Probability 201-1-2391 ASSIGNMENT 4
Conditional probability, independence
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Problem 1

Give an example of two independent events.

Problem 2

Give an example of:
a) Three events that are pairwise independent but NOT independent.
b) Three events that satisfy $p(A \cap B \cap C) = p(A) \cdot p(B) \cdot p(C)$, but are NOT independent.

Problem 3

In a certain country there are equal numbers of Men and of women. 20 percent of the men are color blind and 2 percent of the women are color blind.
a) What is the probability that a person chosen at random in this country is color blind?
b) A person was randomly chosen and was found to be color blind. What is the probability that the person is a man?

Problem 4

We randomly distribute 7 balls into 7 cells.
a) What is the probability that there are exactly 2 empty cells and there is a cell containing exactly 3 balls?
b) What is the probability that there are 2 empty cells?
c) What is the probability that there is a cell containing 3 balls given that there are 2 empty cells?

Problem 5

We toss 5 times a balanced coin. Each time: if we get H we put into a jar a white ball, otherwise we put a black ball.
a) What is the probability that the jar contains only white balls?
b) Answer the question in a, given that we pulled out of the jar 5 balls (returning the pulled out ball each time), and found out that only white balls came.

Problem 6

A basketball player tries to shoot the ball 5 times from a long distance. The probability for a hit is 1/5. Assuming independence of events:
a) What is the probability to hit at least twice?
b) The same question, given that the player already hit once.

Problem 7

There are N children in a family. The probability for a child to be a son is 1/2. We define A - the family has boys and girls. B - the family has at most one girl.For which values of N the events A and B are independent?

Problem 8

A person is about to try his luck by gambling 3 times. At the front of him there are 2 machines. The probability to win on machine A is $p_A = 0.6$ and that on machine B is $p_B = 0.3$. The first time he chooses a machine randomly. At every stage (after the first) if he wins, he will try the same machine again. Otherwise he switches to the other machine.
a) What is the probability that on the second time he will use machine A?
b) The same question, given that on the third time machine A was used?

Problem 9

A jar contains 3 red balls, 4 white balls and 2 black balls. Balls are being pulled out and returned back to the jar forever. What is the probability that the first red ball that was pulled out came out before the first white ball that was pulled out?

Problem 10

There are 3 types of pine trees in the forest, A, B and C. Because of a disease 40 percent of the trees in the forest were damaged. 1/3 of type A, 1/2 of type B, 1/4 of type C. 1/4 of the damaged trees are pines of type A. What is the percentage of pines of type B in the forest?

Problem 11

Two jars contain (each) black and white balls.The probability to pull out a white ball from jar A is p, and the probability to pull out a white ball from jar B is q. Balls are pulled with placing them back into the jars A ans B. At each stage, if the ball pulled out is white, the next ball will be pulled out of the same jar. Otherwise we switch to the other jar. The first ball is pulled out of jar A.
a) Let $\alpha(n)$ be the probability that the n'th ball is pulled out of jar A. Show that for $n \geq 2$ we have:

$$\alpha(n) = \alpha(n-1) \cdot (p+q-1) + (1-q).$$

b) Prove that for $n \geq 1$ we have:

$$\alpha(n) = \frac{1-q}{2-p-q} + \frac{1-p}{2-p-q} \cdot (p+q-1)^{n-1}.$$

Problem 12

Two candidates A and B raced each other in an election campaign. Candidate A accumulated a votes and candidate B accumulated b votes. Assuming $a > b$ what is the probability that candidate A was leading all the way?

Problem 13

Prove that for any two events A, B we have: $R(A, B) = -R(A^c, B) = R(A^c, B^c)$.

Problem 14

A jar contains $a$vwhite balls and b black balls. Two players pull out the balls in turn (one by one). The first who pulls out a white ball wins the game. What is the probability that the first player wins the game?