Artificial Intelligence (AI) For Investments

Lesson 2: Portfolio Performance Evaluation – One-Parameter Measures

Introduction

- Introduction to portfolio performance evaluation
- One-parameter measures
  - Sharpe ratio
  - Treynor's measure
  - Jensen's measure (α)
  - Information ratio (IR) measure
- Performance measurement with downside risk: Sortino's ratio
- Summary and concluding remarks

Portfolio Performance Evaluation

Portfolio Performance Evaluation

While evaluating the performance of a portfolio, the following questions are asked

- What are the policies that the fund has pronounced for itself, and how well those policies are followed?
- How diversified is the fund?
- What is the asset allocation?

- The portfolios being evaluated must be comparable
- For example, if a fund has restricted that its managers should invest only in AA-rated instruments or better should not compare with those funds that invest in funds that have no such restrictions

Portfolio Performance Evaluation

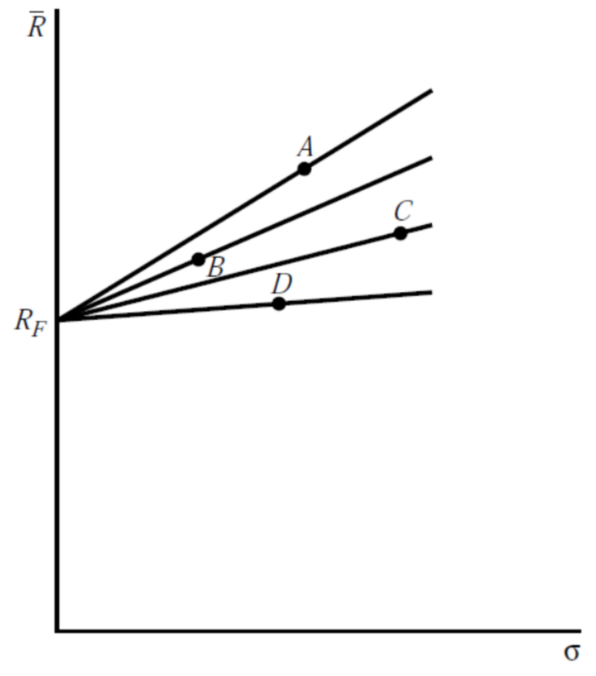
- Therefore, the return earned is directly linked to the amount of risk borne by the fund
- But problems arise where the funds that are compared have different risk levels
- In the ensuing discussions, we will focus on one-parameter measures that are most commonly employed in the literature

One-Parameter Measures: Sharpe Ratio

Sharpe Ratio

Sharpe ratio of point $A = \frac{\overline{R_A} - R_f}{\sigma_A}$

- The ratio measures excess return over risk-free rate against the risk borne by the fund
- The portfolios on line joining the investment A and R_f offer the highest slope and, therefore, the best Sharpe ratio measure of performance



Reilly and Brown, *Investment Analysis and Portfolio Management*, 10th edition (Chapter 25)

Sharpe Ratio

Sharpe ratio of point *A*= $\frac{\overline{R_A}-R_f}{\sigma_A}$

- Compare the examples of three portfolios that follow the Sharpe measure

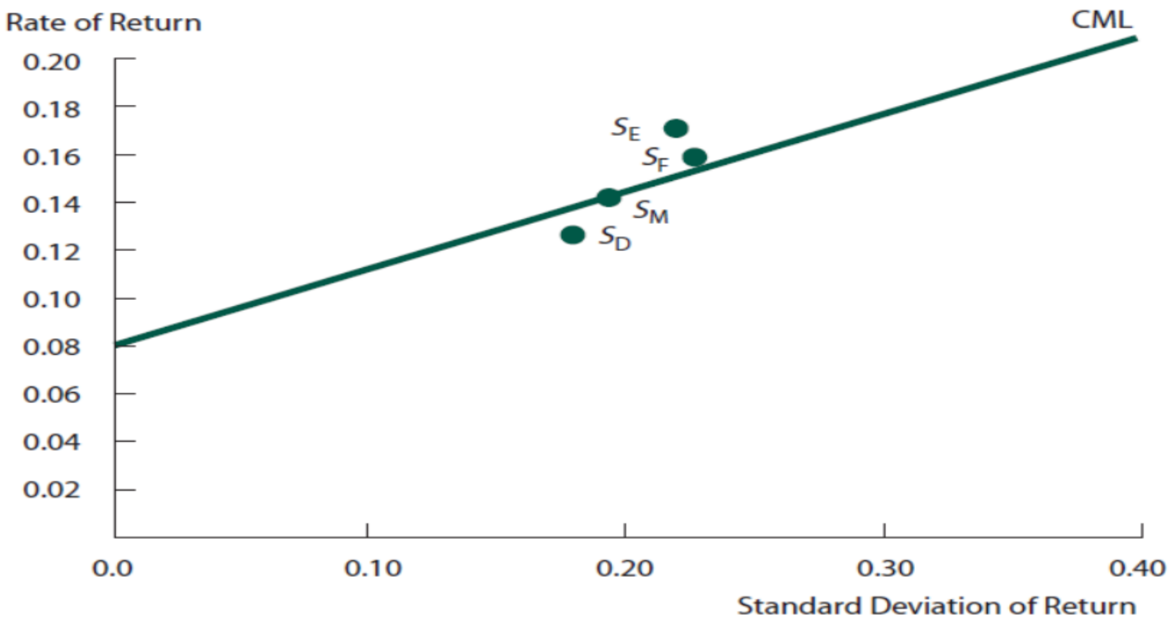
Portfolio	Average Annual Rate of Return	SD	Sharpe Measure
D	13%	0.18	(0.13−0.08)/0.18 = 0.278
E	17%	0.22	(0.17−0.08)/0.22 = 0.409
F	16%	0.23	(0.16−0.08)/0.23 = 0.348
Market	14%	0.20	(0.14−0.08)/0.20 = 0.300
Risk-free	8%		

- Here, portfolio *D* performs the worst, even as compared to the market portfolio
- Portfolio *E* performs best

Sharpe Ratio

The performance of these portfolios can be plotted on the capital market line (CML)

Portfolios *E* and *F* are above the CML line, indicating better risk-adjusted performance



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Sharpe Ratio

- The measure of risk considered here is the standard deviation, i.e., total risk of the fund
- This includes the market risk (systematic risk) and stock-specific risk
- Please note that if the fund is well diversified, then most of the fund’s risk will be systematic risk
- In most of the situations, the investors invested in the fund are small retail investors, who invest a sizable portion of their risk in the fund

Sharpe Ratio

- For the investor, the entire risk of the fund is important, not only the market risk part of it
- Since these investors rely precisely on the ability of the fund to diversify on behalf of them
- The Sharpe measure looks at the decision from the point of view of an investor choosing a mutual fund to represent the majority of his investment

Sharpe Ratio

- An investor choosing a mutual fund to represent a large part of her wealth would likely be concerned with the full risk of the fund, and the standard deviation is a measure of that risk
- The measure computes risk-premium earned per unit of total risk
- This measure uses CML to compare portfolios

One-Parameter Measures: Treynor’s Measure

Treynor’s Measure

Treynor’s measure = $\frac{\overline{R_p} - R_f}{\beta}$

- Treynor’s measure examines excess return with risk measure being beta
- For the diversified investors who only consider the systematic risk for performance evaluation, Treynor’s measure is the appropriate measure
- Treynor’s measure is applicable to majority of the investors irrespective of their risk preferences
- Treynor argues that rational, risk-averse investors would always prefer the portfolios on security market line

Treynor's Measure

Treynor's measure = $\frac{\overline{R_p} - R_f}{\beta}$

- Treynor argues that rational, risk-averse investors would always prefer the portfolios on security market line
- That is, risk-free assets combined with risky portfolios with the largest slope, in order to achieve the highest indifference curve
- The slope of this curve is Treynor's measure
- The risk measure here is the systematic risk component, i.e., beta

Treynor's Measure

Treynor's measure = $\frac{\overline{R_p} - R_f}{\beta}$

- The measure assumes a diversified portfolio and that all investors are risk-averse and would like to maximize this value
- The measure for the standard market portfolio will be $\frac{\overline{R_M} - R_f}{\beta_M}$ where $\beta_M = 1$
- For any portfolio in general: $\frac{\overline{R_P} - R_f}{\beta_P} = (\overline{R_M} - R_f)$ from security market line (SML)
- The equation of SML is shown here: $\overline{R_P} = R_f + \beta_p * (\overline{R_M} - R_f)$

Treynor's Measure

Treynor's measure = $\frac{\overline{R_p} - R_f}{\beta}$

- The equation of SML is shown here: $\overline{R_P} = R_f + \beta_p * (\overline{R_M} - R_f)$
- Combined with Rf, this portfolio will generate the SML
- Any higher value would indicate that the portfolio offers excess risk-adjusted returns and plots above SML

Treynor's Measure

Consider the information about three investment managers below (w, x, and y). In addition, we are given the market rate of return and the risk-free rate

Investment Manager	Average Annual Rate of Return	Beta	Treynor's Measure
w	12%	0.90	$(0.12 - 0.08)/0.90 = 0.044$
x	16%	1.05	$(0.16 - 0.08)/1.05 = 0.076$
y	18%	1.20	$(0.18 - 0.08)/1.20 = 0.083$
Market	14%	1.00	$(0.14 - 0.08)/1.00 = 0.060$
Risk-free	8%	0.00	

- These results indicate that Manager w not only performed worst among the three managers but performed worse than the market as well, on a risk-adjusted basis

Treynor’s Measure

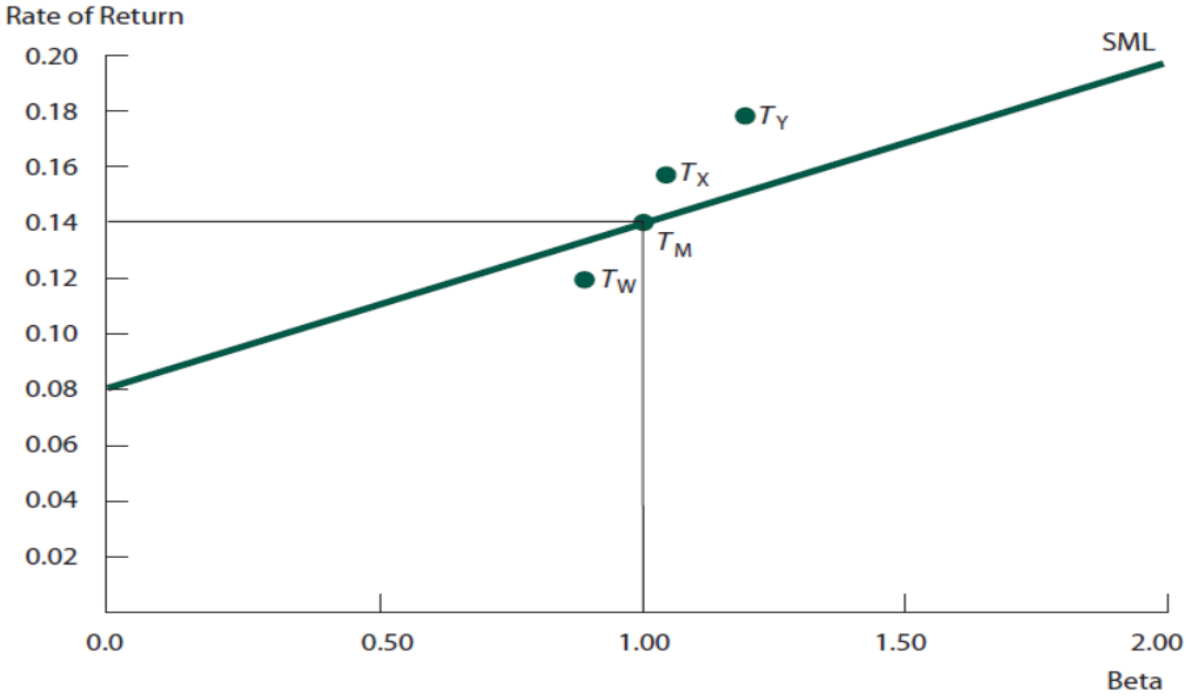
These results indicate that Manager w not only performed worst across the three managers, but performed worse than the market as well, on risk-adjusted basis

- While x and y performed better than the market, y performed best

Investment Manager	Average Annual Rate of Return	Beta	Treynor’s Measure
w	12%	0.90	(0.12 − 0.08)/0.90 = 0.044
x	16%	1.05	(0.16 − 0.08)/1.05 = 0.076
y	18%	1.20	(0.18 − 0.08)/1.20 = 0.083
Market	14%	1.00	(0.14 − 0.08)/1.00 = 0.060
Risk-free	8%	0.00	

Treynor’s Measure

Their performance on SML can be plotted as follows



Reilly and Brown, *Investment Analysis and Portfolio Management*, 10th edition (Chapter 25)

Treynor's Measure

What is the challenge with this measure?

- Consider two portfolios: one, which offers a return that is below risk-free (although with the positive beta)
- The negative measure would indicate poor performance
- Even when plotted on SML, this point would indicate a very poor performance
- Second, consider a security with a negative beta that offers a very high return above the risk-free rate
- This would also offer a negative measure despite good performance

Treynor's Measure

What is the challenge with this measure?

- For example, a portfolio of gold mining stocks with a beta of −2 performs well and offers a 10% return
- Then the measure would be (0.10 − 0.08)/(− 0.2) = − 0.10
- However, if plotted on SML, this will be above SML and indicate exceptional returns
- See for example. $E\left(R_{\text{gold}}\right) = R_f + \beta_{\text{gold}}(R_M - Rf)$= 0.08 + (− 0.2)*(0.14 − 0.08) = 6.8% expected returns, which is lower than the actual return of 10%. Thus, the point will be above SML

One-Parameter Measures: Jensen's Measure (α)

Jensen's Measure (α)

- Jensen's measure is the differential in the return as predicted by the CAPM model
- $R_p = \alpha_p + \bar{R}_p = \alpha_p + R_f + (\bar{R}_M - R_f)\beta_p$
- $\bar{R}_p$ is the expected return. Then $R_p - \bar{R}_p$, this differential return is called the Jensen's measure of performance
- The key assumption here is that CAPM is the guiding model

Jensen's Measure (α)

- $R_{pt} - R_f = \alpha_p + \beta_j\left[R_{mt} - R_f\right] + e_{jt}$
- In this model, we expected $\alpha_p = 0$
- Presence of positive intercept (constant term) α_p would indicate the ability of security selection or predicting the market performance by a portfolio manager
- A negative Alpha would indicate poor performance

One-Parameter Measures: Information Ratio (IR) Measure

Information Ratio (IR) Measure

Information ratio (IR) measure: $\frac{\overline{R_P} - \overline{R_b}}{\sigma_{ER}} = \frac{\overline{\mathrm{ER}_B}}{\sigma_{ER}}$

- Here, $\overline{R_P}$ is the return on a portfolio, $\overline{R_b}$ is the return on the benchmark portfolio
- $\overline{\mathrm{ER}_B}$ is the excess return. σ_{ER} is the standard deviation of excess returns
- The numerator here measures the ability of the portfolio manager to perform better than a given benchmark (e.g., Nifty)

Information Ratio (IR) Measure

Information ratio (IR) measure: $\frac{\overline{R_P} - \overline{R_b}}{\sigma_{ER}} = \frac{\overline{\mathrm{ER}_B}}{\sigma_{ER}}$

- The denominator measures the residual (or incremental) risk that the manager took to obtain these excess returns
- Thus, IR can be interpreted as a benefit to cost ratio
- It evaluates the quality of information with the manager (or stock selection ability) adjusted by the non-systematic taken by the investor

Information Ratio (IR) Measure

- Consider a set of quarterly returns below
- Compute IR= $\frac{\overline{R_P}-\overline{R_b}}{\sigma_{\mathrm{ER}}} = \frac{\overline{\mathrm{ER}_B}}{\sigma_{\mathrm{ER}}}$

Quarter	Portfolio Returns	Benchmark Returns	Difference
1	2.30%	2.70%	−0.40%
2	−3.60%	−4.60%	?
3	11.20%	10.10%	?
4	1.20%	2.20%	?
5	1.50%	0.40%	?
6	3.20%	2.80%	?
7	8.90%	8.10%	?
8	−0.80%	0.60%	?
Average	?	?	$\overline{R_P} - \overline{R_b}$ =?
SD			σ_{ER} =?

Information Ratio (IR) Measure

- IR = 0.2%/1%=0.20; this represents the manager's incremental performance (Alpha, relative to the index) per unit of risk incurred in the pursuit of those active returns
- IR will be only positive when the manager outperforms his benchmark

Quarter	Portfolio Returns	Benchmark Returns	Difference
1	2.30%	2.70%	−0.40%
2	−3.60%	−4.60%	1.00%
3	11.20%	10.10%	1.10%
4	1.20%	2.20%	−1.00%
5	1.50%	0.40%	1.10%
6	3.20%	2.80%	0.40%
7	8.90%	8.10%	0.80%
8	−0.80%	0.60%	−1.40%
Average	2.99%	2.79%	$\overline{R_P} - \overline{R_b}$=0.20%
SD			σ_{ER} =1.00%

Performance Measurement With Downside Risk: Sortino’s Ratio

Sortino's Ratio

Sortino's ratio: $\frac{\overline{R_p}-T}{D_R}$

- Here, D_R is the downside risk. Total risk, i.e., SD, includes upside and downside both risks
- T is the target rate of return
- In most of the computations, $T = R_f$ (risk-free rate) or some target return as set by fund management
- $D_R = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\mathrm{Min}(0, R_i - \mathrm{MAR})\right)^2}$

Sortino's Ratio

Sortino's ratio : $\frac{\overline{R_p}-T}{D_R}$; $D_R = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\mathrm{Min}(0, R_i - \mathrm{MAR})\right)^2}$

- MAR is the minimum acceptable rate of return, often considered as the target return
- Also, in most of the computations, MAR = Average returns ($\overline{R_p}$)
- Risk-free returns (R_f), also in some cases, MAR can be = 0
- Sortino's measure measures returns in excess of a pre-defined target rate

Sortino's Ratio

Sortino's ratio : $\frac{\overline{R_p}-T}{D_R}$; $D_R = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\mathrm{Min}(0, R_i - \mathrm{MAR})\right)^2}$

- This excess return is not adjusted by the total risk (SD) but only by the downside risk
- The downside risk is computed against some minimum acceptable returns
- This kind of downside risk is often considered more appropriate because the downside volatility is often associated with a shortfall
- Thus, this downside risk can be considered to capture the fear of investors more efficiently

Sortino's Ratio: Example

Sortino's Ratio

Consider the example below, where we compare the Sharpe and Sortino measures with a risk-free rate of 2%

Year (R_f = 2%)	Portfolio A Return (%)	Portfolio B Return (%)
1	−5	−1
2	−3	−1
3	−2	−1
4	3	−1
5	3	0
6	6	4
7	7	4
8	8	7
9	10	13
10	13	16
Average	4	4
SD	5.60	5.92

Sortino's Ratio

The Sharpe ratios of portfolios A and B are computed as follows

- $S_A = \frac{4-2}{5.60} = 0.357$ and $S_B = \frac{4-2}{5.92} = 0.338$
- Based on these numbers, it appears that portfolio A outperformed portfolio B
- Let us see what happens when we only consider the downside risk
- Let us use the average return of 4% as MAR to compute the downside return, and target return T as a risk-free return

Sortino's Ratio

The Sortino's ratios of portfolios A and B are computed as follows

- All the positive returns are considered zero
- Sortino's ratio: $\frac{\overline{R_p} - T}{D_R}$
- $D_R = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\mathrm{Min}(0, R_i - \mathrm{MAR})\right)^2}$

Sortino's Ratio

The Sharpe ratios of portfolios A and B are computed as follows

Assume a target rate of 4% (average return)

MAR = risk-free = 2%

$$DR_A = \sqrt{\frac{[(-5-4)^2+\cdots..+\cdots...+\cdots..+\cdots....]}{10}} = ?$$

$$DR_B = \sqrt{\frac{[(-1-4)^2+\cdots..+\cdots...+\cdots...+.....^2]}{10}} = ?$$

Year (R_f = 2%)	Portfolio A Return (%)	Portfolio B Return (%)
1	−5	−1
2	−3	−1
3	−2	−1
4	3	−1
5	3	0
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Average	4	4
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Sortino's Ratio

The Sharpe ratios of portfolios A and B are computed as follows

Assume the target rate of 4% (average return)

MAR = risk-free = 2%

Sortino's ratios for both of these funds are computed as

$ST_A =: \frac{\overline{R_A} - T}{DR_A} = ?$ and $ST_B = \frac{\overline{R_B} - T}{DR_B} = ?$

Year (R_f = 2%)	Portfolio A Return (%)	Portfolio B Return (%)
1	−5	−1
2	−3	−1
3	−2	−1
4	3	−1
5	3	0
6	6	4
7	7	4
8	8	7
9	10	13
10	13	16
Average	4	4
SD	5.60	5.92

Sortino's Ratio

Let us see what happens when we only consider the downside risk

- Let us use the average return of 4% as MAR to compute the downside return

- All the positive returns are considered zero

- $DR_A = \sqrt{\frac{[(-5-4)^2+(-3-4)^2+(-2-4)^2+(3-4)^2+(3-4)^2+]}{10}} = 4.10$

- $DR_B = \sqrt{\frac{[(-1-4)^2+(-1-4)^2+(-1-4)^2+(-1-4)^2+(0-4)^2]}{10}} = 3.41$

Sortino's Ratio

Let us see what happens when we only consider the downside risk

- Let us use the average return of 4% as MAR to compute the downside return
- All the positive returns are considered zero
- Sortino's ratios for both of these funds are computed as
- $ST_A = \frac{4-2}{4.10} = 0.488$ and $S_B = \frac{4-2}{3.41} = 0.587$

Sortino's Ratio

With Sortino's ratio, portfolio B appears to perform better

- This happens because portfolio A appears to have more extreme negative returns
- Various risk-averse investors would be uncomfortable with this aspect of portfolio A

Year (R_f = 2%)	Portfolio A Return (%)	Portfolio B Return (%)
1	−5	−1
2	−3	−1
3	−2	−1
4	3	−1
5	3	0
6	6	4
7	7	4
8	8	7
9	10	13
10	13	16
Average	4	4
SD	5.60	5.92

Summary and Concluding Remarks

Summary and Concluding Remarks

- If portfolios are well diversified, then Treynor's measure and Sharpe give the same results
- However, for poorly diversified portfolios, one can get a high rank on Treynor's measure (as it ignores the systematic risk), despite performing poorly on Sharpe's measure
- Also, to be noted that these measures provide comparisons and produce relative rankings, not absolute performance rankings
- In this regard, the advantage of Jensen's Alpha is that it produces an absolute measure

Summary and Concluding Remarks

- For example, an Alpha value of 2% would indicate that manager generated an excess return of 2% per period, more than the expected returns
- Also, the result from Jensen’s Alpha has certain statistical significance
- Moreover, Jensen’s Alpha has the flexibility to compute the Alpha with respect to any given model
- Another class of measures capture the downside risk dimension only: Sortino’s ratio

Thanks!