Probability 201-1-2391 ASSIGNMENT 2
The elementary probability space
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Problem 1

a) Prove the following distributive laws:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C), \quad A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

b) Prove that complementing a set is an involution: $(A^c)^c = A$. Here $A^c = X - A$ and $A \subseteq X$.
c) Prove that: $(A \cup B)^c = A^c \cap B^c$, $(A \cap B)^c = A^c \cup B^c$.

Problem 2

Which is the greatest? :
a) The probability to obtain at least one 6 in a throw of 4 dice.
b) The probability to obtain at least once $(6, 6)$ in 24 throws of 2 dice (in each throw).

Problem 3

Here is a solution to Problem 2 above: When we throw one dice there are 6 possible outcomes of which only one is a "success" (getting 6). When we throw 2 (a pair) dice there are 36 possible outcomes of which only one is a "success" (getting $(6, 6)$). So for the probability of success to be equal in the two spaces the number of throws of 2 dice should be 6 times as large as that of throwing just one, i.e. $4 \times 6 = 24$. Hence the answer to problem 2 is that the 2 probabilities are equal.
Discuss this argument and express your opinion.

Problem 4

a) A person has 2 match boxes, each containing n matches. When he needs a match he pulls one randomly out of one of his 2 boxes. When he finds out that one of his boxes in empty, what is the probability that his other box contains exactly k matches? Here $0 \leq k \leq n$.
b) Prove the identity:

$$\sum_{k=0}^{n} \frac{1}{2^{2n-k}} \left(\begin{array}{c} 2n-k \\ n \end{array} \right) = 1.$$

Problem 5

k married couples (pairs) arrived at a party. An even number m of the $2k$ people were chosen at random. Compute the probability that:
a) Among the chosen ones there is no married couple.
b) Among the chosen ones there is exactly one married couple.
c) Among the chosen ones there are exactly 4 married couples, assuming of course that $m \geq 8$.
d) All the chosen ones are $m/2$ married couples.

Problem 6

5 dice are thrown. What is the probability that at least two will show identical results?

Problem 7

What is the probability that in distributing n (distinguishable) balls in m cells, cell number 1 will contain exactly k of the balls?

Problem 8

4 balls are randomly distributed into 3 cells (3^4 possibilities of equal probability). We define the events A to G below:
A - cell number 1 is empty, B - only cell number 1 is empty, C - there is exactly one empty cell, D - there is an empty cell, E - there is a non-empty cell, F - cell number 3 contains 2 balls, G - there is a cell that contains exactly 2 balls.
Compute the probabilities of these events, $p_A, p_B, p_C, p_D, p_E, p_F$ and p_G.

Problem 9

A jar contains 10 balls numbered: $1, 2, 3, \ldots, 10$. 5 balls are pulled out randomly. What is the probability that ball number 7 is the second in size out of the balls that were pulled out?

Problem 10

k dice are thrown. Compute the probabilities of the following two events:
A - the largest number that occurred is i,
B - the smallest number that occurred is i.

Problem 11

4 married couples arrived at a party. The 8 people were divided randomly into pairs.
a) What is the probability of the event in which each man found himself with his wife?
b) What is the probability that each pair contains a man and a woman?

Problem 12

n married couples sit randomly around a round table. What is the probability that no woman sits besides her husband?

Problem 13

Out of a deck of cards we randomly choose k cards, $k \geq 13$. What is the probability that each number is represented in the sample?

Problem 14

A dice is thrown 36 times. What is the probability to get each number 6 times?