Ramakrishna Mission Vivekananda Educational and Research Institute
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School of Mathematical Sciences
Department of Computer Science

MSc BDA : Batch 2020-22, Semester III, MidSem Exam
DA311: Time Series
Dr. Sudipta Das

Student Name (in block letters):	Date: 18 Nov 2021
Student Roll No:	Max Marks: 80
Signature:	Time: 2.5hrs

Answers must be properly justified to deserve full credits.

1. (16 points) Explain why
   (a) (4 points) the log differencing is sometimes, preferred over ordinary differencing in time series analysis
   (b) (4 points) ACF and PACF plots are useful in time series analysis.
   (c) (4 points) unit root test is used in time series analysis
   (d) (4 points) one should not remove deterministic trend of a time series by the method of differencing.

2. (16 points) If A and B are uncorrelated random variables with mean 0 and variance 1 and ω is a fixed frequency in the interval $[0, \pi]$, then show that the process

$$X_t = Acos(\omega t) + Bsin(\omega t), t = 0, \pm 1, \ldots,$$

is stationary and find its mean and autocovariance function.

3. (16 points) Suppose that $\{X_t\}$ is the noninvertible MA(1) process

$$X_t = Z_t + \theta Z_{t-1},\ \{Z_t\} \sim WN(0, \sigma^2),$$

where $|\theta| > 1$. Define a new process $\{W_t\}$ as

$$W_t = \sum_{j=0}^{\infty} (-\theta)^{-j} X_{t-j}.$$

and
   (a) (8 points) Show that $\{W_t\} \sim WN(0, \sigma_w^2)$ and express σ_w^2 in terms of θ and σ^2.
   (b) (8 points) Show that $\{X_t\}$ has the invertible representation (in terms of $\{W_t\}$)

$$X_t = W_t + \frac{1}{\theta} W_{t-1}.$$

4. (16 points) Show that the value at lag 2 of the partial ACF of the MA(1) process

$$X_t = Z_t + \theta Z_{t-1},\ t = 0, \pm 1, \ldots,$$

where $\{Z_t\} \sim WN(0, \sigma^2)$, is $\alpha(2) = -\theta^2/(1 + \theta^2 + \theta^4)$.

5. (16 points) Given two observations x_1 and x_2 from the causal AR(1) process satisfying

$$X_t = \phi X_{t-1} + Z_t,\ \{Z_t\} \sim WN(0, \sigma^2),$$

and assuming that $|x_1| \neq |x_2|$, find the maximum likelihood estimates of ϕ and σ^2.

This exam has total 5 questions, for a total of 80 points and 0 bonus points.
Best of luck!!