# Ramakrishna Mission Vivekananda University

Belur Math, Howrah, West Bengal

**School of Mathematical Sciences, Department of Data Science**

M.Sc. in Big Data Analytic 2017, Mid Semester Exam — Date: 19 Sep 2017

Course : **DA***: Time Series** — Time: $1\frac{1}{2}$ hrs

*Instructor : Dr. Sudipta Das* — *Max marks: 30*

Student signature and Id:

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1. Select the right answers

   (a) What does autocovariance measure

   i. Linear dependence between multiple points on the different series observed at different times

   ii. Quadratic dependence between two points on the same series observed at different times

   iii. Linear dependence between two points on different series observed at same time

   iv. Linear dependence between two points on the same series observed at different times

   (b) Which of the following is necessary condition for weakly stationary time series?

   i. Mean is constant and does not depend on time

   ii. Autocovariance function depends on $s$ and $t$ only through their difference $|s-t|$ (where $t$ and $s$ are moments in time)

   iii. The time series under considerations is a finite variance process

   iv. Time series is Gaussian

   (c) Consider the following set of data:

   $$23.32, 32.33, 32.88, 28.98, 33.16, 26.33, 29.88, 32.69, 18.98, 21.23, 26.66, 29.89$$

   What is the lag-one sample autocorrelation of the time series?

   i. 0.26

   ii. 0.52

   iii. 0.13

   iv. 0.07

   (d) Which of the following is true for white noise?

   i. Mean =0

   ii. Zero autocovariances

   iii. Zero autocovariances except at lag zero

   iv. Quadratic Variance

   (e) Consider the following $AR(1)$ model with the disturbances having zero mean and unit variance.

   $$y_t = 0.4 + 0.2y_{t-1} + z_t$$

   The (unconditional) variance of $y_t$ is

i. 0.40
ii. 1.00
iii. 1.04
iv. 1.15

(f) Second differencing in time series can help to eliminate

i. Linear Trend
ii. Quadratic Trend
iii. Seasonality
iv. Noise

(g) The partial autocorrelation function is necessary for distinguishing between

i. An AR and MA model
ii. An AR and an ARMA
iii. An MA and an ARMA
iv. Different models from within the ARMA family

(h) For the following $MA(3)$

i. ACF = 1 at lag 0
ii. ACF = 0 at lag 2
iii. ACF = 0 at lag 3
iv. ACF = 0 at lag 5

(i) Sum of weights in exponential smoothing is

i. $e$
ii. $> e$
iii. $< e$
iv. $\ln e$

(j) Which of the following ARMA processes are causal as well as invertible. (In each case $\{Z_t\}$ denotes white noise)

i. $X_t + 1.9X_{t-1} + 0.88X_{t-2} = Z_t + 0.2Z_{t-1} + 0.7Z_{t-2}$
ii. $X_t + 0.6X_{t-1} = Z_t + 1.2Z_{t-1}$
iii. $X_t + 1.8X_{t-1} + 0.81X_{t-2} = Z_t$
iv. $X_t + 1.6X_{t-1} = Z_t - 0.4Z_{t-1} + 0.04Z_{t-2}$

$[1 \times 10 = 10]$

2. Let $\{Z_t\}$ be iid$\sim N(0,1)$ noise and define

$$X_t = \begin{cases} Z_t, & \text{if } t \text{ is even,} \\ (Z_{t-1}^2 - 1)/\sqrt{2} & \text{if } t \text{ is odd.} \end{cases}$$

(a) Show that $\{X_t\}$ is WN$(0,1)$ but not iid$(0,1)$ noise.

(b) Find $E(Xn+1|X_1,...,X_n)$ for $n$ odd and $n$ even and compare the results.

[4+4=8]

3. Let $Y_t$ be the AR(1) plus noise time series defined by

$$Y_t = X_t + W_t,$$

where $\{W_t\} \sim WN(0,\sigma_w^2)$, $\{X_t\}$ is the AR(1) process, i.e.,

$$X_t - \phi X_{t-1} = Z_t, Z_t \sim WN(0,\sigma_z^2),$$

and $E(W_s Z_t) = 0$ for all $s$ and $t$.

(a) Show that $\{Y_t\}$ is stationary and find its autocovariance function.

(b) Show that the time series $U_t = Y_t - \phi Y_{t-1}$ is an MA(1) process.

[4+4=8]

4. If $m_t = \sum_{k=0}^{p} c_k t^k, t = 0, \pm 1, \pm 2, \ldots$, show that $\triangledown^{p+1} m_t = 0$. [4]

This exam has total 4 questions, for a total of 30 points and 0 bonus points.