# Optimization Assignment 4
# School Timetabling Optimization Problem

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## Sets

- $S$: Set of subjects, $S = \{1, 2, \cdots, 10\}$
- $C$: Set of sections, $C = \{1, 2, 3, 4\}$
- $D$: Set of days, $D = \{1, 2, 3, 4, 5\}$
- $T$: Timeslots of a day, $T = \{1, 2, \cdots, 8\}$
- $S_{\text{sci}}$: Science subjects
- $S_{\text{nonsci}}$: Non-science subjects
- $T_{\text{morning}} = \{1,2,3,4\}$
- $T_{\text{afternoon}} = \{5,6,7,8\}$

## Parameter

- $M = 5$ (= no. of sections + 1): A sufficiently large constant (big-M) used for linearization.

## Decision Variable

- $x_{s,c,d,t}$: Binary variable, 1 if subject $s$ is assigned to section $c$ on day $d$ at timeslot $t$, 0 otherwise
- $p_{s,d,t}$: Binary variable, 1 if practical session for subject $s$ is assigned on day $d$ at timeslot $t$, 0 otherwise
- $y^{\text{sci}}_{c,d,t}$: Binary variable, 1 if a science subject is assigned to section $c$ on day $d$ at times $t$, 0 otherwise
- $y^{\text{nonsci}}_{c,d,t}$: Binary variable, 1 if a non-science subject is assigned to section $c$ on day $d$ at times $t$, 0 otherwise
- $z^{\text{sci}}_{c,d,t}(t \in \{1, 2, \cdots, 7\})$: Binary variable, 1 if consecutive science subjects were assigned to section $c$ on day $d$ in timeslots $t$ and $t + 1$, otherwise 0
- $z^{\text{nonsci}}_{c,d,t}(t \in \{1, 2, \cdots, 7\})$: Binary variable, 1 if consecutive science subjects were assigned to section $c$ on day $d$ in timeslots $t$ and $t + 1$, otherwise 0

- $n_{s,t}$: Number of times subject $s$ is assigned to timeslot $t$ in the week
- $u_s$: Maximum number of times subject $s$ is assigned to a single timeslot

**Explanation:** The $x$-variables represent the assignment of subjects to timeslots across sections. The $p$-variables represent the allocation of practicals for the subjects. The $y$-variables represents if the allocated subjects are science or non-science. The $y$ and $z$ variables are used to penalize consecutive science and non-science allocation. And finally the $n$ and $u$ variables are used to promote the diversity of timeslots for the subjects.

## Objective Function

We want to :

1. Minimize the number of practicals before lunch
2. Minimize the number of consecutive science and non-science classes
3. Diversify the time slots allocated for a subject

A weighted objective function is given by

$$\min \left( \alpha_1 \sum_{s\in S} \sum_{d\in D} \sum_{t\in T_{\text{morning}}} p_{s,d,t} + \alpha_2 \sum_{s\in S} \sum_{d\in D} \sum_{t=1}^{7} \sum_{c\in C} \left( z^{\text{sci}}_{c,d,t} + z^{\text{nonsci}}_{c,d,t} \right) + \alpha_3 \sum_{s\in S} u_s \right)$$

where $\alpha_1, \alpha_2$ and $\alpha_3$ are positive weights reflecting the importance of each objective.

**Explanation:** The first term penalizes the allocation of practicals before lunch, the second term penalizes consecutive science and non-science classes and the third term penalizes the number of times a subject is allocated to a particular time slots across the days of the week.

## Constraints

1. Each subject has 4 classes in a week:

$$\sum_{d\in D} \sum_{t\in T} x_{s,c,d,t} = 4 \quad \forall s \in \mathcal{S}, \forall c \in \mathcal{C}$$

   **Explanation:** Ensures each subject has 4 classes in a week.

2. Every science subject to have one practical in a week:

$$\sum_{d\in D} \sum_{t\in T} p_{s,d,t} = 1 \quad \forall s \in S_{\text{sci}}$$

   **Explanation:** Ensures each science subject has exactly one practical in a week.

3. Only science subjects have practicals

$$p_{s,d,t} = 0 \quad \forall s \in S_{\text{nonsci}}$$

   **Explanation:** Ensures that non-science have no practicals.

4. At most one class per subject per day:

$$\sum_{t \in T} x_{s,c,d,t} \leq 1 \quad \forall s \in S, \forall c \in C, \forall d \in D$$

**Explanation:** Each subject has atmost one class in a day for all the sections.

5. Atmost one practical per section per day:

$$\sum_{s \in \mathcal{S}_{\text{sci}}} \sum_{t \in \mathcal{T}} p_{s,d,t} \leq 1 \quad \forall c \in C, \forall d \in D$$

**Explanation:** Ensures that we have atmost one practical session in a day.

6. Practical session should take place in one of the scheduled class

$$p_{s,d,t} \leq x_{s,c,d,t} \quad \forall s \in S, c \in C, d \in D, t \in T$$

**Explanation:** Ensures practical session for a subject is scheduled in one of the timeslots where that subject is already scheduled.

7. Atmost one subject in a timeslot for a section

$$\sum_{s \in S} x_{s,c,d,t} \leq 1 \quad \forall c \in C, d \in D, t \in T$$

**Explanation:** Ensures that no overlap of subjects occurs for a timeslot for any Section.

8. Sections don't have same subject class simultaneously unless practical

$$\sum_{c \in C} x_{s,c,d,t} \geq 4 \times p_{s,d,t} \quad \forall s \in S, d \in D, t \in T$$

$$\sum_{c \in C} x_{s,c,d,t} \leq 1 + M \times p_{s,d,t} \quad \forall s \in S, d \in D, t \in T$$

**Explanation:**

- The first constraint ensures that if a practical for a subject is scheduled at a particular timeslot, it is scheduled for all the sections.
- The second constraint ensures that if no practical for a subject is scheduled, atmost one of the sections can have a class of that subject.

9. Consecutive science / non-science penalty

$$y^{\text{sci}}_{c,d,t} = \sum_{s \in S_{\text{sci}}} x_{s,c,d,t} \quad \forall c \in C, t \in T, d \in D$$
$$y^{\text{nonsci}}_{c,d,t} = \sum_{s \in S_{\text{nonsci}}} x_{s,c,d,t} \quad \forall c \in C, t \in T, d \in D$$
$$z^{\text{sci}}_{c,d,t} \geq y^{\text{sci}}_{c,d,t} + y^{\text{sci}}_{c,d,t+1} - 1 \quad \forall c \in C, t \in T, d \in D$$
$$z^{\text{nonsci}}_{c,d,t} \geq y^{\text{nonsci}}_{c,d,t} + y^{\text{nonsci}}_{c,d,t+1} - 1 \quad \forall c \in C, t \in T, d \in D$$

**Explanation:**

- The first two constraints ensures that $y^{\text{sci}}_{c,d,t}$ (or $y^{\text{nonsci}}_{c,d,t}$) is 1 if a science (or non-science) is scheduled for section $c$ on day $d$ for timeslot $t$ and 0 otherwise.
- The last two constraints ensures that $z^{\text{sci}}_{c,d,t}$ (or $z^{\text{nonsci}}_{c,d,t}$) is 1 if two consecutive science (or non-science) classes are scheduled for section $c$ on day $d$ for timeslots $t$ and $t+1$. Otherwise, $z^{\text{sci}}_{c,d,t}$ (or $z^{\text{nonsci}}_{c,d,t}$) is 0.

10. Timeslot diversity penalty

$$\begin{aligned} n_{s,t} &= \sum_{c\in C}\sum_{d\in D} x_{s,c,d,t} \\ u_s &\geq n_{s,t} \quad \forall s \in S, t \in T \end{aligned}$$

**Explanation:**

- The first constraint ensures that $n_{s,t}$ is equal to the number of times subject $s$ is scheduled for timeslot $t$ in the week.
- The second constraint ensures that $u_s$ is equal to $\max_{t\in T} n_{s,t}$.

## Necessary Assumptions

- Every subject requires exactly 4 sessions per week, with science subjects including exactly one practical.
- Lunch break perfectly divides the day into morning and afternoon blocks.
- All sections have identical timetabling requirements and must satisfy the same constraints.
- Soft penalties (consecutive runs, timeslot diversity) are weighted and balanced via $\alpha_i$'s chosen by the planner.