Assignment 9 Solution

The correct answer is in bold font

1. The following is a correct statement in the context of portfolio construction and optimization.

   (a) A risk-free asset has a beta of one. [Hint: Risk-free asset is insensitive to market movements]
   (b) Market indices have a beta of zero [Hint: Market indices that are well-diversified capture the market movements accurately]
   (c) Simply adding securities to a portfolio leads to diversification [Hint: Just by increasing the number of securities, one can eliminate the stock-specific idiosyncratic component of risk.]
   (d) One can obtain the largest reduction in risk if the securities are perfectly correlated. [The lower the correlation across the securities, the higher the diversification of risk.]

2. Given that the extreme event has taken place, the following probabilities are provided for three extreme loss events along with the corresponding returns: $P_1 = 0.5, \bar{R}_1 = -5\%$; $P_2 = 0.3, \bar{R}_2 = -8\%$; $P_3 = 0.2, \bar{R}_3 = -10\%$. Assuming that these are the only possibilities (i.e., $P_1 + P_2 + P_3 = 1$) , what is the correct interval in which conditional VaR (CVaR) or expected shortfall (ES) for the portfolio will fall?

   (a) **- 8% to - 6% Hint: CVaR (or ES)=** $\boldsymbol{P_1 * \bar{R}_1 + P_2 * \bar{R}_2 + P_3 * \bar{R}_3 = 0.5 * -5\% + 0.3 * -8\% + 0.2 * -10\%}$
   (b) - 6% to - 4% Hint: CVaR (or ES)= $P_1 * \bar{R}_1 + P_2 * \bar{R}_2 + P_3 * \bar{R}_3$
   (c) - 4% to - 2% Hint: CVaR (or ES)= $P_1 * \bar{R}_1 + P_2 * \bar{R}_2 + P_3 * \bar{R}_3$
   (d) - 2% to - 0% Hint: CVaR (or ES)= $P_1 * \bar{R}_1 + P_2 * \bar{R}_2 + P_3 * \bar{R}_3$

3. Find the expected returns ($\bar{R}_{AB}$) of a two-stock portfolio with Securities A (Expected return ($\bar{R}_A$= 8%, w_A=0.5) and B ($\bar{R}_B$= 18.8%, w_B=0.5) Here w_A and w_B are proportionate weights invested in the two securities (A and B)?

   (a) 8%-10%. Hint $\bar{R}_{AB} = w_A * R_A + w_B * R_B$
   (b) 10%-12%. Hint $\bar{R}_{AB} = w_A * R_A + w_B * R_B$
   (c) 12%-14%. Hint $\bar{R}_{AB} = w_A * R_A + w_B * R_B = 0.5 * 8\% + 0.5 * 18.8\% = 13.4\%$
   (d) 14%-16%. Hint $\bar{R}_{AB} = w_A * R_A + w_B * R_B$

4. Find the risk (σ_{AB}) of a two-stock portfolio with Securities A (standard deviation= 25%, w_A=0.5) and B (standard deviation= 45%, w_B= 0.5) and correlation $\rho_{AB} = 1$. Here w_A and w_B are proportionate weights invested in the two securities (A and B)?

(a) 5%-15%. Hint $\sigma_{AB}^2 = w_A^2 * \sigma_A^2 + w_B^2 * \sigma_B^2 + 2 * w_A * w_B * \rho_{AB} * \sigma_A * \sigma_B$.

(b) 15%-25%. Hint $\sigma_{AB}^2 = w_A^2 * \sigma_A^2 + w_B^2 * \sigma_B^2 + 2 * w_A * w_B * \rho_{AB} * \sigma_A * \sigma_B$.

(c) 25%-35%. Hint $\boldsymbol{\sigma_{AB}^2 = w_A^2 * \sigma_A^2 + w_B^2 * \sigma_B^2 + 2 * w_A * w_B * \rho_{AB} * \sigma_A * \sigma_B}$.; for $\boldsymbol{\rho_{AB} = 1, \sigma_{AB} = w_A * \sigma_A + w_B * \sigma_B = 25\% * 0.5 + 45\% * 0.5 = 35\%}$

(d) 40%-50%. Hint $\sigma_{AB}^2 = w_A^2 * \sigma_A^2 + w_B^2 * \sigma_B^2 + 2 * w_A * w_B * \rho_{AB} * \sigma_A * \sigma_B$.

5. A portfolio consists of two stocks. The expected return of stock A is 7%, and its standard deviation is 4%, while the expected return of stock B is 5%, and its standard deviation is 2%. There is a correlation of 0 between the two stocks. Find the amount invested in the minimum variance portfolio.

(a) $X_A = 0.25, X_B = 0.75$ Hint: $X_A = [\sigma_B^2 - \rho_{AB}\sigma_A\sigma_B] / [\sigma_A^2 + \sigma_B^2 - 2\rho_{AB}\sigma_A\sigma_B]$

(b) $\boldsymbol{X_A = 0.20, X_B = 0.80}$ Hint :$\boldsymbol{X_A = [\sigma_B^2 - \rho_{AB}\sigma_A\sigma_B] / [\sigma_A^2 + \sigma_B^2 - 2\rho_{AB}\sigma_A\sigma_B] = \frac{\sigma_B^2}{\sigma_A^2 + \sigma_B^2} = \frac{2^2}{4^2 + 2^2} = 0.20}$

(c) $X_A = 0.35, X_B = 0.65$ Hint: $X_A = [\sigma_B^2 - \rho_{AB}\sigma_A\sigma_B] / [\sigma_A^2 + \sigma_B^2 - 2\rho_{AB}\sigma_A\sigma_B]$

(d) $X_A = 0.30, X_B = 0.70$ Hint: $X_A = [\sigma_B^2 - \rho_{AB}\sigma_A\sigma_B] / [\sigma_A^2 + \sigma_B^2 - 2\rho_{AB}\sigma_A\sigma_B]$

6. In the previous question (5), determine the variance of the portfolio

(a) 5%-8% $Hint: \sqrt{(w_A^2 * \sigma_A^2 + w_B^2 * \sigma_B^2)}$

(b) **1%-3%** $\boldsymbol{Hint: \sqrt{(w_A^2 * \sigma_A^2 + w_B^2 * \sigma_B^2)} = 1.78\%}$

(c) 3%-5% $Hint: \sqrt{(w_A^2 * \sigma_A^2 + w_B^2 * \sigma_B^2)}$

(d) 8%-10% $Hint: \sqrt{(w_A^2 * \sigma_A^2 + w_B^2 * \sigma_B^2)}$

7. In the previous question (5), if the correlation $\rho_{AB} = -1$, find the amount invested in the minimum variance portfolio

(a) $X_A = 0.35, X_B = 0.65$ Hint: $X_A = [\sigma_B^2 - \rho_{AB}\sigma_A\sigma_B] / [\sigma_A^2 + \sigma_B^2 - 2\rho_{AB}\sigma_A\sigma_B]$

(b) $X_A = 0.30, X_B = 0.70$ Hint: $X_A = [\sigma_B^2 - \rho_{AB}\sigma_A\sigma_B] / [\sigma_A^2 + \sigma_B^2 - 2\rho_{AB}\sigma_A\sigma_B]$

(c) $X_A = 0.25, X_B = 0.75$ Hint: $X_A = [\sigma_B^2 - \rho_{AB}\sigma_A\sigma_B] / [\sigma_A^2 + \sigma_B^2 - 2\rho_{AB}\sigma_A\sigma_B]$

(d) $\boldsymbol{X_A = 0.33, X_B = 0.67}$ Hint: $\boldsymbol{X_A = [\sigma_B^2 - \rho_{AB}\sigma_A\sigma_B] / [\sigma_A^2 + \sigma_B^2 - 2\rho_{AB}\sigma_A\sigma_B] = \sigma_B(\sigma_B + \sigma_A) / (\sigma_A + \sigma_B)\hat{}2 = \sigma_B / \sigma_B + \sigma_A = 1/3}$

8. In the previous question (5), determine the variance of the portfolio if the correlation $\boldsymbol{\rho_{AB} = -1}$

(a) **0-2%** $\boldsymbol{Hint\!: w_A\sigma_A - w_B\sigma_B = \frac{1}{3} * 4 - \frac{2}{3} * 2 = 0}$
(b) 2%-4% $Hint\!: w_A\sigma_A - w_B\sigma_B$
(c) 6%-8% $Hint\!: w_A\sigma_A - w_B\sigma_B$
(d) 4%-6% $Hint\!: w_A\sigma_A - w_B\sigma_B$

9. A portfolio has a covariance with the market (σ_{im}) = 180 and variance of (σ_m^2)=95. What is the beta of the security?

(a) 0.25-0.75. Hint: Beta is the ratio of covariance with the market to that of variance of the market.
(b) 0.75-1.25. Hint: Beta is the ratio of covariance with the market to that of variance of the market.
(c) 1.25-1.75. Hint: Beta is the ratio of covariance with the market to that of variance of the market
(d) **1.75-2.25. Hint:** $\boldsymbol{\beta_i = \frac{\sigma_{im}}{\sigma_m^2} = 1.8.}$

10. Historical Return distribution of daily returns is provided to you. You are asked to compute 5% VaR for your portfolio. You divide these returns into 100 equally spaced intervals (X1…X2….X101) from extremely positive return quantiles (X1 onwards) at the bottom and extremely negative returns (losses) at the top (ending at X101). For example, X5 would be the beginning and X6 would be the end of the 5^{th} interval. Similarly, X100 would be the beginning and X101 would be the end of the 100^{th} interval. Identify the return position of 5% VaR.

(a) X6 Hint: X6 would indicate the return observation corresponding to the top 5 percentile on the higher side. We are looking at the 5 percentiles from the lowest returns.
(b) X95 Hint: X95 would indicate 94 percentiles from the top (or 6% VaR)
(c) X100 Hint: X100 would indicate 99 percentiles from the top (or 1% VaR)
(d) **X96 Hint: X96 would indicate 95 percentiles from the top (or 5% VaR)**