Probability 201-1-2391 ASSIGNMENT 6 -answers
One dimensional distributions and random variables
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Problem 1

A balanced dice is thrown twice. Let X denote the maximal of the two results. Find the distribution function of X.

$$F(k) = \begin{cases} 0 & , \quad k < 1 \\ (k/6)^2 & , \quad 1 \le k \le 6 \\ 1 & , \quad 6 < k \end{cases}$$

Problem 2

A balanced dice is thrown 5 times. Let X be the number of times that a number smaller than 3 had shown up.
a) Find the distribution function of X.

$$p_5(m) = \begin{pmatrix} 5 \\ m \end{pmatrix} \left(\frac{1}{3}\right)^m \left(\frac{2}{3}\right)^{5-m}, \quad 0 \le m \le 5.$$

b) Find the distribution function of $Y = 2X - 1$.

$$p(Y = k) = \begin{pmatrix} 5 \\ (k+1)/2 \end{pmatrix} \left(\frac{1}{3}\right)^{(k+1)/2} \left(\frac{2}{3}\right)^{(9-k)/2}$$

Problem 3

An athlete gets 5 opportunities to jump over the bar. If he succeeds on a certain jump he does not try again. If he fails all the 5 chances then he is out. A certain jumper tries to jump over 2.15 meters and it is known that the probability that he passes the bar is r. Assuming that the jumps are independent, find the distribution of X - the number of jumps that he will do.

$p(\text{succeeds in } j) = (1-r)^{j-1} \cdot r, \;\; j = 1, 2, 3, 4.\, p(\text{don't care about } 5) = (1-r)^4$.Else 0.

Problem 4

A jar contains 15 black balls and 30 white balls. The balls are randomly being pulled out of the jar till the first white ball is pulled out. Let X be the number of the balls pulled out. Compute the distribution of X.

$$p(X = k) = \frac{15!(45-k)! \cdot 30}{45!(16-k)!}.$$

Problem 5

Same as in Problem 4 above, but balls are being pulled out till the second white ball is pulled out.

$$p(X = k) = \left\{ \binom{30}{1} \binom{15}{k-2} \cdot 29 \right\} / \left\{ \binom{45}{k-1} \cdot (46-k) \right\}.$$

Problem 6

Let X be a random variable that assumes all the non-negative integral values. We are given that for any positive integer: $k \cdot p(X = k) = 10 \cdot p(X = k-1)$. Find the distribution of X.

$$p(X = k) = \frac{10^k}{k!} e^{-10}.$$

Problem 7

A jar contains a white balls and b black balls. Two opponents pull out (with returning the balls back) balls one by one. The first who pulls out a white ball, wins.
a) What is the probability that the first player wins?

$$p(\text{The first player wins}) = \frac{a+b}{a+2b}.$$

b) What is the distribution of the number of pull outs till the determination of the winner (inclusive)?

$$p(X = k) = \left(\frac{b}{a+b}\right)^{k-1} \left(\frac{a}{a+b}\right).$$

Problem 8

A robot is located at point 0 on the real axis. Then it forms N steps where in each step (which is independent of the others) it moves rightwards once with probability p, and leftwards once with probability $q = 1 - p$. Let X be the number on the real axis at which the robot arrives after N steps. What is the distribution of X?

$$p(X = k) = \binom{N}{(k+N)/2} p^{(k+N)/2} q^{(N-k)/2}.$$

Problem 9

There are 20 notes in a hat that are numbered with the numbers $1, 2, 3, \ldots, 20$. One number is written on each note.You pull out the notes one by one till you reach 13. Let X be the number of notes pulled out. What is the distribution of X?

$$p(X = k) = \frac{1}{20}.$$