Ramakrishna Mission Vivekananda University

Belur Math, Howrah, West Bengal

School of Mathematical Sciences, Department of Data Science

M.Sc. in Big Data Analytic 2018, Mid Semester Exam — Date: 15 Sep 2018

Course : **DA311: Time Series** — Time: $1\frac{3}{4}$ hrs

Instructor : Dr. Sudipta Das — *Max marks: 40*

Student signature and Id:

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1. Select the right answers

   (a) Which of the following is necessary condition for weakly stationary time series?

      i. Mean is constant and does not depend on time

      ii. Autocovariance function depends on s and t only through their difference $|s-t|$ (where t and s are moments in time)

      iii. The time series under considerations is a finite variance process

      iv. Time series is Gaussian

   (b) Which of the following is true for white noise?

      i. Mean =0

      ii. Autocorrelation function is constant at zero

      iii. Zero autocovariances except at lag zero

      iv. Quadratic Variance

   (c) Second differencing in time series can help to eliminate

      i. Linear Trend

      ii. Quadratic Trend

      iii. Seasonality

      iv. Noise

   (d) The partial autocorrelation function is necessary for distinguishing between

      i. An AR and MA model

      ii. An AR and an ARMA

      iii. An MA and an ARMA

      iv. Different models within the ARMA family

   $[1 \times 4 = 4]$

2. For an iid sequence $Y_1, \ldots, Y_n$, let S be the number of values of i such that $Y_i > Y_{i-1}$, $i = 2, \ldots, n$. Find the expectation and variance of S. [1+3=4]

3. Let X and Y be two random variables with $EY^2 < \infty$. Deduce that the random variable $f(X)$ that minimizes $E(Y - f(X))^2$ is $f(X) = E[Y|X]$. [4]

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4. Let $\{Z_t\}$ be a sequence of independent normal random variables, each with mean 0 and variance σ^2. Is the following process stationary, $X_t = Z_t \cos(ct) + Z_{t-1} \sin(ct)$? [4]

5. If $m_t = \sum_{k=0}^{p} c_k t^k, t = 0, \pm 1, \pm 2, \ldots$, show that $\nabla^{p+1} m_t = 0$. [4]

6. Is the following ARMA process causal as well as invertible. ($\{Z_t\}$ denotes white noise)

$$X_t - .75X_{t-1} + .5625X_{t-2} = Z_t + 1.25Z_{t-1}$$

[2+2=4]

7. Show that the two MA(1) processes

$$X_t = Z_t + \theta Z_{t-1}, \ \{Z_t\} \sim WN(0, \sigma^2)$$

and

$$Y_t = \tilde{Z}_t + \frac{1}{\theta}\tilde{Z}_{t-1}, \ \{\tilde{Z}_t\} \sim WN(0, \sigma^2\theta^2),$$

where $0 < |\theta| < 1$, have the same autocovariance functions. [4]

8. For an MA(1), $X_t = Z_t + \theta Z_{t-1}$, what can be the maximum value of $|\rho_X(1)|$ for any real θ. For which values of θ does $\rho_X(1)$ attain its maximum and minimum? ($\{Z_t\}$ denotes white noise and $\rho_X(1)$ is the auto-correlation of X_t at lag 1) [3+3=6]

9. Show that the value at lag 2 of the partial ACF of the MA(1) process

$$X_t = Z_t + \theta Z_{t-1}, t = 0, \pm 1, \ldots,$$

where $\{Z_t\} \sim WN(0, \sigma^2)$, is $\phi_{22} = -\theta^2/(1 + \theta^2 + \theta^4)$. [6]

This exam has total 9 questions, for a total of 40 points and 0 bonus points.