Artificial Intelligence (AI) for Investments

Lesson 6 : Introduction to Risk and Return

Introduction

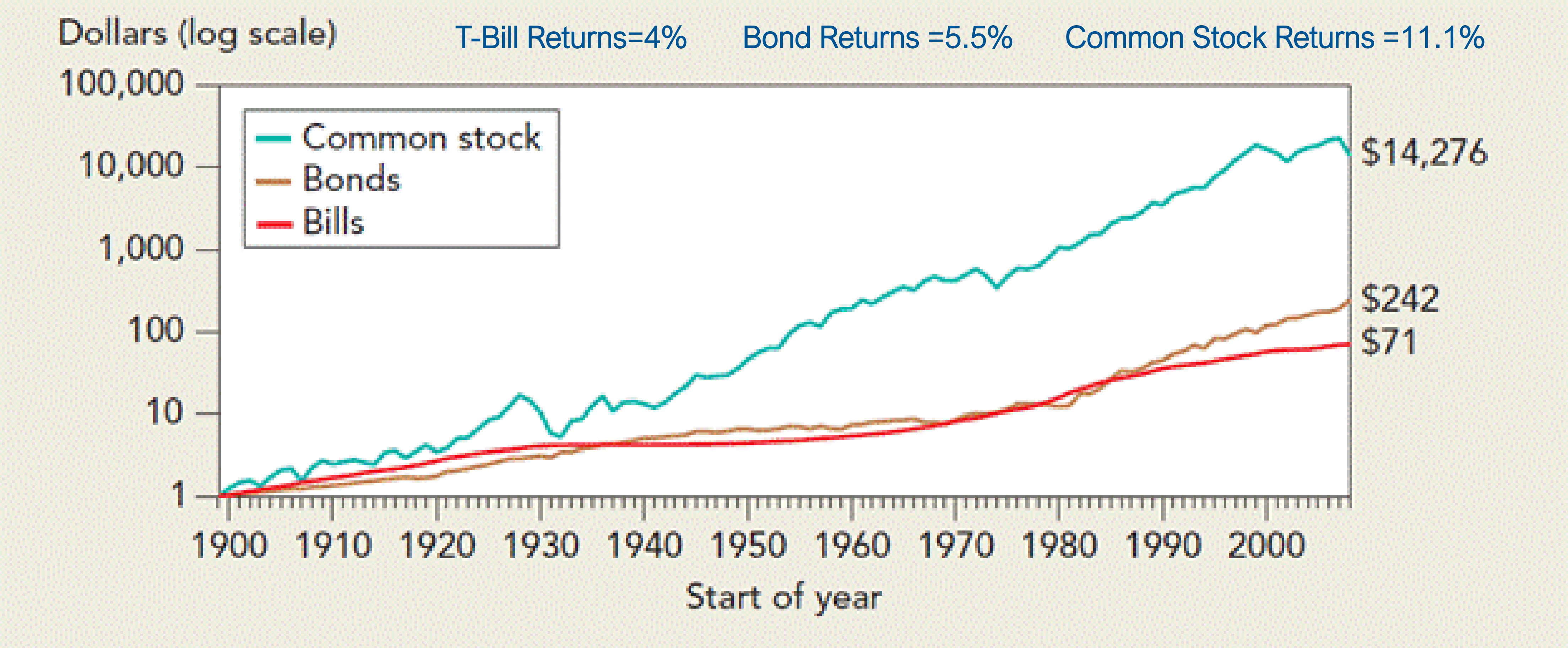
In this lesson, we will cover the following topics:

- Basics of risk-return framework
- Measures of risk
- Diversification of risk
- Computing portfolio risk
- Impact of individual securities on portfolio risk
- Summary and concluding remarks

Basics of Risk-Return Framework

- Consider three instruments: T-Bills, government bonds, and common stock.
- T-Bills are short maturity instrument with almost no risk of default.
- Bond is a rather long-term instrument and fluctuates with the interest rates.
- Common stocks are infinite maturity instruments.

Basics of Risk-Return Framework



Basics of Risk-Return Framework

- Notice the difference between the returns on T-Bills and common stocks is 11.1 - 4 = 7.1 percent.
- This additional return can also be called the risk-premium received by investors.
- If on a given year T-Bill rate was 0.2 percent, and you are asked to estimate the expected return on common stocks. A reasonable estimate would be obtained by adding this to 7.1 percent to obtain the total return of 7.30 percent.
- However, this assumes that there is a stable risk premium on the common stock portfolio, that is, future risk premium can be measured by the average past risk premium.
- But (a) Economic and financial conditions change over time; (b) Risk perceptions change; (c) Investors' risk tolerance and return expectations also change over time.

Basics of Risk-Return Framework

- Consider a stock with \$12 dividend expected by the end of the year
- Investors are expecting a 10% return on this stock
- P$V = \frac{DIV_1}{r-g} = \frac{12}{0.10-0.07} = \$400$; Dividend yield = 12/400 = 3%
- If dividend yield changes to 2%, and investors demand an expected return = 2% + 7% = 9%
- $PV = \frac{12}{0.09-0.07} = \$600$
- Expected returns on the stock reflect the dividend yields and the growth rate of dividends: $r = \frac{DIV_1}{P_0} + g$
- Risk-premium= $r - r_f$; this risk premium can change overtime
- Often dividend yield is a good indicator of risk-premium

Measures of Risk

- A very prominent statistical measure of risk is variance (or standard deviation)
- $Variance\ (r_t) = Expected\ value\ of\ (r_t - \bar{r})^2$
- Where r_t is the actual return and $\bar{r}$ is the expected returns
- $Standard\ deviation\ (SD) = \sqrt{Variance\ (r_t)}$
- Standard deviation is often denoted by the symbol σ and variance by σ^2

Measures of Risk

- Let us understand this concept with a small coin toss game
- The following probabilities are observed
  - (a) H + H: Gain 40%; (b) H + T: Gain 10%;
  - (c) T + H: Gain 10%; (d) T + T: Lose 20%
- Thus, there is a 25% chance that your return will be 40%, 50% chance that your return will be 10%, and 25% chance that you will lose 20%
- Expected return : $\bar{r}$=0.25*40% + 0.5*10% + 0.25*(-20%) = 10%.

Measures of Risk

- Now let us compute the variance and standard deviation of these returns.

Returns (%)	Mean Deviation $\mathbf{r_t - \bar{r}}$	Mean Square Deviation $(\mathbf{r_t - \bar{r}})^2$	Probability	Probability Squared Deviation
40	30	900	0.25	225
10	0	0	0.50	0
-20	-30	900	0.25	225
			Total	450

- Variance = 225 + 225 = 450 and Standard Deviation (σ) = $\sqrt{(450)}$ = 21%
- An event is considered risky if there are many possibilities of outcomes associated with it.
- As these possibilities increase, that is, the spread of possible outcomes increases, the event is said to have become riskier.
- Standard deviation or variance is a summary measure of these possibilities, that is spread in the possible outcome.

Measures of Risk

- The risk of an asset can be completely expressed, by writing all the possible outcomes and the possible payoffs associated with each of the outcomes.
- If the outcome was certain, that is, it had no risk, then the standard deviation would have been zero
- One of the challenges in performing such computations is the estimation of probability associated with each outcome
- One way to go about this is to observe past variability
- For example, consider the historical volatilities of three different kinds of securities.

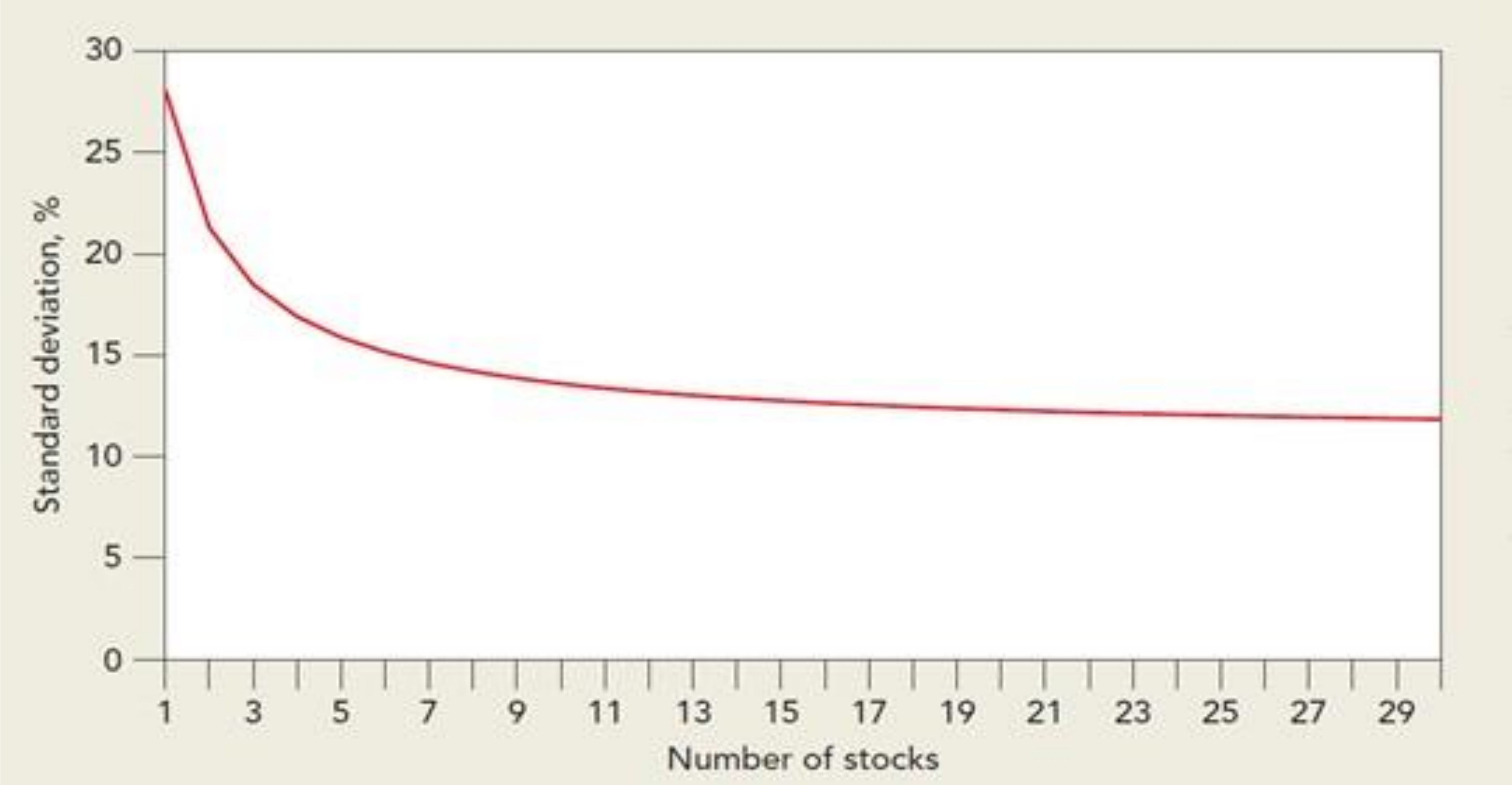
Portfolio	Standard Deviation (σ)	Variance (σ^2)
Treasury Bills	2.8	7.7
Government Bonds	8.3	69.3
Common Stocks	20.2	406.4

- It appears that T-Bills are the least variable and common stocks are the most variable

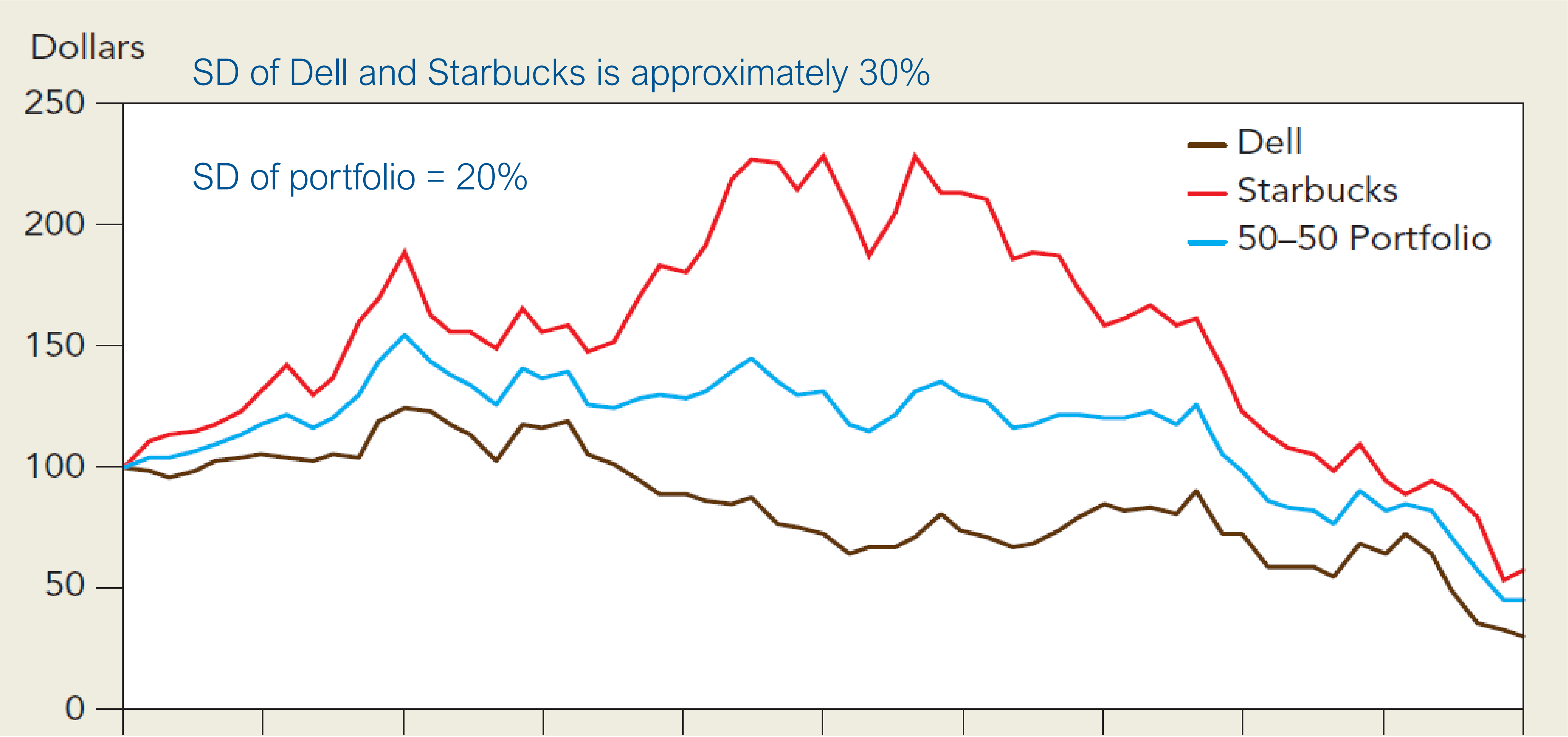
Diversification of Risk

- One can compute the measure of variability for individual securities as well as a portfolio of securities.
- The standard deviation of selected U.S. Common stocks (2004-08) such as Amazon (50.9%), Ford (47.2%), Newmont (36.1%), Dell (30.9%), and Starbucks (30.3%) was much less than the standard deviation of a market portfolio, that is, 13% during this period.
- It is well known that individual stocks are more volatile than the market indices.
- The variability of market doesn't reflect or is the same as the variability of individual stock components.
- The simple answer to this question is that diversification reduces variability.

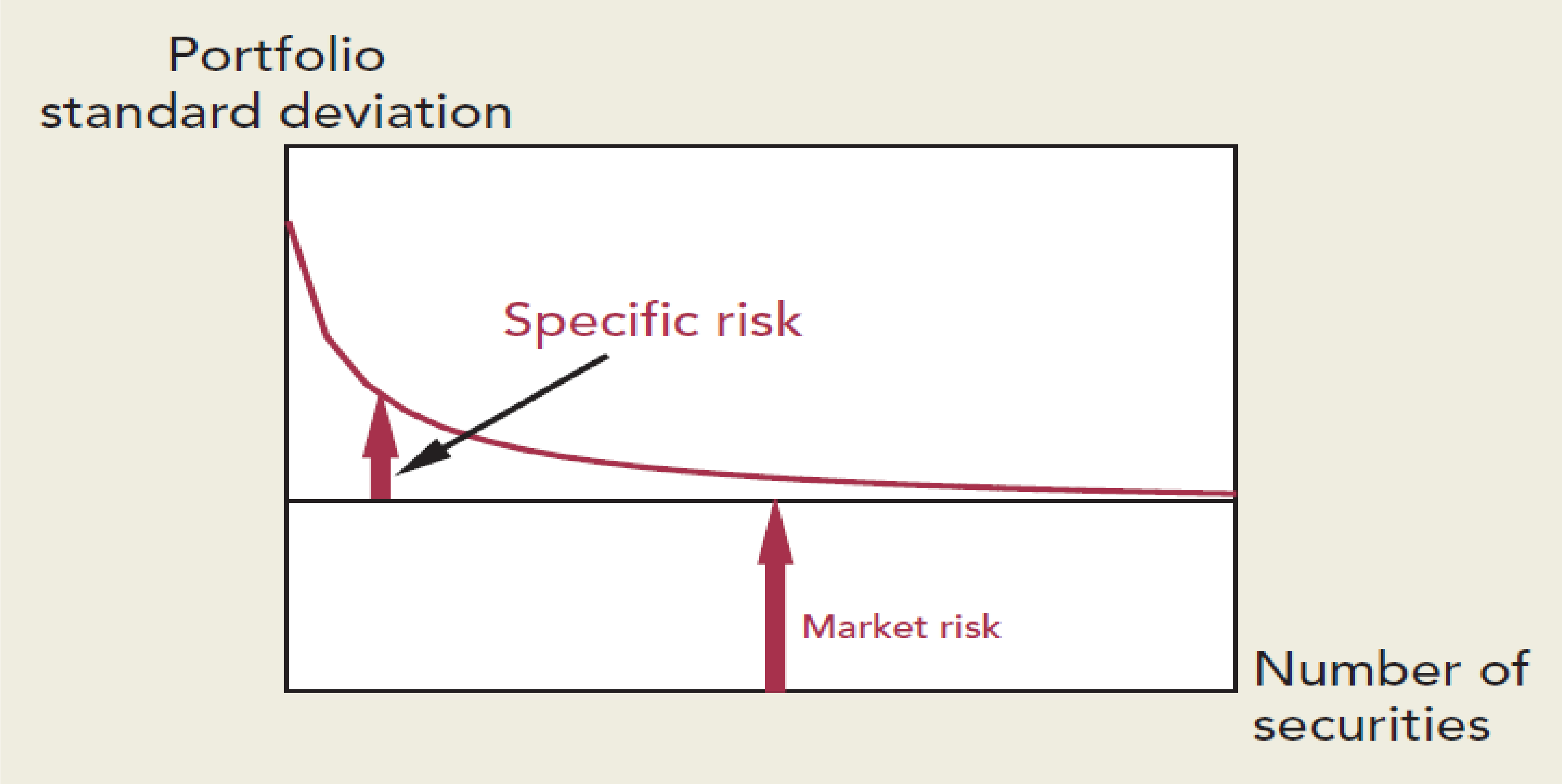
Diversification of Risk



Diversification of Risk



Diversification of Risk



Computing Portfolio Risk

- We now know that diversification reduces the risk of a portfolio
- Consider a portfolio comprising two stocks, A (60%) and B (40%)
- A has expected returns of 3.1%, and B has expected returns of 9.5%
- $Expected\ portfolio\ return = 0.6 * 3.1 + 0.40 * 9.5 = 5.7\%$
- Standard deviation of A is observed as 15.8% for A and 23.7% for B
- Therefore, the standard deviation of this portfolio: 0.6*15.8% + 0.4*23.7% = 19.0%?
- This would be incorrect

Computing Portfolio Risk

$$Portfolio\ Variance = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2(x_1 x_2 \rho_{12}\sigma_1\sigma_2)$$

$$Covariance\ (\sigma_{12}) = \rho_{12}\sigma_1\sigma_2$$

Correlation coefficient $(\boldsymbol{\rho_{12}})$ $= \rho_{12}\sigma_1\sigma_2$

	Stock 1	Stock 2
Stock 1	$x_1^2\sigma_1^2$	$x_1x_2\sigma_{12} = x_1x_2\rho_{12}\sigma_1\sigma_2$
Stock 2	$x_1x_2\sigma_{12} = x_1x_2\rho_{12}\sigma_1\sigma_2$	$x_2^2\sigma_2^2$

Computing Portfolio Risk

- Let us fill the above box with some numbers; assume a correlation coefficient of 1

	Stock A	Stock B
Stock A	$x_1^2\sigma_1^2 = 0.6^2 * 15.8^2$	$x_1 x_2 \sigma_1 \sigma_2 = 0.6 * 0.4 * 1 * 15.8 * 23.7$
Stock B	$x_1 x_2 \sigma_1 \sigma_2 = 0.6 * 0.4 * 1 * 15.8 * 23.7$	$x_2^2\sigma_2^2 = 0.4^2 * 23.7^2$

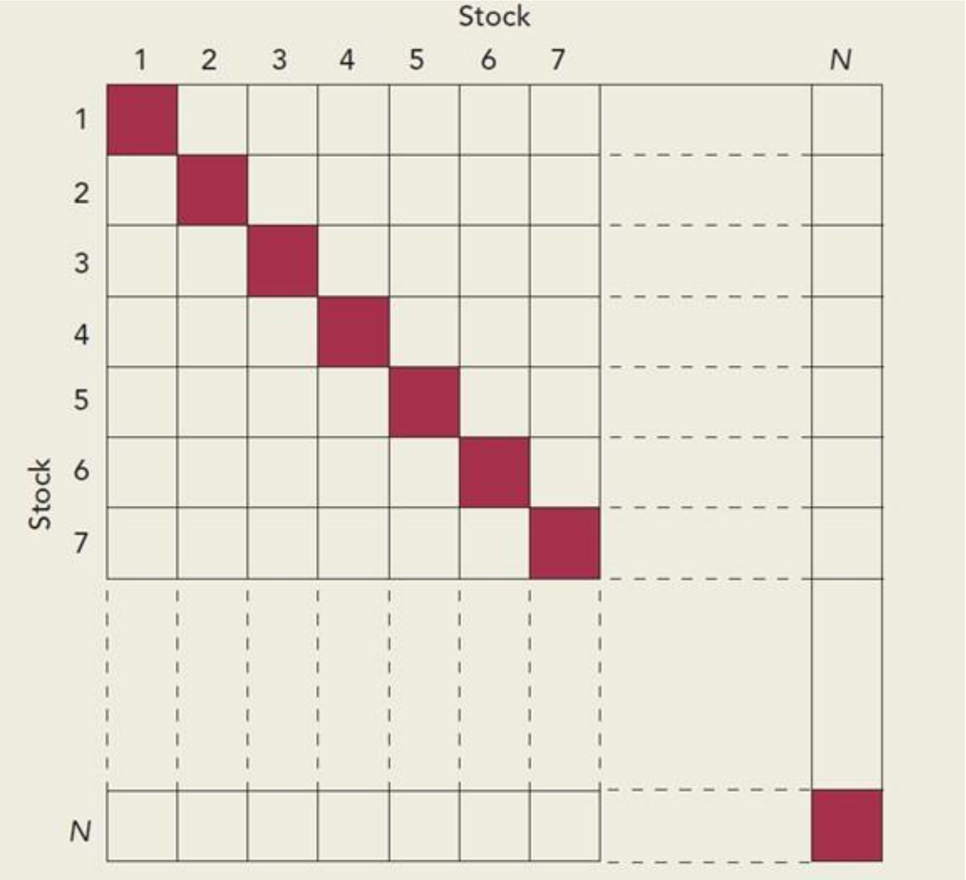
- Portfolio variance = $0.6^2 * 15.8^2 + 2 * 0.6 * 0.4 * 1 * 15.8 * 23.7 + 0.4^2 * 23.7^2 = 359.5$
- The standard deviation is $\sqrt{359.5} = 19\%$
- Let us now assume a correlation coefficient of $\rho_{12} = 0.18$
- Portfolio variance= $0.6^2 * 15.8^2 + 2 * 0.6 * 0.4 * 0.18 * 15.8 * 23.7 + 0.4^2 * 23.7^2 = 212.1$
- The standard deviation is $\sqrt{212.1} = 14.6\%$

Computing Portfolio Risk

- Let us consider a very hypothetical case of extreme negative correlation $\rho_{12} = -1$
- Portfolio variance = $0.6^2 * 15.8^2 + 2 * 0.6 * 0.4 * (-1) * 15.8 * 23.7 + 0.4^2 * 23.7^2 = 0!$
- However, perfect negative correlations do not exist in real markets.

Computing Portfolio Risk

- Variances in diagonal boxes $(x^2\sigma^2)$
- Covariance terms in off-diagonal $(x_i x_j \sigma_{ij})$
- Let us consider a case of N securities and equal investment in all the securities $(\frac{1}{N})$
- Portfolio variance can be computed in the form of two components. That is, variance component and covariance component



Computing Portfolio Risk

- There will be N variance terms; then portfolio variance can be simply written as $N * \frac{1}{N^2} * (Average\ variance)$
- Remember $w_1 * w_2 * \sigma^2. Here\ w_1 = w_2 = \frac{1}{N}; and\ \sigma = average\ variance = \sigma_{Avg}$
- Also, $N^2 - N$ covariance terms where average covariance term =$\sigma_{Cov-Avg}$
- The sum of covariance terms is $(N^2 - N) * \frac{1}{N^2} * \sigma_{Cov-Avg}$
- $Portfolio\ Variance = N * \left(\frac{1}{N^2}\right) * (Average\ Varinace) + (N^2 - N) * \left(\frac{1}{N^2}\right) * (Average\ Covariance)$
- As the number of securities, N, in the portfolio increase, the specific-risk term, $N * \left(\frac{1}{N^2}\right) * (Average\ Varinace)$, approaches to a value of zero

Computing Portfolio Risk

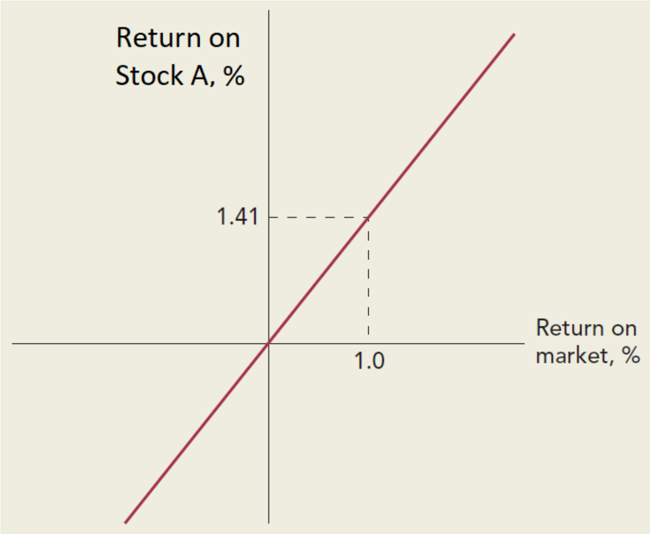
- Thus, the overall portfolio variance approaches the average covariance term
- This is also often referred to as portfolio diversification
- Thus, if these securities have very low correlation, then one can obtain a portfolio with very low risk
- That is, just by increasing the number of securities in a portfolio, one can eliminate the idiosyncratic (specific or diversifiable risk)
- The remaining risk is often called market risk or non-diversifiable risk
- That is why, this market risk (or average covariance or non-diversifiable risk) is what constitutes the bedrock of risk, that is risk that is there after eliminating all the specific risk

Impact of Individual Securities on Portfolio Risk

- Investors usually add many securities in their portfolio to diversify the stock-specific idiosyncratic risk
- It is not the risk of a security held individually but in a portfolio that is important
- To measure the impact of a security to the risk of portfolio, one needs to measure the market risk component of the security
- The market risk of a security is measured through its beta
- Stocks with beta of more than 1.0 tend to amplify the movements of market
- Stocks with beta between 0 to 1.0 tend to move in the same direction as market, but are considered less sensitive
- The market portfolio has a beta of 1.0 and reflects the average movement of all the stocks in the market

Impact of Individual Securities on Portfolio Risk

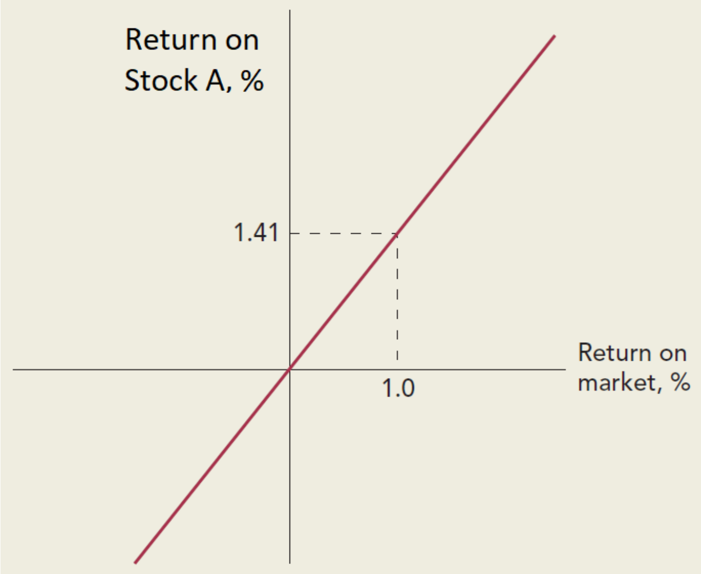
- Consider a stock A with beta of 1.41 over a given time-horizon
- This means that, on average, when market rises by 1%, stock A will rise by 1.41%
- The stock would also have some stock-specific risk



- When a stock is added to a well-diversified portfolio, the movements on account of idiosyncratic factors are expected to cancel each other out
- Therefore, for this portfolio what matters is only these systematic market related effects

Impact of Individual Securities on Portfolio Risk

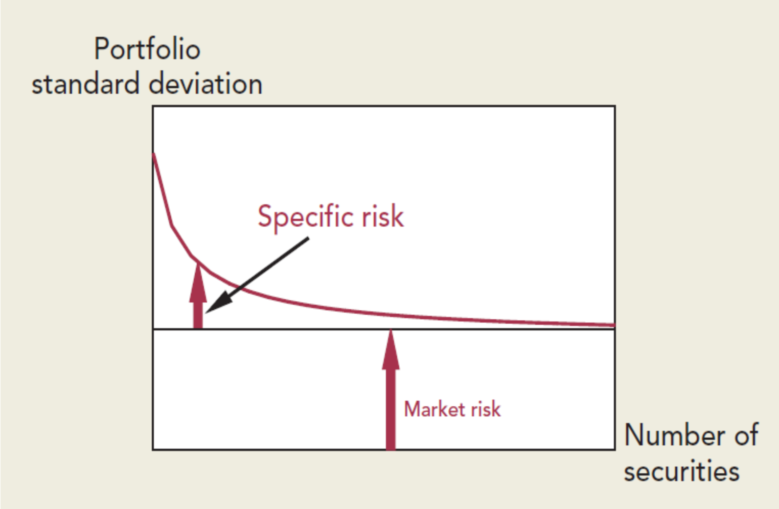
- Stocks like Stock A with high beta will have steep straight curve
- Stocks with small beta (e.g., beta = 0.3), the straight-line plot will be less steep
- A stock with high beta may also have less idiosyncratic risk and a stock with low beta may also have high idiosyncratic risk



- For example, a stock of gold-mining firm may have low beta and a very high idiosyncratic stock specific risk
- When added to a well-diversified portfolio, the idiosyncratic risk of this gold-firm will not matter

Impact of Individual Securities on Portfolio Risk

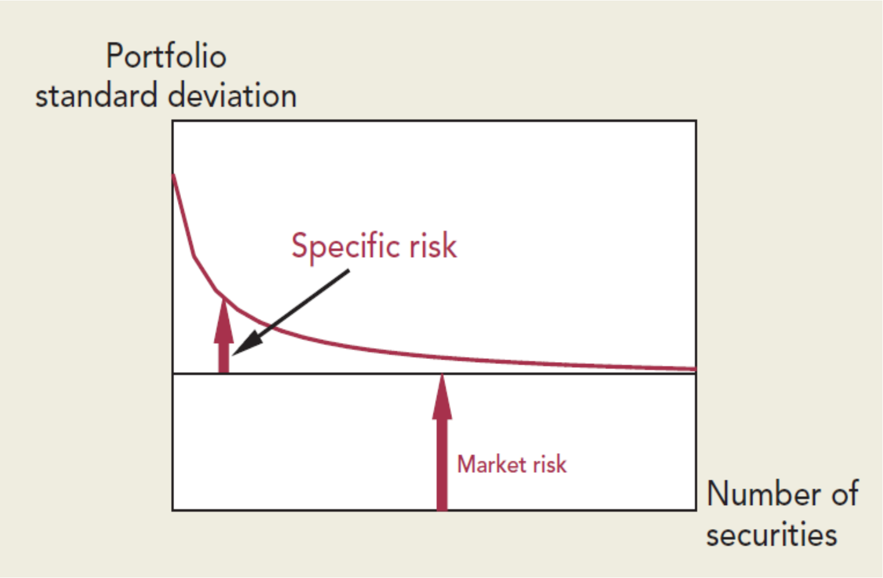
- So, let us now answer this question how security betas affect the portfolio risk
- Market risk accounts for most of the risk of a well-diversified portfolio
- Beta of an individual security measures its sensitivity to market movements



- Examine the figure shown here: the standard deviation (total risk) of the portfolio depends on the number of securities in the portfolio
- As the number of securities increase in the portfolio, more diversification is achieved

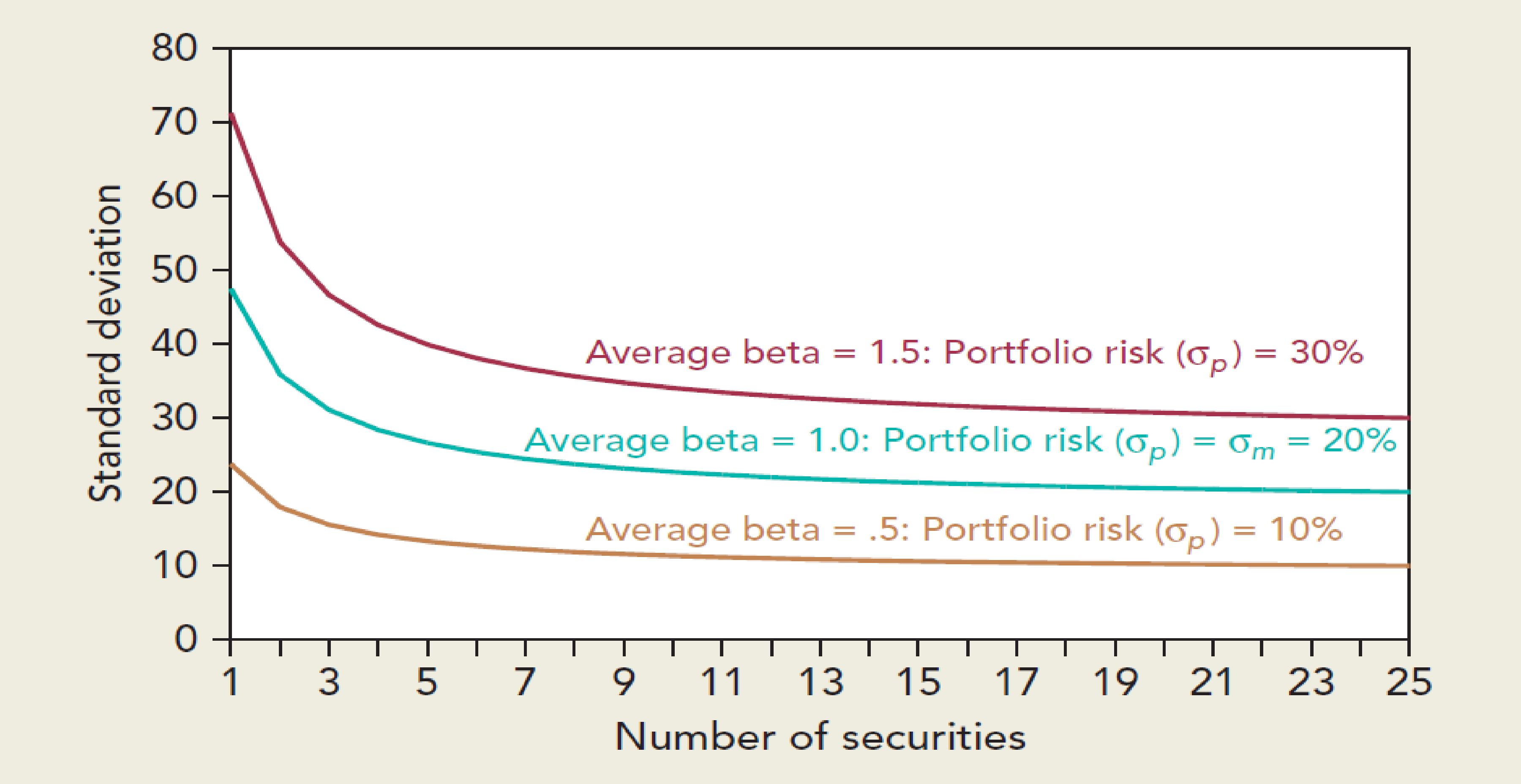
Impact of Individual Securities on Portfolio Risk

- With addition of more and more securities, the specific risk declines until all the stock specific risk is eliminated and only market risk remains
- Market risk depends on the average beta of the securities, that is, the portfolio beta
- If one selects a fairly large number of securities from a market, you diversify all the idiosyncratic risk



- Thus, you get the market portfolio with beta = 1.0
- If the market portfolio has a standard deviation of 20%, then this portfolio is expected to have a standard deviation of close to 20%

Impact of Individual Securities on Portfolio Risk



Impact of Individual Securities on Portfolio Risk

- Beta of a stock 'i' can be computed using the following formula. $\beta_i = \sigma_{im}/\sigma_m^2$. Here σ_{im} is the covariance between the stock returns and market returns. σ_m^2 is the variance of the returns on the market.

1	2	3	4	5	6	7
Month	Market Return (%)	Deviation in Market Returns	Squared Market Deviation	Stock A	Deviation in Stock A Returns	Deviation Product (3*6)
1	-8	-10	100	-11	-13	130
2	4	2	4	8	6	12
3	12	10	100	19	17	170
4	-6	-8	64	-13	-15	120
5	2	0	0	3	1	0
6	8	6	36	6	4	24
	Avg.= 2		Sum=304	Avg.= 2		Sum=456
Variance= $\sigma_m^2 = \frac{304}{6} = 50.67$						
Co-variance= $\sigma_{im} = \frac{456}{6} = 76$						
Beta= $\beta_i = \frac{\sigma_{im}}{\sigma_m^2} = \frac{76}{50.67} = 1.5$						

Impact of Individual Securities on Portfolio Risk

- Can we say that a diversified firm is more attractive to investors than an undiversified firm
- If diversification is a good objective for a firm to pursue then each new project's contribution to the firm's diversification should also add value to the firm.
- This seems to be not consistent with what we have studied about present values
- Investors can diversify for themselves more easily than firms.
- If investors can diversify on their own, they would not be paying anything extra to a firm for this diversification.
- The present value of any number of assets is equal to the present value of their parts. That is, PV (ABC) = PV(A) + PV(B) + PV(C): Value Additivity

Summary and Concluding Remarks

- Returns to investor vary depending upon the risk borne by them.
- Very safe instruments such as treasury securities provide the lowest returns.
- Equity securities are considered more riskier asset class and offer higher expected returns.
- Accordingly, the discount rates applied to a safe project versus risky project will also differ.
- Risk of a security means that there are many possible return outcomes for that security…
- The total risk of a stock has two components: stock-specific risk and systematic (or market) risk.

Summary and Concluding Remarks

- Investors eliminate a sizable portion of their specific (or diversifiable) risk, simply by adding more securities to their portfolio.
- A well diversified portfolio is only exposed to market risk.
- A security's contribution to a well diversified portfolio measured as the sensitivity of the security to market movements, that is, beta (β).
- A stock with high beta is more sensitive to market movements and vice-versa.
- Investors can diversify on their personal account; they do not want firms to pursue the diversification objective.

Thanks!