Optimization Assignment 4
School Timetabling Optimization Problem

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Sets

- S: Set of subjects, $S = \{1, 2, \cdots, 10\}$
- C: Set of sections, $C = \{1, 2, 3, 4\}$
- D: Set of days, $D = \{1, 2, 3, 4, 5\}$
- T: Timeslots of a day, $T = \{1, 2, \cdots, 8\}$
- S_{sci}: Science subjects
- S_{nonsci}: Non-science subjects
- $T_{\text{morning}} = \{1,2,3,4\}$
- $T_{\text{afternoon}} = \{5,6,7,8\}$

Decision Variable

- $x_{s,c,d,t}$: Binary variable, 1 if subject s is assigned to section c on day d at timeslot t, 0 otherwise.
- $p_{s,c,d,t}$: Binary variable, 1 if practical session for subject s is assigned to section c on day d at timeslot t, 0 otherwise.
- $y^{\text{sci}}_{c,d,t}$: Binary variable, 1 if a science subject is assigned to section c on day d at times t, 0 otherwise.
- $y^{\text{nonsci}}_{c,d,t}$: Binary variable, 1 if a non-science subject is assigned to section c on day d at times t, 0 otherwise.
- $z^{\text{sci}}_{c,d,t} (t \in \{1, 2, \cdots, 7\})$: Binary variable, 1 if consecutive science subjects were assigned to section c on day d in timeslots t and $t + 1$, otherwise 0.
- $z^{\text{nonsci}}_{c,d,t} (t \in \{1, 2, \cdots, 7\})$: Binary variable, 1 if consecutive science subjects were assigned to section c on day d in timeslots t and $t + 1$, otherwise 0.
- $n_{s,t}$: Number of times subject s is assigned to timeslot t in the week.
- u_s: Maximum number of times subject s is assigned to a single timeslot.

Explanation: The x-variables represent the assignment of subjects to timeslots across sections. The p-variables represent the allocation of practicals for the subjects. The y-variables represents if the allocated subjects are science or non-science. The z variables are used to penalize consecutive science and non-science allocation. And finally the n and u variables are used to promote the diversity of timeslots for the subjects.

Objective Function

We want to create an optimal timetable by minimizing:

1. Number of practicals scheduled before lunch.
2. Number of consecutive science/non-science classes.
3. Repetitive scheduling of subjects at the same timeslot.

A weighted objective function is given by,

$$\min \left(\alpha_1 \sum_{s \in S} \sum_{d \in D} \sum_{c \in C} \sum_{t \in T_{\text{morning}}} p_{s,c,d,t} + \alpha_2 \sum_{s \in S} \sum_{d \in D} \sum_{t=1}^{7} \sum_{c \in C} \left(z^{\text{sci}}_{c,d,t} + z^{\text{nonsci}}_{c,d,t} \right) + \alpha_3 \sum_{s \in S} u_s \right)$$

where α_1, α_2 and α_3 are positive weights reflecting the importance of each objective.
Explanation: The first term penalizes the allocation of practicals before lunch, the second term penalizes consecutive science and non-science classes and the third term penalizes the number of times a subject is allocated to a single time slot across the days of the week.

Constraints

1. Weekly Class Requirements

$$\sum_{d \in D} \sum_{t \in T} x_{s,c,d,t} = 4 \quad \forall s \in S, \forall c \in C$$

Explanation:

Each subject must have exactly 4 classes per week for each section.

2. Science Subjects' Practical Requirements

$$\sum_{d \in D} \sum_{t \in T} p_{s,c,d,t} = 1 \quad \forall s \in S_{\text{sci}}$$

Explanation:

Ensures each science subject has exactly one practical session per week for each section.

3. Non-Science Subjects Have No Practicals

$$p_{s,c,d,t} = 0 \quad \forall s \in S_{\text{nonsci}}, c \in C, d \in D, t \in T$$

Explanation:

Ensures that non-science have no practicals.

4. At most one class per subject per day for each section:

$$\sum_{t \in T} x_{s,c,d,t} \leq 1 \quad \forall s \in S, \forall c \in C, \forall d \in D$$

Explanation:

Ensure that each subject has atmost one class in a day for all the sections.

5. Atmost one practical per section per day:

$$\sum_{s \in S_{\text{sci}}} \sum_{t \in T} p_{s,c,d,t} \leq 1 \quad \forall c \in C, \forall d \in D$$

Explanation:

Ensures that each section has atmost one practical session in a day.

6. Practical Sessions Must Be Scheduled During Regular Classes

$$p_{s,c,d,t} \leq x_{s,c,d,t} \quad \forall s \in S, c \in C, d \in D, t \in T$$

Explanation:

Ensures that practical session of a subject is scheduled in one of the regular classes for each section.

7. No overlapping subjects

$$\sum_{s \in S} x_{s,c,d,t} \leq 1 \quad \forall c \in C, d \in D, t \in T$$

Explanation:

Ensures that each section can have atmost one subject in any timeslot.

8. Laboratory Capacity Constraints

$$\sum_{c \in C} p_{s,c,d,t} \leq 1 \quad \forall s \in S_{\text{sci}}, d \in D, t \in T$$

Explanation:

Since there's only one practical laboratory per science subject, no more than one section can have a practical for the same subject at the same time.

9. No consecutive practical classes

$$\sum_{s \in S_{\text{sci}}} (p_{s,c,d,t} + p_{s,c,d,t+1}) \leq 1 \quad \forall c \in C, d \in D, t \in \{1, 2, \cdots, 7\}$$

Explanation:

Ensures that no section has consecutive practical sessions.

10. Track consecutive science and non-science classes

$$\begin{aligned}
y^{\text{sci}}_{c,d,t} &= \sum_{s \in S_{\text{sci}}} x_{s,c,d,t} \quad \forall c \in C, t \in T, d \in D \\
y^{\text{nonsci}}_{c,d,t} &= \sum_{s \in S_{\text{nonsci}}} x_{s,c,d,t} \quad \forall c \in C, t \in T, d \in D \\
z^{\text{sci}}_{c,d,t} &\geq y^{\text{sci}}_{c,d,t} + y^{\text{sci}}_{c,d,t+1} - 1 \quad \forall c \in C, t \in T, d \in D \\
z^{\text{nonsci}}_{c,d,t} &\geq y^{\text{nonsci}}_{c,d,t} + y^{\text{nonsci}}_{c,d,t+1} - 1 \quad \forall c \in C, t \in T, d \in D
\end{aligned}$$

Explanation:

- The first two constraints ensures that $y^{\text{sci}}_{c,d,t}$ (or $y^{\text{nonsci}}_{c,d,t}$) is 1 if a science (or non-science) is scheduled for section c on day d for timeslot t and 0 otherwise.

  $y^{\text{sci}}_{c,d,t}$ and $y^{\text{nonsci}}_{c,d,t}$ basically track which type of subject is scheduled.

- The last two constraints ensures that $z^{\text{sci}}_{c,d,t}$ (or $z^{\text{nonsci}}_{c,d,t}$) is 1 if two consecutive science (or non-science) classes are scheduled for section c on day d for timeslots t and $t+1$. Otherwise, $z^{\text{sci}}_{c,d,t}$ (or $z^{\text{nonsci}}_{c,d,t}$) is 0.

  $z^{\text{sci}}_{c,d,t}$ and $z^{\text{nonsci}}_{c,d,t}$ basically identify when two consecutive periods have the same type of subjects for a section.

11. Tracking timeslot usage

$$\begin{aligned} n_{s,t} &= \sum_{c \in C} \sum_{d \in D} x_{s,c,d,t} \\ u_s &\geq n_{s,t} \quad \forall s \in S, t \in T \end{aligned}$$

Explanation:

- The first constraint ensures that $n_{s,t}$ is equal to the number of times subject s is scheduled for timeslot t in the week.
- The second constraint ensures that u_s is equal to $\max_{t \in T} n_{s,t}$ i.e. the maximum number of times subject s is scheduled in a single timeslot.

Necessary Assumptions

- Each subject requires exactly 4 sessions per week.
- Science subjects require exactly 1 practical session out of their 4 weekly sessions.
- Lunch break occurs between timeslots 4 and 5.
- All sections have identical timetabling requirements and must satisfy the same constraints.
- All constraints are equally applicable across all sections.
- The weights in the objective function $(\alpha_1, \alpha_2, \alpha_3)$ can be adjusted to prioritize different aspects of the schedule.