Optimization in Industry Assignment 1
Mathematical Formulation for Round Robin Match Scheduling

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Sets

- T: Set of teams, indexed by i, j, where $i, j \in T$.
- D: Set of days in the tournament, indexed by d.

Parameters

- $I_{i,j}$ $(i \neq j)$: Binary parameter, 1 if the match between team i and j is interesting, 0 otherwise
- W_d: Binary parameter, 1 if d is a weekend, 0 otherwise.
- $\text{Dist}_{i,j}$: Distance between home stadiums of teams i and j.
- $E_{d,i}$: Binary parameter, 1 if there is a big event in the city of home stadium of team i on day d, 0 otherwise.
- $\text{MinGap}, \text{MaxGap}$: Minimum and maximum gap between two consecutive matches for a team.
- MaxConsecHome, MaxConsecAway: Integer parameter, maximum number of consecutive home matches and away matches.
- MaxNightMatches, MinNightMatches: Integer parameter, maximum and minimum number of night matches to be played by each team.
- MaxSlot: Integer parameter, maximum number of matches allowed to be played simultaneouly in a single time slot (day/night).
- M is the Big - M constant for our formulation.

Decision Variables

- $x_{i,j,d}$ $(i \neq j)$: Binary variable, 1 if match (day or night) is scheduled on day d at home stadium of i between team i amd j, 0 otherwise.
- $N_{i,j,d}$ $(i \neq j)$: Binary variable, 1 if there is a night match between team i and j at home stadium of i in day d, 0 otherwise.

- $\text{count}_{i,d}$: Integer variable to count the number of matches played by team i as of day d
- $\text{count\_home}_{i,d}$: Integer variable to count the number of home matches played by team i as of day d
- $\text{count\_away}_{i,d}$: Integer variable to count the number of away matches played by team i as of day d
- $L_{i,j,n}$: Binary variable, 1 if team j played their nth match of the round robin in the home stadium of team i.
- $\text{Travel}_{i,j,k,n}(n \geq 2)$: Binary variable, 1 if team k travel from home stadium of team i to home stadium of team j to play their nth match, i.e. $\text{Travel}_{i,j,k,n} = L_{i,k,n-1} \cdot L_{j,k,n}$.
- $\text{Match\_diff}_{i,j,d}$: Continuous variable, which takes the value $|\text{count}_{i,d} - \text{count}_{j,d}|$
- $\text{Max\_match\_difference}_d, \text{Min\_match\_difference}_d$: Continuous variable, Max and Min difference in the number of matches played by two teams as of day d.
- $\text{Home\_away\_diff}_{i,d}$: Continuous variable, which takes the value $|\text{count\_away}_{i,d} - \text{count\_home}_{i,d}|$
- $\text{Max\_home\_away\_diff}_d, \text{Min\_home\_away\_diff}_d$: Continuous variable, Max and Min differences between the number of home and away matches played by a team as of day d.
- MaxDist, MinDist: Continuous variable, maximum and minimum distance covered by a team.
- max_night_played, min_night_played: Integer variable, max and min number of night matches played by a team.
- $s_{i,j,d}$: Binary variable, 1 if $\text{count}_{j,d} \geq \text{count}_{i,d}$, else 0.
- $t_{i,d}$: Binary variable, 1 if $\text{count\_home}_{i,d} \geq \text{count\_away}_{i,d}$, else 0.
- $\text{XN}_{i,j,d}(i \neq j)$: Binary variable, 1 if $x_{i,j,d} \cdot N_{i,j,d} = 1$, else 0.
- $\text{XC1}_{i,j,d}(i \neq j)$: Integer variable, equal to $x_{i,j,d} \cdot \text{count}_{j,d}$
- $\text{XC2}_{j,i,d}(i \neq j)$: Integer variable, equal to $x_{j,i,d} \cdot \text{count}_{j,d}$

Objective Function

Maximize:

$$\alpha_1 \cdot \sum_{i,j \in T; i \neq j} \sum_{d \in D} I_{i,j} \cdot x_{i,j,d} \cdot W_d - \alpha_2 \cdot \sum_{n=2}^{2(|T|-1)} \sum_{k \in T} \sum_{i,j \in T; i \neq j} \text{dist}_{i,j} \text{Travel}_{i,j,k,n}$$
$$- \alpha_3 \cdot \sum_{d \in D} (\text{Max\_match\_difference}_d - \text{Min\_match\_difference}_d) - \alpha_4 \cdot (\text{MaxDist} - \text{MinDist})$$
$$- \alpha_5 \cdot \sum_{d \in D} (\text{Max\_home\_away\_diff}_d - \text{Min\_home\_away\_diff}_d)$$
$$- \alpha_6 \cdot (\text{max\_night\_played} - \text{min\_night\_played})$$

where $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$ are coefficients which determine the individual weights of the individual objective functions and $\alpha_i \geq 0, \forall 1 \leq i \leq 6$.

Constraints

1. **Each team plays all others twice (one home and one away):**

$$\sum_{d \in D} x_{i,j,d} = 1, \forall j \in T \quad (i \text{ plays } j \text{ exactly once at home})$$
$$\sum_{d \in D} x_{j,i,d} = 1, \forall j \in T \quad (i \text{ plays } j \text{ exactly once away})$$

2. **Each team plays at most one match per date**

$$\sum_{j \in T; i \neq j} (x_{i,j,d} + x_{j,i,d}) \leq 1, \forall i \in T \quad (i \text{ plays atmost one match each day, home or away})$$

3. **Number of matches in a single day/night slot:**

$$\mathrm{XN}_{i,j,d} \leq x_{i,j,d} \quad \forall i, j \in T, i \neq j; \forall d \in D$$
$$\mathrm{XN}_{i,j,d} \leq N_{i,j,d} \quad \forall i, j \in T, i \neq j; \forall d \in D$$
$$\mathrm{XN}_{i,j,d} \geq x_{i,j,d} + N_{i,j,d} - 1 \quad \forall i, j \in T, i \neq j; \forall d \in D$$
$$\sum_{i,j \in T, i \neq j} (x_{i,j,d} - \mathrm{XN}_{i,j,d}) \leq \mathrm{MaxSlot}, \forall d \in D \quad (\text{Bound on number of day matches on day d})$$
$$\sum_{i,j \in T, i \neq j} \mathrm{XN}_{i,j,d} \leq \mathrm{MaxSlot}, \forall d \in D \quad (\text{Bound on number of night matches on day d})$$

For the above constraints,

- The first three constraints ensure that $\mathrm{XN}_{i,j,d} = x_{i,j,d} \cdot N_{i,j,d}$
- The last two constraints ensure that $\sum_{i,j \in T; i \neq j} x_{i,j,d}(1 - N_{i,j,d}) \leq \mathrm{MaxSlot}$ and $\sum_{i,j \in T; i \neq j} x_{i,j,d} \cdot N_{i,j,d} \leq \mathrm{MaxSlot}$

4. **Number of Day and Night matches played by a team should be balanced across teams:**

$$\sum_{d \in D} \sum_{j \in T; j \neq i} (N_{i,j,d} + N_{j,i,d}) \leq \text{max\_night\_played} \quad \forall i \in T$$
$$\sum_{d \in D} \sum_{j \in T; j \neq i} (N_{i,j,d} + N_{j,i,d}) \geq \text{min\_night\_played} \quad \forall i \in T$$

5. **Night Match is not possible if there is no match on that day**

$$N_{i,j,d} \leq x_{i,j,d} \quad \forall i, j \in T, d \in D \text{ and } i \neq j$$

6. **No overlapping of interesting matches:**

$$\sum_{i,j \in T, i \neq j} I_{i,j} \mathrm{XN}_{i,j,d} \leq 1 \quad \forall d \in D \quad (\text{At most one interesting night match on a weekend})$$
$$\sum_{i,j \in T, i \neq j} I_{i,j} (x_{i,j,d} - \mathrm{XN}_{i,j,d}) \leq 1 \quad \forall d \in D \quad (\text{At most one interesting day match on a weekend})$$

7. **Count of Home and Away matches**

$$\text{count\_home}_{i,d} = \sum_{d' \leq d} \sum_{j \in T; j \neq i} x_{i,j,d'} \quad \forall i \in T, \forall d \in D$$
$$\text{count\_away}_{i,d} = \sum_{d' \leq d} \sum_{j \in T; j \neq i} x_{j,i,d'} \quad \forall u \in T, \forall d \in D$$
$$\text{count}_{i,d} = \text{count\_home}_{i,d} + \text{count\_away}_{i,d} \quad \forall i \in T, \forall d \in D$$

8. **Number of matches played by teams should be equally distributed at any point of time in the Round Robin**

$$\text{Match\_diff}_{i,j,d} \geq \text{count}_{i,d} - \text{count}_{j,d} \quad \forall i,j \in T, i \neq j; d \in D$$
$$\text{Match\_diff}_{i,j,d} \geq \text{count}_{j,d} - \text{count}_{i,d} \quad \forall i,j \in T, i \neq j; d \in D$$
$$\text{Match\_diff}_{i,j,d} \leq \text{count}_{i,d} - \text{count}_{j,d} + Ms_{i,j,d} \quad \forall i,j \in T, i \neq j; d \in D$$
$$\text{Match\_diff}_{i,j,d} \leq \text{count}_{j,d} - \text{count}_{i,d} + M(1 - s_{i,j,d}) \quad \forall i,j \in T, i \neq j; d \in D$$
$$\text{Match\_diff}_{i,j,d} \leq \text{Max\_match\_difference}_{d} \quad \forall i,j \in T, i \neq j; d \in D$$
$$\text{Match\_diff}_{i,j,d} \geq \text{Min\_match\_difference}_{d} \quad \forall i,j \in T, i \neq j; d \in D$$

9. **Bounds on gaps between consecutive matches:**

$$\text{count}_{i,d'} - \text{count}_{i,d} = 0 \quad \forall i \in T \text{ and } \forall d, d' \in D \text{ such that } d' \geq d \text{ and } d' - d \leq \text{MinGap}$$
$$\text{count}_{i,d'} - \text{count}_{i,d} \geq 0 \quad \forall i \in T \text{ and } \forall d, d' \in D \text{ such that } d' \geq d \text{ and } d' - d \geq \text{MinGap}$$
$$\text{count}_{i,d+\text{MaxGap}} - \text{count}_{i,d} \geq 1 \quad \forall i \in T \text{ and } \forall d \in D \setminus \{d : d > \text{end\_date} - \text{MaxGap}\}$$

10. **Bound on number of consecutive home and away matches played:**

$$\text{count\_home}_{i,d} - \text{count\_away}_{i,d} \leq \text{MaxConsecHome} - 1 \quad \forall i \in T, \forall d \in D$$
$$\text{count\_away}_{i,d} - \text{count\_home}_{i,d} \leq \text{MaxConsecAway} - 1 \quad \forall i \in T, \forall d \in D$$

11. **Home and away matches played by a team should be distributed in time:**

$$\text{Home\_away\_diff}_{i,d} \geq \text{count\_away}_{i,d} - \text{count\_home}_{i,d} \quad \forall i \in T, \forall d \in D$$
$$\text{Home\_away\_diff}_{i,d} \geq \text{count\_home}_{i,d} - \text{count\_away}_{i,d} \quad \forall i \in T, \forall d \in D$$
$$\text{Home\_away\_diff}_{i,d} \leq \text{count\_away}_{i,d} - \text{count\_home}_{i,d} + Mt_{i,d} \quad \forall i \in T, \forall d \in D$$
$$\text{Home\_away\_diff}_{i,d} \leq \text{count\_home}_{i,d} - \text{count\_away}_{i,d} + M(1 - t_{i,d}) \quad \forall i \in T, \forall d \in D$$
$$\text{Home\_away\_diff}_{i,d} \leq \text{Max\_home\_away\_diff}_{d} \quad \forall i \in T; \forall d \in D$$
$$\text{Home\_away\_diff}_{i,d} \geq \text{Min\_home\_away\_diff}_{d} \quad \forall i \in T; \forall d \in D$$

12. **Avoid matches on restricted days:**

$$x_{i,j,d} \leq 1 - E_{d,i} \quad \forall d \in D, \forall i,j \in T, i \neq j$$

13. **Match relocation day for incase of rain/other disruptions:**

$$x_{i,j,d} + x_{i,j,d+1} \leq 1 \quad \forall d \in D \setminus \{\text{end\_date}\}, \forall i,j \in T, i \neq j$$

14. **Bound on number of Night matches:**

$$\sum_{d\in D}\sum_{j\in T:j\neq i}(N_{i,j,d}+N_{j,i,d}) \leq \text{MaxNightMatches} \quad \forall i\in T$$
$$\sum_{d\in D}\sum_{j\in T:j\neq i}(N_{i,j,d}+N_{j,i,d}) \geq \text{MinNightMatches} \quad \forall i\in T$$

15. **Match order Constraints**

$$\begin{aligned}
&\text{XC1}_{i,j,d} \leq \text{count}_{j,d} + M(1-x_{i,j,d}) \quad \forall i,j\in T, i\neq j; \forall d\in D\\
&\text{XC1}_{i,j,d} \geq \text{count}_{j,d} - M(1-x_{i,j,d}) \quad \forall i,j\in T, i\neq j; \forall d\in D\\
&\text{XC1}_{i,j,d} \leq Mx_{i,j,d} \quad \forall i,j\in T, i\neq j; \forall d\in D\\
&\text{XC2}_{j,i,d} \leq \text{count}_{j,d} + M(1-x_{j,i,d}) \quad \forall i,j\in T, i\neq j; \forall d\in D\\
&\text{XC2}_{j,i,d} \geq \text{count}_{j,d} - M(1-x_{j,i,d}) \quad \forall i,j\in T, i\neq j; \forall d\in D\\
&\text{XC2}_{j,i,d} \leq Mx_{j,i,d} \quad \forall i,j\in T, i\neq j; \forall d\in D
\end{aligned}$$
$$\sum_{d\in D}(\text{XC1}_{i,j,d} - (n-1)x_{i,j,d}) \leq 1 + M(1-L_{i,j,n}) \quad \forall i,j\in T, i\neq j, \forall 1\leq n\leq |T|-1$$
$$\sum_{d\in D}(\text{XC1}_{i,j,d} - (n-1)x_{i,j,d}) \geq 1 - M(1-L_{i,j,n}) \quad \forall i,j\in T, i\neq j, \forall 1\leq n\leq |T|-1$$
$$\sum_{i\in T;i\neq j}\sum_{d\in D}(\text{XC2}_{j,i,d} - (n-1)x_{j,i,d}) \leq 1 + M(1-L_{j,j,n}) \quad \forall j\in T, \forall 1\leq n\leq |T|-1$$
$$\sum_{i\in T;i\neq j}\sum_{d\in D}(\text{XC2}_{j,i,d} - (n-1)x_{j,i,d}) \geq 1 - M(1-L_{j,j,n}) \quad \forall j\in T, \forall 1\leq n\leq |T|-1$$

For the above constraints,

- The first six constraints ensure $XC1_{i,j,d} = x_{i,j,d}\cdot \text{count}_{j,d}$ and $XC2_{j,i,d} = x_{j,i,d}\cdot \text{count}_{j,d}$.
- The next two constraints ensure that $\sum_{d\in D} x_{i,j,d}\cdot(\text{count}_{j,d} - n + 1) = 1$, only if $L_{i,j,n} = 1$.
- The last two constraints ensure that $\sum_{i\in T;i\neq j}\sum_{d\in D} x_{j,i,d}\cdot(\text{count}_{j,d} - n + 1) = 1$, only if $L_{j,j,n} = 1$.

16. **Travel Constraints**

$$\begin{aligned}
&\text{Travel}_{i,j,k,n} \leq L_{i,k,n-1} \quad \forall i,j,k\in T, 2\leq n\leq 2(|T|-1)\\
&\text{Travel}_{i,j,k,n} \leq L_{j,k,n} \quad \forall i,j,k\in T, 2\leq n\leq 2(|T|-1)\\
&\text{Travel}_{i,j,k,n} \geq L_{i,k,n-1} + L_{j,k,n} - 1 \quad \forall i,j,k\in T, 2\leq n\leq 2(|T|-1)
\end{aligned}$$

The above constraints ensures that $\text{Travel}_{i,j,k,n} = L_{i,k,n-1}\cdot L_{j,k,n}$

17. **Distance travelled by all teams should be balance for all teams**

$$\sum_{n=2}^{2(|T|-1)}\sum_{i,j\in T;i\neq j}\text{dist}_{i,j}\text{Travel}_{i,j,k,n} \leq \text{MaxDist} \quad \forall k\in T$$
$$\sum_{n=2}^{2(|T|-1)}\sum_{i,j\in T;i\neq j}\text{dist}_{i,j}\text{Travel}_{i,j,k,n} \geq \text{MinDist} \quad \forall k\in T$$

Assumptions

- Interesting matches are known in advanced and depends on which two teams are playing only.
- Stadiums are uniformly distributed across zones.
- Travel distances between stadiums are known and constant.
- Each team has only one home stadium.
- MinGap and MaxGap values are set based on league regulations.
- MaxConsecHome and MaxConsecAway are set based on league regulations.
- Day and night slots for matches of a day do not overlap with each other.
- MaxNightMatches and MinNightMatches are set in advanced based on league regulations.
- The days from the start and end date of the round robin are coded as positive integers i.e. $D = \{1, 2, 3, \cdots\}$.
- $\forall d \in D, d + 1,$ is the immediate next day and so on.
- end_date is the last date in D.
- The Big-M constant, M is sufficiently large enough.
- $\text{dist}_{i,i} = 0 \quad \forall i \in T$.
- After playing a match at a venue, the teams directly travel to the venue where their next match is scheduled, or don't travel if there is next match is in the same venue. This assumption ensures that $\text{Travel}_{i,j,k,n} = L_{i,k,n-1} \cdot L_{j,k,n}$
- All teams are in the venue where their first match is scheduled at the start of the round robin. The total distance considered is the distance travelled after the first match till the end of the round robin.