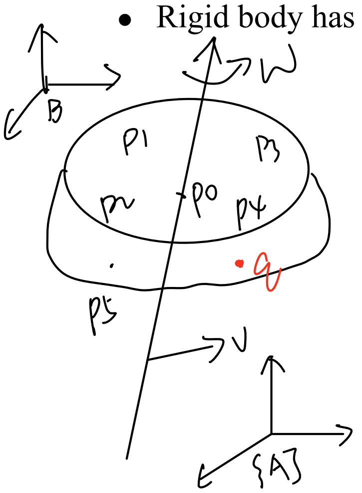
认为 body 以 V_{0B} 运动
并绕 w 旋转

$^B\mathcal{V} = \begin{bmatrix} w \\ V_{0B} \end{bmatrix}$

Extra Note
Tutorial on Twist/Spatial Velocity and Screw Axis

1. What is Spatial Velocity and Twist?

- Twist/spatial velocity is the velocity of the whole rigid body not velocity of a particular point
- Rigid body has infinitely many points with different velocities



$V_{p_1} = g(p_1, \text{parameters})$

$V_{p_2} = g(p_2, \text{parameters})$

$V_{p_3} = g(p_3, \dots)$

$V_{p_4} = g(p_4, \dots)$

common for the entire body

- All these velocities V_{p_i} are not independent
- Can be expressed by same set of parameters
- twist / spatial velocity is one such parameters.

1. Assume p_0 is on the rotation axis / body-fixed. the any other body-fixed points:

$$V_{p_1} = V_{p_0} + w \times (\overrightarrow{p_0p_1})$$

$$V_{p_i} = V_{p_0} + w \times (\overrightarrow{p_0p_i}) \quad \cdots (1)$$

para

location of p_i

2°: What if we use V_q as the refence velocity, for arbitrary body-fixed point q (may not be on the rotation axis)

we still have the same expression $V_{p_i} = V_q + w \times (\overrightarrow{qp_i}) \quad \cdots (2)$

para

point location

Why: use "p_0" as intermediate variable?

q: body-fixed by (1)

$$V_q = V_{p_0} + w \times (\overrightarrow{p_0q})$$
$$= V_{p_i} - w \times \overrightarrow{p_0p_1} + w \times \overrightarrow{p_0q}$$

3° Let's consider a frame $\{A\}$ with origin a

3.1° Assume $\{A\}$ is body-fixed frame, moves with the body (in this case, let $O_A = q$ in by (2))

$$V_{p_i} = V_{OA} + w \times (\overrightarrow{O_Ap_i}) \quad \cdots (3)$$

In $\{A\}$ coordinate sys: $\quad {}^AV_{p_i} = {}^AV_{OA} + {}^Aw \times {}^Ap_i \quad (V_p = V_{OA} + w \times p)$

3-2°

Assume {A} is not body-fixed / $\begin{cases} \{A\} \text{ does not move} \\ \text{or moves in other way} \end{cases}$

let q – body-fixed point such that $\boxed{q(t) = O_A}$ ⌢ ⋯ (*)

by (2) $\quad V_{pi} = V_{q(t)} + W \times \vec{q p_i}$

by (*) $\quad V_{pi} = V_{OA} + W \times \overrightarrow{O_A p_i}$ ⋯ (4) , in {A} $\quad {}^A V_{pi} = {}^A V_{OA} + {}^A W \times {}^A p_i$

(3) & (4) are the same if we define V_{OA} as

velocity of the $\boxed{\text{"body-fixed" point currently coincides with } O_A}$

means equ (*)

$V_{OA} \Rightarrow$ 无论{A}是否移动

— Summary : Given the twist $\mathcal{V} = \begin{bmatrix} W \\ V_r \end{bmatrix}$ $\boxed{\text{不论是否body-fixed!!!}}$

velocity of body-fixed point currently coincides with r

1°

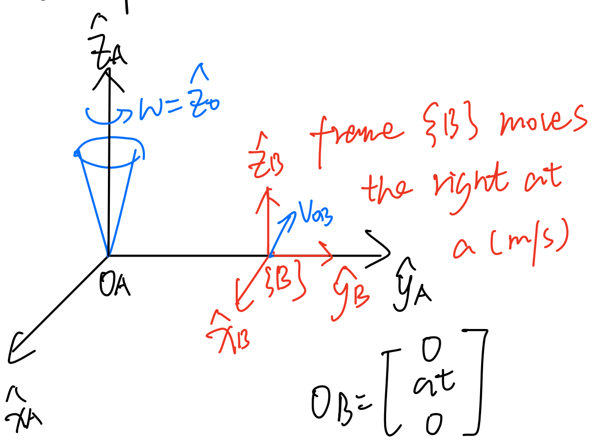
For any body-fixed point pi

$V_{pi} = V_r + W \times (\vec{r p_i})$ = if r point is origin of frame {A}

$^A V_{pi} = {}^A V_r + {}^A W \times {}^A p_i$

2°: we can think : the body is translating at velocity V_r, while rotating at velocity W about axis passing through r.

Example



$O_B = \begin{bmatrix} 0 \\ at \\ 0 \end{bmatrix}$

$^B \mathcal{V}_{top} = \begin{bmatrix} ^B W \\ ^B V_{OB} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ \\ \\ \end{bmatrix}$

By definition $\quad {}^B V_{OB} = {}^B V_{OA} + {}^B W \times {}^B(\overrightarrow{O_A O_B})$

$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ at \\ 0 \end{bmatrix}$

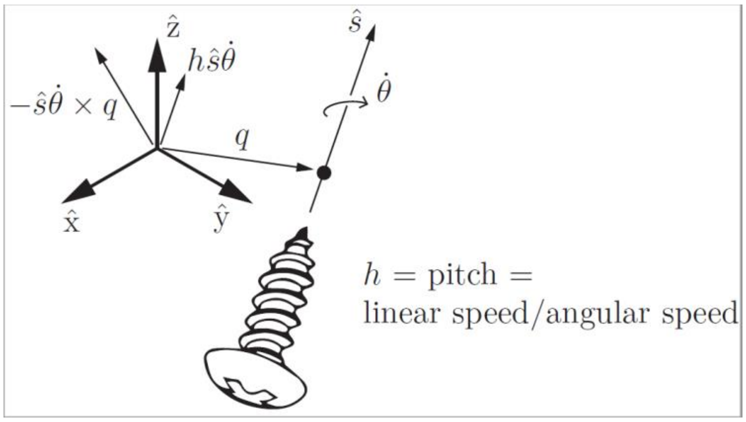
"coordinate free"

V_{OA}: $\boxed{\text{body-fixed}}$ point coincides with OA

↳ $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ zero vector $\Rightarrow {}^B V_{OA} = 0$

$= \begin{bmatrix} -at \\ 0 \\ 0 \end{bmatrix}$ 与时间t无关

2. What is Screw Motion and Axis?



Screw motion: combined angular and linear motion

- motion driven by rotation
- parameters $(\hat{s}, h, q)$ and $\dot{\theta}$

☆ $(\hat{s}, q)$: determines the rotation axis

h: linear speed / angular speed (rotation induces linear motion)

↳ "due to the thread" on the screw

$h=0 \Rightarrow$ pure rotation $h=\infty \Rightarrow$ pure translation

Overall, ☆ $(\hat{s}, q, h) \Leftarrow$ screw axis (rotation axis + pitch)

$\dot{\theta}$ = how fast screw rotates

1° Screw motion is a special rigid body motion, so it has a twist.

- pick a frame $\{A\}$: $(\hat{s}, q, h) \oplus \dot{\theta} \Rightarrow \begin{bmatrix} {}^{A}\omega \\ {}^{A}v \end{bmatrix}$, $\boxed{{}^{A}\omega = \hat{s}\dot{\theta}}$

$\boxed{{}^{A}v = h\dot{\theta}\,{}^{A}\hat{s} - {}^{A}\omega \times {}^{A}q}$ --- (5)

remark: $h\dot{\theta}\,{}^{A}\hat{s}$ only the velocity relative to the axis

use q as the reference point.

$V_{OA} = V_q + \omega \times (\vec{qOA})$

$= (h\dot{\theta})\,{}^{A}\hat{s} + {}^{A}\omega \times (-{}^{A}q)$

$\mathcal{V}$ = Screw to Twist $(\hat{s}, q, h, \dot{\theta})$

eq(5)

2° Any rigid body motion can be viewed as screw motion

Given any twist $\mathcal{V}$, we can always find $(\hat{s}, q, h), \dot{\theta}$ such that

eq(5) holds.

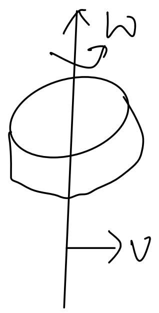
3° By (5), we know $\mathcal{V}$ = Screw to Twist $(\hat{s}, q, h, 1)\,\dot{\theta} = S\dot{\theta}$

$\triangleq$ $S \leftarrow$ the twist correspond to screw motion $(\hat{s}, q, h)$, $\dot{\theta} = 1$

e.g $\omega = \hat{\omega}\dot{\theta}$ $\quad$ $\hat{\omega}$: rotation axis

or angular velocity when rotates about $\hat{\omega}$ at $\dot{\theta} = 1$

Summarize: Given any rigid body motion $\mathcal{V} = \begin{bmatrix} \omega \\ v \end{bmatrix}$



$$\boxed{\begin{array}{l} \text{angular direction } \hat{\omega} \\ \text{linear motion direction } \hat{v} \end{array}} \Longrightarrow S = \begin{bmatrix} \omega_{\text{screw axis}} \\ v_{\text{screw axis}} \end{bmatrix}$$

screw axis "direction"

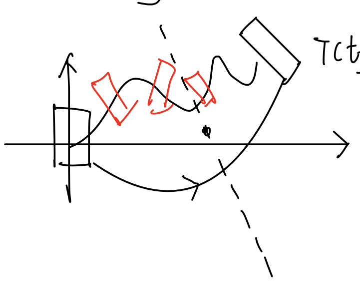
$\updownarrow$

$(\hat{S}, h, q)$

$\mathcal{V} = S \cdot \dot{\theta}$

What does the 2° means?

For any $T \in SE(3)$, $\Rightarrow T = e^{[S]\theta}$



$T(t_f) \in SE(3)$