Optimization in Industry Assignment 1 Mathematical Formulation for Round Robin Match Scheduling

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Sets

- T: Set of teams, indexed by i, j, where $i, j \in T$.

- D: Set of days in the tournament, indexed by d.

Parameters

- $I_{i,j}$ $(i \neq j)$: Binary parameter, 1 if the match between team i and j is interesting, 0 otherwise

- W_d: Binary parameter, 1 if d is a weekend, 0 otherwise.

- $\text{Dist}_{i,j}$: Distance between home stadiums of teams i and j.

- $E_{d,i}$: Binary parameter, 1 if there is a big event in the city of home stadium of team i on day d, 0 otherwise.

- $\text{MinGap}, \text{MaxGap}$: Minimum and maximum gap between two consecutive matches for a team.

- $\text{MaxConsecHome}, \text{MaxConsecAway}$: Integer parameter, maximum number of consecutive home matches and away matches.

- $\text{MaxNightMatches}, \text{MinNightMatches}$: Integer parameter, maximum and minimum number of night matches to be played by each team.

Decision Variables

- $x_{i,j,d}$ $(i \neq j)$: Binary variable, 1 if match (day or night) is scheduled on day d at home stadium of i between team i amd j, 0 otherwise.

- $N_{i,j,d}$ $(i \neq j)$: Binary variable 1 if there is a night match between team i and j at home stadium of i in day d, 0 otherwise.

- $\text{count}_{i,d}$: Integer variable to count the number of matches played by team i as of day d

- count_home$_{i,d}$: Integer variable to count the number of home matches played by team i as of day d

- count_away$_{i,d}$: Integer variable to count the number of away matches played by team i as of day d

- $L_{i,j,d}$ $(i \neq j)$: Binary variable 1 if team j is in home location of team i on day d.

Objective Function

$$\text{Maximize:} \quad \sum_{i,j \in T; i \neq j} \sum_{d \in D} I_{i,j} \cdot x_{i,j,d} \cdot W_d$$

Constraints

1. **Each team plays all others twice (one home and one away):**

$$\sum_{d \in D} x_{i,j,d} = 1, \forall j \in T \quad (i \text{ plays } j \text{ exactly once at home})$$

$$\sum_{d \in D} x_{j,i,d} = 1, \forall j \in T \quad (i \text{ plays } j \text{ exactly once away})$$

2. **Each team plays at most one match per date**

$$\sum_{j \in T; i \neq j} x_{i,j,d} \leq 1, \forall i \in T \quad (i \text{ plays atmost one match each day})$$

3. **Only one match in a single day/night slot:**

$$\sum_{i,j \in T, i \neq j} x_{i,j,d}(1 - N_{i,j,d}) \leq 1, \forall d \in D$$

$$\sum_{i,j \in T, i \neq j} x_{i,j,d} N_{i,j,d} \leq 1, \forall d \in D$$

4. **Night Match is not possible if there is no match on that day**

$$N_{i,j,d} \leq x_{i,j,d} \quad \forall i, j \in T, d \in D \text{ and } i \neq j$$

5. **No overlapping of interesting matches:**

$$\sum_{i,j \in T, i \neq j} x_{i,j,d} I_{i,j} N_{i,j,d} \leq 1 \quad \forall d \in D$$

$$\sum_{i,j \in T, i \neq j} x_{i,j,d} I_{i,j}(1 - N_{i,j,d}) \leq 1 \quad \forall d \in D$$

6. **Count of Home and Away matches**

$$\text{count\_home}_{i,d} = \sum_{d' \leq d} \sum_{j \in T; j \neq i} x_{i,j,d'}$$

$$\text{count\_away}_{i,d} = \sum_{d' \leq d} \sum_{j \in T; j \neq i} x_{j,i,d'}$$

$$\text{count}_{i,d} = \text{count\_home}_{i,d} + \text{count\_away}_{i,d}$$

7. **Location of the team:**

$$L_{i,j,d} = x_{i,j,d} \quad \forall i, j \in T; i \neq j \text{ and } d \in D.$$

8. **Bounds on gaps between consecutive matches:**

$$\text{count}_{i,d'} - \text{count}_{i,d} = 0 \quad \forall i \in T \text{ and } \forall d, d' \in D \text{ such that } d' \geq d \text{ and } d' - d \leq \text{MinGap}$$
$$\text{count}_{i,d'} - \text{count}_{i,d} \geq 0 \quad \forall i \in T \text{ and } \forall d, d' \in D \text{ such that } d' \geq d \text{ and } d' - d \geq \text{MinGap}$$
$$\text{count}_{i,d+\text{MaxGap}} - \text{count}_{i,d} \geq 1 \quad \forall i \in T \text{ and } \forall d \in D \setminus \{d : d \geq \text{end\_date} - \text{MaxGap}\}$$

9. **Home/away match balance:**

$$\text{count\_home}_{i,d} - \text{count\_away}_{i,d} \leq \text{MaxConsecHome}$$
$$\text{count\_away}_{i,d} - \text{count\_home}_{i,d} \leq \text{MaxConsecAway}$$

10. **Avoid matches on restricted days:**

$$x_{i,j,d} \leq 1 - E_{d,i} \quad \forall d \in D, \forall i, j \in T, i \neq j$$

11. **Match relocation day for incase of rain:**

$$x_{i,j,d} + x_{i,j,d+1} \leq 1 \quad \forall d \in D \setminus \{\text{end\_date}\}$$

12. **Day and night match balance:**

$$\sum_{d \in D} \sum_{j \in T : j \neq i} N_{i,j,d} \leq \text{MaxNightMatches}$$
$$\sum_{d \in D} \sum_{j \in T : j \neq i} N_{i,j,d} \geq \text{MinNightMatches}$$

Assumptions

- Interesting matches are known in advanced and depends on which two teams are playing only.

- Stadiums are uniformly distributed across zones.

- Travel distances between stadiums are known and constant.

- Each team has only one home stadium.

- MinGap and MaxGap values are set based on league regulations.

- MaxConsecHome and MaxConsecAway are set basedon league regulations.

- Day and night slots for matches of a day do not overlap with each other.

- MaxNightMatches and MinNightMatches are set in advanced based on league regulations.

- The days from the start and end date of the round robin are coded as integers i.e. $D = \{1, 2, 3, \cdots\}$

- $\forall d \in D, d + 1$, is the immediate next day

- end_date is the last date in D