Ramakrishna Mission Vivekananda Educational & Research Institute

Belur Math, Howrah, West Bengal

School of Mathematical Sciences, Department of Computer Science

M.Sc. in Big Data Analytic 2019, Mid Semester Exam — Date: 04 Sep 2019

Course : **DA311: Time Series** — Time: 2 hrs

Instructor : Dr. Sudipta Das — *Max marks: 50*

Student signature and Id:

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1. Define strict stationary process and (weak) stationary process. [1+3=4]

2. Explain why

   (a) The log differencing is sometimes, preferred over ordinary differencing in time series analysis

   (b) One should not remove deterministic trend of a time series by the method of differencing.

   (c) An MA(q) process with i.i.d. Z_ts, is strictly stationary. $[2 \times 3 = 6]$

For the following problems, consider $\{Z_t\} \sim WN(0, 1)$.

3. Check the stationarity, causality and invertibility of the following process [5]

$$X_t - X_{t-1} + 0.1X_{t-2} = Z_t - 0.3Z_{t-1}.$$

4. Let $X_t = \sum_{k=0}^{\infty} Z_{t-k}$, for $t = 0, \pm 1, \pm 2, \ldots$. Show that the best linear predictor of X_{t+k} based on $\{X_u : u \leq t\}$ is X_t. [5]

5. For the following MA(1) process

$$X_t = Z_t + \theta Z_{t-1}, t = 0, \pm 1, \ldots,$$

the best linear predictor of X_{n+1} based on $X_1, \ldots, X_n$ is $X_{n+1}^{(n)} = \boldsymbol{\phi_n}'\mathbf{X_n}$, where $\boldsymbol{\phi_n}$ satisfies $\Gamma_n \boldsymbol{\phi_n} = \boldsymbol{\gamma_n}$. Show that for $1 \leq j < n$,

$$\phi_{n,n-j} = (-\theta)^{-j}(1 + \theta^2 + \cdots + \theta^{2j})\phi_{nn}.$$

Hence, find ϕ_{nn}, that the value at lag n of the partial ACF of this MA(1) process. [10]

P.T.O.

6. Consider the following linear process

$$Y_t = Z_t + \sum_{j=1}^{\infty} \frac{Z_{t-j}}{j}.$$

Show that $\lim_{h \to \infty} \gamma(h) = 0$, where $\gamma(h)$ is the autocovariance at lag h. [10]

(Hint: $\displaystyle\sum_{j=1}^{h} \frac{1}{j} = \mathcal{C} + \log h + \mathcal{O}\left(\frac{1}{h}\right)$, where $\mathcal{C} = 0.5722\ldots$, the Euler' constant)

7. The sample autocorrelation at lag 1, from a set of 498 observations, is calculated as -0.66 and the maximum of the correlations from lag 2 to lag 20 is .08. Assuming that the sample size is large, test the claim that the sample is from an MA(1) process, at a 5% level of significance. [10]

(Bartlett's formula: $w_{ij} = \displaystyle\sum_{k=1}^{\infty} \{\rho(k+i) + \rho(k-i) - 2\rho(i)\rho(k)\}\{\rho(k+j) + \rho(k-j) - 2\rho(j)\rho(k)\}$)

This exam has total 7 questions, for a total of 50 points and 0 bonus points.