Probability 201-1-2391 ASSIGNMENT 3
The Borel system
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Problem 1

We defined a field of events β as a family $\beta \subseteq 2^{\Omega}$ that satisfies the following 3 axioms:
I. $\Omega \subseteq \beta$.
II. If $A, B \in \beta$ then $A - B \in \beta$.
III. If $A_1, A_2, \ldots \in \beta$ is countable then the union $\bigcup_{n=1}^{\infty} A_n \in \beta$

Let us state the following axiom:
II'. If $A \in \beta$, then $A^c \in \beta$ where A^c is the complimentary set of A.
Prove that the three axioms I,II,III are equivalent to I,II',III.

Problem 2

Using the definition we gave to a Borel system (Ω, β, p), prove (using the axioms):
1. $p(\emptyset) = 0$.
2. If $A, B \in \beta$ satisfy $A \subseteq B$ then $p(A) \leq p(B)$.

Problem 3

Give an example of a probability space (Ω, β, p) such there are $A, B \in \beta$, $A \neq \emptyset$ and $B \neq \Omega$, but $p(A) = 0$ and $p(B) = 1$.

Problem 4

Give an example of a probability space (Ω, β, p) such that Ω is an infinite sample space but $|\beta| < \infty$, or prove that no such a probability space can exist.

Problem 5

Give an example of a probability space (Ω, β, p) such that $|\beta| = 3$ <u>or</u> prove that no such a probability space can exist.

Problem 6

Give an example of a probability space (Ω, β, p) such that $|\beta| = 4$ <u>or</u> prove that no such a probability space can exist.

Problem 7

Give an example of a probability space (Ω, β, p) such that $|\beta|$ is an odd integer <u>or</u> prove that no such a probability space can exist.