# Econometrics, Semester II, 2024-25
# Homework II (100 points)

Instructor: M.A. Rahman

Deadline: 5:00 pm, February 28, 2025.

**Please read the instructions carefully and follow them while writing answers.**

- *Solutions to homework should be typed in LaTeXor written in A4 size loose sheets.*
- *Questions should be answered in order as they appear in the homework. Every new question should begin in a new page. Please number all the pages of your homework solution.*
- *Please leave a margin of one inch from top and one inch from left. Staple the sheets on the top-left.*
- *Matlab assignments (if any) and written answers should be together and in order.*
- *Please write your name and names of your group members on the first page of your answer script.*

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**1. (10+10+10+10+30 = 70 points.)**. Consider the ordinal regression model discussed in class:

$$\begin{aligned} z_i &= x_i'\beta + \epsilon_i, && \forall\, i = 1, \cdots, n, \\ \gamma_{j-1} < z_i \leq \gamma_j \;&\Rightarrow\; y_i = j, && \forall i,\; j = 1, \cdots, J, \end{aligned} \tag{1}$$

where (in the first equation) $z_i$ is the latent variable for individual $i$, $x_i$ is a $k \times 1$ vector of covariates, $\beta$ is a $k \times 1$ vector of unknown parameters, and $n$ denotes the number of observations. The second equation shows how $z_i$ is related to the observed discrete response $y_i$, where $-\infty = \gamma_0 < \gamma_1 \cdots < \gamma_{J-1} < \gamma_J = \infty$ are the cut-points (or thresholds) and $y_i$ is assumed to have $J$ categories or outcomes.

(a) Assuming the error follows a standard normal distribution (i.e., $\epsilon_i \sim N(0,1)$ for $i = 1, 2, \cdots, n$), find the probability of success $\Pr(y_i = j)$? Derive the likelihood function for the ordinal probit model.

(b) Assuming the error follows a standard logistic distribution (i.e., $\epsilon_i \sim \mathcal{L}(0,1)$ for $i = 1, 2, \cdots, n$), find the probability of success $\Pr(y_i = j)$? Derive the likelihood function for the ordinal logit model.

(c) Consider the ordinal probit model. Show that adding a constant $c$ to the cut-point $\gamma_j$ and the mean $x_i'\beta$, does not change the outcome probability. How do we solve this first identification problem?

(d) Once again, consider the ordinal probit model. Show that rescaling the parameters $(\gamma_j, \beta)$ and the scale of the distribution by some arbitrary constant $d$ lead to same outcome probabilities. How do we solve the second identification problem?

(e) Consider the data present in the file `Feb14Data.xlsx`. This file contains 1,492 observations from the February 2014 Political Survey conducted during February 14-23, 2014, by the Princeton Survey Research Associates and sponsored by the Pew Research Center for the People and the Press. Based on this data, do the following. (i) Present a descriptive summary of the data as presented in Table 1 of the lecture slides. (ii) Please use the data to analyze public opinion on extent marijuana legalization in the US i.e., estimate Model 8 and replicate the results presented in Table 2 in lecture slides. (iii) Compute the covariate effects for variables presented in Table 3.

**2. (10+10+10 = 30 points)**. Consider the investment data present in the file `Grunfeld220obs.xlsx`. We will not use data for the firm, "American Steel" The objective is to study investment pattern of the remaining 10 firms. So, our dependent variable is `inv`.

(a) Estimate a pooled effects model on the covariates: `value` and `capital`. Summarize the results and interpret the coefficients.

(b) Now, consider the panel structure of the data. Estimate a fixed-effects model, using the `plm` function, by regressing `inv` on `value` and `capital`. Do not ignore the indexing of data by `firm` and `year`. Summarize the results and interpret the coefficients.

(c) Once again, consider the panel structure of the data. Estimate a random-effects model, using the `plm` function, by regressing `inv` on `value` and `capital`. Do not ignore the indexing of data by `firm` and `year`. Summarize the results and interpret the coefficients.