Probability 201-1-2391 ASSIGNMENT 8
Expectation, variance and covariance
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Problem 1

There are 6 lottery cards in a hat. Three are marked with 0, two are marked with 20 and one is marked with 40. A gambler invests 20 Dollars in order to participate. He pulls out of the hat two cards and gets an amount that equals the average of his cards. Compute the distribution of the amount that the gambler will get, the expectation of the gain and its variance.
We define: X is the amount that the gambler gets, Y is the gain of the gambler. So $Y = -20 + X$. Then:

$$p(X = 0) = \frac{3}{15}, \; P(X = 10) = \frac{6}{15}, \; P(X = 20) = \frac{4}{15}, \; P(X = 30) = \frac{2}{15}.$$

So:

$$E(X) = \frac{4}{3}, \; E(Y) = -\frac{2}{3}, \; E(X^2) = \frac{4000}{15}, \; \sigma^2(Y) = \frac{800}{9}.$$

Problem 2

Two dice are thrown 1000 times. Let X be the number of throws in which the sum is at least 11. Compute the expectation and the variance of X.
$E(X) = 83\frac{1}{3}$, $\sigma^2(X) = 76\frac{7}{18}$.

Problem 3

In a TV game, handles of three machines can be pulled down. In machine 1 the probability to win 500 Dollars is 1/2, in machine 2 the probability to win 1000 Dollars is 0.6, and in machine 3 the probability to win 1500 Dollars is 0.7. Let X be the total win. Find $E(X)$.
$E(X) = 1900$.

Problem 4

Let X, Y be two random variables that assume (each) the values $0, 1$. Write down formulas for $E(X)$, $Var(X)$ and $E(X \cdot Y)$.
$E(X) = P(X = 1)$, $\sigma^2(X) = P(X = 1) \cdot P(X = 0)$, $E(X \cdot Y) = P(X = 1 \cap Y = 1)$.

Problem 5

Out of 10 people 3 are randomly chosen, returning back (each time). Let X be the number of different people chosen in a sample. Find the expectation and the variance of X.
$E(X) = 2.71$, $\sigma^2(X) = 0.2259$.

Problem 6

n numbers are randomly chosen from $1, 2, \ldots, N$ (without returning). Let X be the largest number chosen. For $n < N$ prove that:

$$E(X) = \frac{n}{n+1} \cdot (N + 1).$$

Problem 7

n balls that are numbered $1, 2, \ldots, n$ are randomly distributed into n cells that are also numbered $1, 2, \ldots, n$. Every cell has a room for exactly 1 ball. Let X be the number of balls that were put into cells with identical numbers (i.e. the ball and it's cell have identical numbers). Compute $E(X)$ and $Var(X)$.
$E(X) = 1$, $\sigma^2(X) = 1$.

Problem 8

A dice is thrown n times. Let X be the number of different faces that came up. What is $E(X)$?

$$E(X) = 6 - \frac{5^n}{6^{n-1}}.$$

Problem 9

A dice is thrown again and again till all the faces show up. Let Y be the number of throws. What is $E(Y)$?
$E(Y) = 14.7$.

Problem 10

There are n students in the class. Out of all the

$$\binom{n}{2}$$

pairs of students we let X be the number of pairs in which the two students were born in the same day of the year. Compute $E(X)$ and $Var(X)$.

$$E(X) = \binom{n}{2} \cdot \frac{1}{365}, \ \sigma^2(X) = \binom{n}{2} \cdot \frac{364}{365^2}.$$

Problem 11

A balanced coin is tossed N times. Let X be the number of sequences of three adjacent H's. Compute $E(X)$ and $Var(X)$.

$$E(X) = \frac{N-2}{8}, \ \sigma^2(X) = \frac{15N-40}{64}.$$

Problem 12

X is distributed $B(n,p)$. $Y|X$ is distributed $U(X,m)$. What is $E(Y)$?

$$E(Y) = \frac{np+m}{2}.$$

Problem 13

A dice is thrown N times. Let X be the number of times 6 showed. Let Y be the number of times 5 showed. Compute $Cov(X,Y)$.

$$Cov(X,Y) = -\frac{N}{36}.$$

Problem 14

A red dice is thrown, and a blue dice is thrown. Let X be the result shown on the red dice, and let Y be the maximal result. Compute $Cov(X, Y)$.

$$Cov(X, Y) = \frac{35}{24}.$$