INDIAN INSTITUTE OF TECHNOLOGY KANPUR

Artificial Intelligence (AI) for Investments

Lesson 2: Cash Flow Discounting

Introduction

In this lesson we will cover the following topics:

- Time value of money and cash flow discounting
- Discount rates and opportunity cost of capital
- Present value of annuities and perpetuities
- Valuation of growing annuities and perpetuities
- Impact of compounding frequency on present value computation
- Summary and concluding remarks

Time value of money

- Receive $100 today or one year later
- What would be that additional amount for which you would agree to delay your consumption of this $100 by a year
- This is called time-value of money
- A dollar worth today is more than the dollar worth tomorrow: but how much more

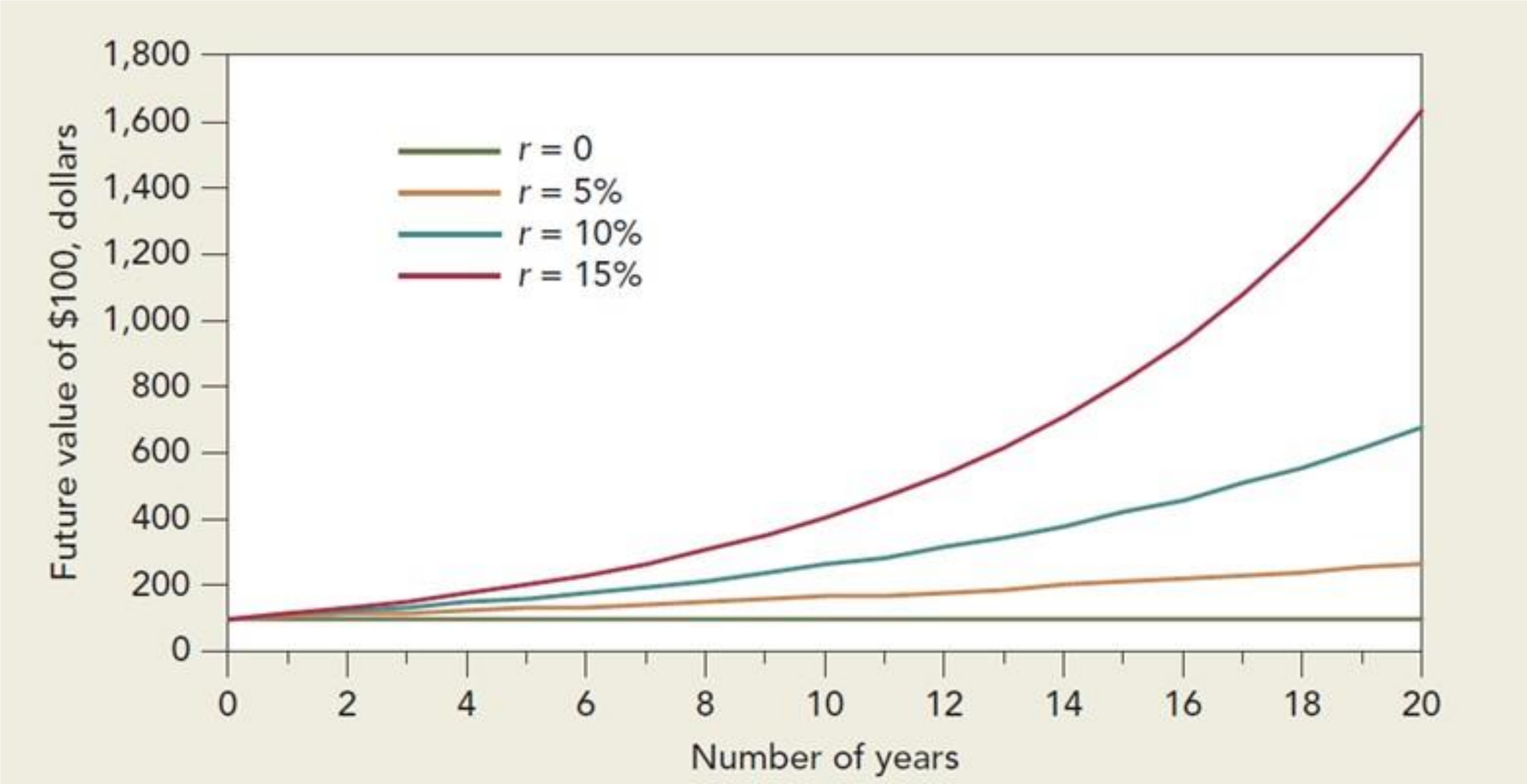
Time value of money

- An investment of \$100 into fixed deposit at 7% interest will grow to become \$107 in one year and \$114.49 in two years

- In second year, you earn interest on principal as well as on the interest earned in second year: power of compounding

- Similarly \$100, invested for 20 years at 10% will grow to become $100 * 1.10^{20} = \$672.75$

Time value of money



Assume an interest rate of r and a period of t. The future value of $100 invested today, will be: $100 * (1 + r)^t$

20 years, $100 invested at 10% interest. It will grow to become $100 * 1.10^{20} = \$672.75$

At 5% the money will grow to become $100 * 1.05^{20} = \$265.33$

Time value of money

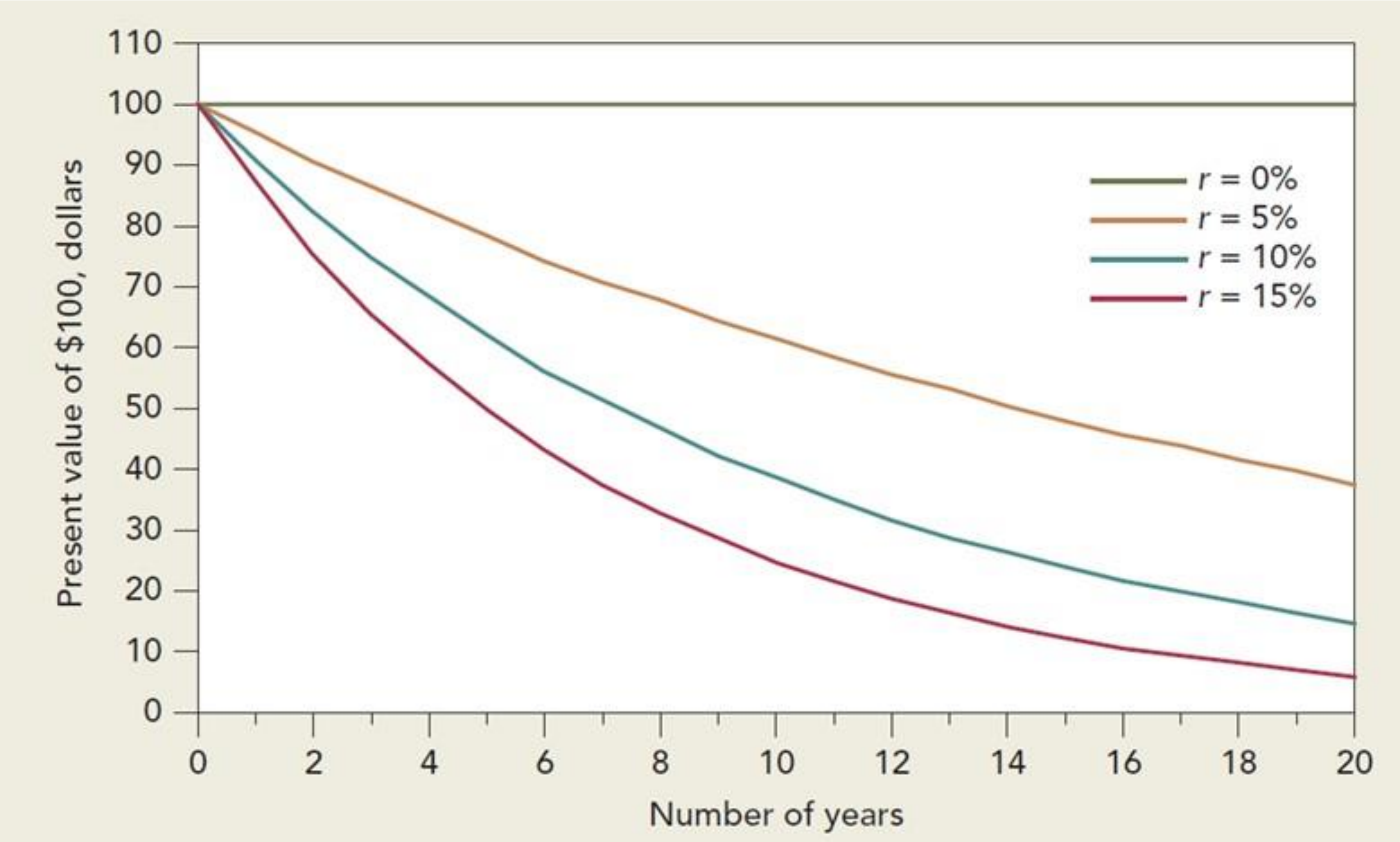
- \$100 invested for two years at 7% will grow to a future value of $100 * 1.07^2 = \$114.49$

- So, if appropriate interest is 7%, then the present value of $\$114.49$ to be received two years from now is \$100 today!

- This can be simply computed as follows: Present Value (PV)= $\frac{114.49}{1.07^2}$=\$100

- Therefore the formula of present value can also be written simply as follows: $PV = \frac{C_t}{(1+r)^t}$.

Time value of money



A payment worth $100 to be received in 20 years at an interest rate of 5% has a PV of $\frac{100}{1.05^{20}} = \$37.69$

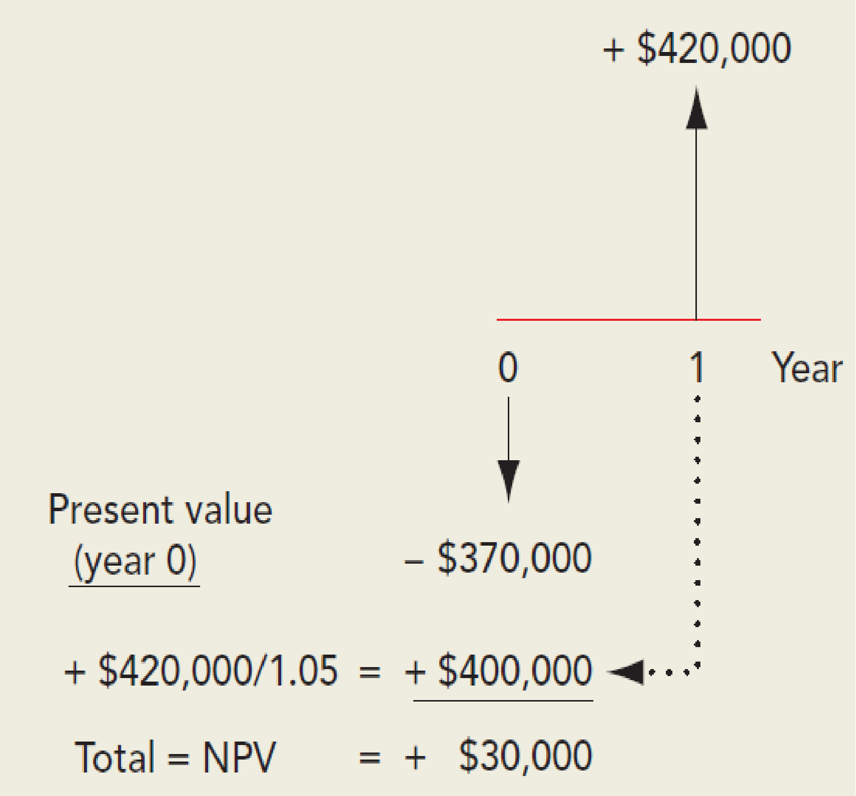
If the interest rate increases to 10%, the PV falls to $\frac{100}{1.10^{20}} = \$14.86$. A decline of more than 50%.

Time value of money

- A simple project: purchase of office space a cost of $370000
- Your advisor tells you that it is a sure thing with $42000 expected by the end of the year
- What is the appropriate opportunity cost: prevailing risk-free rate of r=5%
- The present value of this investment can be computed as $PV = \frac{420000}{1.05} = \$400000$
- Net-present value of this investment (NPV)=PV-investment=400000-370000=$30000

Time value of money

- This NPV formula can be easily represented as $NPV = C_0 + \frac{C_1}{1+r}$
- Accept the project if NPV>0 and reject the project if NPV is less than 0
- It is useful to perform the analysis through time-lines, as shown in the diagram here



Time value of money

- Let us now introduce risk
- In our previous office space example, now consider that you are not so certain about the revenues
- You consider it to be a risky venture and a 12% interest rate to be an appropriate opportunity cost
- The new NPV computation: $NPV = PV - 370000 = \frac{420000}{1.12} - 370000 = 5000$
- NPV of the project has come down as it has become riskier for you
- The present value of the office space has two aspects (1) The timelines of the cash flows; and (2) The risk of the cash flow

Time value of money

- There is another decision rule for evaluating such projects: Rate of return rule

- $\mathbf{Return} = \frac{\mathbf{Profit}}{\mathbf{Investment}} = \frac{\mathbf{420000 - 370000}}{\mathbf{370000}} = \mathbf{13.5\%}$

- Opportunity cost of capital> Project return then accept the project and vice-versa

- So now we have two rules for making investment decisions:

    - Net-present value rule (NPV) rule: Accept the investments that have positive NPVs

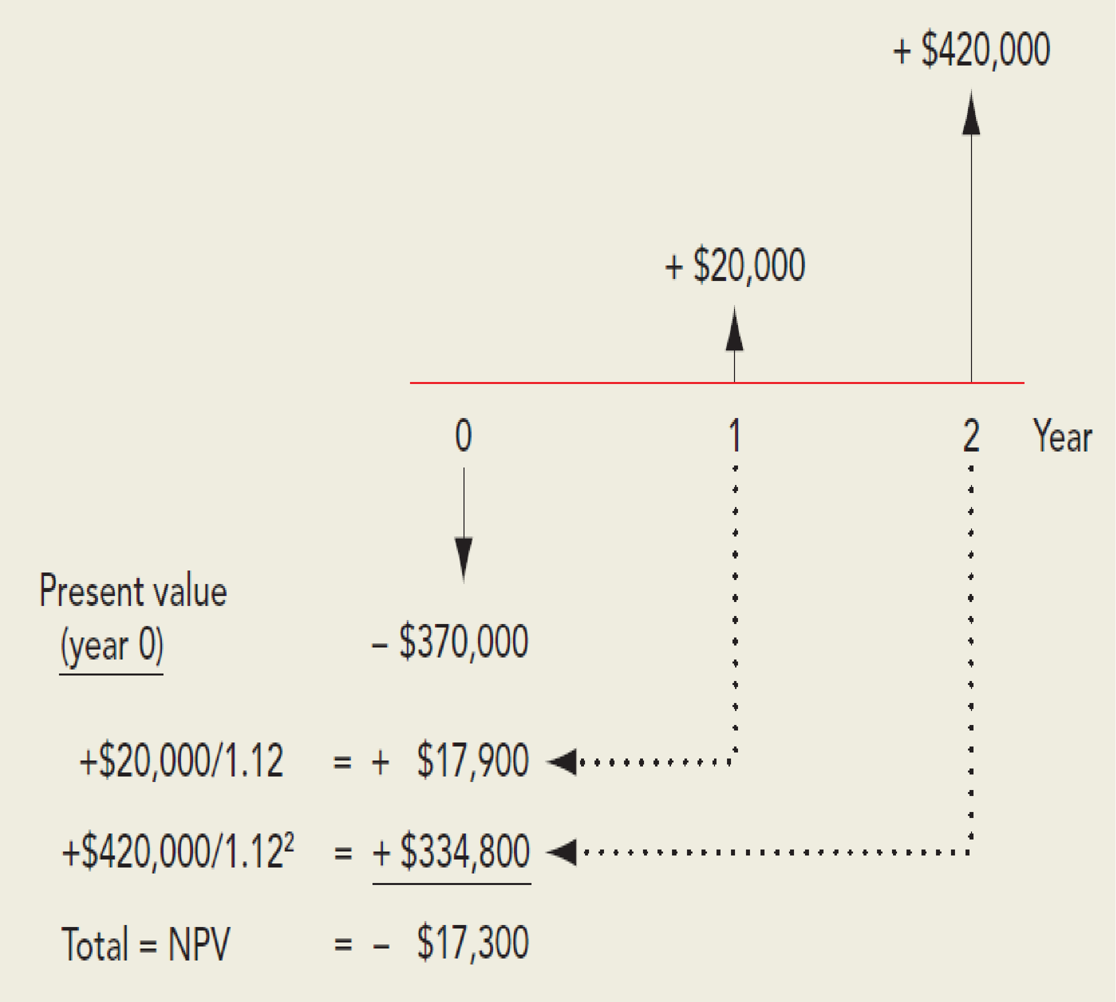
    - Rate of return rule: Accept the investments that have rate of returns higher than their opportunity cost of capital.

Computing NPVs with multiple cash flows

- Present values can be simply added up
- Suppose that a cash flow stream spread over ‘t’ years is provided as follows, $C_i, for\ i = 1\ to\ T$. Also assume a discount rate ‘r’
- $PV = \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \cdots + \frac{C_\mathrm{T}}{(1+r)^T} = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t}$
- $NPV = C_0 + PV = C_0 + \sum_{t=1}^{T} \frac{C_t}{(1+r)^t}$

Computing NPVs with multiple cash flows

- You are planning to rent out your recently purchased office premises for $20000 a year for two years (acquired at a cost of $370000)
- Also at the end of second year you plan to sell the premise and receive an expected cash flow of $400000 at the end of the year
- The appropriate discount rate is 12%. The timelines for these cash flows are shown in the Figure here

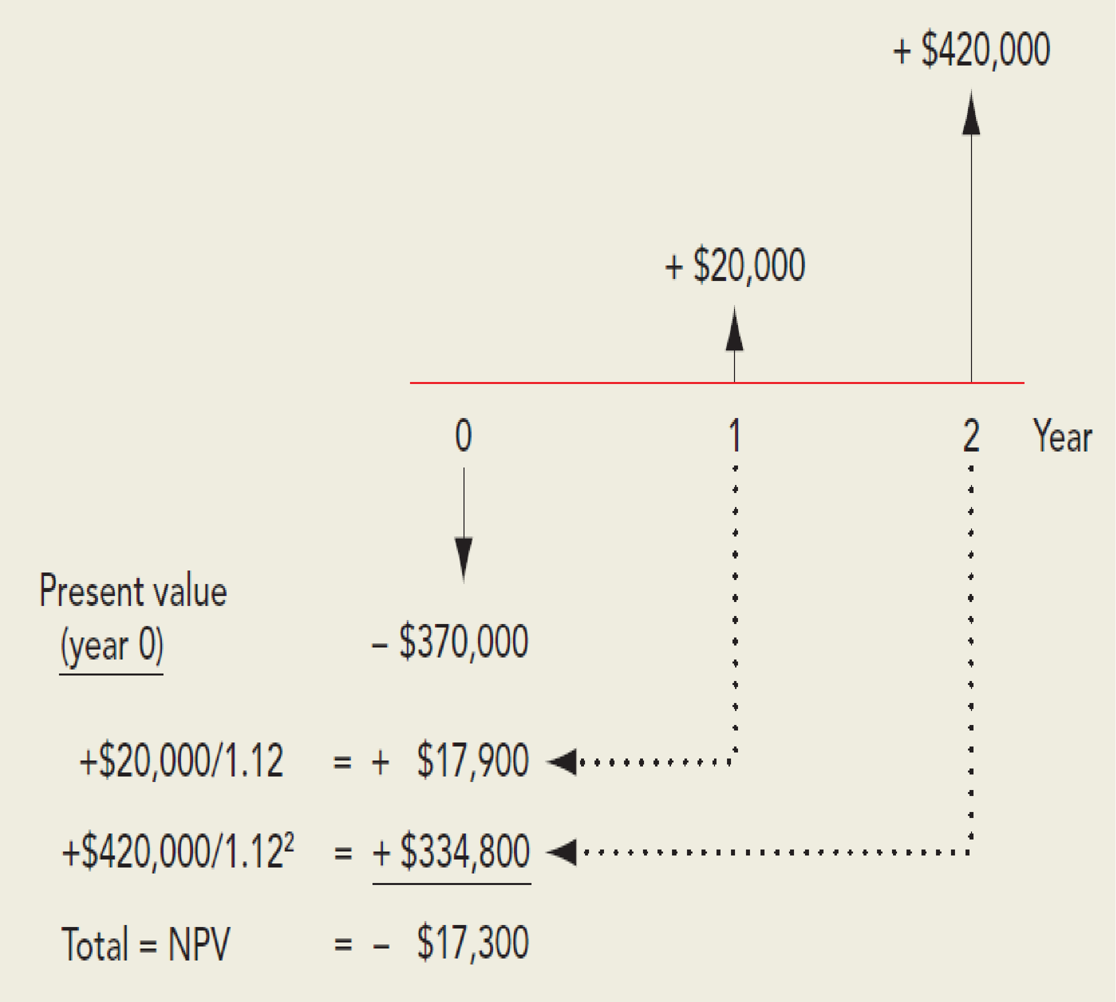


Computing NPVs with multiple cash flows

- You expect to earn \$20000 in the first year and \$420000 at the end of the second year. The present value of these cash flows can be computed as per the following scheme.

- $PV = \frac{20000}{1.12} + \frac{420000}{1.12^2} = 17900 + 334800 = 352700$

- The NPV of this investment is = $352700 - 370000 = -17300$.



Valuing perpetuities and annuities

- A perpetuity is a security that pays periodic cash flows over infinite time intervals
- Consider a perpetuity with annual cash flows amounting to ‘C’ and an appropriate discounting rate of r
- The present value of this perpetuity is provided here: $PV = \frac{C}{r}$
- Consider a simple example as follows. You are a billionaire and would like to fund the education at your alma-mater with \$1 Mn each year in perpetuity, starting with next year. If the interest rate is 10%, you would need to provide the following amount: $\frac{1}{0.1} = \$10Mn$
- In case you want this perpetuity to start right now immediately. Then you would need to shell-out an additional \$1Mn, i.e., \$11 Mn total.

Valuing perpetuities and annuities

- Annuity has a finite life of a specified number of years
- The annuity computation formula can be derived with the help of perpetuity formula as shown in the diagram here

	Cash flow						
Year:	1	2	3	4	5	6 . . .	Present value
1. Perpetuity A	$1	$1	$1	$1	$1	$1 . . .	$\frac{1}{r}$
2. Perpetuity B				$1	$1	$1 . . .	$\frac{1}{r(1+r)^3}$
3. Three-year annuity (1 − 2)	$1	$1	$1				$\frac{1}{r} - \frac{1}{r(1+r)^3}$

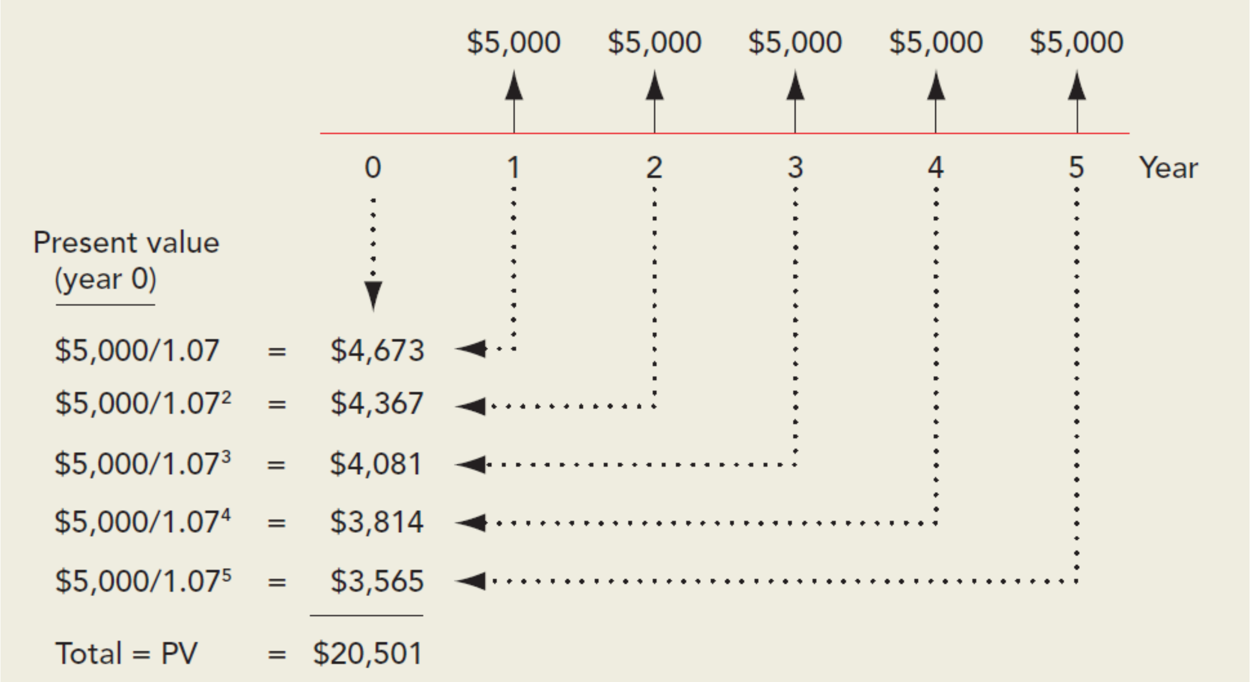
- Similarly, we can value an annuity that pays C amount at the end of year for each of the t years, starting from the year end. This will be : $\frac{C}{r}[1 - \frac{1}{(1+r)^t}]$

Valuing perpetuities and annuities

- Consider an example of an annuity that pays $5000 a year, paid at the end of year, for each of the next five years

- If the appropriate discount rate is 7%, what is the present value of this annuity.

- $PV = \frac{5000}{0.07}\left[1 - \frac{1}{1.075^5}\right] = \$20501$



Valuing perpetuities and annuities

- Often these cash flows do not remain constant and exhibit a certain growth rate.
- If a perpetuity is growing at a rate of 'g' the simple formula for this perpetuity becomes: $\frac{C}{r-g}$; where r>g
- A \$1Mn perpetuity, starting from the year end, that grows at an interest of 4%. If the appropriate discount rate is 10%, the present value of this perpetuity will be $\frac{1}{0.10-0.04} = \$16.67\ Mn$
- The simple formula for annuities (C) growing at a *rate* 'g' for 't' years is provided below: $PV = \frac{C}{r-g}[1 - \frac{(1+g)^t}{(1+r)^t}]$
- Consider a 3-year \$5000 annuity with 10% discount rate and a growth rate of 6%
- Then its PV would be $PV = \frac{5000}{0.10-0.06}\left[1 - \frac{1.06^3}{1.10^3}\right] = \$13146$

A short lesson on compounding

- Sometimes cash flows are not received annually but at higher frequencies, e.g., quarterly, weekly, monthly
- If you get an interest of 10% per annum on $100. You will get an interest amount of $10
- However, if a 5% interest is paid at 6-monthly intervals you get an overall amount of 1.05*105= $110.25 by the end of this year
- Therefore, 10% interest compounded semi-annually results in an effective interest of $1.05^2 - 1 = 10.25\%$
- The compounding frequency increases to m periods, the resulting formula becomes: $\left[1 + \left(\frac{r}{m}\right)\right]^m$
- if m-> ∞, then the resulting formula becomes e^r, where is e=2.718

Summary and Concluding remarks

- Cash flows are discounted for two simple reasons
- (1) Dollar is worth more today than a dollar tomorrow
- (2) A safe dollar is worth more than a risky dollar
- Managers can maximize the firm-value by accepting projects with positive net present values (NPVs)
- $NPV = C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} \ldots\ldots$
- C_0 here is the initial investment, which is expected to be negative, that is outflow of cash. Discount rate ‘r’ is obtained by examining the prevailing interest rates in financial markets, on the instruments with the same risk

Thanks!