Probability 201-1-2391 ASSIGNMENT 5-answers
Asymptotic laws: The theorems of DeMoivre, Laplace, Bernoulli and Poisson
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Problem 1

Give an estimate, using normal approximation, to the probability that in a 1000 tosses of a coin with a probability 0.2 for an H, we will get at least 180 H's.

$$p(N(0,1) \geq \frac{179.5-200}{\sqrt{160}}) = 1-\Phi(\frac{-20.5}{\sqrt{160}}) = 0.9747.$$

Problem 2

A balanced coin is tossed 300 times. Compute the probability that it will show 140 times 'H':
(a) Using the Stirling asymptotic formula, and

$$p = \left(\begin{array}{c} 300 \\ 140 \end{array}\right)\left(\frac{1}{2}\right)^{300} \approx \frac{300^{300}}{140^{140}\cdot 160^{160}}\sqrt{\frac{300}{2\pi\cdot 140\cdot 160}}\left(\frac{1}{2}\right)^{300}$$

(b) Assuming the normal distribution.
What is the relative deviation between the two methods?

$$\text{By DeMoivre } p \approx \frac{1}{\sqrt{75}}\phi\left(\frac{140-150}{\sqrt{75}}\right) = \frac{1}{\sqrt{75}\sqrt{2\pi}}e^{-(-10/\sqrt{75})^2/2}.$$

Problem 3

We do 15000 independent trials with the probability 1/3 for a success. Compute the probability that the number of successes is between 4950 and 5050.

$$n = 15000,\ p = \frac{1}{3},\ q = \frac{2}{3},\ \mu = np = 5000,\ \sigma = \sqrt{npq} = \frac{100}{\sqrt{3}}$$

$$a = 4950,\ b = 5050,\ X_a = \frac{4950-5000}{100/\sqrt{3}},\ X_b = \frac{5050-5000}{100/\sqrt{3}}$$

So, answer (using Laplace Theorem): $p(4950 \leq m \leq 5050) \approx \Phi(X_b) - \Phi(X_a)$.

Problem 4

A balanced die is thrown 24000 times one after the other. The face 5 appeared in m of these throws. Estimate the probability of the event $3900 \leq m \leq 4050$.
Use Laplace as in Problem 3.

Problem 5

Find the smallest natural number x that satisfies the condition: If in k out of 10000 deliveries male babies were born then

$$P(5000 - x \leq k \leq 5000 + x) \geq 0.95.$$

$$n = 10000,\ p = q = \frac{1}{2},\ \mu = np = 5000,\ \sigma = \sqrt{npq} = 50.$$

$$\text{By Laplace}: p(5000 - x \leq k \leq 5000 + x) \approx \Phi(X_{5000+x}) - \Phi(X_{5000-x}).$$

$$\text{We have}: X_{5000-x} = \frac{(5000 - x) - 5000}{50} = -\frac{x}{50},\ X_{5000+x} = \frac{x}{50}.$$

$$\text{SO}: 2\Phi\left(\frac{x}{50}\right) - 1 = 0.95,\ \text{hence}: x = 50 \times \Phi^{-1}\left(\frac{1.95}{2}\right)$$

Problem 6

Find the probability that in a group of 600 people there will be m birthdays on the New Years day.

Problem 7

In a certain typing process the probability for a typo is 0.001. Each line contains 50 characters. Compute the probability for two typos to occur
(a) Using the exact distribution.
(b) Using Poisson's approximation.
What is the relative deviation between the two results?

Problem 8

There is 1 percent of sick people in a population. Estimate the probability that among 200 of the people we will find at least 4 sick.

Problem 9

A certain population has 0.5 percents color blinds. We choose out of the population n people in a blind sampling (a person might be chosen more than once). Find the minimal value of n if the probability to find a color blind man in this sample is at least 0.95.