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In [2]:
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import random
import time
import tracemalloc
import matplotlib.pyplot as plt
from random import sample
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In [3]:
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random.seed(1)
sequence = [i for i in range(100000)]
random.shuffle(sequence)
#print(sequence)
```

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In [4]:
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class Node:

    # Constructor to create a new node
    def __init__(self, key):
        self.key = key
        self.left = None
        self.right = None

    # A utility function to do inorder traversal
    def inorder(root):
        if root is not None:
            inorder(root.left)
            print(root.key, end=" ")
            inorder(root.right)

    # A utility function to insert a new node with given key
    def insert(node, key):

        # If the tree is empty, return a new node
        if node is None:
            return Node(key)

        # Otherwise recur down the tree
        if key < node.key:
            node.left = insert(node.left, key)
        else:
            node.right = insert(node.right, key)
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# return the (unchanged) node pointer
return node

# Given a binary tree and a key, this function delete the key and returns the new root
def deleteNode(root, key):

    # Base Case
    if root is None:
        return root

    # Recursive calls for ancestors of node to be deleted
    if key < root.key:
        root.left = deleteNode(root.left, key)
        return root

    elif (key > root.key):
        root.right = deleteNode(root.right, key)
        return root

    # We reach here when root is the node to be deleted.

    # If root node is a leaf node

    if root.left is None and root.right is None:
        return None

    # If one of the children is empty

    if root.left is None:
        temp = root.right
        root = None
        return temp

    elif root.right is None:
        temp = root.left
        root = None
        return temp

    # If both children exist

    succParent = root

    # Find Successor
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succ = root.right

while succ.left != None:
    succParent = succ
    succ = succ.left

# Delete successor. Since successor is always left child of its parent
# we can safely make successor's right right child as left of its parent.
# If there is no succ, then assign succ->right to succParent->right
if succParent != root:
    succParent.left = succ.right
else:
    succParent.right = succ.right

# Copy Successor Data to root

root.key = succ.key

return root

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In [5]:

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root = None
for i in range(len(sequence)):
    root = insert(root, sequence[i])
inorder(root)

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2 99423 99424 99425 99426 99427 99428 99429 99430 99431 99432 99433 99434 99435 99436 99437 99438 99439 99  
440 99441 99442 99443 99444 99445 99446 99447 99448 99449 99450 99451 99452 99453 99454 99455 99456 99457  
99458 99459 99460 99461 99462 99463 99464 99465 99466 99467 99468 99469 99470 99471 99472 99473 99474 9947  
5 99476 99477 99478 99479 99480 99481 99482 99483 99484 99485 99486 99487 99488 99489 99490 99491 99492 99  
493 99494 99495 99496 99497 99498 99499 99500 99501 99502 99503 99504 99505 99506 99507 99508 99509 99510  
99511 99512 99513 99514 99515 99516 99517 99518 99519 99520 99521 99522 99523 99524 99525 99526 99527 9952  
8 99529 99530 99531 99532 99533 99534 99535 99536 99537 99538 99539 99540 99541 99542 99543 99544 99545 99  
546 99547 99548 99549 99550 99551 99552 99553 99554 99555 99556 99557 99558 99559 99560 99561 99562 99563  
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1 99582 99583 99584 99585 99586 99587 99588 99589 99590 99591 99592 99593 99594 99595 99596 99597 99598 99

```
599 99600 99601 99602 99603 99604 99605 99606 99607 99608 99609 99610 99611 99612 99613 99614 99615 99616  
99617 99618 99619 99620 99621 99622 99623 99624 99625 99626 99627 99628 99629 99630 99631 99632 99633 9963  
4 99635 99636 99637 99638 99639 99640 99641 99642 99643 99644 99645 99646 99647 99648 99649 99650 99651 99  
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99670 99671 99672 99673 99674 99675 99676 99677 99678 99679 99680 99681 99682 99683 99684 99685 99686 9968  
7 99688 99689 99690 99691 99692 99693 99694 99695 99696 99697 99698 99699 99700 99701 99702 99703 99704 99  
705 99706 99707 99708 99709 99710 99711 99712 99713 99714 99715 99716 99717 99718 99719 99720 99721 99722  
99723 99724 99725 99726 99727 99728 99729 99730 99731 99732 99733 99734 99735 99736 99737 99738 99739 9974  
0 99741 99742 99743 99744 99745 99746 99747 99748 99749 99750 99751 99752 99753 99754 99755 99756 99757 99  
758 99759 99760 99761 99762 99763 99764 99765 99766 99767 99768 99769 99770 99771 99772 99773 99774 99775  
99776 99777 99778 99779 99780 99781 99782 99783 99784 99785 99786 99787 99788 99789 99790 99791 99792 9979  
3 99794 99795 99796 99797 99798 99799 99800 99801 99802 99803 99804 99805 99806 99807 99808 99809 99810 99  
811 99812 99813 99814 99815 99816 99817 99818 99819 99820 99821 99822 99823 99824 99825 99826 99827 99828  
99829 99830 99831 99832 99833 99834 99835 99836 99837 99838 99839 99840 99841 99842 99843 99844 99845 9984  
6 99847 99848 99849 99850 99851 99852 99853 99854 99855 99856 99857 99858 99859 99860 99861 99862 99863 99  
864 99865 99866 99867 99868 99869 99870 99871 99872 99873 99874 99875 99876 99877 99878 99879 99880 99881  
99882 99883 99884 99885 99886 99887 99888 99889 99890 99891 99892 99893 99894 99895 99896 99897 99898 9989  
9 99900 99901 99902 99903 99904 99905 99906 99907 99908 99909 99910 99911 99912 99913 99914 99915 99916 99  
917 99918 99919 99920 99921 99922 99923 99924 99925 99926 99927 99928 99929 99930 99931 99932 99933 99934  
99935 99936 99937 99938 99939 99940 99941 99942 99943 99944 99945 99946 99947 99948 99949 99950 99951 9995  
2 99953 99954 99955 99956 99957 99958 99959 99960 99961 99962 99963 99964 99965 99966 99967 99968 99969 99  
970 99971 99972 99973 99974 99975 99976 99977 99978 99979 99980 99981 99982 99983 99984 99985 99986 99987
```

In [6]:

```
def complexity(data, num):  
    root = data  
    start = time.time()  
    tracemalloc.start()  
    for i in range(num):  
        root = deleteNode(root, i+10)  
    end = time.time()  
    runtime = end - start  
    current, peak = tracemalloc.get_traced_memory()  
    usage = current / 10**6  
    print(f"Current memory usage is {usage}MB; Peak was {peak / 10**6}MB")  
    tracemalloc.stop()  
    print(f"Runtime of the snippet is {runtime}")  
    return runtime, usage
```

In [7]:

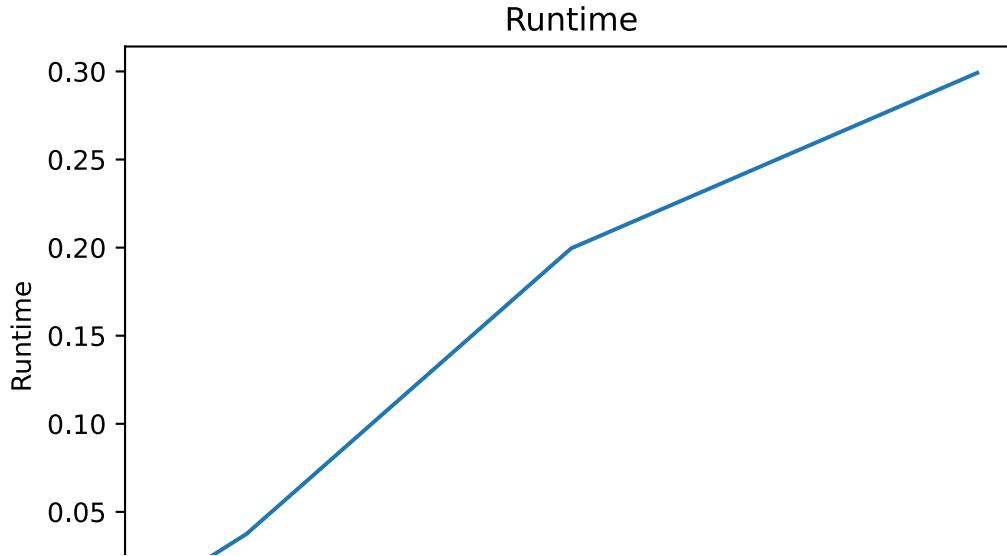
```
cost = []  
seqRange = [50, 100, 500, 1000, 5000, 10000, 50000, 100000]  
for i in range(8):  
    print(f"{i+1}: for {seqRange[i]} items\n-----")  
    cost.append(complexity(root, seqRange[i]))  
runtime, usage = map(list, zip(*cost))
```

```
1: for 50 items
-----
Current memory usage is 0.002736MB; Peak was 0.002736MB
Runtime of the snippet is 0.0009965896606445312
2: for 100 items
-----
Current memory usage is 0.000169MB; Peak was 0.000201MB
Runtime of the snippet is 0.0
3: for 500 items
-----
Current memory usage is 2.8e-05MB; Peak was 0.000112MB
Runtime of the snippet is 0.001993894577026367
4: for 1000 items
-----
Current memory usage is 2.8e-05MB; Peak was 0.000112MB
Runtime of the snippet is 0.002995014190673828
5: for 5000 items
-----
Current memory usage is 0.000283MB; Peak was 0.000343MB
Runtime of the snippet is 0.02293848991394043
6: for 10000 items
-----
Current memory usage is 0.00047MB; Peak was 0.000552MB
Runtime of the snippet is 0.037462711334228516
7: for 50000 items
-----
Current memory usage is 0.0089MB; Peak was 0.010901MB
Runtime of the snippet is 0.19961905479431152
8: for 100000 items
-----
Current memory usage is 0.003585MB; Peak was 0.008991MB
Runtime of the snippet is 0.2992093563079834
```

In [18]:

```
fig, ax = plt.subplots()
ax.plot(seqRange, runtime, label='Runtime')
ax.set_xlabel('Items')
ax.set_ylabel('Runtime')
ax.set_title("Runtime")
```

Out[18]: Text(0.5, 1.0, 'Runtime')



In [19]:

```
fig, ax = plt.subplots()
ax.plot(seqRange, usage, label='Memory usage')
ax.set_xlabel('Items')
ax.set_ylabel('Memory usage')
ax.set_title("Memory usage")
```

Out[19]: Text(0.5, 1.0, 'Memory usage')

Memory usage

