Artificial Intelligence (AI) for Investments

Lesson 4: Valuation of fixed income securities

Introduction

In this lesson we will cover the following topics:

- Introduction to fixed income securities (FIS)
- Valuation of FIS through DCF methods
- Theories of term structure of interest rates
- Concept of yield to maturity
- Duration of an FIS and interest rate risk
- Summary and concluding remarks

Simple valuation of fixed income securities (FIS)

- If you own a fixed income security like a bond you are entitled to fixed set of payoffs called interest or coupons; and at maturity you get the face value or the principal
- Consider a simple bond that pays 8.5% interest. If you have invested \$100, you will get \$8.50 annually, if the coupons are annual, and at maturity you will also get the principal amount, i.e., total \$108.5. Also assume a 3% discount rate
- The PV of this bond can be easily computed as provided here
- $PV = \frac{8.50}{1.03} + \frac{8.50}{1.03^2} + \frac{8.50}{1.03^3} + \frac{108.50}{1.03^4} = \$120.44$; or in the manner provided below
- PV (Bond)=PV (annuity of bond coupon payments)+PV(Principal payment)
- PV (Bond)=$\frac{8.5}{0.03} * \left(1 - \frac{1}{1.03^4}\right) + \frac{100}{1.03^4} = 31.59 + 88.85 = \$120.44$

Simple valuation of fixed income securities (FIS)

- Another very important concept for FIS is yield-to-maturity (YTM)
- In the previous example, if the bond under discussion, has a present value of $120.44 then what is the current interest rate or yield of this bond to the buyer
- This YTM can be easily computed as with expression shown here
- $120.44 = \frac{8.50}{1+ytm} + \frac{8.50}{(1+ytm)^2} + \frac{8.50}{(1+ytm)^3} + \frac{108.50}{(1+ytm)^4}$
- In this case, the answer is ytm=3%, since we assumed the discount rate of 3% at the beginning

Simple valuation of fixed income securities (FIS)

- Consider the example of a simple bond with the following cash-flow profile

T=6m	T=12m	T=18m	T=24m	T=30m	T=36m
24.375	24.375	24.375	24.375	24.375	1024.375

- If the bond is currently trading at \$1107.95, then the current ytm of the bond can be simply computed from this equation provided here

- Coupons amounting to \$24.375 are paid semi-annually and at the end of the period, a principal payment of \$1000 is paid at the end of 3-years

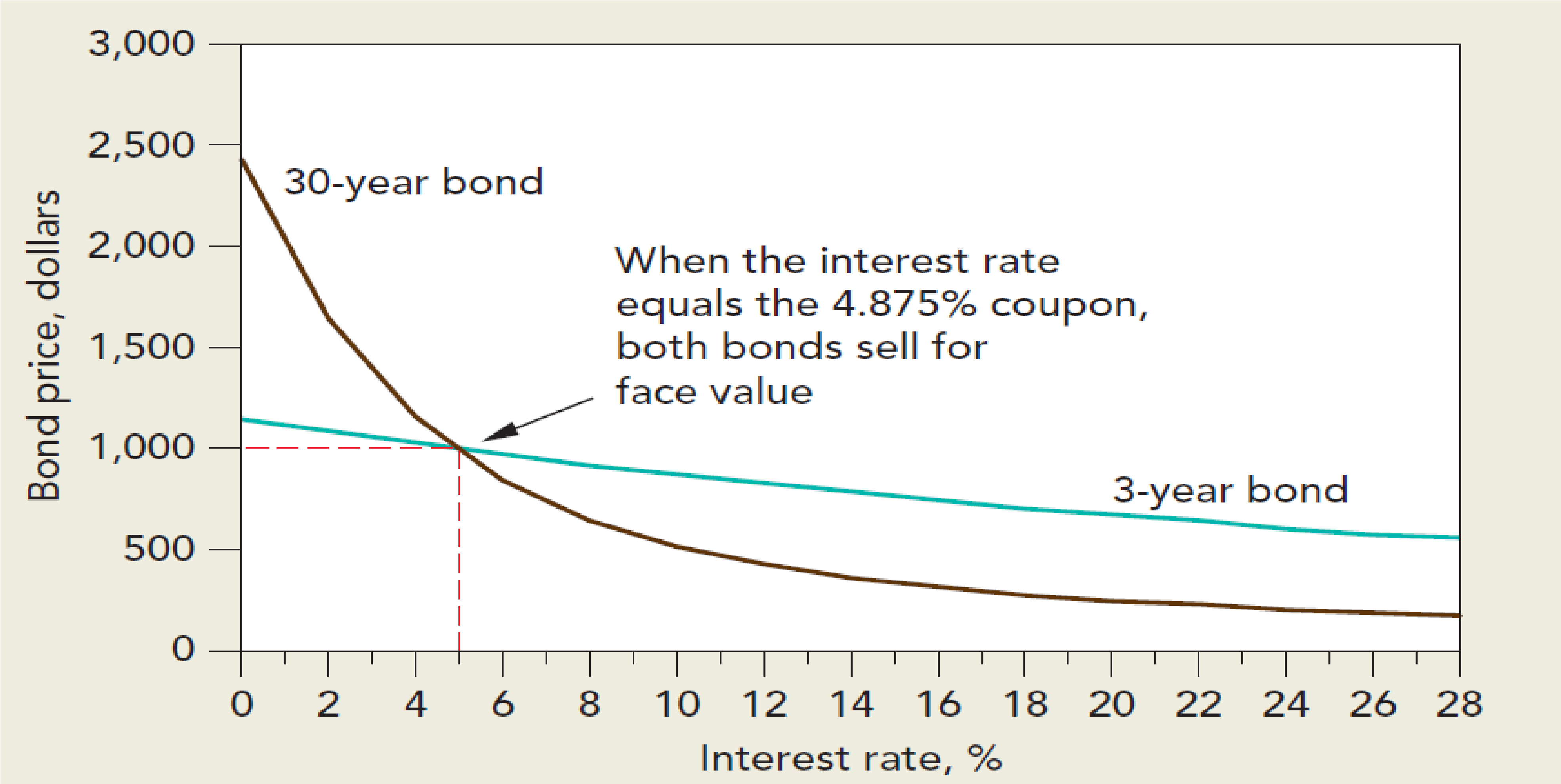
- $PV = \frac{24.375}{1+\frac{ytm}{2}} + \frac{24.375}{\left(1+\frac{ytm}{2}\right)^2} + \frac{24.375}{\left(1+\frac{ytm}{2}\right)^3} + \frac{24.375}{\left(1+\frac{ytm}{2}\right)^4} + \frac{24.375}{\left(1+\frac{ytm}{2}\right)^5} + \frac{1024.375}{\left(1+\frac{ytm}{2}\right)^6}$; here ytm/2=0.6003%; ytm=1.2006%

- The effective annual yield (EAF) would be (1.6003)^2-1=1.2042%

Bond prices and interest rates

- Bond prices change with interest rates
- In the previous example, where the semi-annual yield was 0.6003%, assume investors start demanding a semi-annual yield of 4%, that is annual percentage quoted rate of 8%
- The price of this bond will fall to reflect this change in yield, as per the computation shown here
- $PV = \frac{24.375}{1.04} + \frac{24.375}{(1.04)^2} + \frac{24.375}{(1.04)^3} + \frac{24.375}{(1.04)^4} + \frac{24.375}{(1.04)^5} + \frac{1024.375}{(1.04)^6} = \$918.09$

Bond prices and interest rates



Duration of a bond

- We saw that changes in interest rates have greater impact on the prices of long-term bonds that short-term bonds
- Separate Trading of Registered Interest and Principal Securities (STRIPS) are special instruments, created by stripping the cash flows from treasury instruments and government securities
- These are often called as zero-coupon bonds, and have the maturity same as duration

Duration of a bond

- Consider three bonds, one strip and two coupon paying bonds with cash flow profile as provided here

	Price (%)	Cash payments %			
Bond	**Feb. 2015**	**Aug. 2015**	**Feb. 2016. . .**	**. . . Aug. 2016**	**Feb. 2021**
Strip for Feb. 2015	88.74	0	0 . . .	. . . 0	100.00
Feb. 2015 (4% p.a.)	111.26	2.00	2.00 . . .	. . . 2.00	102.00
Feb. 2015 (11.25% p.a.)	152.05	5.625	5.625 . . .	. . . 5.625	105.625

- All of these bonds have a ytm of 2%
- The two coupon paying bonds offer a considerable proportion of their cash flows earlier than maturity. Thus it is very easy to observe that the strip has the longest duration
- Bond with 11.25% coupon (i.e., 5.625% semi-annual coupon) offers a larger proportion of cash flows earlier than maturity, as compared to the bond with lower coupon of 4% (i.e., 2% semi-annual coupon)

Duration of a bond

- However, we need a more concrete measure of duration
- The duration measure also indicates the sensitivity of a fixed income security to interest rate changes
- The simple measure of duration is computed as a weighted average of times, with weights being the present value of cash flows received at these times
- Consider a bond with a maturity of T years. The corresponding cash flows in each of these years being $C_1, C_2 \ldots . C_T$ being received at the end of year 1, 2, 3,…,T
- $Duration = 1 * \frac{PV(C_1)}{PV} + 2 * \frac{PV(C_2)}{PV} + 3 * \frac{PV(C_3)}{PV} + \cdots + T * \frac{PV(C_T)}{PV}$

Duration of a bond

- Let us understand this through one example
- Consider a fixed income security with coupons of $8.5 paid at the end of each year and a final principal payment in the final year, that is fourth year
- Also assume the appropriate interest rate of 3%

Year (t)	1	2	3	4	
Cash payment (C t)	8.5	8.5	8.5	108.5	PV
PV(Ct) at 3%	8.25	8.01	7.78	96.4	120.44
Fraction of total value [PV(Ct)/PV]	0.069	0.067	0.065	0.8	Total=Duration
Year x Fraction of total value [t x PV(Ct)/PV]	0.069	0.134	0.195	3.2	3.6 years

- $Modified\ Duration = \frac{Duration}{(1+yield)}$

Duration of a bond

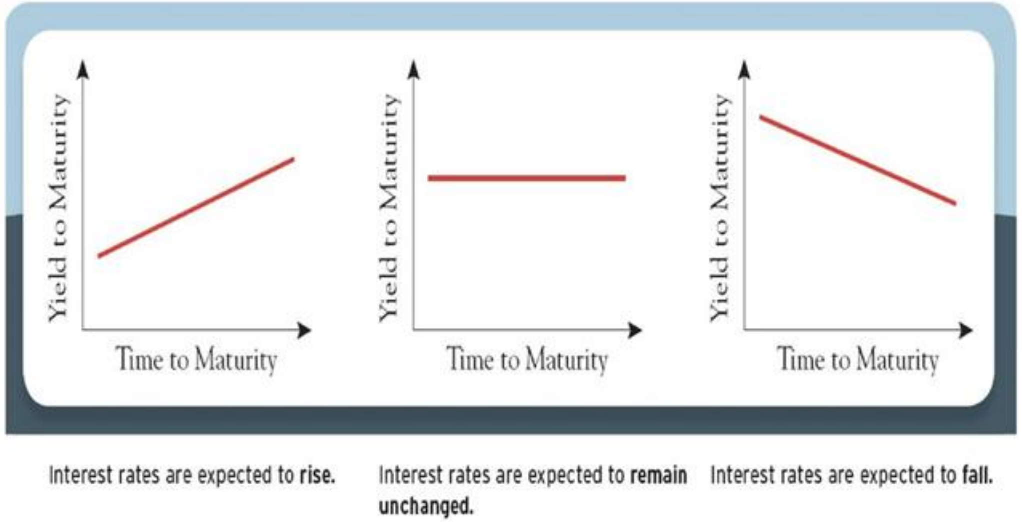
- This modified duration measures the percentage change in price a one percentage change in yield (or interest rates)
- For our bond of duration 3.6 years. This measure works out to 3.6/1.03= 3.49%
- Now consider a scenario where interest rates rise by 0.5% and fall by the same amount

Year (*t*)	1	2	3	4	PV	Change (%)
Cash payment (*C t*)	8.5	8.5	8.5	108.5		
PV(*Ct*) at 3%	8.25	8.01	7.78	96.4	120.44	
PV(*Ct*) at 3.5%	8.21	7.93	7.67	94.55	118.37	-1.72%
PV(*Ct*) at 2.5%	8.29	8.09	7.89	98.30	122.57	1.77%

- The total magnitude of change works out to 1.72%+1.77%= 3.49%
- This is the same amount as our modified duration measure

Term structure of interest rates

- Interest rates vary over different tenors, and short-term interest rates are different from long-term interest rates
- This variation in interest rates over short-term and long-term and across periods, is often referred to as term structure of interest rates



Term structure of interest rates

- Consider a term structure of interest rates $r_1, r_2, r_3, \dots r_t$ for time periods 1, 2, 3,…,t

- A simple cash-inflow of $1 in the first year will have a value of $PV = \frac{1}{1+r_1}$. Here r_1, would be called the one-year spot rate

- Similarly, a loan that pays $1 at the end of two years, will have a present value of $PV = \frac{1}{(1+r_2)^2}$

- For simple illustration purposes assume that $r_1 = 3\%$ and $r_2 = 4\%$. A security that offers only these two cash flows will have a present value of $PV = \frac{1}{1.03} + \frac{1}{(1.04)^2} = 1.895$

- P$V = \frac{1}{1+ytm} + \frac{1}{(1+ytm)^2} = 1.895$

- Solving for this equation, we get ytm= 3.67%

Term structure of interest rates

- In a well-functioning liquid and efficient markets, all safe (that is risk-free cash flows) must be discounted at the same risk-free spot: Law of one price

		1	2	3	4	Bond Price (PV)	Ytm
	Spot rates	3.50%	4%	4.20%	4.40%		
	Discount factor	0.97	0.92	0.88	0.84		
A	8% coupon-2year	80	1080				
	PV	77.29	998.52	-	-	1,075.82	3.98%
B	11%-coupon-3year	110	110	1110			
	PV	106.28	101.70	981.11	-	1,189.10	4.16%
C	6% coupon-4year	60	60	60	1060		
	PV	57.97	55.47	53.03	892.29	1,058.76	4.37%
D	STRIP				1000		
					841.78	841.78	4.40%

Term structure of interest rates

- A 10-year strip with face-value of $1000 at then end of maturity is selling at $714.18
- $F_0 = \frac{1}{(1+r_{10})^{10}} = 0.71418$; solving for this, $r_{10} = 3.42\%$
- **Expectations theory of term structure:** Term-structure of interest rates reflect the expectation of interest rates in future
- Assume that the spot rate for year 1, r_1 is 5% and spot rate for year 2, r_2 is 7%
- If you invest $100 for one year, you get $5 for interest. If you invest it for two years, you get $100 * 1.07^2$, that is, $114.49 after two years
- The extra return that you earn in second year can be computed as noted here. $\frac{1.07^2}{1.05}$-1=9.0%
- This means that if you invest for two years, you will get 5% in year 1 and 9% in year 2

Term structure of interest rates

- If you expect that bond prices in the year 2 will yield more, then you would prefer to invest at 1-year spot and then invest in second year at prevailing rate

- In equilibrium, long term spot rates are a combination of short-term spot and a series of forward rates

- Forward rates are future rates booked (contracted) today. For example, rate of interest for period T=1 to T=2 booked at T=0; or interest rate for T=2 to T=4 booked at T=0

- **Liquidity preference theory** suggests that investors prefer to invest in short-term as they fear the additional volatility, risk, and uncertainty associated with the long-term instruments

Summary and concluding remarks

- Fixed income securities like bonds are simply long-term loans
- These instruments include regular interest (or coupon) payments and at the maturity you get back the face-value (or principal)
- These instruments can be easily valued through discounting cash flow valuation method
- Also, it is appropriate to discount each of these cash flows with its on spot rate corresponding to the duration of the cash flows
- The spot rate is observed on the term structure of interest rates. The term structure of interest rates is computed using the STRIPs

Summary and concluding remarks

- Once the present value of a bond is computed, using bond cash flows, one can also calculate the ytm of the bond
- Duration reflects the average time associated with cash-flows of a fixed income security
- The expectations theory of interest rates suggests that rising interest rates reflect the future expectations of investors
- The theory of liquidity preference suggests that investors prefer to hold short-term instruments as compared to long-term instruments

Thanks!