Probability 201-1-2391 ASSIGNMENT 1-answers
Combinatorial Analysis

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Problem 1

15 cubes are ordered in a row. 5 cubes are red, 7 are white and 3 are green. How many different arrangements are there?

$$\frac{15!}{5!7!3!} \text{ or } \binom{15}{5}\binom{10}{7}\binom{3}{3}$$

Problem 2

In how many ways can we arrange n people in a row so that:
a) John and Jane stand next to one another?

$$2(n-1)!$$

b) John, Jane and Mary stand together?

$$3!(n-2)!$$

Problem 3

How many different 4 digits numbers can we produce from the following leggo units: $8, 8, 6, 5, 3, 2$?

$$\binom{4}{2} \cdot 4 \cdot 3 + 5!$$

Problem 4

In how many ways can a group of $2n$ people be divided into n pairs?

$$\frac{(2n)!}{n!2^n} \text{ or } \frac{1}{n!}\binom{2n}{2}\binom{2n-2}{2}\cdots\binom{2}{2}$$

Problem 5

A box contains 50 bulbs, 15 of which are defected. A person pulls out randomly 3 bulbs from the box. How many different ways are there in which that person will have at least one non-defected bulb?

$$\binom{50}{3} - \binom{15}{3} \text{ or } \binom{35}{3}\binom{15}{0} + \binom{35}{2}\binom{15}{1} + \binom{35}{1}\binom{15}{2}$$

Problem 6

A person gets 13 cards from a deck of cards. How many different possibilities he has, so that his 13 cards are: 5 hearts, 4 diamonds, 2 spades and 2 leaves?

$$\binom{13}{5}\binom{13}{4}\binom{13}{2}\binom{13}{2}$$

Problem 7

In a lost ship there are 20 children. Those children do not remember their birthdays and would like to be assigned birthdays.
a) In how many ways this can be done so that exactly 2 children will get identical day, and the other 18 will get each a separate day?

$$\binom{20}{2} \cdot \frac{365!}{346!}$$

b) What is the number of possibilities to assign, so that at least one day of the year will belong to at least 2 children?

$$365^{20} - \frac{365!}{345!}$$

Problem 8

a) 12 children want to march in a row. In how many ways this can be done so that between John and Jeff there will be exactly 5 children?

$$6 \cdot 2 \cdot 10!$$

b) The same question, but this time the children want to dance in a circle?

$$10!$$

Problem 9

a) How many different "words" could be created using all the characters of ABRACADABRA?

$$\frac{11!}{5!2!2!} \text{ or } \binom{11}{5}\binom{6}{2}\binom{4}{2}\binom{2}{1}\binom{1}{1}$$

b) In how many of these there are no 2 A's one next to the other?

$$\frac{6!}{2!2!}\cdot\binom{7}{5}$$

c) In how many of the "words" in a) there are no 2 identical characters one next to the other?

$$\frac{6!}{2!2!}\cdot\binom{7}{5}-\frac{5!}{2!}\binom{6}{5}-\frac{5!}{2!}\binom{6}{5}+4!\cdot\binom{5}{5}$$

Problem 10

How many different solutions are there for the equation $X_1+X_2+\ldots+X_k=n$, where:

a) For each i, $X_i \geq 0$ is integral.

$$\binom{n+k-1}{n}=\binom{n+k-1}{k-1}$$

b) For each i, $X_i=1$ or $X_i=2$ $(k \leq n)$.

$$\binom{k}{n-k}$$

c) For each i, $X_i \geq 0$ is integral and there are exactly 4 X_i's which equal 0?

$$\binom{k}{4}\binom{n-1}{n-k+4}$$

Problem 11

Prove the identity:

$$\sum_{k=0}^{n}\binom{n}{k}=2^n.$$

Problem 12

Prove the identity:

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}.$$

Problem 13

Prove the identity:

$$\binom{n}{n} + \binom{n+1}{n} + \ldots + \binom{N}{n} = \binom{N+1}{n+1}, \quad (N \geq n).$$

Problem 14

Prove the identity:

$$\binom{n+m}{k} = \sum_{i=0}^{k} \binom{n}{i} \binom{m}{k-i}.$$

Proof. The left hand side is the total number of ways to choose a committee of k out of a group of $n+m$ people. This is also the number on the right hand side, for if we split our group of people into two sub-groups, one of size n and a second of size m then

$$\binom{n}{i} \binom{m}{k-i}$$

is the number of committees of size k, which have exactly i members from the first sub-group.

Problem 15

Prove the identity:

$$\binom{n}{0} + \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} + \ldots + \frac{1}{n+1}\binom{n}{n} = \frac{2^{n+1}-1}{n+1}.$$

Problem 16

n identical balls are distributed in m different cells.
a) In how many of these distributions there will be exactly k empty cells?

$$\binom{m}{k} \cdot \binom{n-1}{n-m+k}$$

b) Prove the following identity:

$$\sum_{k=0}^{m-1} \binom{m}{k} \binom{n-1}{n-m+k} = \binom{n+m-1}{n}.$$