Lab Assignment: Lab 3: Energy of a Pendulum

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Lab 3: Energy of a Pendulum

Introduction

In this lab, we will explore the dynamics of a pendulum's energy during its oscillations following release from a large initial angle. Our objective is to ascertain the rate at which energy dissipates over time by determining the value of τ, representing the characteristic timescale for energy loss.

Understanding energy conservation is a fundamental principle in physics, giving us valuable insights into how physical systems behave. This experiment specifically looks at conserving mechanical energy, focusing on a swinging pendulum. Our main goal is to explore how energy gradually dissipates over time as the pendulum swings. We will determine what factors cause this slow energy loss during the oscillations of the pendulum. In this report, we'll carefully examine the basic theories, important equations for the pendulum's movement, and the setup we used for collecting data. The goal of this lab is to provide a clear understanding of how the energy is transformed in this basic physical system.

We will be using the law of conservation of energy to analyze the pendulum. The mechanical energy, E, of the pendulum is given by:

$$E = U + K \quad \textbf{(Eq. 3.1)}$$

where U is the gravitational potential energy (in J), and K is the kinetic energy (in J). The potential energy, U, and the kinetic energy, K, can be calculated using the following equations:

$$U = m \cdot g \cdot y \quad \textbf{(Eq. 3.2)}$$

$$K = \frac{1}{2} \cdot m \cdot v^2 \quad \textbf{(Eq. 3.3)}$$

where m is the mass of the Bob attached to the pendulum (in kg), g is the gravitational acceleration of free-fall (in m/s^2), y is the vertical distance from the lowest point of the Bob in the experiment (in m), and v is the velocity of the Bob that can be calculated using its horizontal and vertical velocity by:

$$v = \sqrt{v_x^2 + v_y^2} \quad \textbf{(Eq. 3.4)}$$

Since we will be considering the energy lost to the environment due to non-conservative work over the course of oscillations in our experiment, we will model the amount of energy (in J) in the system over time (in s), E(t), using a damping model:

$$E(t) = E_i \cdot e^{\frac{t_i - t}{\tau}} \quad \textbf{(Eq. 3.5)}$$

where E_i is the initial energy (in J), t_i is the time when the initial energy is measured (in s), and τ is the characteristic timescale for energy loss (in s). We can linearize (Eq. 3.5) which will help us obtain the value for τ:

$$\ln\left[\frac{E(t)}{E_i}\right] = -\frac{1}{\tau}(t - t_i) \quad \textbf{(Eq. 3.6)}$$

$$y = m(x)$$

Methods

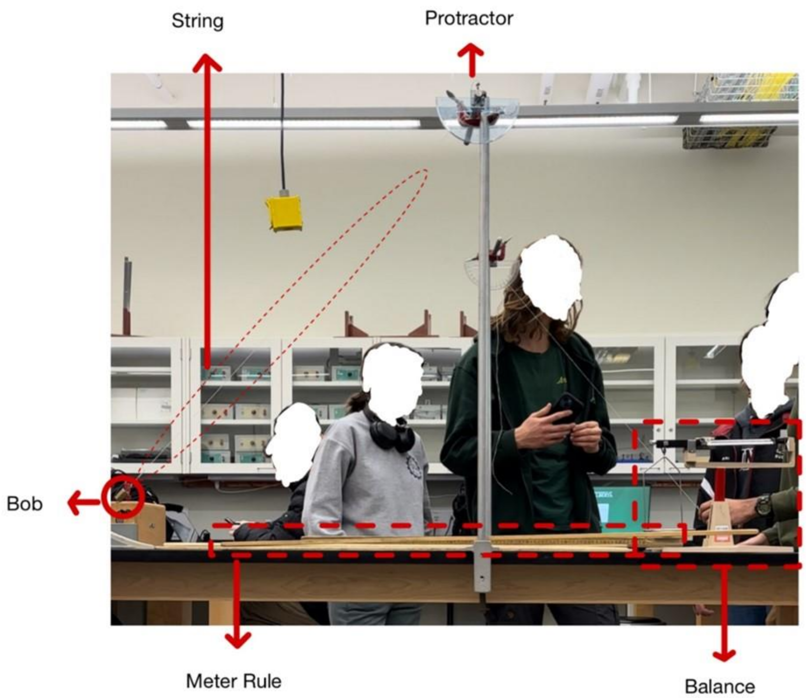

Fig. 3.1: A labelled photo of the experimental setup used to analyze the motion of the pendulum. A protractor was used to measure the "large" angle the bob was supposed to be released from. The string holds the bob from the clamped stand throughout its motion. The meter rule is placed in the experiment site to act as a calibration stick when tracking data on the Tracker software. Balance was used to measure the mass of the bob accurately.

The key part of this experiment was to transform our measured values from a video into standard variables we use to describe energy such as kinetic and potential energy. To do this, we needed to measure the efficiency of the conserved energy of the swing, so we measured the kinetic and potential energy of the bob at every point of the experiment. This would later help us measure the rate at which non-conservative work was done on the system.

Firstly, we set up a pendulum system as shown in Fig. 3.1 ensuring that the length of string was greater than 0.6 m for adequate motion of the bob. Next, we set up a phone camera that was level with the height of the pendulum to film its motion, placing it sufficiently far away to minimize distortion in the field, and at a position to avoid parallax error. A meter rule was placed in the experiment site to act as a calibration stick when tracking data on the Tracker software. We measured the mass of the bob using a balance to be able to calculate the energies. One of the lab partners released the bob by pulling it at an angle greater than 45 degrees right after another lab partner started filming its motion. We recorded the first five oscillations of the pendulum, making sure the bob would not hit anything and only oscillate back and forth in a vertical plane. Finally, we transferred the film to our respected computers and recorded the positions of the bob in at least three complete oscillations using the Tracker software.

Results

<u>Raw data:</u> Mass of bob measured using balance, $m = (2.084 \pm 0.001) \times 10^{-2}$ kg

Acceleration of free-fall due to gravity, $g = 9.81$ m/s^2

Length of string measured using the meter rule, $l = (9.17 \pm 0.01) \times 10^{-1}$ m

Angle from which bob was released measured using protractor, $\theta = (45.0 \pm 0.5)^\circ$

Sample data table – **Table A.1 –** can be found in the appendix.

Sample Calculations:

For row 2 of Table A.1:

Velocity = $\sqrt{(X - \text{velocity})^2 + (Y - \text{velocity})^2} = \sqrt{(0.153)^2 + (0)^2} = 0.153$ m/s

Potential Energy, U = m · g · y = (0.02084) × (9.81) × (0.297) = 0.0607 J

Since $\Delta g = 0$ and $\Delta y = 0$, $\Delta U = \sqrt{\left(\frac{\partial U}{\partial m}\Delta m\right)^2} = \sqrt{\left((y \cdot g) \cdot \Delta m\right)^2} = 3 \times 10^{-5}$ J

Kinetic Energy, $K = \frac{1}{2} \cdot m \cdot v^2 = \frac{1}{2} \cdot (0.02084) \cdot (0.153)^2 = 0.0002439$ J

Since $\Delta v = 0$, $\Delta K = \sqrt{\left(\frac{\partial K}{\partial m}\Delta m\right)^2} = \sqrt{\left(\left(\frac{v^2}{2}\right) \cdot \Delta m\right)^2} = 1 \times 10^{-7}$ J

Total Energy, E = U + K = (0.0607) + (0.0002439) = 0.0610 J

$\Delta E = \sqrt{\left(\frac{\partial E}{\partial U}\Delta U\right)^2 + \left(\frac{\partial E}{\partial K}\Delta K\right)^2} = \sqrt{\left((1) \cdot \Delta U\right)^2 + \left((1) \cdot \Delta k\right)^2} = 3 \times 10^{-5}$

Therefore, For row 2, Velocity = 0.153 m/s, Potential Energy, U = $(6.072 \pm 0.003) \times 10^{-2}$ J, Kinetic Energy, K = $(2.439 \pm 0.001) \times 10^{-4}$ J, Total Energy, E = $(6.094 \pm 0.003) \times 10^{-2}$ J.

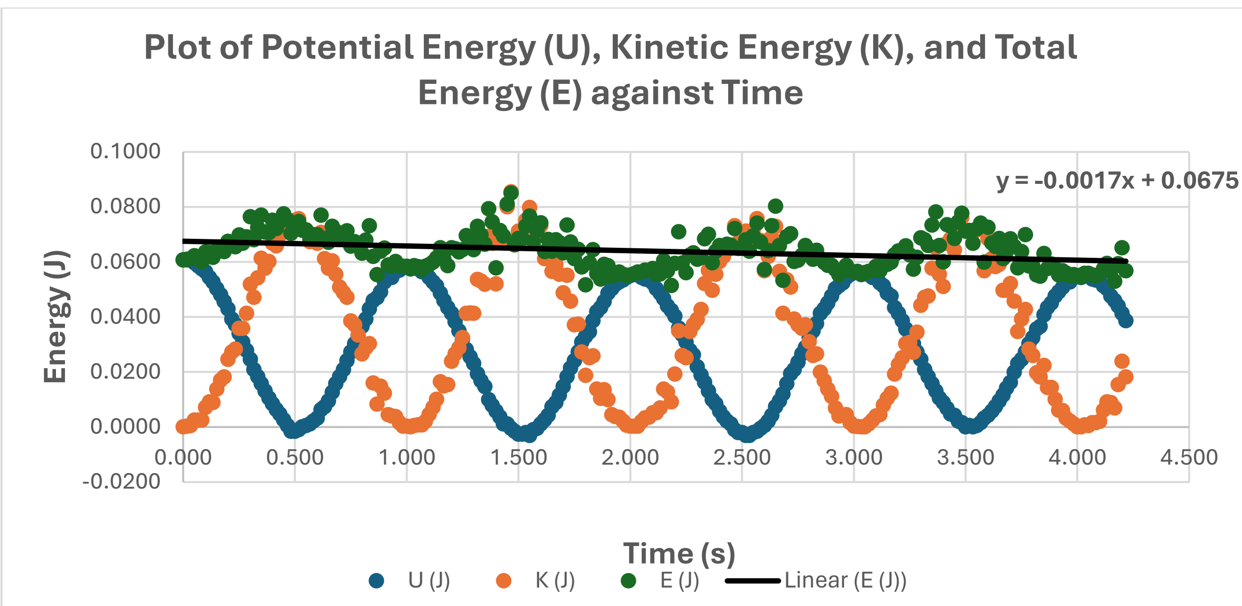

Fig. 3.2: A plot of Potential Energy U (in J), Kinetic Energy K (in J), and Total Energy E (in J) on the y-axis is plotted against Time (in s) on the x-axis. A trendline of best-fit representing Total Energy is also displayed with the equation y = -0.0017x + 0.0675. Using the LINEST function in MS Excel gives us the slope = (-0.0017 ± 0.0003) s^{-1} and the y-intercept = (0.0675 ± 0.0008) J.

Using Eq. 3.6, slope of Fig. 3.2 = $\frac{-1}{\tau}$

Therefore $\tau = \frac{-1}{-0.0017} = 588.2352941$, $\Delta\tau = \sqrt{\left(\frac{\partial \tau}{\partial \text{slope}}\Delta \text{slope}\right)^2} = \sqrt{\left(\left(\frac{1}{(\text{slope})^2}\right) \cdot \Delta \text{slope}\right)^2} = 1 \times 10^2$ s

Hence, $\tau = (6 \pm 1) \times 10^2$ s

Time for Energy to reach $\frac{1}{100}$th of its original value:

$E_f = \frac{1}{100}E_i$, and manipulating Eq. 3.6 gives us

$t = t_i - \tau\cdot\ln(\frac{Ef}{Ei}) = 0 - (600)\cdot\ln(\frac{1}{100}) = 2763.1s$

Therefore, it takes 2760 seconds for energy to reach $\frac{1}{100}$th of its original value.

Discussion

Rate at which energy dissipated can be determined using the value of $\tau = (6 \pm 1) \times 10^2$ s.

It takes 2760 seconds for energy to reach $\frac{1}{100}$th of its original value.

Implications of the slope and y-intercept values from Fig. 3.2:

The slope of Fig. 3.2, obtained through linear regression, represents the rate of change of the total energy of the pendulum over time. Here, the negative slope indicates a decrease in energy with time, which proves that energy is being dissipated. The y-intercept, on the other hand, corresponds to the initial total energy of the pendulum. These values together reveal that the pendulum loses energy over time, conforming to our expectations.

Uncertainties/Improvements:

A reason for errors in our value could be influenced by factors such as frame rate limitations and human error while tracking motion of particle in the video analysis section. This could be improved by repeating the experiment several times by different individuals and comparing results to give more accurate data.

Another reason for the errors could be due to the external factors of the experiment site such as air currents that influence motion of the pendulum. A possible improvement could be to conduct the experiment in a controlled environment that minimizes external influences.

Conclusion

In summary, our experiment regarding the energy of the pendulum yielded several key findings. The experiment demonstrated that the pendulum dissipates energy over time, as evidenced by the negative slope in Fig. 3.2. The characteristic timescale (τ) represents the temporary aspect of this energy dissipation, indicating the duration over which the pendulum loses energy significantly.

These results highlight how important it is to think about energy loss in things that oscillate. It helps us see the practical limits of keeping mechanical energy. What we learned here gives us a better idea of how pendulums move, and it teaches us more about how energy conservation and loss work in different physical systems.

References

[1] *Lab Manual EN PH 131*. Edmonton: University of Alberta, Department of Physics.

Acknowledgements

I would like to express my gratitude to my lab partners Arshiya, Zeeshan, and Samhita for their invaluable assistance in recording the short film of the oscillating pendulum. Additionally, I extend my appreciation to our lab Teaching Assistant, Mr. Kiril Kolevski, for providing consistent guidance and support throughout the entire laboratory experiment.

Appendix

Table A.1. Sample data table of the Time (in seconds), Y-position (in meters), X-velocity and Y-velocity (both in m/s), velocity (in m/s), Potential energy U, Kinetic energy K, and Total energy E (all three in Joules). Time, Y-position, X-velocity and Y-velocity were tracked using the tracker software based on the relative motion of the pendulum. Velocity was calculated using Eq. 3.4. Potential energy, U, was calculated using Eq. 3.2. Kinetic energy, K, was calculated using Eq. 3.3. Total energy, E, was calculated using Eq. 3.1. The "Time (s)" column contains the recorded data plotted on the x-axis of Fig. 3.2 and the "Potential Energy, U (J)", "Kinetic Energy, K (J)" and "Total Energy, E (J)" columns contain the calculated data plotted on the y-axis of Fig. 3.2.

Time (s)	Y-position (m)	X-Velocity (m/s)	Y-Velocity (m/s)	Velocity (m/s)	Potential Energy, U (J)	Kinetic Energy, K (J)	Total Energy, E (J)
0.000	0.297	0.000	0.000	0.000	0.0607	0.0000	0.0607
0.017	0.297	0.153	0.000	0.153	0.0607	0.0002	0.0610
0.033	0.297	0.153	-0.153	0.216	0.0607	0.0005	0.0612
0.050	0.292	0.384	-0.307	0.492	0.0597	0.0025	0.0622
0.067	0.286	0.384	-0.307	0.492	0.0585	0.0025	0.0610
0.083	0.281	0.384	-0.307	0.492	0.0574	0.0025	0.0600
0.100	0.276	0.691	-0.460	0.830	0.0564	0.0072	0.0636
0.117	0.266	0.767	-0.537	0.936	0.0544	0.0091	0.0635
0.133	0.258	0.691	-0.614	0.924	0.0527	0.0089	0.0616
0.150	0.246	0.921	-0.691	1.151	0.0503	0.0138	0.0641
0.167	0.235	1.074	-0.691	1.277	0.0480	0.0170	0.0650
0.183	0.223	1.074	-0.767	1.320	0.0456	0.0181	0.0637
0.200	0.210	1.228	-0.921	1.535	0.0429	0.0246	0.0675
0.217	0.192	1.381	-0.844	1.618	0.0393	0.0273	0.0665
0.233	0.182	1.458	-0.767	1.647	0.0372	0.0283	0.0655
0.250	0.166	1.611	-0.921	1.856	0.0339	0.0359	0.0698
0.267	0.151	1.611	-0.921	1.856	0.0309	0.0359	0.0668
0.283	0.136	1.765	-0.921	1.991	0.0278	0.0413	0.0691
0.300	0.120	1.995	-0.997	2.230	0.0245	0.0518	0.0764
0.317	0.102	1.918	-0.921	2.128	0.0209	0.0472	0.0680
0.333	0.090	2.148	-0.767	2.281	0.0183	0.0542	0.0725
0.350	0.077	2.302	-0.767	2.426	0.0157	0.0613	0.0770
0.367	0.064	2.225	-0.767	2.353	0.0131	0.0577	0.0708
0.383	0.051	2.302	-0.691	2.403	0.0105	0.0602	0.0707
0.400	0.041	2.455	-0.614	2.531	0.0084	0.0667	0.0751
0.417	0.031	2.455	-0.537	2.513	0.0063	0.0658	0.0721
0.433	0.023	2.532	-0.537	2.588	0.0047	0.0698	0.0745
0.450	0.013	2.609	-0.614	2.680	0.0026	0.0749	0.0775
0.467	0.003	2.609	-0.614	2.680	0.0005	0.0749	0.0754
0.483	-0.008	2.609	-0.307	2.627	-0.0016	0.0719	0.0000
0.500	-0.008	2.685	0.077	2.686	-0.0016	0.0752	0.0736
0.517	-0.005	2.685	0.230	2.695	-0.0010	0.0757	0.0746
0.533	0.000	2.609	0.230	2.619	0.0000	0.0715	0.0715
0.550	0.003	2.609	0.153	2.613	0.0005	0.0712	0.0717
0.567	0.005	2.532	0.230	2.542	0.0010	0.0674	0.0684
0.583	0.010	2.532	0.307	2.551	0.0021	0.0678	0.0699
0.600	0.015	2.455	0.614	2.531	0.0031	0.0667	0.0699
0.617	0.031	2.532	0.614	2.605	0.0063	0.0707	0.0770
0.633	0.036	2.379	0.460	2.423	0.0073	0.0612	0.0685
0.650	0.046	2.225	0.767	2.353	0.0094	0.0577	0.0671
0.667	0.061	2.225	0.921	2.408	0.0125	0.0604	0.0730
0.683	0.077	2.148	0.844	2.308	0.0157	0.0555	0.0712
0.700	0.090	2.072	0.767	2.209	0.0183	0.0509	0.0692

0.717	0.102	1.995	0.844	2.166	0.0209	0.0489	0.0697
0.733	0.118	1.918	0.921	2.128	0.0241	0.0472	0.0713
0.750	0.133	1.688	0.921	1.923	0.0272	0.0385	0.0657
0.767	0.148	1.688	0.844	1.887	0.0303	0.0371	0.0674
0.783	0.161	1.535	0.921	1.790	0.0329	0.0334	0.0663
0.800	0.179	1.304	0.921	1.596	0.0366	0.0266	0.0632
0.817	0.192	1.381	0.921	1.660	0.0393	0.0287	0.0680
0.833	0.210	1.381	0.997	1.703	0.0429	0.0302	0.0732
0.850	0.225	1.074	0.614	1.237	0.0460	0.0159	0.0619
0.867	0.230	0.767	0.460	0.894	0.0470	0.0083	0.0554
0.883	0.240	0.844	0.844	1.194	0.0491	0.0148	0.0639
0.900	0.258	0.767	0.767	1.085	0.0527	0.0123	0.0650
0.917	0.266	0.614	0.230	0.656	0.0544	0.0045	0.0589
0.933	0.266	0.537	0.307	0.619	0.0544	0.0040	0.0584
0.950	0.276	0.460	0.384	0.599	0.0564	0.0037	0.0602
0.967	0.279	0.307	0.230	0.384	0.0570	0.0015	0.0586
0.983	0.284	0.077	0.153	0.171	0.0581	0.0003	0.0584
1.000	0.284	0.153	-0.077	0.171	0.0581	0.0003	0.0584
1.017	0.281	0.077	0.000	0.077	0.0574	0.0001	0.0575
1.033	0.284	-0.230	0.000	0.230	0.0581	0.0006	0.0586
1.050	0.281	-0.153	-0.230	0.276	0.0574	0.0008	0.0582
1.067	0.276	-0.230	-0.153	0.276	0.0564	0.0008	0.0572
1.083	0.276	-0.307	-0.307	0.434	0.0564	0.0020	0.0584
1.100	0.266	-0.384	-0.537	0.660	0.0544	0.0045	0.0589
1.117	0.258	-0.691	-0.460	0.830	0.0527	0.0072	0.0599
1.133	0.251	-0.767	-0.614	0.982	0.0513	0.0101	0.0614
1.150	0.238	-0.997	-0.767	1.258	0.0487	0.0165	0.0651
1.167	0.225	-0.921	-0.767	1.199	0.0460	0.0150	0.0610
1.183	0.212	-0.997	-0.691	1.213	0.0433	0.0153	0.0587
1.200	0.202	-1.304	-0.767	1.513	0.0413	0.0238	0.0651
1.217	0.187	-1.304	-0.921	1.596	0.0382	0.0266	0.0648
1.233	0.171	-1.381	-0.921	1.660	0.0350	0.0287	0.0637
1.250	0.156	-1.458	-0.997	1.766	0.0319	0.0325	0.0644
1.267	0.138	-1.765	-0.921	1.991	0.0282	0.0413	0.0695
1.283	0.125	-1.841	-0.767	1.994	0.0256	0.0414	0.0670
1.300	0.113	-1.765	-0.921	1.991	0.0231	0.0413	0.0644
1.317	0.095	-2.072	-0.921	2.267	0.0193	0.0536	0.0729
1.333	0.082	-2.148	-0.691	2.256	0.0167	0.0531	0.0698
1.350	0.072	-1.995	-0.997	2.230	0.0146	0.0518	0.0665
1.367	0.049	-2.379	-0.997	2.579	0.0099	0.0693	0.0793
1.383	0.038	-2.455	-0.614	2.531	0.0078	0.0667	0.0746
1.400	0.028	-2.148	-0.614	2.234	0.0058	0.0520	0.0578
1.417	0.018	-2.455	-0.691	2.550	0.0037	0.0678	0.0714
1.433	0.005	-2.532	-0.384	2.561	0.0010	0.0683	0.0694
1.450	0.005	-2.762	-0.230	2.772	0.0010	0.0800	0.0811
1.467	-0.003	-2.839	-0.384	2.865	-0.0005	0.0855	0.0850
1.483	-0.008	-2.532	-0.307	2.551	-0.0016	0.0678	0.0662
1.500	-0.013	-2.609	-0.153	2.613	-0.0026	0.0712	0.0686
1.517	-0.013	-2.609	0.077	2.610	-0.0026	0.0710	0.0684
1.533	-0.010	-2.685	-0.077	2.686	-0.0021	0.0752	0.0731
1.550	-0.015	-2.762	0.153	2.766	-0.0031	0.0797	0.0766
1.567	-0.005	-2.532	0.384	2.561	-0.0010	0.0683	0.0673
1.583	-0.003	-2.609	0.307	2.627	-0.0005	0.0719	0.0714
1.600	0.005	-2.609	0.460	2.649	0.0010	0.0731	0.0742
1.617	0.013	-2.379	0.460	2.423	0.0026	0.0612	0.0638
1.633	0.020	-2.379	0.691	2.477	0.0042	0.0639	0.0681

1.650	0.036	-2.225	0.691	2.330	0.0073	0.0566	0.0639
1.667	0.043	-2.302	0.614	2.382	0.0089	0.0591	0.0680
1.683	0.056	-2.148	0.844	2.308	0.0115	0.0555	0.0670
1.700	0.072	-1.918	0.997	2.162	0.0146	0.0487	0.0633
1.717	0.090	-2.072	0.997	2.299	0.0183	0.0551	0.0734
1.733	0.105	-1.918	0.844	2.095	0.0215	0.0458	0.0672
1.750	0.118	-1.688	0.844	1.887	0.0241	0.0371	0.0612
1.767	0.133	-1.611	0.997	1.895	0.0272	0.0374	0.0646
1.783	0.151	-1.381	0.844	1.618	0.0309	0.0273	0.0582
1.800	0.161	-1.228	0.537	1.340	0.0329	0.0187	0.0516
1.817	0.169	-1.304	0.844	1.553	0.0346	0.0251	0.0597
1.833	0.189	-1.151	1.074	1.574	0.0386	0.0258	0.0645
1.850	0.205	-0.921	0.691	1.151	0.0419	0.0138	0.0557
1.867	0.212	-0.844	0.537	1.000	0.0433	0.0104	0.0538
1.883	0.223	-0.844	0.767	1.140	0.0456	0.0136	0.0591
1.900	0.238	-0.691	0.691	0.977	0.0487	0.0100	0.0586
1.917	0.246	-0.460	0.460	0.651	0.0503	0.0044	0.0547
1.933	0.253	-0.460	0.384	0.599	0.0517	0.0037	0.0555
1.950	0.258	-0.537	0.230	0.584	0.0527	0.0036	0.0563
1.967	0.261	-0.384	0.230	0.448	0.0534	0.0021	0.0554
1.983	0.266	-0.077	0.230	0.242	0.0544	0.0006	0.0550
2.000	0.269	-0.077	0.153	0.171	0.0550	0.0003	0.0553
2.017	0.271	-0.077	0.153	0.171	0.0554	0.0003	0.0557
2.033	0.274	0.077	-0.153	0.171	0.0560	0.0003	0.0563
2.050	0.266	0.307	-0.307	0.434	0.0544	0.0020	0.0563
2.067	0.263	0.537	-0.153	0.558	0.0538	0.0032	0.0570
2.083	0.261	0.537	-0.230	0.584	0.0534	0.0036	0.0569
2.100	0.256	0.537	-0.460	0.707	0.0523	0.0052	0.0575
2.117	0.246	0.460	-0.537	0.707	0.0503	0.0052	0.0555
2.133	0.238	0.614	-0.537	0.816	0.0487	0.0069	0.0556
2.150	0.228	0.921	-0.691	1.151	0.0466	0.0138	0.0604
2.167	0.215	0.921	-0.614	1.107	0.0440	0.0128	0.0567
2.183	0.207	0.767	-0.537	0.936	0.0423	0.0091	0.0515
2.200	0.197	0.997	-0.921	1.357	0.0403	0.0192	0.0595
2.217	0.176	1.535	-0.997	1.830	0.0360	0.0349	0.0709
2.233	0.164	1.381	-0.767	1.580	0.0335	0.0260	0.0595
2.250	0.151	1.304	-0.844	1.553	0.0309	0.0251	0.0560
2.267	0.136	1.688	-0.691	1.824	0.0278	0.0347	0.0625
2.283	0.128	1.688	-0.844	1.887	0.0262	0.0371	0.0633
2.300	0.107	1.611	-1.074	1.936	0.0219	0.0391	0.0609
2.317	0.092	1.841	-0.844	2.025	0.0188	0.0427	0.0616
2.333	0.079	2.072	-0.844	2.237	0.0162	0.0522	0.0684
2.350	0.064	2.148	-0.921	2.337	0.0131	0.0569	0.0700
2.367	0.049	2.072	-0.691	2.184	0.0099	0.0497	0.0596
2.383	0.041	2.225	-0.614	2.308	0.0084	0.0555	0.0639
2.400	0.028	2.302	-0.691	2.403	0.0058	0.0602	0.0659
2.417	0.018	2.379	-0.537	2.439	0.0037	0.0620	0.0656
2.433	0.010	2.455	-0.384	2.485	0.0021	0.0643	0.0664
2.450	0.005	2.455	-0.460	2.498	0.0010	0.0650	0.0661
2.467	-0.005	2.609	-0.460	2.649	-0.0010	0.0731	0.0721
2.483	-0.010	2.532	-0.153	2.537	-0.0021	0.0670	0.0650
2.500	-0.010	2.532	-0.153	2.537	-0.0021	0.0670	0.0650
2.517	-0.015	2.609	-0.153	2.613	-0.0031	0.0712	0.0680
2.533	-0.015	2.532	0.153	2.537	-0.0031	0.0670	0.0639
2.550	-0.010	2.609	0.230	2.619	-0.0021	0.0715	0.0694
2.567	-0.008	2.685	0.230	2.695	-0.0016	0.0757	0.0741

2.583	-0.003	2.532	0.307	2.551	-0.0005	0.0678	0.0673
2.600	0.003	2.302	0.384	2.334	0.0005	0.0568	0.0573
2.617	0.010	2.379	0.537	2.439	0.0021	0.0620	0.0641
2.633	0.020	2.455	0.767	2.572	0.0042	0.0689	0.0731
2.650	0.036	2.532	0.767	2.646	0.0073	0.0729	0.0803
2.667	0.046	2.225	0.691	2.330	0.0094	0.0566	0.0660
2.683	0.059	1.765	0.921	1.991	0.0120	0.0413	0.0533
2.700	0.077	1.995	1.074	2.266	0.0157	0.0535	0.0692
2.717	0.095	2.072	0.767	2.209	0.0193	0.0509	0.0702
2.733	0.102	1.841	0.614	1.941	0.0209	0.0392	0.0601
2.750	0.115	1.688	0.844	1.887	0.0235	0.0371	0.0606
2.767	0.130	1.688	0.767	1.854	0.0266	0.0358	0.0624
2.783	0.141	1.688	0.844	1.887	0.0288	0.0371	0.0659
2.800	0.159	1.458	0.921	1.725	0.0325	0.0310	0.0635
2.817	0.171	1.381	0.767	1.580	0.0350	0.0260	0.0610
2.833	0.184	1.304	0.921	1.596	0.0376	0.0266	0.0642
2.850	0.202	1.151	0.767	1.383	0.0413	0.0199	0.0612
2.867	0.210	1.151	0.537	1.270	0.0429	0.0168	0.0597
2.883	0.220	0.921	0.691	1.151	0.0450	0.0138	0.0588
2.900	0.233	0.691	0.767	1.032	0.0476	0.0111	0.0587
2.917	0.246	0.614	0.537	0.816	0.0503	0.0069	0.0572
2.933	0.251	0.537	0.307	0.619	0.0513	0.0040	0.0553
2.950	0.256	0.460	0.460	0.651	0.0523	0.0044	0.0567
2.967	0.266	0.460	0.460	0.651	0.0544	0.0044	0.0588
2.983	0.271	0.307	0.230	0.384	0.0554	0.0015	0.0569
3.000	0.274	0.077	0.153	0.171	0.0560	0.0003	0.0563
3.017	0.276	-0.077	-0.077	0.109	0.0564	0.0001	0.0565
3.033	0.271	0.000	-0.077	0.077	0.0554	0.0001	0.0555
3.050	0.274	0.000	0.000	0.000	0.0560	0.0000	0.0560
3.067	0.271	-0.230	-0.230	0.325	0.0554	0.0011	0.0565
3.083	0.266	-0.460	-0.153	0.485	0.0544	0.0024	0.0568
3.100	0.266	-0.537	-0.307	0.619	0.0544	0.0040	0.0584
3.117	0.256	-0.460	-0.460	0.651	0.0523	0.0044	0.0567
3.133	0.251	-0.691	-0.537	0.875	0.0513	0.0080	0.0593
3.150	0.238	-0.844	-0.537	1.000	0.0487	0.0104	0.0591
3.167	0.233	-0.767	-0.767	1.085	0.0476	0.0123	0.0599
3.183	0.212	-0.997	-0.921	1.357	0.0433	0.0192	0.0625
3.200	0.202	-1.304	-0.691	1.476	0.0413	0.0227	0.0640
3.217	0.189	-1.304	-0.844	1.553	0.0386	0.0251	0.0638
3.233	0.174	-1.381	-0.997	1.703	0.0356	0.0302	0.0658
3.250	0.156	-1.535	-0.767	1.716	0.0319	0.0307	0.0626
3.267	0.148	-1.458	-0.691	1.613	0.0303	0.0271	0.0574
3.283	0.133	-1.611	-0.844	1.819	0.0272	0.0345	0.0617
3.300	0.120	-1.841	-0.921	2.059	0.0245	0.0442	0.0687
3.317	0.102	-1.918	-0.921	2.128	0.0209	0.0472	0.0680
3.333	0.090	-1.995	-0.767	2.137	0.0183	0.0476	0.0659
3.350	0.077	-2.225	-0.767	2.353	0.0157	0.0577	0.0734
3.367	0.064	-2.379	-0.767	2.500	0.0131	0.0651	0.0782
3.383	0.051	-2.225	-0.614	2.308	0.0105	0.0555	0.0660
3.400	0.043	-2.148	-0.537	2.214	0.0089	0.0511	0.0600
3.417	0.033	-2.455	-0.614	2.531	0.0068	0.0667	0.0735
3.433	0.023	-2.532	-0.384	2.561	0.0047	0.0683	0.0730
3.450	0.020	-2.455	-0.384	2.485	0.0042	0.0643	0.0685
3.467	0.010	-2.609	-0.384	2.637	0.0021	0.0725	0.0746
3.483	0.008	-2.685	-0.307	2.702	0.0016	0.0761	0.0777
3.500	0.000	-2.609	-0.153	2.613	0.0000	0.0712	0.0712

3.517	0.003	-2.609	0.000	2.609	0.0005	0.0709	0.0715
3.533	0.000	-2.532	0.000	2.532	0.0000	0.0668	0.0668
3.550	0.003	-2.609	0.230	2.619	0.0005	0.0715	0.0720
3.567	0.008	-2.609	0.384	2.637	0.0016	0.0725	0.0740
3.583	0.015	-2.302	0.384	2.334	0.0031	0.0568	0.0599
3.600	0.020	-2.532	0.384	2.561	0.0042	0.0683	0.0725
3.617	0.028	-2.379	0.460	2.423	0.0058	0.0612	0.0669
3.633	0.036	-2.302	0.537	2.364	0.0073	0.0582	0.0655
3.650	0.046	-2.302	0.614	2.382	0.0094	0.0591	0.0686
3.667	0.056	-2.072	0.691	2.184	0.0115	0.0497	0.0612
3.683	0.069	-2.072	0.691	2.184	0.0141	0.0497	0.0638
3.700	0.079	-2.072	0.844	2.237	0.0162	0.0522	0.0684
3.717	0.097	-1.841	0.997	2.094	0.0199	0.0457	0.0655
3.733	0.113	-1.688	0.691	1.824	0.0231	0.0347	0.0578
3.750	0.120	-1.841	0.614	1.941	0.0245	0.0392	0.0638
3.767	0.133	-1.841	0.844	2.025	0.0272	0.0427	0.0699
3.783	0.148	-1.458	0.767	1.647	0.0303	0.0283	0.0585
3.800	0.159	-1.381	0.767	1.580	0.0325	0.0260	0.0585
3.817	0.174	-1.228	0.614	1.373	0.0356	0.0196	0.0552
3.833	0.179	-1.074	0.767	1.320	0.0366	0.0181	0.0547
3.850	0.199	-1.074	0.997	1.465	0.0407	0.0224	0.0631
3.867	0.212	-0.997	0.614	1.171	0.0433	0.0143	0.0576
3.883	0.220	-0.997	0.614	1.171	0.0450	0.0143	0.0593
3.900	0.233	-0.767	0.614	0.982	0.0476	0.0101	0.0577
3.917	0.240	-0.614	0.460	0.767	0.0491	0.0061	0.0552
3.933	0.248	-0.537	0.537	0.759	0.0507	0.0060	0.0567
3.950	0.258	-0.460	0.307	0.553	0.0527	0.0032	0.0559
3.967	0.258	-0.384	0.230	0.448	0.0527	0.0021	0.0548
3.983	0.266	-0.077	0.307	0.316	0.0544	0.0010	0.0554
4.000	0.269	0.000	0.000	0.000	0.0550	0.0000	0.0550
4.017	0.266	-0.077	0.000	0.077	0.0544	0.0001	0.0544
4.033	0.269	0.230	0.153	0.276	0.0550	0.0008	0.0558
4.050	0.271	0.153	-0.153	0.216	0.0554	0.0005	0.0559
4.067	0.263	0.230	-0.307	0.384	0.0538	0.0015	0.0553
4.083	0.261	0.384	-0.077	0.392	0.0534	0.0016	0.0550
4.100	0.261	0.307	-0.307	0.434	0.0534	0.0020	0.0553
4.117	0.251	0.614	-0.460	0.767	0.0513	0.0061	0.0574
4.133	0.246	0.767	-0.537	0.936	0.0503	0.0091	0.0594
4.150	0.233	0.691	-0.614	0.924	0.0476	0.0089	0.0565
4.167	0.225	0.614	-0.537	0.816	0.0460	0.0069	0.0529
4.183	0.215	0.997	-0.691	1.213	0.0440	0.0153	0.0593
4.200	0.202	1.304	-0.767	1.513	0.0413	0.0238	0.0651
4.217	0.189	1.074	-0.767	1.320	0.0386	0.0181	0.0568
4.233	0.176	1.151	-0.691	1.342	0.0360	0.0188	0.0548
4.250	0.166	1.228	-0.691	1.409	0.0339	0.0207	0.0546
4.267	0.153	1.381	-0.921	1.660	0.0313	0.0287	0.0600
4.283	0.136	1.765	-0.844	1.956	0.0278	0.0399	0.0677
4.300	0.125	1.841	-0.691	1.966	0.0256	0.0403	0.0658
4.317	0.113	1.841	-0.767	1.994	0.0231	0.0414	0.0645
4.333	0.100	1.841	-0.844	2.025	0.0204	0.0427	0.0631
4.350	0.084	2.072	-0.844	2.237	0.0173	0.0522	0.0694
4.367	0.072	2.148	-0.844	2.308	0.0146	0.0555	0.0701
4.383	0.056	1.995	-0.844	2.166	0.0115	0.0489	0.0604
4.400	0.043	2.148	-0.460	2.197	0.0089	0.0503	0.0592
4.417	0.041	2.302	-0.307	2.322	0.0084	0.0562	0.0646
4.433	0.033	2.302	-0.614	2.382	0.0068	0.0591	0.0659

4.450	0.020	2.455	-0.614	2.531	0.0042	0.0667	0.0709
4.467	0.013	2.609	-0.307	2.627	0.0026	0.0719	0.0745
4.483	0.010	2.455	-0.153	2.460	0.0021	0.0630	0.0651
4.500	0.008	2.532	-0.307	2.551	0.0016	0.0678	0.0694
4.517	0.000	2.455	-0.153	2.460	0.0000	0.0630	0.0630
4.533	0.003	2.379	0.000	2.379	0.0005	0.0590	0.0595
4.550	0.000	2.532	0.077	2.533	0.0000	0.0669	0.0669
4.567	0.005	2.532	0.307	2.551	0.0010	0.0678	0.0688
4.583	0.010	2.379	0.307	2.399	0.0021	0.0600	0.0620
4.600	0.015	2.532	0.307	2.551	0.0031	0.0678	0.0709
4.617	0.020	2.455	0.384	2.485	0.0042	0.0643	0.0685
4.633	0.028	2.302	0.460	2.348	0.0058	0.0574	0.0632
4.650	0.036	2.302	0.537	2.364	0.0073	0.0582	0.0655
4.667	0.046	2.302	0.614	2.382	0.0094	0.0591	0.0686
4.683	0.056	1.995	0.767	2.137	0.0115	0.0476	0.0591
4.700	0.072	1.995	0.767	2.137	0.0146	0.0476	0.0622
4.717	0.082	2.072	0.844	2.237	0.0167	0.0522	0.0689
4.733	0.100	1.765	0.691	1.895	0.0204	0.0374	0.0578
4.750	0.105	1.765	0.691	1.895	0.0215	0.0374	0.0589
4.767	0.123	1.918	1.074	2.198	0.0251	0.0504	0.0755
4.783	0.141	1.535	0.767	1.716	0.0288	0.0307	0.0595
4.800	0.148	1.074	0.691	1.277	0.0303	0.0170	0.0473
4.817	0.164	1.458	0.767	1.647	0.0335	0.0283	0.0618
4.833	0.174	1.381	0.691	1.544	0.0356	0.0248	0.0604
4.850	0.187	1.074	0.767	1.320	0.0382	0.0181	0.0564
4.867	0.199	0.997	0.767	1.258	0.0407	0.0165	0.0572
4.883	0.212	0.921	0.691	1.151	0.0433	0.0138	0.0572
4.900	0.223	0.921	0.767	1.199	0.0456	0.0150	0.0606
4.917	0.238	0.691	0.537	0.875	0.0487	0.0080	0.0566
4.933	0.240	0.460	0.307	0.553	0.0491	0.0032	0.0523
4.950	0.248	0.537	0.384	0.660	0.0507	0.0045	0.0552
4.967	0.253	0.384	0.307	0.492	0.0517	0.0025	0.0542
4.983	0.258	0.230	0.153	0.276	0.0527	0.0008	0.0535
5.000	0.258	0.153	0.000	0.153	0.0527	0.0002	0.0530
5.017	0.258	0.000	0.000	0.000	0.0527	0.0000	0.0527
5.033	0.258	0.000	0.000	0.000	0.0527	0.0000	0.0527
5.050	0.258	-0.153	0.077	0.171	0.0527	0.0003	0.0531
5.067	0.261	-0.307	-0.077	0.316	0.0534	0.0010	0.0544
5.083	0.256	-0.384	-0.307	0.492	0.0523	0.0025	0.0549
5.100	0.251	-0.614	-0.537	0.816	0.0513	0.0069	0.0582
5.117	0.238	-0.537	-0.537	0.759	0.0487	0.0060	0.0547
5.133	0.233	-0.691	-0.460	0.830	0.0476	0.0072	0.0548
5.150	0.223	-0.767	-0.460	0.894	0.0456	0.0083	0.0539
5.167	0.217	-0.844	-0.537	1.000	0.0444	0.0104	0.0548
5.183	0.205	-1.304	-0.691	1.476	0.0419	0.0227	0.0646
5.200	0.194	-1.151	-0.691	1.342	0.0397	0.0188	0.0584
5.217	0.182	-0.997	-0.844	1.306	0.0372	0.0178	0.0550
5.233	0.166	-1.151	-0.767	1.383	0.0339	0.0199	0.0539
5.250	0.156	-1.458	-0.767	1.647	0.0319	0.0283	0.0602
5.267	0.141	-1.918	-0.844	2.095	0.0288	0.0458	0.0746
5.283	0.128	-1.827	-0.438	1.879	0.0262	0.0368	0.0629
5.302	0.125	-1.611	-0.691	1.753	0.0256	0.0320	0.0576
5.317	0.105	-1.918	-0.921	2.128	0.0215	0.0472	0.0686
5.335	0.095	-1.827	-0.658	1.942	0.0193	0.0393	0.0586
5.352	0.082	-1.918	-0.614	2.014	0.0167	0.0423	0.0590
5.368	0.074	-2.148	-0.997	2.368	0.0152	0.0584	0.0736

5.385	0.049	-2.072	-1.074	2.334	0.0099	0.0568	0.0667
5.402	0.038	-2.148	-0.844	2.308	0.0078	0.0555	0.0633
5.418	0.020	-2.225	-0.921	2.408	0.0042	0.0604	0.0646
5.435	0.008	-2.455	-0.537	2.513	0.0016	0.0658	0.0674
5.452	0.003	-2.685	-0.153	2.689	0.0005	0.0754	0.0759
5.468	0.003	-2.455	-0.077	2.456	0.0005	0.0629	0.0634
5.485	0.000	-2.379	-0.307	2.399	0.0000	0.0600	0.0600
5.502	-0.008	-2.762	-0.384	2.789	-0.0016	0.0810	0.0795
5.518	-0.013	-2.455	-0.077	2.456	-0.0026	0.0629	0.0602
5.535	-0.010	-2.455	0.153	2.460	-0.0021	0.0630	0.0610
5.552	-0.008	-2.685	0.230	2.695	-0.0016	0.0757	0.0741
5.568	-0.003	-2.532	0.307	2.551	-0.0005	0.0678	0.0673
5.585	0.003	-2.532	0.307	2.551	0.0005	0.0678	0.0683
5.602	0.008	-2.532	0.384	2.561	0.0016	0.0683	0.0699
5.618	0.015	-2.379	0.614	2.457	0.0031	0.0629	0.0660
5.635	0.028	-2.072	0.460	2.122	0.0058	0.0469	0.0527
5.652	0.031	-2.148	0.537	2.214	0.0063	0.0511	0.0574
5.668	0.046	-2.072	0.767	2.209	0.0094	0.0509	0.0603
5.685	0.056	-2.072	0.844	2.237	0.0115	0.0522	0.0637
5.702	0.074	-2.072	0.997	2.299	0.0152	0.0551	0.0703
5.718	0.090	-1.841	0.844	2.025	0.0183	0.0427	0.0610
5.735	0.102	-1.765	0.767	1.924	0.0209	0.0386	0.0594
5.752	0.115	-1.611	0.691	1.753	0.0235	0.0320	0.0555
5.768	0.125	-1.841	0.844	2.025	0.0256	0.0427	0.0683
5.785	0.143	-1.688	0.921	1.923	0.0292	0.0385	0.0678
5.802	0.156	-1.151	0.691	1.342	0.0319	0.0188	0.0507
5.818	0.166	-1.304	0.691	1.476	0.0339	0.0227	0.0566
5.835	0.179	-1.304	0.691	1.476	0.0366	0.0227	0.0593
5.852	0.189	-0.997	0.844	1.306	0.0386	0.0178	0.0564
5.868	0.207	-0.921	0.767	1.199	0.0423	0.0150	0.0573
5.885	0.215	-0.691	0.460	0.830	0.0440	0.0072	0.0511
5.902	0.223	-0.691	0.767	1.032	0.0456	0.0111	0.0567
5.918	0.240	-0.767	0.537	0.936	0.0491	0.0091	0.0582
5.935	0.240	-0.614	0.153	0.633	0.0491	0.0042	0.0532
5.952	0.246	-0.537	0.460	0.707	0.0503	0.0052	0.0555
5.968	0.256	-0.460	0.230	0.514	0.0523	0.0028	0.0551

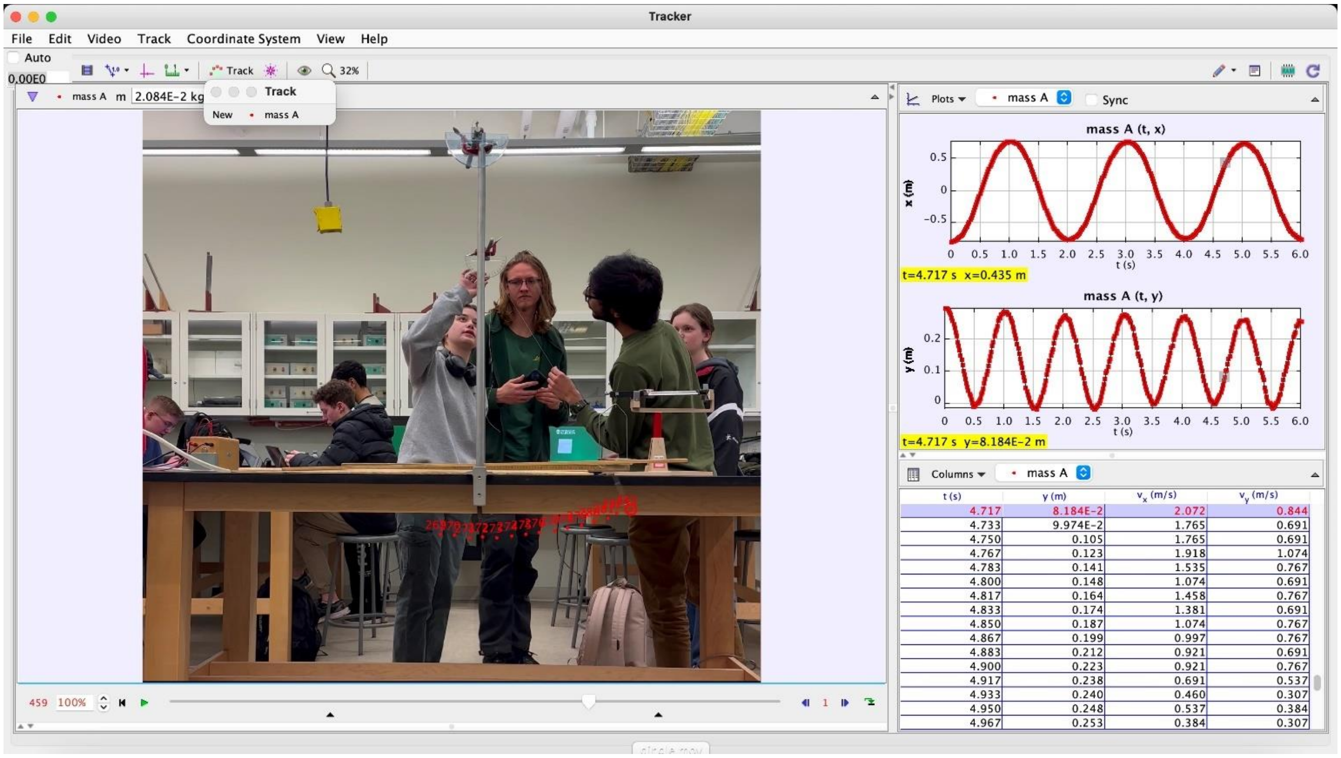

Fig. A.1: Screenshot of a frame taken on the Tracker software when analyzing a bob oscillating in a simple pendulum system. Meter rule is used as a calibration stick. Tracked points of frame 269 to frame 283 are denoted by a red dot in the figure.