Artificial Intelligence (AI) for Investments

Lesson 1: Economic theory of choice

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Introduction

In this lesson, we will cover the following topics:

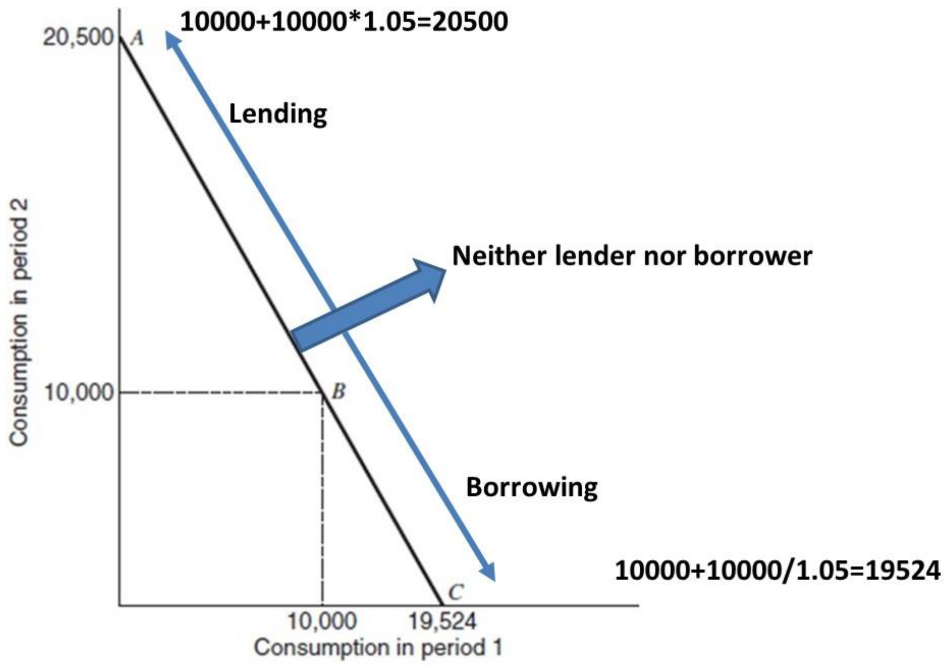
- Economic theory of choice and interest rates
- Indifference curves and utility of wealth
- Risk-return framework in financial markets
- Expected return and actual returns
- Measures of risk and their computation

Economic Theory of Choice Under Certainty

Economic Theory of Choice Under Certainty

What should be the consumption pattern of an investor?

- Consider the example of an investor who will receive \$10000 at the end of years 1 and 2. Also, assume that only investment available offers 5% rate and only borrowing available is at 5% rate.

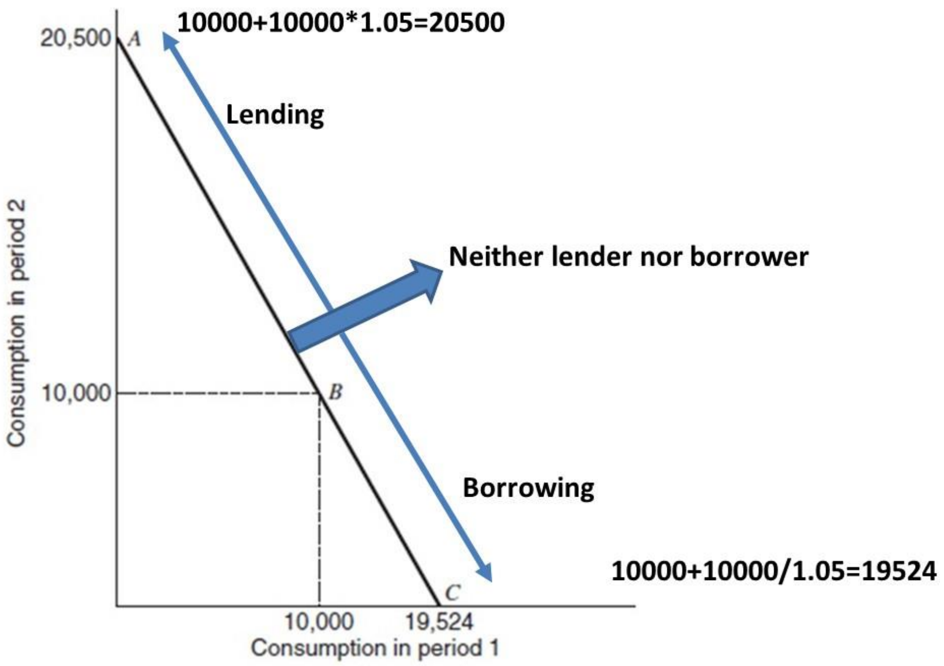


- Let us define his opportunity set.
- One extreme of the options available is to consume \$10000 in each period and save nothing.

Economic Theory of Choice Under Certainty

What should be the consumption pattern of an investor?

- Invest the income in period 1, which will be received in period 2. Then, consume all in period 2: $10000 + 1000 \times 1.05 =$ \$20500

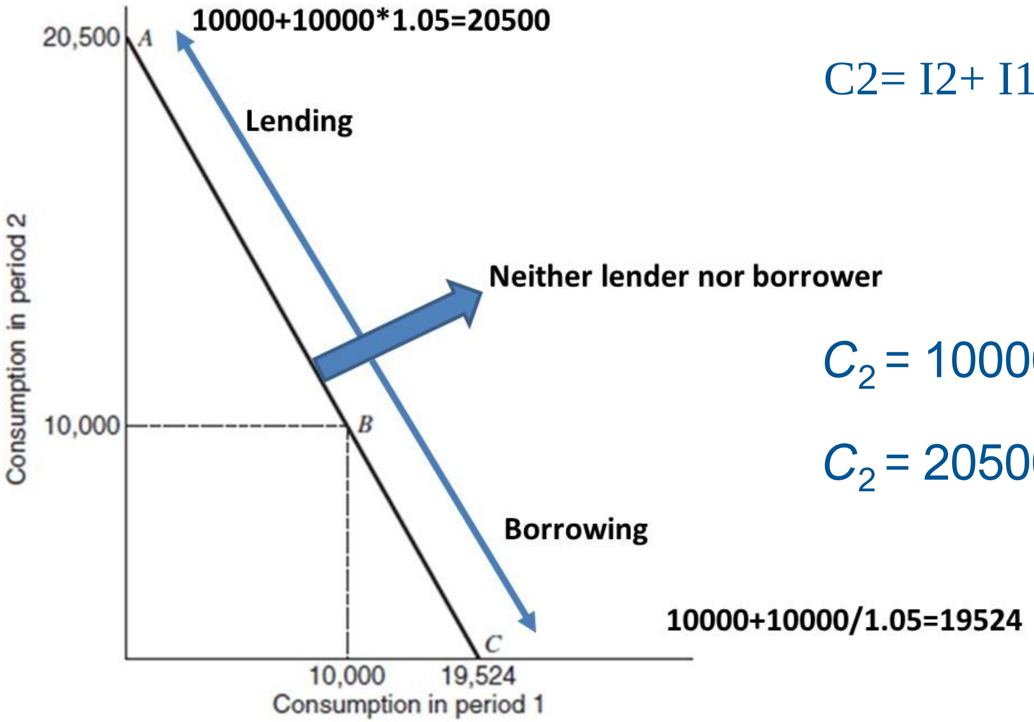


OR

- Another extreme is to borrow an amount against the income in period 2, and consume all in period 1: $10000 + 10000/1.05 =$ \$19524

Economic Theory of Choice Under Certainty

And so it appears that all the choices available to the investor can be represented on this straight line (AC)



C2= I2+ I1*1.05-C1*1.05, here I1=I2=10000

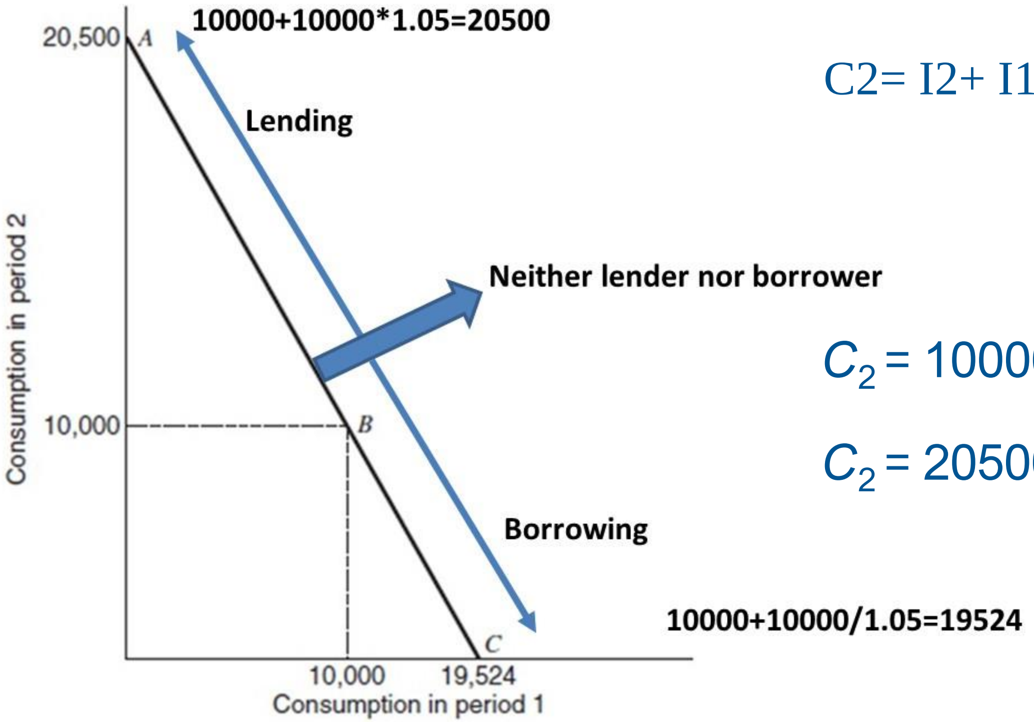
$$C_2 = 10000 + (10000 - C_1) \times 1.05 \qquad \text{(Eq. 1)}$$

$$C_2 = 20500 - 1.05C_1 \qquad \text{(Eq. 2)}$$

Elton, Gruber, Brown, Goetzmann, *Modern Portfolio Theory and Investment Analysis* , 9th edition (Chapter 1)

Economic Theory of Choice Under Certainty

Easy to understand that the consumption of the investor is constrained by his income



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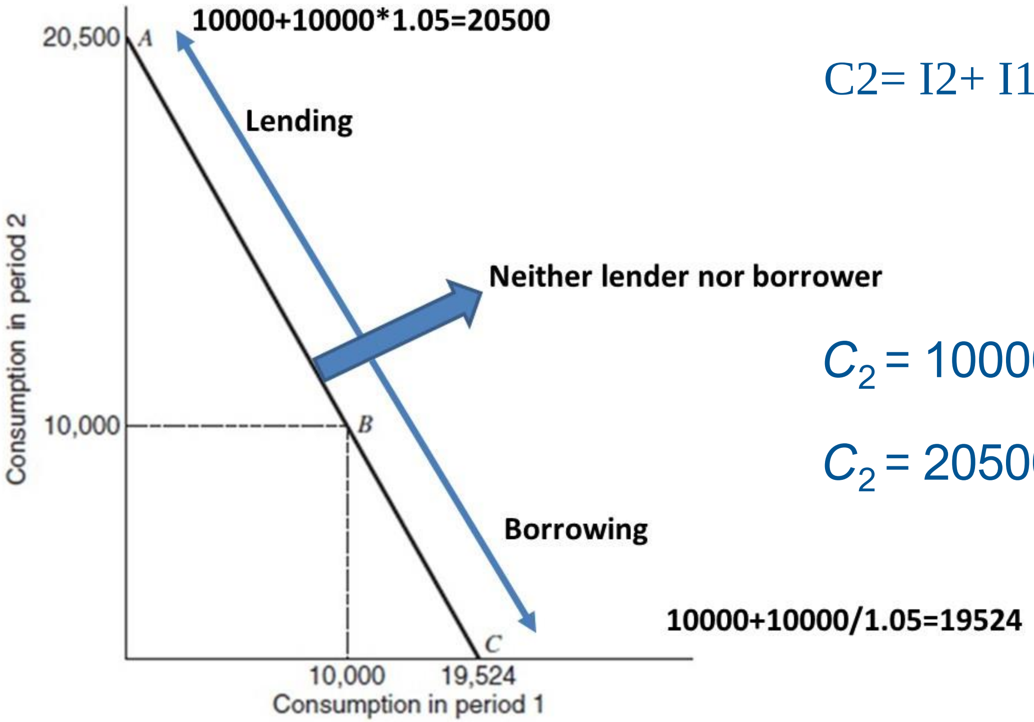
Economic Theory of Choice Under Certainty

If the consumption in period 2 is C2, and in period 1 is C1, we can write a simple equation that defines the relationship between income (I1, I2) and consumption (C1, C2)



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Economic Theory of Choice Under Certainty

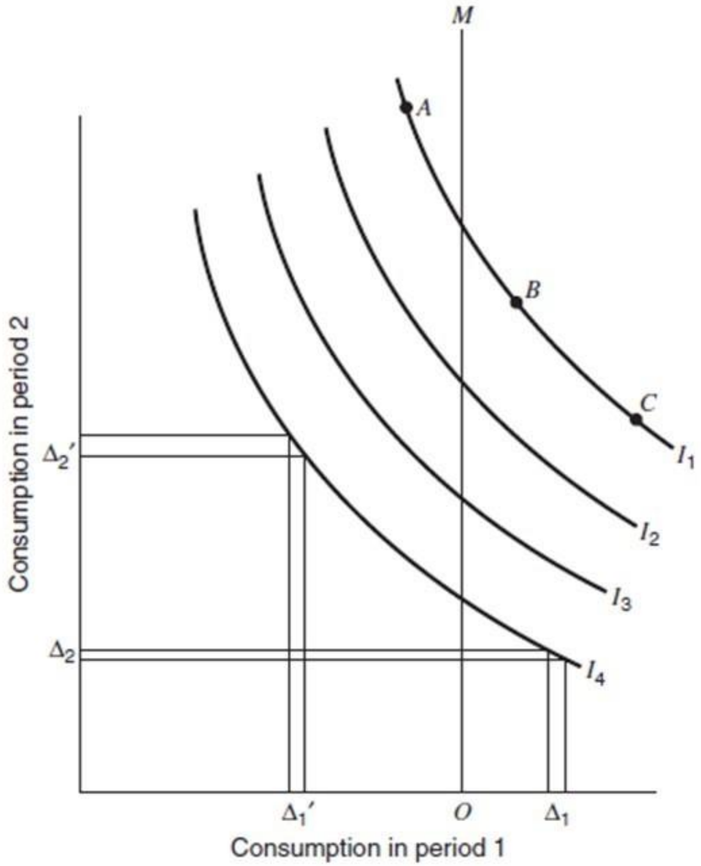
The consumption pattern or opportunity set of the investor is defined by a simple straight line equation given in (Eq. 1).

- Slope of -1.05 is because of the interest rate of 5%.
- If one delays a consumption of 1 unit today, he gets to consume 1.05 more in the next period.
- But the happiness/utility of consuming 1 unit today may or may not be the same as 1.05 in the next period! How?

Indifference Curves and Interest Rates

Indifference or Utility Curves

- Indifference curves, also called utility functions, represent those points on the consumption region where the consumer derives the same utility moving on a curve.
- For an investor, utility curves are drawn, i.e., I_1, I_2, I_3, and I_4. That investor on the curve I_1 is equally happy or has the same utility whether he is on *A*, *B*, or *C*.



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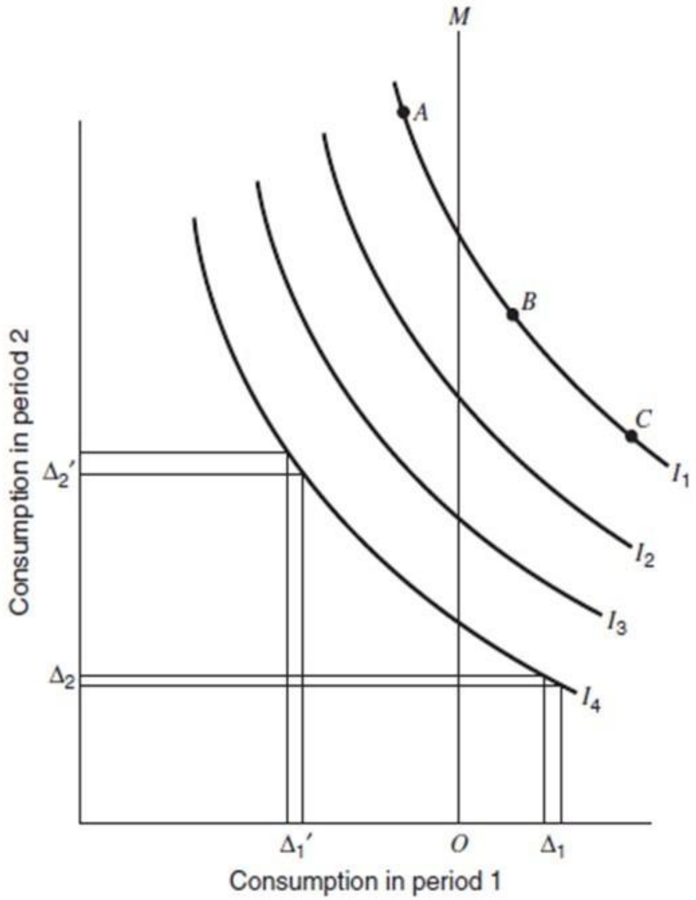
Indifference or Utility Curves

- However, if he moves to I_2, his utility is reduced. If he moves to I_3, his utility is further reduced. This ordering $I_1 > I_2 > I_3 > I_4$ assumes that more is preferred to less.
- Also, please remember if we are consuming more and more in a period, then the marginal utility of consumption in that period, as compared to the other period, comes down.



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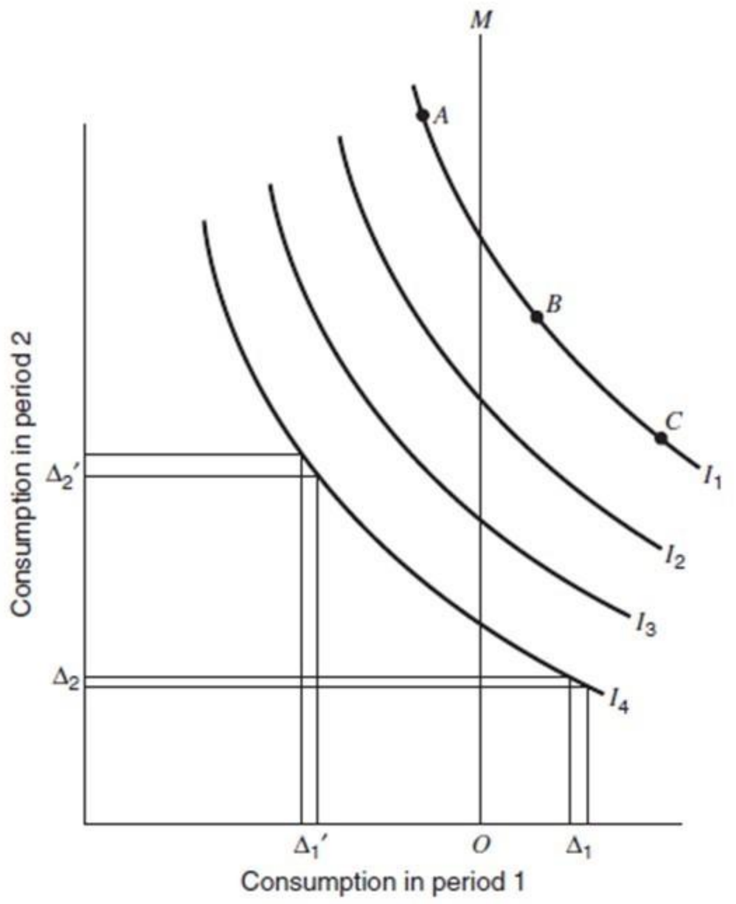
Indifference or Utility Curves

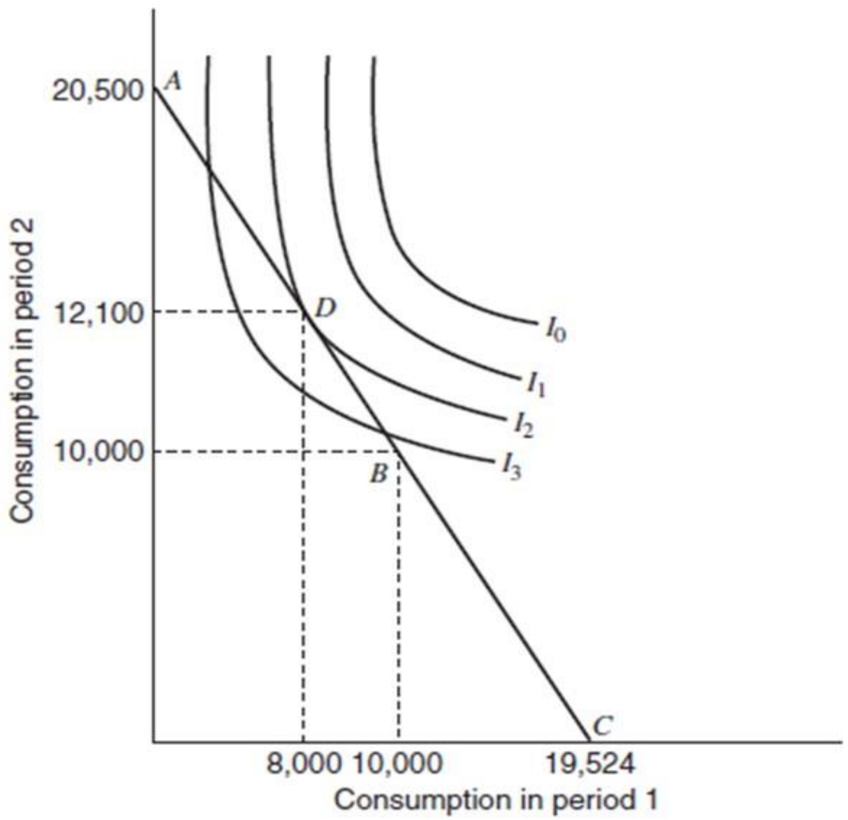
For example, consider a situation when we are heavily consuming in period 1.

- If we increase or decrease the consumption by Δ_1, then the corresponding change in period 2 to maintain the same utility is much lower.
- Similarly, if we are consuming much less in period 1, then any increase or decrease in consumption by Δ_1' in period 1 would require a much higher or lower decrease or increase in period 2.



Indifference or Utility Curves: Solution

- On the opportunity set, we aspire to achieve maximum utility. This is obtained at point *D*, which is at the point of tangency between the opportunity set and indifference curves.
- One interesting observation is to be made here. If the optimum consumption is closer to point *A*, that is, higher consumption in period 2, then the investor is effectively a lender at 5%.



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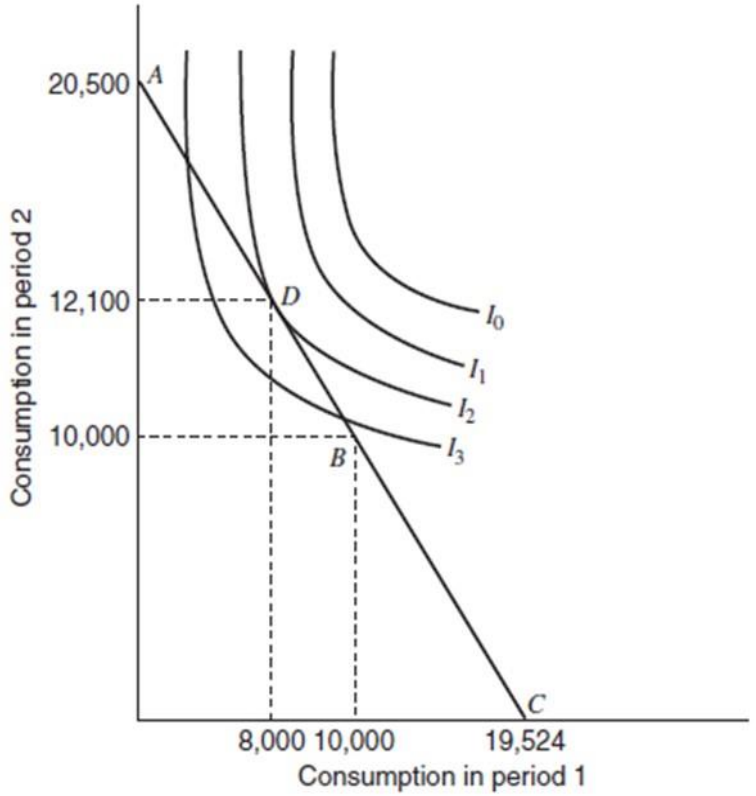
Indifference or Utility Curves

- Similarly, if the optimum consumption point is closer to point *C*, then the investor is a borrower at 5%.
- Somewhere in between (say point *B*) investor is neither a borrower or lender at 5%.
- Now, summing across all the investors who wish to lend provides one point on the supply curve.



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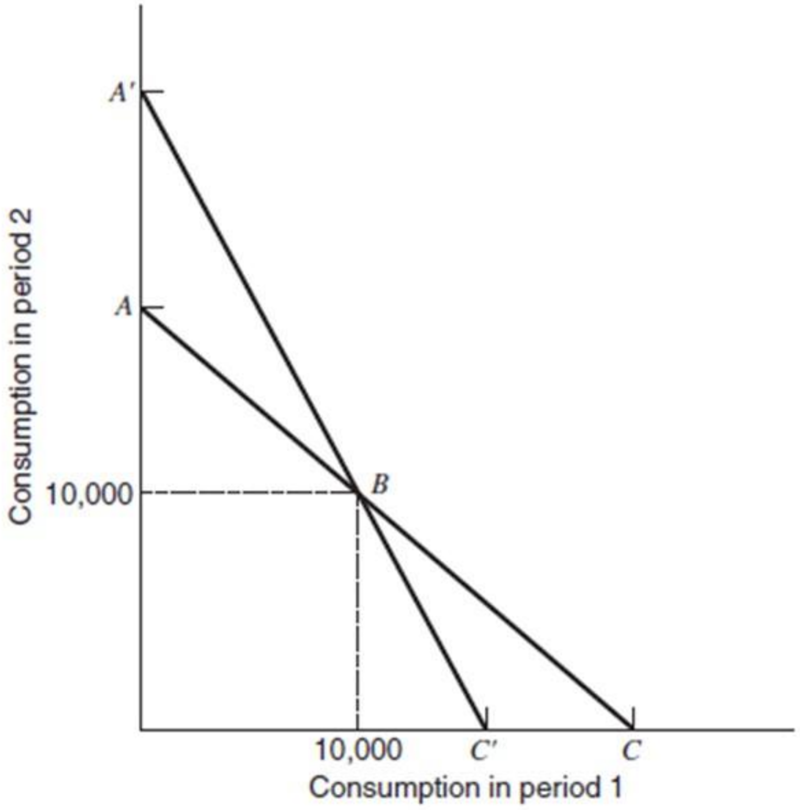
Indifference or Utility Curves

- Similarly, summing across all the investors who wish to borrow provides one point on the demand curve.
- As interest rates vary, full demand and supply curves are generated. Thus, the equilibrium interest rate is obtained when the amount supplied is equal to the amount demanded – called as market clearing condition.
- Therefore, two key factors, i.e., investors' income and "taste and preferences," lead to the determination of interest rates in the market.

Problem with Multiple Securities and Interest Rates

What if there were two assets?

- The first asset had 5% rate (for lending and borrowing), whereas the second asset had 10% rate.
- Now, investors would prefer to lend at 10% and borrow at 5%.



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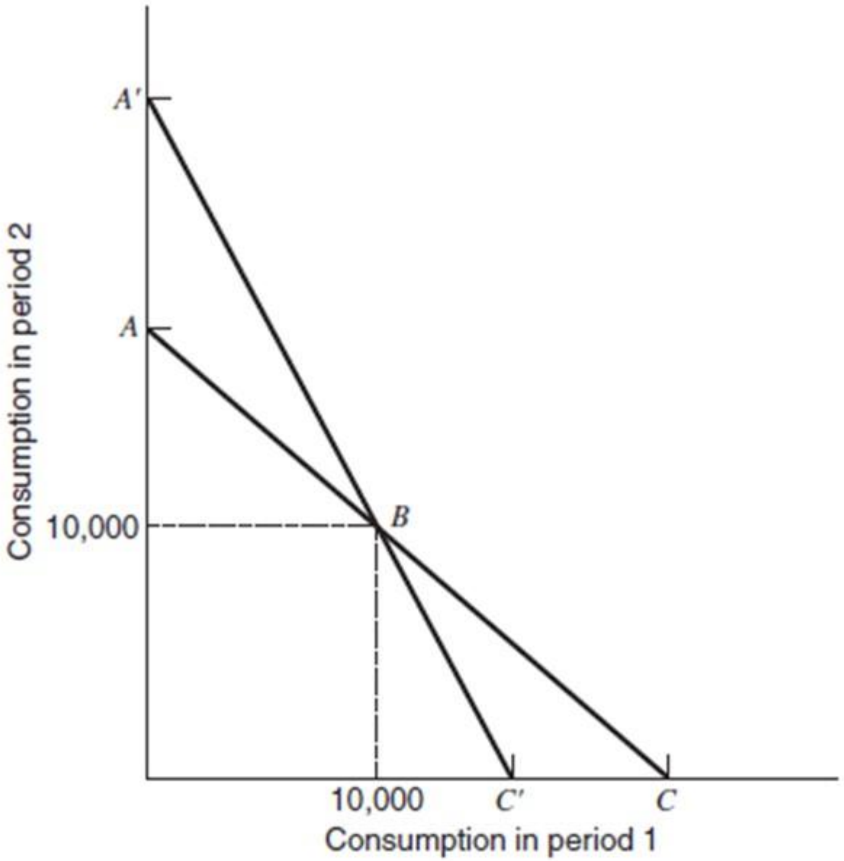
Problem with Multiple Securities and Interest Rates

What if there were two assets?

- Common sense would suggest that this situation is unstable. Nobody would like to invest at 5% and borrow at 10%.
- However, we do observe different interest rates. This has to do with the risk associated with different interest rate instruments.



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Expected and Actual Returns

Risk and Return in Financial Markets

How do we understand the framework of risk and return in financial markets?

- How much return we should expect from SBI-FD (Government Bank) vs Mutual Funds.
- You would expect a higher returns from Mutual fund as compared to a FD in government bank. Why?
- Government FD is a safer instrument, with a lot of surety about the principal and interest amount.
- Mutual funds are risky.

Risk and Return in Financial Markets

How do we understand the framework of risk and return in financial markets?

- Computation of returns: (1) interest income, and (2) capital appreciation
- $\text{Return} = \dfrac{\text{Capital appreciation} + \text{Interest income}}{\text{Initial investment}}$
- Consider a stock at price P_0 = 10, held for 1 year. The price at the end of the year is P_1 = 15. Also, during the year, it gave a dividend of 5.
- Return : $\dfrac{5+(15-10)}{10}$=1, or 100% annual return.

Risk and Return in Financial Markets

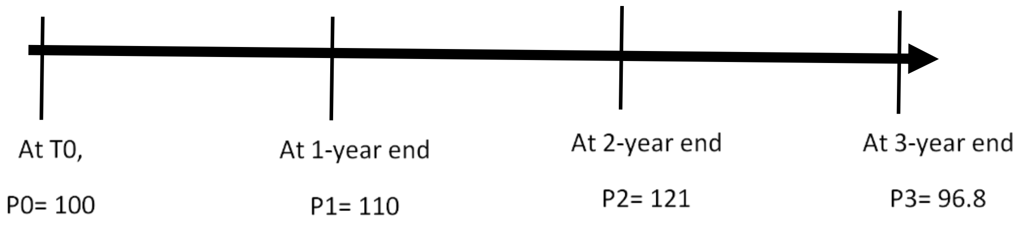
How do we understand the framework of risk and return in financial markets?

- Stock A offers 10% return and stock B offers 20% return
- Which one is the better investment?
- Do we need some more information

Risk and Return in Financial Markets

Arithmetic averages and geometric averages

- Consider an investment (e.g., stock) that is held for three years with the following closing prices (no interest/dividend is offered)
- Let us see what are the returns for different holding periods, i.e., T0–T1, T1–T2, and T2–T3
- R_{T0-T1} = (110 − 100)/100 = 10%
- R_{T1-T2} = (121 − 110)/110 = 10%
- R_{T2-T3} = (96.8 − 121)/121= −20%



Risk and Return in Financial Markets

Arithmetic averages and geometric averages

- Average return from the investment = $\frac{10\% + 10\% - 20\%}{3} = 0\%?$
- The intuition that he is wrong comes from the fact that you are left with 96.8, which is 3.2 less than your original investment
- Total return from the investment = $\frac{96.8-100}{100} = -3.2\%.$
- This return is for three years, should we divide it by 3

Risk and Return in Financial Markets

Arithmetic averages and geometric averages

- So we move to our friend that is compounded returns
- So we move to our friend that is compounded returns: $(1 + \bar{R})^3 - 1 = -3.2\%$, here $\bar{R}$ is the average return.
- $\left(1 + \left(-\frac{1.07825}{100}\right)\right)^3 - 1$ should be equal to 0.968
- $\bar{R}$ works out to be $= -1.07825\%$.
- Also the negative return seems fair

Returns: Expected Returns

Different from actual returns: builds an expectation of future

- A more general way to represent the expected returns would be like this
- $E(R_{i,t}) = \sum_{t=1}^{T} P_t \times R_t$.

S. No. (t)	Probabilities (P_i)	Expected Returns
1	0.1	40%
2	0.2	20%
3	0.3	0%
4	0.2	−20%
5	0.2	−30%

Returns: Expected Returns

Different from actual returns: builds an expectation of future

- What is our expected outcome in this game?
- *E* (return from the game)
- $= 0.1 \times 40\% + 0.2 \times 20\% + 0.3 \times 0\% + 0.2 \times (-20\%) + 0.2 \times (-30\%) = -2\%$
- A more general way to represent the expected returns would be like this: $E(R_{i,t}) = \sum_{t=1}^{T} P_t \times R_t$
- Here, $R_{i,t}$'s are the returns of a security 'i' for the period 't' with a probability of P_t

Returns: Expected Returns

What if we do not have these probabilities, and we only have past returns and all returns are equally likely?

- I.e., $P_1 = P_2 = P_3 = P_4 \ldots \ldots . = P_T$. Since we know that $\sum_{i=1}^{T} P_i = 1$
  Therefore, $P_1 = P_2 = P_3 = P_4 \ldots \ldots . = P_T = \frac{1}{T}$
- $E(R_i) = \bar{R_i} = \frac{1}{T}\sum_{t=1}^{T} R_t$
- This suggests that averaging of observed returns to obtain expected average return is a special case, where all the return outcomes are assigned equal probabilities

Returns: Expected returns

Different from actual returns: builds an expectation of future

- A more general way to represent the expected returns would be like this
- $E(R_{i,t})=\frac{1}{T}\sum_{t=1}^{T} R_t$=(1/5) × (40% + 20% + 0% − 20% − 30%)=2%

S. No. (t)	Expected Returns
1	40%
2	20%
3	0%
4	−20%
5	−30%

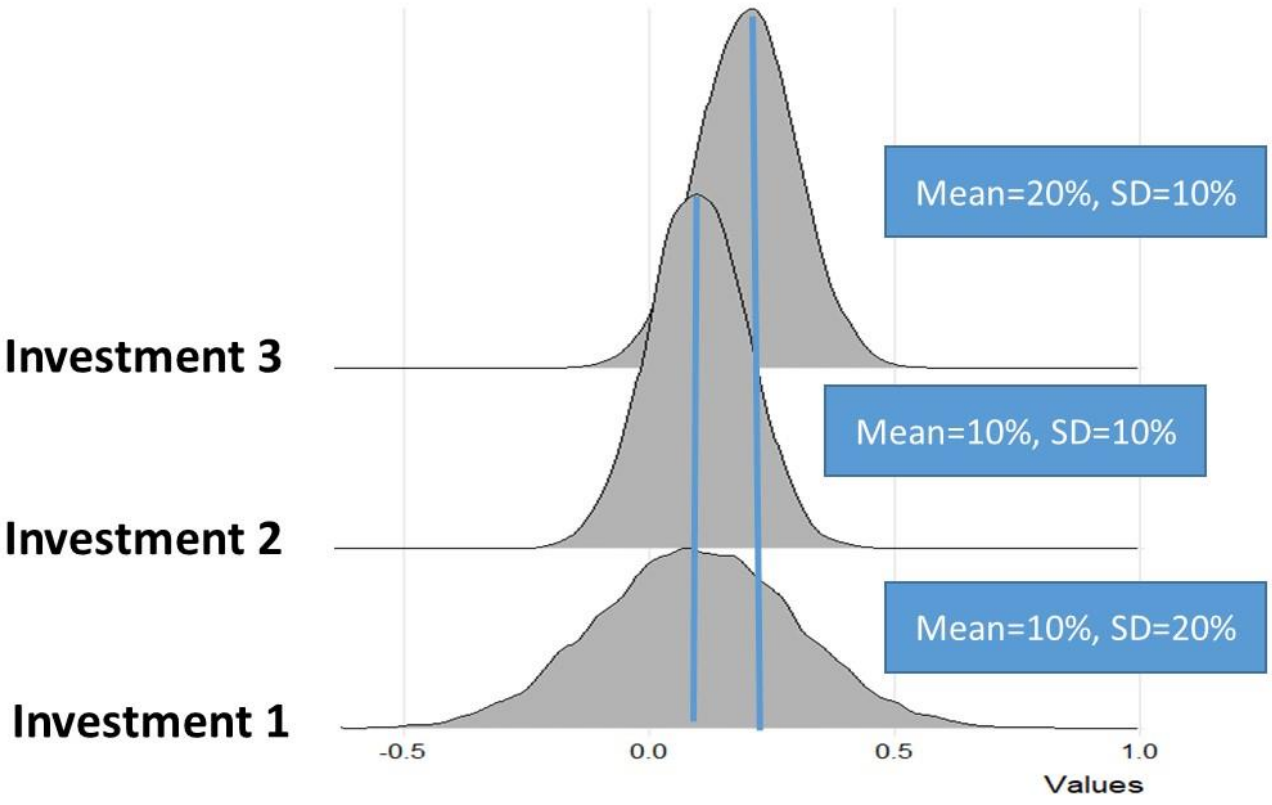
- But what are these expectations based upon? Risk

Risk: What Is Risk and How to Measure It?

Distribution of Returns

Between investments 1, 2, and 3, which on to choose

- Between 1 & 2, 1 is preferred: compare the risk
- Between 2 & 3, 3 is preferred: compare the expected returns



Risk: What Is Risk and How to Measure It?

Risk in financial markets:

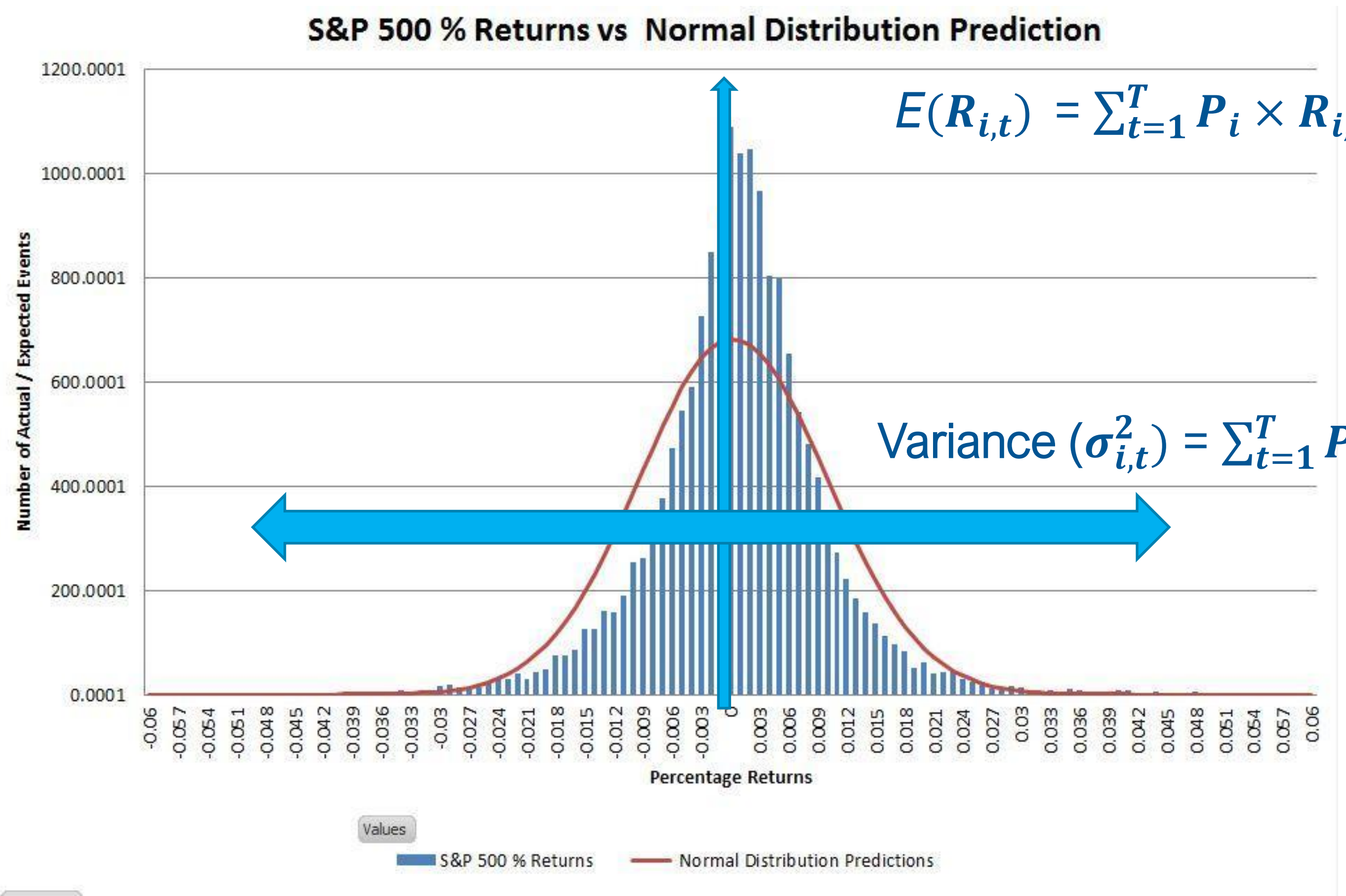
- Uncertain outcomes lead to risk. If the outcome is certain (SBI FD), then there is no risk. Risk-free assets.
- A person can be risk-averse, risk-taking, and risk-neutral.
- How to measure risk: variance $(\sigma_{i,t}^2)$ or standard deviation $(\sigma_{i,t})$
- $E(\sigma_{i,t}^2)$= Variance $(\sigma_{i,t}^2)$= $\sum_{t=1}^{T} P_i(R_{i,t} - \bar{R}_i\ )^2$

Risk: What Is Risk and How to Measure It?

E($\sigma^2_{i,t}$)= Variance ($\sigma^2_{i,t}$)= $\sum_{t=1}^{T} P_i(R_{i,t} - \bar{R}_i)^2$

- Again, for past observations that are equally likely
- I.e., $P_1 = P_2 = P_3 = P_4 \ldots \ldots . = P_T$. Since we know that $\sum_{i=1}^{T} P_i$ = 1. Therefore, $P_1 = P_2 = P_3 = P_4 \ldots \ldots . = P_T = \frac{1}{T}$
- Variance ($\bar{\sigma}_i^2$)= $\frac{1}{T}\sum_{t=1}^{T}(R_{i,t} - \bar{R}_i)^2$
- Often, while working with samples, we use $\bar{\sigma}_i^2 = \frac{1}{T-1}\sum_{t=1}^{T}(R_{i,t} - \bar{R}_i)^2$.

Risk: A Few Words on Normal Distribution



$E(R_{i,t}) = \sum_{t=1}^{T} P_i \times R_{i,t}$

Variance $(\sigma_{i,t}^{2}) = \sum_{t=1}^{T} P_i (R_{i,t} - \overline{R_i})^2$

https://seekingalpha.com/article/3959933-predicting-stock-market-returns-lose-normal-and-switch-to-laplace

Risk: What Is Risk and How to Measure It?

- Let us go back to our example "Game" for which we computed expected returns with given probabilities. Now, we will compute the expected variance for the same example using probabilities.
- $E(\sigma_{i,t}^2)$ = Variance $(\sigma_{i,t}^2)$ = $\sum_{t=1}^{T} P_i(R_{i,t} - \bar{R}_i)^2$

S. No. (t)	Probabilities ($\boldsymbol{P_i}$)	Expected Returns (Mean $\overline{\boldsymbol{R_i}}$ = −2%)
1	0.1	40%
2	0.2	20%
3	0.3	0%
4	0.2	−20%
5	0.2	−30%

Risk: What Is Risk and How to Measure It?

Probabilities (P_i)	Expected Returns (Mean $\overline{R_i}$ = −2%)	Mean deviation ($R_i - \overline{R_i}$)	Squared Deviation $(R_i - \overline{R_i})^2$	Probability × Squared Deviation $P_i \times (R_i - \overline{R_i})^2$
0.1	40%	42%	0.1764	0.01764
0.2	20%	22%	0.0484	0.00968
0.3	0%	−2%	0.0004	0.00012
0.2	−20%	−18%	0.0324	0.00648
0.2	−30%	−28%	0.0784	0.01568

- $E(\sigma_{i,t}^2) = \sum_{t=1}^{T} P_i(R_{i,t} - \overline{R_i})^2$ = 0.01764 + 0.00968 + 0.00012 + 0.00648 + 0.01568 = 0.0496

- The value of standard deviation = $\sqrt{E(\sigma_{i,t}^2} = \sqrt{0.0496}$ = 0.2227 or 22.27%

A Short Note on Compounding of Interest

A Short Note on Compounding of Interest

Interest rates can be sometimes misleading:

- One should carefully examine the frequency of the interest payment and compounding period of interest rates.
- For example, you borrow from a bank at 12%. These quoted rates are usually annual percentage rates (APR)
- Your bank tells you that you have to pay 1% monthly installments. Now, your effective rate becomes $(1.01)^{12} - 1 = 12.6825\%$.

A Short Note on Compounding of Interest

As a general rule, for payment (compounding frequency) of "*m*" times per year with a quoted APR of *r*%, the following formula can be used to compute the effective interest rate: $\left(1+\frac{r}{100\times m}\right)^{m}-1$

- Here, *r* is in %. As the period of compounding becomes smaller, the effective interest becomes longer.
- For a special case, when *m* becomes infinitely large, then this value converges to $e^{r/100}$.
- For a period of "*t*" years, the effective amount will be $e^{rt/100}$ and effective interest will be $e^{rt/100}$ – 1.

A Short Note on Compounding of Interest

- For a special case, when *m* becomes infinitely large, then this value converges to $e^{r/100}$.
- For a period of "*t*" years, the effective amount will be $e^{rt/100}$ and effective interest will be $e^{rt/100} - 1$.
- This is called continuous compounding. In financial markets research, mostly continuous compounding is employed in the computation of returns.
- For example, if opening price $P_0 = 15$ and closing price $P_1 = 20$, then returns under continuous compounding will be computed as follows: $\ln\left(\frac{P_1}{P_0}\right)$.

Example on Compounding of Interest

Compounding Frequencyz
1 year = 1.12
6 months (2 times)
4 months (3 times)
3 months (4 times)
1 month (12 times)
1 day (365 times)
Very small period (less than a day – continuous compounding)

Example on Compounding of Interest

Compounding Frequency	Effective Interest Rate	Excess Over 12%
1 year = 1.12	1.12 – 1 = 12%	0%
6 months (2 times)	$1.06^{2} - 1$ = 12.36%	0.36%
4 months (3 times)	$1.04^{3} - 1$ = 12.4864%	0.4864%
3 months (4 times)	$1.03^{3} - 1$ = 12.551%	0.551%
1 month (12 times)	1.01^{12} – 1 = 12.6825%	0.6825%
1 day (365 times)	$(1 + 0.12/365)^{365}$ – 1 = 12.7475%	0.7475%
Very small period (less than a day – continuous compounding)	$e^{.12}$ = 12.75%	0.75%

Summary and Concluding Remarks

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- Financial markets provide the conduit for individuals to optimize the time pattern of their consumption.
- The demand and supply of funds determine the efficient clearing prices, and, therefore, the interest rate environment prevails in the economy.
- Expectations of interest rate from security are determined based on the risk of the security.

Summary and Concluding Remarks

- Risk of security represents the increased uncertainty in outcomes; the wider the possible outcomes the more the risk.
- For a given level of risk, investors prefer instruments with higher expected returns, and for a given level of expected returns, investors prefer instruments with lower risk.

Thanks!