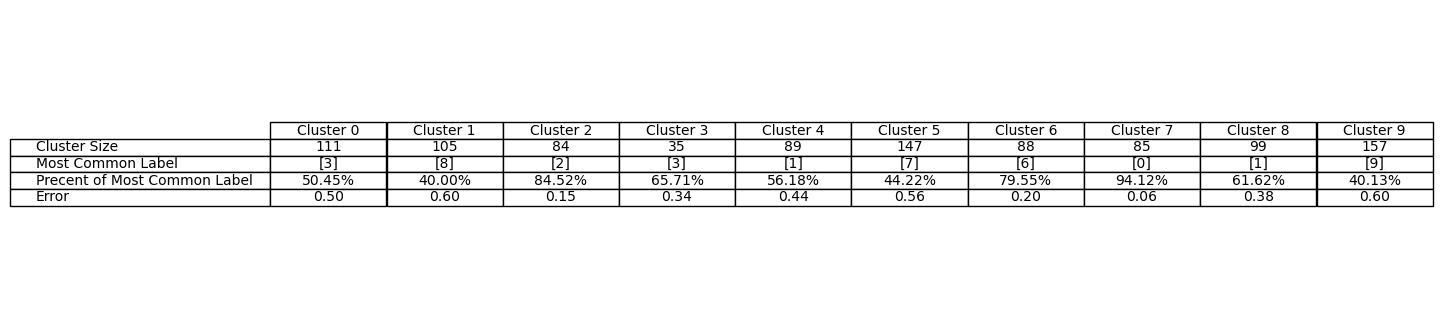
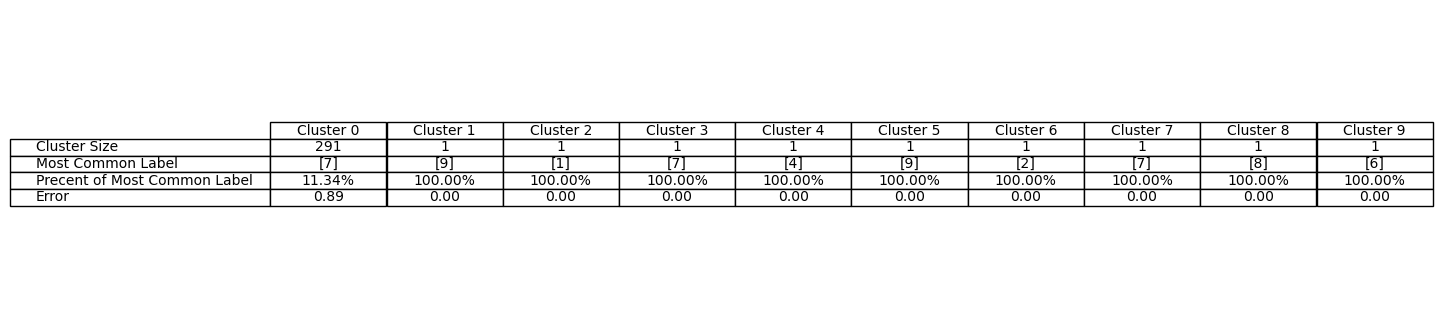
1.

(c)

The error on cluster is calculated by taking the number of wrong labels in the cluster and dividing it by the cluster size

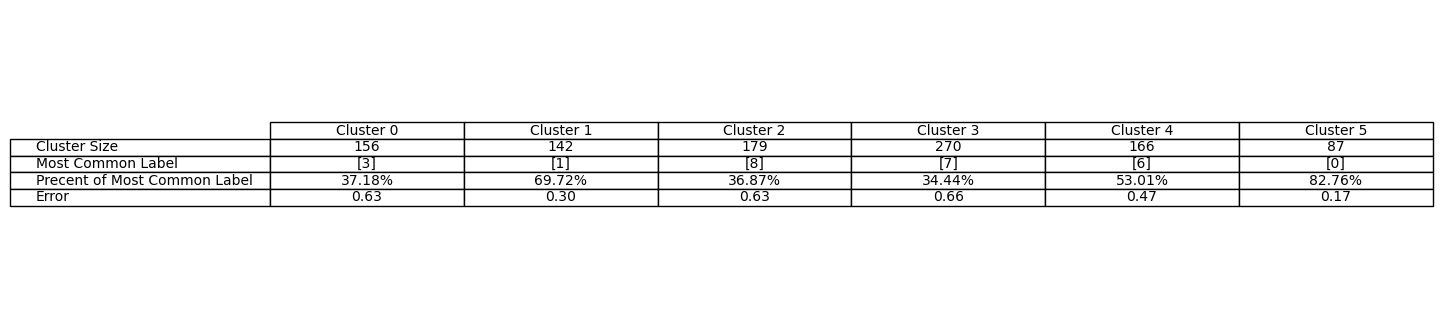
By this calculation we can deduct that the algorithm classified correctly wrong about 419 samples, therefore the error is

(d)

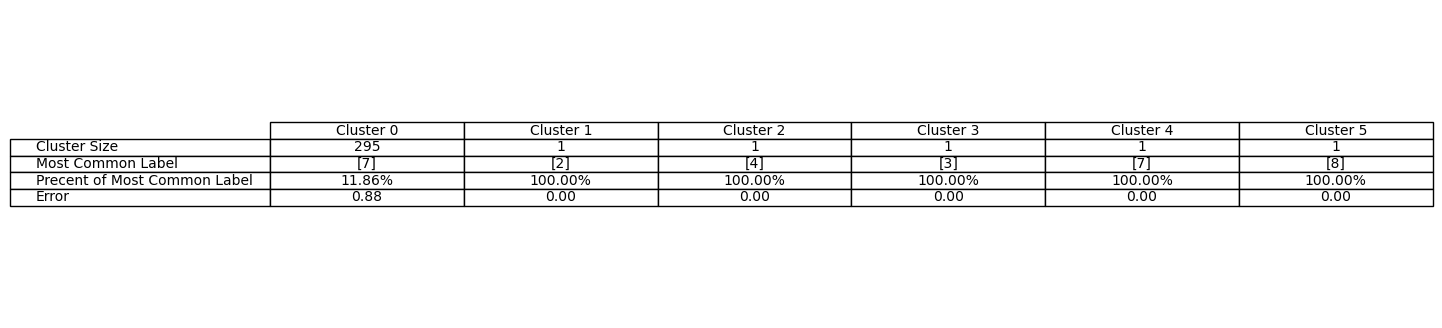
The same way we calculated the error before, now we get that the error is

Clearly k-means worked better for this problem.

(e)

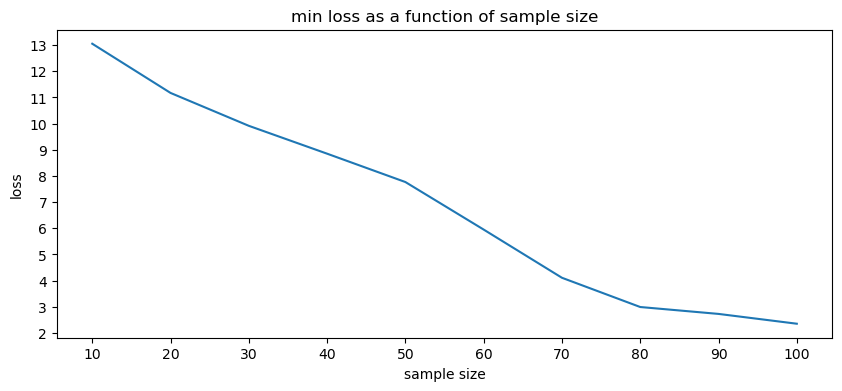
Kmeans:

Error:

Single-linkage

Error:

We can see that the k means algorithm got better results for k=10. It makes sense since we know the data is originally divided into 10 sets, each represent a digit.

2.(a)

(b)

As the training set grows, we expected the loss on the distribution to reduce. Based on what we’ve seen in class: . we used the that minimizes and as grows, gets smaller.

(c) yes, we can see it in the plot.

(d)

We saw in class that the bays-optimal regressor for **squared loss** is .

bays-optimal regressor for **absolute loss** is

And since is a scalar, not random variable:

Where the last equality is based on the fact that and median of gaussian random variable is its mean.

3.(a)

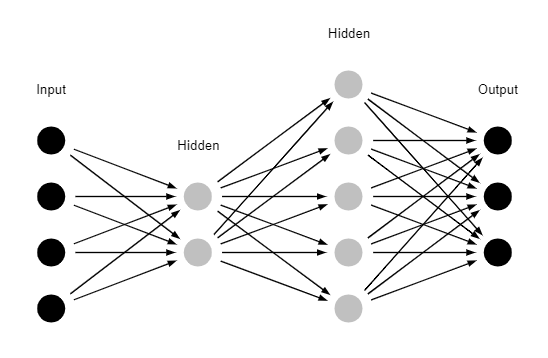
According to GD algorithm:

Let

we will find

Hence,

(b)

4.(a)

(b).

Since the input layer has 4 neurons,

(c)

Since ,

(d)

Then

5.(a)

For each sample there are attributes, and we can test 3 options for , hence there are options for inner vertex (+2 for label). In a tree of depth there are at most nodes, thus

(b)

Danny is wrong, since ID3 is heuristic algorithm and not ERM it does not guarantee minimal error. After performing pruning, we can’t guarantee we will get minimal error. Thus PAC assumption on the sample size does not apply.

6.(a)

The assumption **does not hold**.

Naïve bayes assumption is

But in the given distribution,

and

And

Which means for some

(b)

From previous section, we know we can’t use the naïve bayes assumption.

Hence

For we get:

for the predictor will return

For we get:

for the predictor will return

For we get:

for the predictor will return

For we get:

for the predictor will return

Therefore ,

7.(a)

From the definition of we can see that there’s a linear dependence between the first and second coordinates to the third and fourth coordinates, hence the degree of is 2 and therefore the degree of is 2 as well. Since and the degree of is 2, therefore it has 2 eigenvalues 0.

it has kernel of dimension 2, and from the dimension theorem (

we know that the eigenvalues of are non-negative. thus, the 2 smallest eigenvalues of are both 0 and the distortion is

(b)

Let ,

Note that all satisfy

For

After performing gaussian elimination on we get

We got all the eigenvalues are 1, therefore the distortion is sum of positives and .

8.(a)

Let (there’s a mistake in the question for the definition of so we defined as followed) and

(a)

Let

Thus we can represent as:

We define for :

Thus

=

Thus,

]

And

Now, we’ll demand the derivative to be 0 to find maximum value