Optimization in Industry Assignment 1
Mathematical Formulation for Round Robin Match Scheduling

Utpalraj Kemprai
MDS202352

January 29, 2025

Sets

- T: Set of teams, indexed by i, j, where $i, j \in T$.
- D: Set of days in the tournament, indexed by d.

Parameters

- $I_{i,j}$ $(i \neq j)$: Binary parameter, 1 if the match between team i and j is interesting, 0 otherwise
- W_d: Binary parameter, 1 if d is a weekend, 0 otherwise.
- $\text{Dist}_{i,j}$: Distance between home stadiums of teams i and j.
- $E_{d,i}$: Binary parameter, 1 if there is a big event in the city of home stadium of team i on day d, 0 otherwise.
- MinGap, MaxGap: Minimum and maximum gap between two consecutive matches for a team.
- MaxConsecHome, MaxConsecAway: Integer parameter, maximum number of consecutive home matches and away matches.
- MaxNightMatches, MinNightMatches: Integer parameter, maximum and minimum number of night matches to be played by each team.
- M is the Big - M constant for our formulation.

Decision Variables

- $x_{i,j,d}$ $(i \neq j)$: Binary variable, 1 if match (day or night) is scheduled on day d at home stadium of i between team i amd j, 0 otherwise.
- $N_{i,j,d}$ $(i \neq j)$: Binary variable, 1 if there is a night match between team i and j at home stadium of i in day d, 0 otherwise.

- $\text{count}_{i,d}$: Integer variable to count the number of matches played by team i as of day d
- $\text{count\_home}_{i,d}$: Integer variable to count the number of home matches played by team i as of day d
- $\text{count\_away}_{i,d}$: Integer variable to count the number of away matches played by team i as of day d
- $L_{i,j,n}$ $(i \neq j)$: Binary variable, 1 if team j played their nth match in the home stadium of team i.

Objective Function

Maximize:

$$\alpha \sum_{i,j \in T; i \neq j} \sum_{d \in D} I_{i,j} \cdot x_{i,j,d} \cdot W_d$$

$$- \beta \sum_{2 \leq n \leq |T|-1} \Big(\sum_{k \in T} \sum_{i,j \in T; i \neq j; i,j \neq k} \text{dist}_{i,j} L_{i,k,n-1} L_{j,k,n} + \sum_{j \in T} \sum_{i \in T; i \neq j} \text{dist}_{i,j} L_{i,j,n-1} L_{j,j,n}$$

$$+ \sum_{j \in T} \sum_{i \in T; i \neq j} \text{dist}_{i,j} L_{j,j,n-1} L_{i,j,n} \Big)$$

$+$ Other terms for fair share of distances and match gaps

Constraints

1. **Each team plays all others twice (one home and one away):**

$$\sum_{d \in D} x_{i,j,d} = 1, \forall j \in T \quad (i \text{ plays } j \text{ exactly once at home})$$

$$\sum_{d \in D} x_{j,i,d} = 1, \forall j \in T \quad (i \text{ plays } j \text{ exactly once away})$$

2. **Each team plays at most one match per date**

$$\sum_{j \in T; i \neq j} x_{i,j,d} \leq 1, \forall i \in T \quad (i \text{ plays atmost one match each day})$$

3. **Only one match in a single day/night slot:**

   Non Linearity to convert to Linearity

$$\sum_{i,j \in T, i \neq j} x_{i,j,d}(1 - N_{i,j,d}) \leq 1, \forall d \in D$$

$$\sum_{i,j \in T, i \neq j} x_{i,j,d} N_{i,j,d} \leq 1, \forall d \in D$$

4. **Night Match is not possible if there is no match on that day**

$$N_{i,j,d} \leq x_{i,j,d} \quad \forall i, j \in T, d \in D \text{ and } i \neq j$$

5. **No overlapping of interesting matches:**

   Non Linearity to convert to Linearity

$$\sum_{i,j \in T, i \neq j} x_{i,j,d} I_{i,j} N_{i,j,d} \leq 1 \quad \forall d \in D$$

$$\sum_{i,j \in T, i \neq j} x_{i,j,d} I_{i,j} (1 - N_{i,j,d}) \leq 1 \quad \forall d \in D$$

6. **Count of Home and Away matches**

$$\text{count\_home}_{i,d} = \sum_{d' \leq d} \sum_{j \in T; j \neq i} x_{i,j,d'} \quad \forall i \in T, \forall d \in D$$

$$\text{count\_away}_{i,d} = \sum_{d' \leq d} \sum_{j \in T; j \neq i} x_{j,i,d'} \quad \forall u \in T, \forall d \in D$$

$$\text{count}_{i,d} = \text{count\_home}_{i,d} + \text{count\_away}_{i,d} \quad \forall i \in T, \forall d \in D$$

7. **Bounds on gaps between consecutive matches:**

$$\text{count}_{i,d'} - \text{count}_{i,d} = 0 \quad \forall i \in T \text{ and } \forall d, d' \in D \text{ such that } d' \geq d \text{ and } d' - d \leq \text{MinGap}$$

$$\text{count}_{i,d'} - \text{count}_{i,d} \geq 0 \quad \forall i \in T \text{ and } \forall d, d' \in D \text{ such that } d' \geq d \text{ and } d' - d \geq \text{MinGap}$$

$$\text{count}_{i,d+\text{MaxGap}} - \text{count}_{i,d} \geq 1 \quad \forall i \in T \text{ and } \forall d \in D \setminus \{d : d > \text{end\_date} - \text{MaxGap}\}$$

8. **Home/away match balance:**

$$\text{count\_home}_{i,d} - \text{count\_away}_{i,d} \leq \text{MaxConsecHome} \quad \forall i \in T, \forall d \in D$$

$$\text{count\_away}_{i,d} - \text{count\_home}_{i,d} \leq \text{MaxConsecAway} \quad \forall i \in T, \forall d \in D$$

9. **Avoid matches on restricted days:**

$$x_{i,j,d} \leq 1 - E_{d,i} \quad \forall d \in D, \forall i, j \in T, i \neq j$$

10. **Match relocation day for incase of rain:**

$$x_{i,j,d} + x_{i,j,d+1} \leq 1 \quad \forall d \in D \setminus \{\text{end\_date}\}, \forall i, j \in T, i \neq j$$

11. **Day and night match balance:**

$$\sum_{d \in D} \sum_{j \in T: j \neq i} (N_{i,j,d} + N_{j,i,d}) \leq \text{MaxNightMatches} \quad \forall j \in T$$

$$\sum_{d \in D} \sum_{j \in T: j \neq i} (N_{i,j,d} + N_{j,i,d}) \geq \text{MinNightMatches} \quad \forall j \in T$$

12. **Match sequence Constraints**

$$\sum_{d \in D} x_{i,j,d} (count_{j,d} - n + 1) \leq 1 + M(1 - L_{i,j,n}) \quad \forall i, j \in T, i \neq j, \forall 1 \leq n \leq |T| - 1$$

$$\sum_{d \in D} x_{i,j,d} (count_{j,d} - n + 1) \geq 1 - M(1 - L_{i,j,n}) \quad \forall i, j \in T, i \neq j, \forall 1 \leq n \leq |T| - 1$$

$$\sum_{i \in T; i \neq j} \sum_{d \in D} x_{j,i,d} (count_{j,d} - n + 1) \leq 1 + M(1 - L_{j,j,n}) \quad \forall j \in T, \forall 1 \leq n \leq |T| - 1$$

$$\sum_{i \in T; i \neq j} \sum_{d \in D} x_{j,i,d} (count_{j,d} - n + 1) \geq 1 - M(1 - L_{j,j,n}) \quad \forall j \in T, \forall 1 \leq n \leq |T| - 1$$

Assumptions

- Interesting matches are known in advanced and depends on which two teams are playing only.
- Stadiums are uniformly distributed across zones.
- Travel distances between stadiums are known and constant.
- Each team has only one home stadium.
- MinGap and MaxGap values are set based on league regulations.
- MaxConsecHome and MaxConsecAway are set basedon league regulations.
- Day and night slots for matches of a day do not overlap with each other.
- MaxNightMatches and MinNightMatches are set in advanced based on league regulations.
- The days from the start and end date of the round robin are coded as integers i.e. $D = \{1, 2, 3, \cdots\}$.
- $\forall d \in D, d + 1,$ is the immediate next day
- end_date is the last date in D.
- The Big-M constant M is sufficiently large enough.