MEE5114 Advanced Control for Robotics

Lecture 6: Product of Exponential and Kinematics of Open Chain

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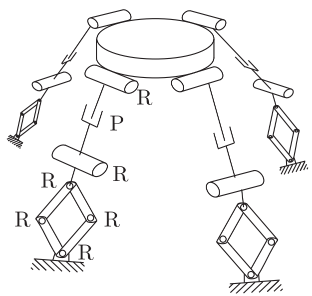
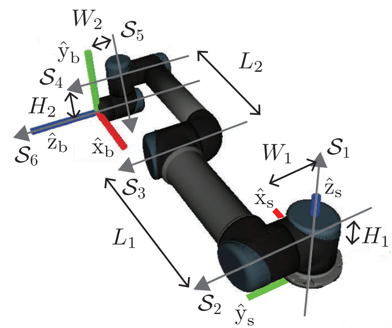
Outline

- Background
- Product of Exponential Formula Derivations
- Example

Kinematics

Kinematics is a branch of classical mechanics that describes the motion of points, bodies (objects), and systems of bodies (groups of objects) without considering the mass of each or the forces that caused the motion





- **Forward Kinematics**: calculation of the configuration $T = (R, p)$ of the end-effector frame from joint variables $\theta = (\theta_1, \ldots, \theta_n)$

- **Velocity Kinematics**: Deriving the Jacobian matrix: linearized map from the joint velocities to the spatial velocity of the end-effector

Basic Setup (1/3)

- Suppose that the robot has n joints and n links. Each joint has one degree of freedom represented by joint variable θ_i, $i = 1, \ldots, n$
  - θ_i: the joint angle (Revolute joint) or joint displacement (Primatic joint)

- Specify a fixed frame $\{s\}$: also referred to as frame $\{0\}$

world frame

- Attach frame $\{i\}$ to link i at joint i, for $i = 1, \ldots, n$

- Attach frame $\{b\}$ at the end-effector: sometimes referred to as frame $\{n+1\}$

- ${}^i\mathcal{S}_i$: screw axis of joint i expressed in frame $\{i\}$

- ${}^0\mathcal{S}_i$: screw axis of joint i expressed in fixed frame $\{0\}$ (i.e. frame $\{s\}$)

Basic Setup (2/3)

- Illustration Example: Kinematis problems aims to find ${}^sT_b$

screw axis:

e.g. ${}^0S_1 = (w_1, v_1)$ $\quad w_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $\quad v_1 = -w_1 \times q_1 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\downarrow$

$(q_1, \hat{s}_1, h=0)$, choose ${}^0q_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

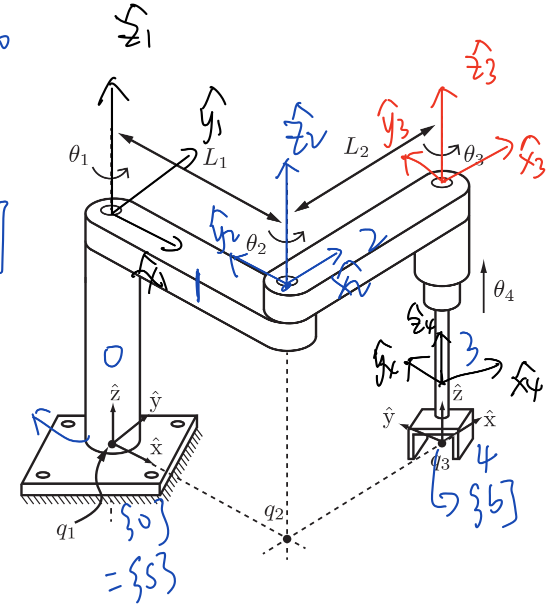
${}^2S_2 = ({}^2w_2, {}^2v_2)$ $\quad {}^2w_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, ${}^2v_2 = -{}^2w_2 \times {}^2q_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

${}^0S_2 = {}^0X_2 \, {}^2S_2$

independent of joint varible

depend on θ_1, θ_2

Similarly we can find ${}^0S_3$, ${}^0S_4$



Basic Setup (3/3)

- For simplicity, we write configuration as T_{sb}, which is the same as ${}^sT_b$. Similarly, $T_{ij} = {}^iT_j$

- Note: ${}^i\mathcal{S}_i$ does not change when the robot moves (i.e. when θ changes), but ${}^0\mathcal{S}_i$ depends on $\theta_1, \ldots, \theta_i$. Sometimes, we write out the dependency explicitly, i.e. ${}^0\mathcal{S}_i(\theta_1, \ldots, \theta_i)$

- Define home position: $\theta_1 = 0, \ldots, \theta_n = 0$. This is the configuration when all the joint angles are zero. One can also choose other *fixed* angles as the home position

- Define ${}^0\bar{\mathcal{S}}_i = {}^0\mathcal{S}_i(0, \ldots, 0)$: the screw axis of joint i expressed in frame $\{0\}$, when the robot is at the home position.

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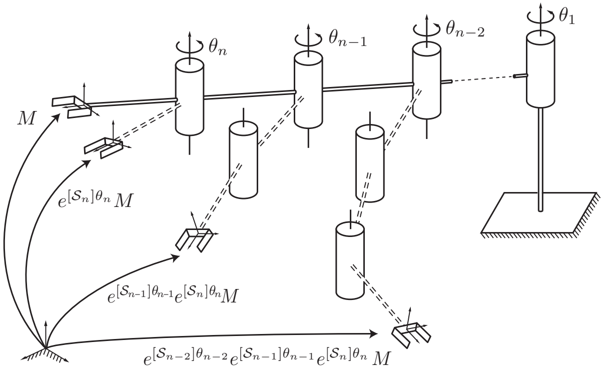
Product of Exponential: Main Idea

- **Goal:** Derive $T_{sb}(\theta_1, \ldots, \theta_n)$
- Compute $M \triangleq T_{sb}(0, \ldots, 0)$: the configuration of end-effector when the robot is at home position



- Apply screw motion to joint n: $T_{sb}(0, \ldots, 0, \theta_n) = e^{[{}^0\bar{\mathcal{S}}_n]\theta_n}M$

- Apply screw motion to joint $n-1$ to obtain:

$$T_{sb}(0, \ldots, 0, \theta_{n-1}, \theta_n) = e^{[{}^0\bar{\mathcal{S}}_{n-1}]\theta_{n-1}}e^{[{}^0\bar{\mathcal{S}}_n]\theta_n}M$$

- After n screw motions, the overall forward kinematics:

$$T_{sb}(\theta_1, \ldots, \theta_n) = e^{[{}^0\bar{\mathcal{S}}_1]\theta_1}e^{[{}^0\bar{\mathcal{S}}_2]\theta_2}\cdots e^{[{}^0\bar{\mathcal{S}}_n]\theta_n}M$$

PoE: Screw Motions in Different Order (1/2)

- PoE was obtained by applying screw motions along screw axes ${}^0\bar{\mathcal{S}}_n$, ${}^0\bar{\mathcal{S}}_{n-1}$, ... What happens if the order is changed?

- For simplicity, assume that $n = 2$, and let us apply screw motion along ${}^0\bar{\mathcal{S}}_1$ first:
  - $T_{sb}(\theta_1, 0) = e^{[{}^0\bar{\mathcal{S}}_1]\theta_1} M$
  - Now screw axis for joint 2 has been changed. The new axis ${}^0\mathcal{S}_2 = {}^0\mathcal{S}_2(\theta_1, 0) \neq {}^0\bar{\mathcal{S}}_2$.

PoE: Screw Motions in Different Order (2/2)

- $T_{sb}(\theta_1, \theta_2) = e^{[{}^0\mathcal{S}_2]\theta_2} T_{sb}(\theta_1, 0)$

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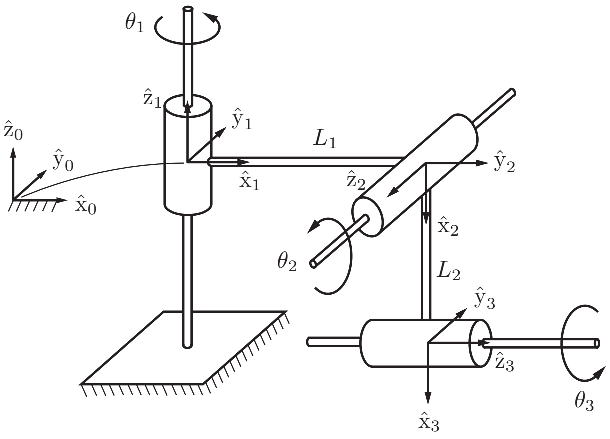
PoE Example: 3R Spatial Open Chain



More Discussions

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