Optimization in Industry
Mid Semester Exam

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Question 1

We will introduce a binary variable $I \in \{0, 1\}$. And set the following linear constraint.

$$x = 15 + 5 \times I$$

This will ensure that x is either 15 or 20.

Question 2

(A) A and B are parallel

- Net Lead Time = max(LT_A, LT_B)
- Net Capacity = Cap_A + Cap_B

Assumption for the parallel case

Both processes A and B must be completed before operation X is considered complete.

(B) A and B are in series

- Net Lead Time = LT_A + LT_B
- Net Capacity = min(Cap_A, Cap_B)

Question 3

As X and Y are integers, 2X + 2Y is an even integer and so cannot be equal to 35. Since 2X+2Y is always even, we can tighten the constraint by replacing 35 with 34 in the original constraint.

So the new tighter constraint is,

$$2X + 2Y \leq 34$$
$$\text{or } X + Y \leq 17$$

Question 4

Sets

- I: set of items indexed by i.
- D: set of days indexed by d, where d ranges from the item's availDate to its shipDate and has been coded as an integer (i.e., $d = 1, 2, \cdots$). Here 1 corresponds to the earliest date in D, 2 to the day immediately after and so on. D contains all the dates from the earliest availDate (of the items) to the latest shipDate (of the items).
- T: set of Trucks available indexed by t.

Parameters

- $w_i > 0$: weight of item i.
- avail_i: earliest possible shipping day for item i.
- ship_i: latest shipping day for item i.
- Truck load bounds: $L_{\text{min},t}$ and $L_{\text{max},t}$, minimum and maximum truck load for truck t.

Decision Variables

- $x_{i,d,t} \in \{0, 1\}$: 1 if item i is shipped on day d by truck t, otherwise 0.
- $y_{t,d} \in \{0, 1\}$: 1 if the truck t is dispatched on day d, otherwise 0.

Constraints

1. **Item Assignment:**

$$\sum_{d=\text{avail}_i}^{\text{ship}_i} \sum_{t \in T} x_{i,d,t} = 1, \quad \forall i \in I$$

$$x_{i,d,t} = 0, \quad d \notin \{\text{avail}_i, \text{avail}_i + 1, \cdots, \text{ship}_i\}, \forall i \in I, \forall t \in T$$

   Explanation

   The above constraints ensure that each item i will be shipped in one of the dates from avail_i to ship_i.

2. **Truck Capacity (if the truck is dispatched):**

$$\sum_{i \in I} w_i \cdot x_{i,d,t} \leq L_{\text{max},t} \cdot y_{t,d}, \quad \forall d \in D, \forall t \in T$$

$$\sum_{i \in I} w_i \cdot x_{i,d,t} \geq L_{\text{min},t} \cdot y_{t,d}, \quad \forall d \in D, \forall t \in T$$

Explanation

The above constraints ensure that for each scheduled dispatch for a truck t, the items assigned exceed the minimum load but do not exceed its maximum load. It also ensures that no items are assigned to a truck if the truck is not dispatched.

Objective

Minimize the cumulative time between the date of shipment of items and their availDate:

$$\min \sum_{d \in D} \sum_{i \in I} \sum_{t \in T} (d - \text{avail}_i) \cdot x_{i,d,t}$$

Question 5

Possible Constraints

- **Limited aircrafts:** Total aircrafts assigned $\leq$ number of available aircrafts.
- **Flight coverage:** Each scheduled flight must be assigned an aircraft.
- **Meeting demand:** Matching aircraft type (90-seat or 180-seat) with flight demand.
- **Maintenance and crew:** Scheduling constraint for maintenance and crew availability. e.g. time between consecutive flights, number of flights in a day etc.
- **Operational Regulations:** Compliance with regulatory requirements.
- **Boarding and Unboarding Constraints:** Time window for the passengers to board and unboard the flights.

Possible Objectives

- **Maximize Revenue/Profit:** By optimally matching aircraft to flight demand.
- **Minimize Costs:** Including fuel, crew and maintenance costs.
- **Enhance Robustness:** Mitigate delays and disruptions while maintaining the schedule.