SET10112 Exam

40274924

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A

1a.

A $\cup$ B = {a, b , c , d, f , h}
A $\cap$ B = {b, d}
Cardinality of A $\cup$ B = 6 and A $\cap$ B = 2

$|A \cap B| \leq |A \cup B|$
2 $\leq$ 6 = True

1b.

If n $\geq$ 2 then $n^2 > 2n$ (*n)
If n $\geq$ 2 then 2n > 4 (*2)

$n \geq 2 \longrightarrow n > 1$
$(n - 1)^2 > 0$
$n^2 - 2n + 1 > 0$
$n^2 + 1 > 2n$
$n^2 > 2n - 1$
$n^2 + 1 > n^2$ (True)

Therefore: $n^2 + 1 > 2n - 1$

2.

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P, P \Rightarrow R, Q}\,False}{P \Rightarrow (P \supset Q), R}\,R \supset \qquad \dfrac{}{R, P \Rightarrow R}\,Ax}{((P \supset Q) \supset R), P \Rightarrow R}\,L \supset \qquad \dfrac{\dfrac{\dfrac{}{P, Q \Rightarrow R, Q}\,Ax}{Q \Rightarrow (P \supset Q), R}\,R \supset \qquad \dfrac{}{R, Q \Rightarrow R}\,Ax}{((P \supset Q) \supset R), Q \Rightarrow R}\,L \supset}{((P \supset Q) \supset R), (P \vee Q) \Rightarrow R}\,L\vee}{((P \supset Q) \supset R) \Rightarrow ((P \vee Q) \supset R)}\,R \supset}{\Rightarrow ((P \supset Q) \supset R) \supset ((P \vee Q) \supset R)}\,R \supset
$$

As the far left branch has come back false, this means the entire formula is false as all branches need to return true for the formula to be true.

3.

Case P is:

$((p \wedge T) \supset \{q := \neg p\}(p \wedge q))$
$\wedge$
$((p \wedge T \supset \{q := q \vee p\}(p \wedge q))$

$((p \wedge T) \supset (p \wedge q)\left[\dfrac{\neg p}{q}\right])$
$\wedge$
$(\neg(p \wedge T) \supset (p \wedge q)\left[\dfrac{q \vee p}{q}\right])$

$\neg(p \wedge T) \equiv \neg \vee \bot$
$p \wedge \neg p \equiv \bot$
$p \wedge T \equiv p$
$p \wedge q \vee p \equiv p$

$((p \wedge T) \supset (p \wedge \neg p))$
$\wedge$
$(\neg(p \wedge T) \supset (p \wedge (q \vee p))$

$((p \wedge T) \supset (p \wedge \neg p)) \wedge (\neg(p \wedge T) \supset (p \wedge (q \vee p))$
$((p) \supset (\bot)) \wedge ((\neg p \vee \bot) \supset p)$ $\quad A \supset B \equiv \neg A \cup B$

$(\neg p \vee \bot) \wedge (\neg(\neg p \vee \bot) \vee p)$
$(\neg p \vee \bot) \wedge (p \wedge T \vee p)$ $\quad \neg p \vee \bot \equiv \neg p$
$p \wedge T \equiv p$

$(\neg p) \wedge (p \vee p)$ $\quad p \vee p \equiv p$
$\neg p \wedge p$ $\quad \neg p \wedge p \equiv \bot$
$\bot$
Therefore the weakest pre-condition is False from the rule: $\neg p \wedge p \equiv \bot$.

B

4a.

i.

The program is using n as an in/out variable, when it actually is set as an in variable. M is also set as an out variable but is being used as an in/out variable.

ii.

Set both n and m on line 5 and 12 to be in/out variables for the program to pass bronze certification

4b.

MyType[1..20]

If m > 4 then n := m + 2; m := n − 3
else n := m + 6; m := n + 1

Expecting m = 1 − 18 and n = 4 − 19
m ≥ 1 ∧ m ≤ 20 − Implicit Post Condition

= (m > 4) ⊃ $\{n := m + 2\} \wedge \{m := n - 3\}(n \geq 1 \wedge n \leq 20)$
∧
$(m \leq 4) \supset \{n := m + 6\} \wedge \{m := n + 1\}(n \geq 1 \wedge n \leq 20)$

Going to do m first
$= (m > 4) \supset \{n := m + 2\}(n \geq 1 \wedge n \leq 20)$
∧
$(m \leq 4) \supset \{n := m + 6\}(n \geq 1 \wedge n \leq 20)$

$$= (m > 4) \supset (n \geq 1 \wedge n \leq 20)\left[\frac{m+2}{n}\right]$$
∧
$$= (m \leq 4) \supset (n \geq 1 \wedge n \leq 20)\left[\frac{m+6}{n}\right]$$

$= (m > 4) \supset (m + 2 \geq 1 \wedge m + 2 \leq 20)$
∧
$= (m \leq 4) \supset (m + 6 \geq 1 \wedge m + 6 \leq 20)$

$P \supset (Q \wedge R) \equiv (P \supset Q) \wedge (P \supset R)$

$= (m > 4) \supset (m + 2 \geq 1)$ (Since adding to 2 to m will always make it bigger than 1, this statement is just true)
∧
$(m > 4) \supset (m + 2 \leq 20)$
∧
$(m \leq 4) \supset (m + 6 \geq 1)$ (Since adding to 6 to m will always make it bigger than 1, this statement

is just true)
$\wedge$
$(m \leq 4) \supset (m+6 \geq 20)$ (Since $(m \leq 4)$ and therefore the biggest number m can be is 4 and $(4+6 < 20)$. This statement is just true)

$= (m > 4) \supset (m \leq 18)$ $\quad$ $P \supset Q \equiv \neg P \vee Q$

$= ((m \leq 4) \vee (m \leq 18)$

$= 1 \leq m \leq 18$
As there was no lower limit for silver, I took the lower limit of the range i.e 1

Working out for n

$(n \geq 1) \wedge (n \leq 20)$

$= (m > 4) \supset \{m := n-3\}(n \geq 1 \wedge n \leq 20)$
$\wedge$
$(m \leq 4) \supset \{m := n+1\}(n \geq 1 \wedge n \leq 20)$

$= (m > 4) \supset (m \geq 1 \wedge m \leq 20) \left[\frac{n-3}{m}\right]$
$\wedge$
$(m \leq 4) \supset (m \geq 1 \wedge m \leq 20) \left[\frac{n+1}{m}\right]$

$P \supset (Q \wedge R) \equiv (P \supset Q) \wedge (P \supset R)$

$= (m > 4) \supset (n-3 \geq 1 \wedge n-3 \leq 20)$
$\wedge$
$(m \leq 4) \supset (n+1 \geq 1 \wedge n+1 \leq 20)$

$= (m > 4) \supset (n-3 \geq 1)$
$\wedge$
$(m > 4) \supset (n-3 \leq 20)$ (As the range of possible values is 1..20, doing $20-3 < 20$ and therefore this statement is true)
$\wedge$
$(m \leq 4) \supset (n+1 \geq 1)$ (Since adding 1 to n will also be ≥ 1 this statement is true)
$\wedge$
$(m \leq 4) \supset (n+1 \leq 20)$

$= (m > 4) \supset (n \geq 4)$
$\wedge$
$(m \leq 4) \supset (n \leq 19)$ $\quad$ $P \supset Q \equiv \neg P \vee Q$

$= ((m \geq 4) \vee (n \geq 4)$
$\wedge$
$((m \leq 4) \vee (n \leq 19)$

$= n \geq 4 \wedge n \leq 19$

$= 4 \leq n \leq 19$

4c.

i.

Post $= m > n$

If $m > 4$ then $n := m + 2; m = n - 3$
else $n := m + 6; m = n + 1$

$= (m > 4) \supset \{m := n - 3\}(m > n)$
$\wedge$
$(m \leq 4) \supset \{m := n + 1\}(m > n)$

$= (m > 4) \supset (m > n) \left[\dfrac{n-3}{m}\right]$
$\wedge$
$= (m \leq 4) \supset (m > n) \left[\dfrac{n+1}{m}\right]$

$= (m > 4) \supset (n + 1 > n)$ (n's cancel out)
$\wedge$
$(m \leq 4) \supset (n - 3 > n)$ (n's cancel out)

$= (m > 4) \supset (-3 > 0)\perp$ (False)
$\wedge$
$(m \leq 4) \supset (1 > 0)T$ (True)

$T \wedge (m > 4) \supset \perp$

$= (m > 4) \supset \perp$ $\qquad\qquad \neg P \equiv P \supset \perp$

$= m \leq 4$

Cant do this for n as the post condition is $m > n$ as n in this case would always be bigger

ii.

$1 \leq m \leq 18 \qquad 4 \leq n \leq 19$

$m \leq 4$

$4 \leq m \leq 18$