |
|---|---|---|---|
| 52.5 | 13.75 | 2.75 | 0.25 |
| 22.5 | 110 | 2.5 | -8 |
| 17.5 | 3.75 | 22.5 | 2.25 |
| 2.5 | -5 | 2.5 | -13 |

1st col 运算

| | | | |
|---|---|---|---|
| 33.125 | 19.375 | 2.75 | 0.25 |
| 66.25 | -43.75 | 2.5 | -8 |
| 17.5 | 3.75 | 22.5 | 2.25 |
| 2.5 | -5 | 2.5 | -13 |

2nd row 运算

| | | | |
|---|---|---|---|
| 49.6875 | -12.1875 | 2.75 | 0.25 |
| -16.5625 | 31.5625 | 2.5 | -8 |
| 17.5 | 3.75 | 22.5 | 2.25 |
| 2.5 | -5 | 2.5 | -13 |

2nd col 运算

final output

3. (30%) For the final 4x4 output of Question 2, what are coded significance map and the outputs of the subordinate passes for the first round of EZW coding?

3.

| | | | |
|---|---|---|---|
| 49.6875 | -12.1875 | 2.75 | 0.25 |
| -16.5625 | 31.5625 | 2.5 | -8 |
| 17.5 | 3.75 | 22.5 | 2.25 |
| 2.5 | -5 | 2.5 | -13 |

| subband | coefficient value | symbol | reconstructed value |
|---|---|---|---|
| LL2 | 49.6875 | P | 48 |
| HL2 | -12.1875 | ZTR | 0 |
| LH2 | -16.5625 | ZTR | 0 |
| HH2 | 31.5625 | ZTR | 0 |

$T_0 = \frac{64}{2} = 32$


⇒ subordinate output : 1

H