1. **(3+3 points)** Modern Robotics: Exercise 3.18

Solution: (a):

$$T_{rs} = T_{ra}T_{ae}T_{es} = T_{ar}^{-1}T_{ea}^{-1}T_{es} \tag{1}$$

(b): We can get

$$\begin{aligned} {}^{e}p_r &= [1,1,1]^T \\ {}^{e}p_s &= [1,1,1]^T \end{aligned} \tag{2}$$

So point r is coincide with point s, so

$${}^{r}p_s = [0,0,0]^T \tag{3}$$

□

2. **(3 points)** Modern Robotics: Exercise 3.26

Solution: See Fig. 1

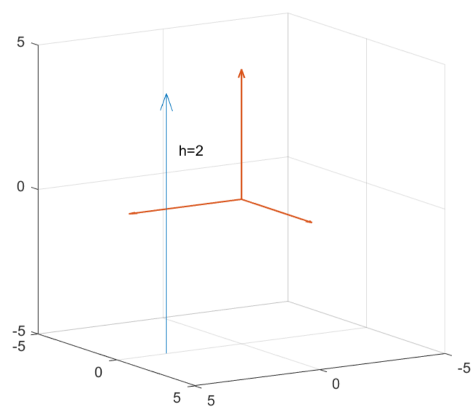

Figure 1: Screw axis for Exercise 3.26

□

3. **(3 points)** Modern Robotics: Exercise 4.14 (only need to find M and $\mathcal{S}_i$ in $\{0\}$, no need to compute the $\{b\}$ frame case)

Solution: The initial configuration is

$$M = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

And the screw axes are

$$\begin{aligned}
\mathcal{S}_1 &= [0, 0, 1, 4, 0, 0]^T \\
\mathcal{S}_2 &= [0, 0, 0, 0, 1, 0]^T \\
\mathcal{S}_3 &= [0, 0, -1, -6, 0, -0.1]^T
\end{aligned} \tag{5}$$

When $\theta = [\pi/2, 3, \pi]$

$$\begin{aligned}
T &= e^{[\mathcal{S}_1]\theta_1} e^{[\mathcal{S}_2]\theta_2} e^{[\mathcal{S}_3]\theta_3} M \\
&= \begin{bmatrix} 0 & 1 & 0 & -5 \\ 1 & 0 & 0 & 4 \\ 0 & 0 & -1 & 2 - 0.1\pi \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned} \tag{6}$$

□

4. **(3+3 points)** Modern Robotics: Exercise 5.6 (Note: part-(a) is modified to compute the twist of frame b expressed in frame b, i.e., the ${}^b\mathcal{V}_b$)

Solution: (a): We have

$$\begin{aligned}
{}^s\mathcal{S}_1 &= [0, 1, 0, 0, 0, 0]^T \\
{}^s\mathcal{S}_2 &= [0, 0, 1, 0, 0, 0]^T
\end{aligned} \tag{7}$$

and

$$M = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -20 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

So

$$\begin{aligned}
J_b &= [Ad^{-1}_{e^{\mathcal{S}_1\theta_1} e^{\mathcal{S}_2\theta_2} M} \mathcal{S}_1, Ad^{-1}_{e^{\mathcal{S}_2\theta_2} M} \mathcal{S}_2] \\
&= \begin{bmatrix} \sin\theta_2 & 0 \\ \cos\theta_2 & 0 \\ 0 & 1 \\ -20\cos\theta_2 & 0 \\ 20\sin\theta_2 & 10 \\ -10\cos\theta_2 & 0 \end{bmatrix}
\end{aligned} \tag{9}$$

We have $\theta_1 = \theta_2 = t$ and $V = J\dot{\theta}$

$${}^b\mathcal{V} = [\sin t, \cos t, 1, -20\cos t, 20\sin t + 10, -10\cos t]^T \tag{10}$$

(b):

$$\begin{aligned}
{}^s\mathcal{V} &= Ad_{T_{sb}} {}^b\mathcal{V} \\
&= [\sin t, 1, \cos t, 0, 0, 0]^T
\end{aligned} \tag{11}$$

$$p(t) = e^{\mathcal{S}_1\theta_1}e^{\mathcal{S}_2\theta_2}p(0) \tag{12}$$

So

$$\dot{p} = \omega \times p + v = \begin{bmatrix} -10\sin 2t - 20\cos t \\ 10\cos t \\ 20\sin t - 10\cos 2t \end{bmatrix} \tag{13}$$

$\square$

5. **(3+3 points)** Modern Robotics: Exercise 5.7-(a)

Solution: (a):

$$\begin{aligned} \mathcal{S}_1 &= [0,0,1,0,0,0]^T \\ \mathcal{S}_2 &= [1,0,0,0,2,0]^T \\ \mathcal{S}_3 &= [0,0,0,0,1,0]^T \end{aligned} \tag{14}$$

$$M = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{15}$$

When $\theta = [90°, 90°, 1]$

$$\begin{aligned} T &= e^{\mathcal{S}_1\theta_1}e^{\mathcal{S}_2\theta_2}e^{\mathcal{S}_3\theta_3}M \\ &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{16}$$

$$\begin{aligned} J_s &= \begin{bmatrix} \mathcal{S}_1 & Ad_{e^{\mathcal{S}_1\theta_1}}\mathcal{S}_2 & Ad_{e^{\mathcal{S}_2\theta_2}e^{\mathcal{S}_2\theta_2}}\mathcal{S}_3 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{17}$$

$\square$

6. **(3+3+3+3 points)** Consider the robot shown in Fig.4.3.

   (a) Use Drake to build this robot model (similar to the example we discussed during class) and show the snapshots of the Meshcat visualization at three different sets of joint positions.

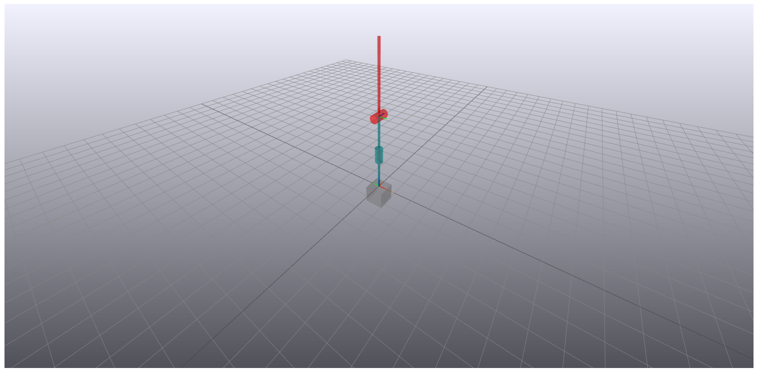

Figure 2: (Answer for problem 5) Config. in $\theta = [90°, 90°, 1]$.

(b) Write your own forward kinematics function (using PoE) to compute the pose of the end-effector frame (i.e. frame $\{3\}$) relative to the world frame $\{0\}$. Test your function for a few different sets of joint positions and compare your results with Drake's built-in function

(c) Write your own function to compute the geometric Jacobian of the end-effector frame (i.e. frame $\{3\}$) expressed in the world frame $\{0\}$. Test your function for a few different sets of joint positions and compare your results with Drake's built-in function

(d) Let q be a point attached to frame $\{3\}$ with local coordinate ${}^{3}q = (1, 2, 3)$.

- Derive the (analytic) Jacobian ${}^{0}J_a(\theta)$, i.e., ${}^{0}\dot{q} = {}^{0}J_a(\theta)\dot{\theta}$. Show all your steps.
- Write a function in Drake to implement your formula. Test your function for a few different sets of joint positions/joint velocities, and compare your results with the Drake's built-in function.

Solution: (d):

$$\begin{aligned} {}^{0}\dot{q} &= v_0 + \omega_0 \times {}^{0}q \\ &= \begin{bmatrix} -[{}^{0}q] & I \end{bmatrix} \mathcal{V} \\ &= \begin{bmatrix} -[{}^{0}q] & I \end{bmatrix} J\dot{\theta} \\ &= E J_{\text{geom}}\dot{\theta} \end{aligned} \tag{18}$$

□