Mathematical Formulation for Round Robin Match Scheduling

Sets

- T: Set of teams, indexed by i, j, where $i, j \in T$.
- D: Set of days in the tournament, indexed by d.
- S: Set of stadiums, indexed by s.
- Z: Set of zones (E/W/N/S), indexed by z.
- M: Set of matches, where each match m is a tuple (i, j), $i \neq j$.

Parameters

- Home(i): Home stadium of team i.
- Interest(m): Interest level of match m, higher values indicate more interesting matches.
- Weekend(d): Binary parameter, 1 if d is a weekend, 0 otherwise.
- Distance(s_1, s_2): Distance between stadiums s_1 and s_2.
- StadiumCapacity(s): Capacity of stadium s.
- CityEvent(d, s): Binary parameter, 1 if there is a big event in the city of stadium s on day d, 0 otherwise.
- StadiumEvent(d, s): Binary parameter, 1 if there is an event at stadium s on day d, 0 otherwise.
- Rain(d): Binary parameter, 1 if it rains on day d, 0 otherwise.
- Zone(s): Zone of stadium s.
- MinGap, MaxGap: Minimum and maximum gap between two matches for a team.

Decision Variables

- $x_{m,d,s}$: Binary variable, 1 if match m is scheduled on day d at stadium s, 0 otherwise.
- $y_{i,d}$: Binary variable, 1 if team i plays a match on day d, 0 otherwise.
- $z_{i,d}$: Binary variable, 1 if team i has an away match on day d, 0 otherwise.
- $w_{i,d}$: Binary variable, 1 if team i has a home match on day d, 0 otherwise.

Objective Function

$$\begin{aligned}\text{Maximize: } & \sum_{m\in M}\sum_{d\in D}\sum_{s\in S}\text{Interest}(m)\cdot x_{m,d,s}\cdot\text{Weekend}(d)\\ & -\lambda_1\sum_{i\in T}\sum_{d\in D}\sum_{s_1,s_2\in S}\text{Distance}(s_1,s_2)\cdot z_{i,d}\cdot z_{i,d+1},\end{aligned}$$

where λ_1 is a weight balancing the importance of minimizing travel distances.

Constraints

1. **Each team plays all others twice (home and away):**

$$\sum_{d\in D}\sum_{s\in S}x_{m,d,s}=1\quad\forall m\in M,\ \text{where team } i \text{ is home.}$$
$$\sum_{d\in D}\sum_{s\in S}x_{m,d,s}=1\quad\forall m\in M,\ \text{where team } j \text{ is home.}$$

2. **Matches on weekends for high-interest games:**

$$\sum_{d\in D}\sum_{s\in S}x_{m,d,s}\cdot\text{Weekend}(d)\geq\beta\cdot\text{Interest}(m)\quad\forall m\in M.$$

3. **No overlapping of interesting matches:**

$$\sum_{m\in M}x_{m,d,s}\leq 1\quad\forall d\in D,\ \text{where Interest}(m)>\theta.$$

4. **Match gaps for teams:**

$$\text{MinGap}\leq d_2-d_1\leq\text{MaxGap}\quad\forall d_1,d_2\in D,\forall i\in T.$$

5. **Minimize consecutive away matches:**

$$z_{i,d}+z_{i,d+1}\leq 1\quad\forall d\in D,\forall i\in T.$$

6. **Home/away match balance:**

$$w_{i,d}+z_{i,d}\leq 1\quad\forall d\in D,\forall i\in T.$$

7. **Avoid matches on restricted days:**

$$x_{m,d,s} = 0 \quad \text{if CityEvent}(d, s) = 1 \text{ or StadiumEvent}(d, s) = 1.$$

8. **Match relocation due to rain:**

$$x_{m,d+1,s} \geq x_{m,d,s} \cdot \text{Rain}(d) \quad \forall m \in M, \forall d \in D.$$

9. **Stadium capacity for smaller matches:**

$$x_{m,d,s} \cdot \text{Interest}(m) \leq \text{StadiumCapacity}(s) \quad \forall m \in M, \forall s \in S.$$

10. **Zone fairness:**

$$\sum_{d \in D} \sum_{s \in S: \text{Zone}(s) = z} x_{m,d,s} \text{ is balanced across zones.}$$

11. **Day and night match balance:**

$$\sum_{d \in D: \text{DayMatch}(d)} y_{i,d} \approx \sum_{d \in D: \text{NightMatch}(d)} y_{i,d} \quad \forall i \in T.$$

Assumptions

- All matches can be rescheduled to the next day if rain occurs.
- Interest levels of matches are pre-determined and quantifiable.
- Stadiums are assigned to zones, and matches are distributed equitably across zones.
- Travel distances between stadiums are known and constant.
- MinGap and MaxGap values are set based on league regulations.
- Day and night matches are clearly defined for all days in D.