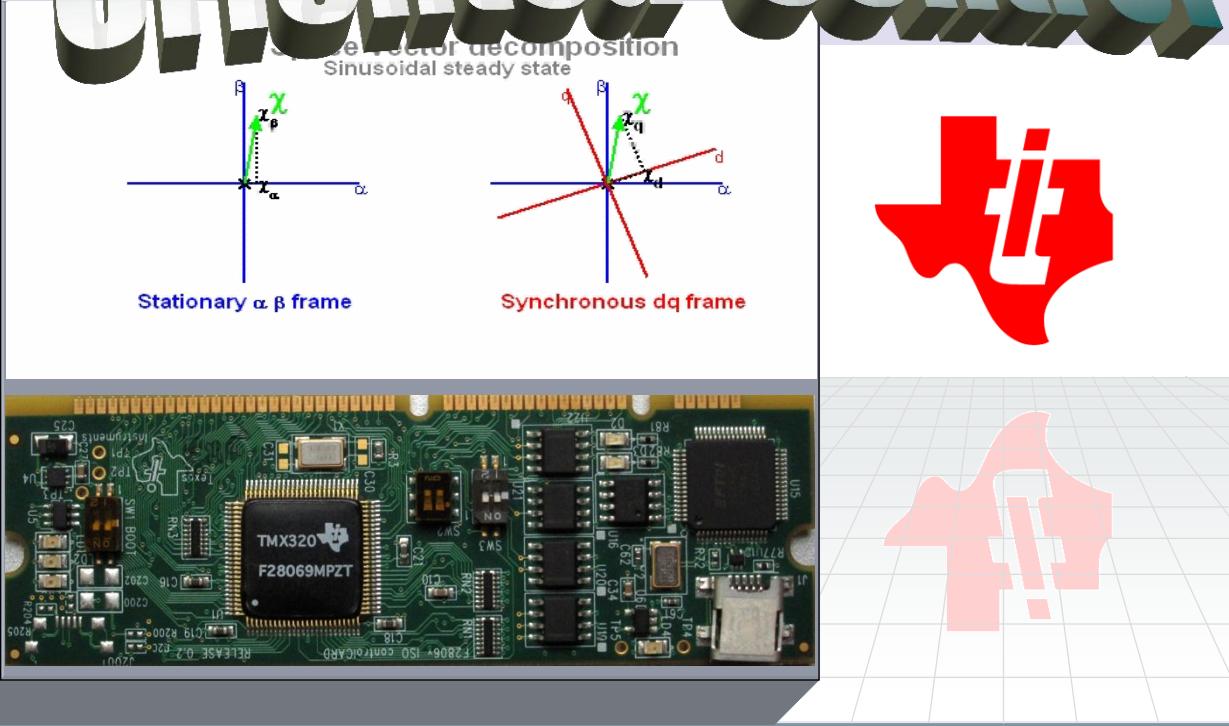
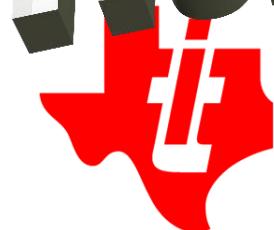
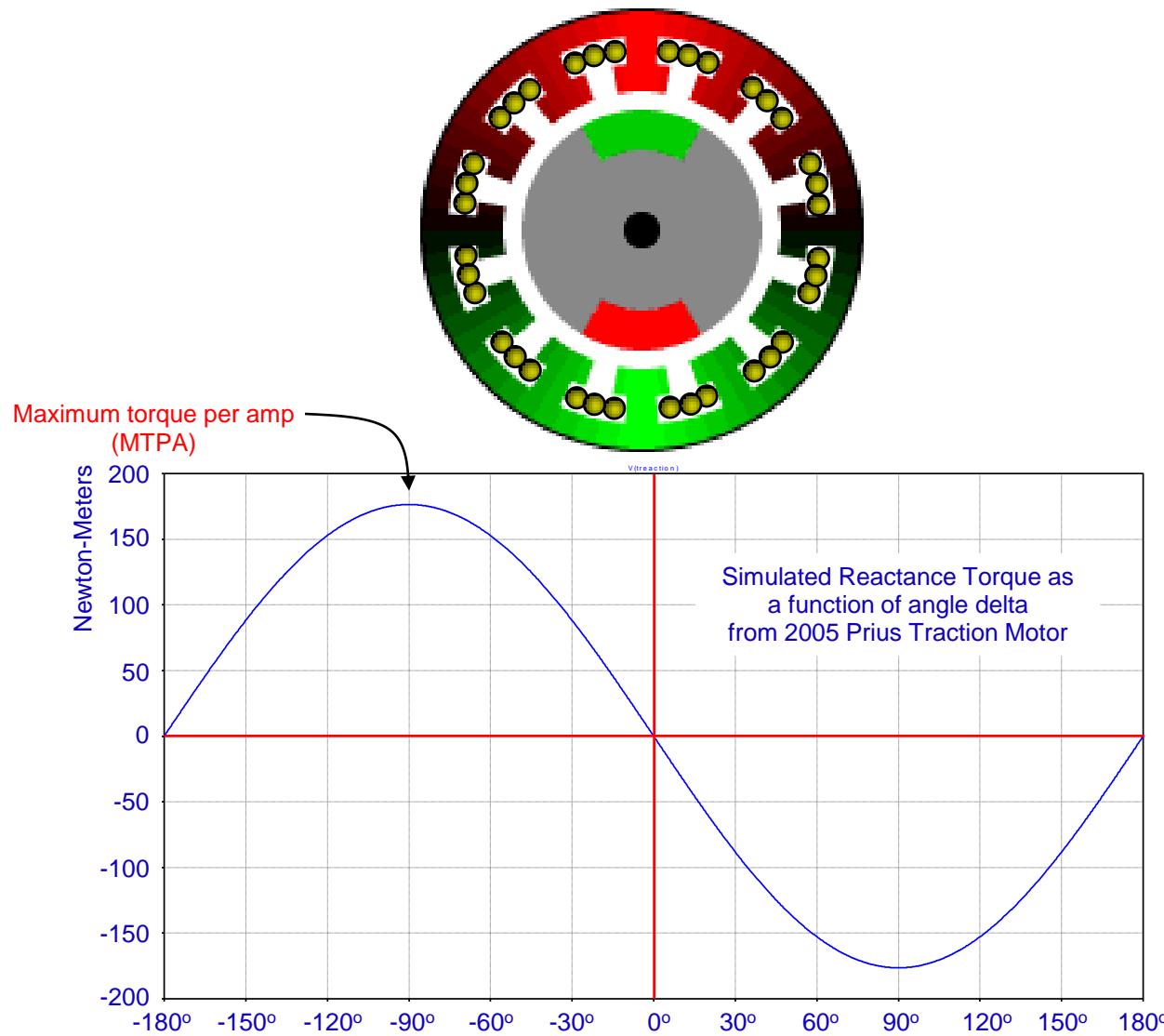


Intro to Field Oriented Control



Dave Wilson

Maximum Torque Per Amp (MTPA)

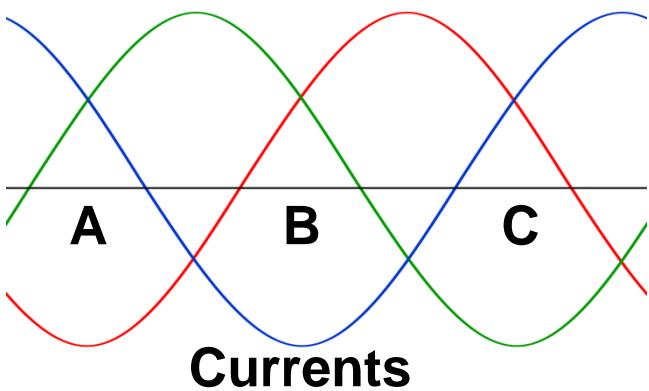


Field Oriented Control in Real Time

Low Torque

Medium Torque

High Torque

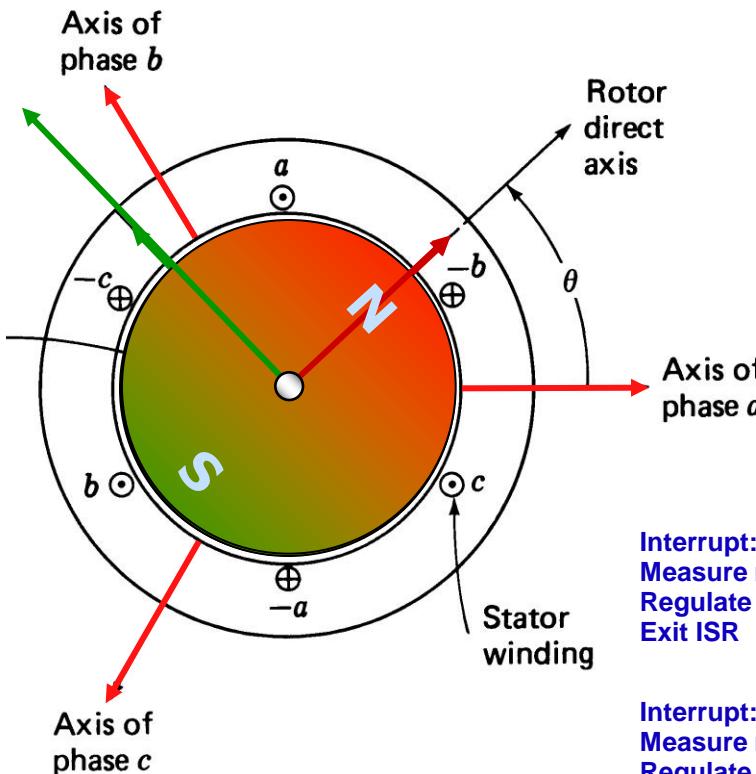


$$Torque = \frac{3}{2} \frac{P}{2} [\lambda_{dr} I_{qs}]^t$$

Annotations:

- Constant (arrow pointing to λ_{dr})
- Constant (for now) (arrow pointing to I_{qs})
- Adjustable (arrow pointing to the superscript t)

^t Torque expression based on amplitude invariant form of Clarke Transform.



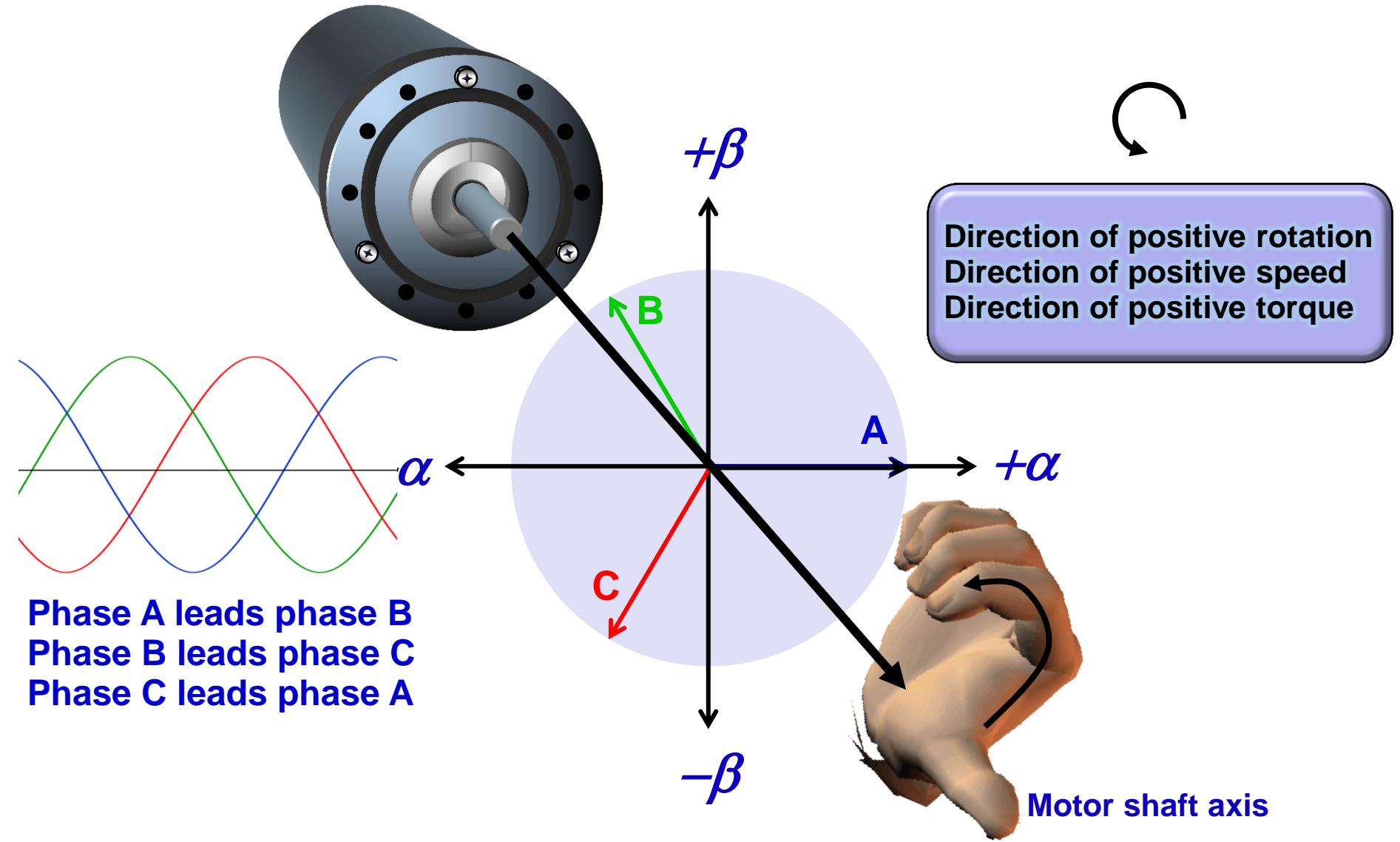
Interrupt:
Measure rotor flux angle
Regulate current vector to be 90° wrt rotor flux
Exit ISR

Interrupt:
Measure new rotor flux angle
Regulate current vector to be 90° wrt rotor flux
Exit ISR

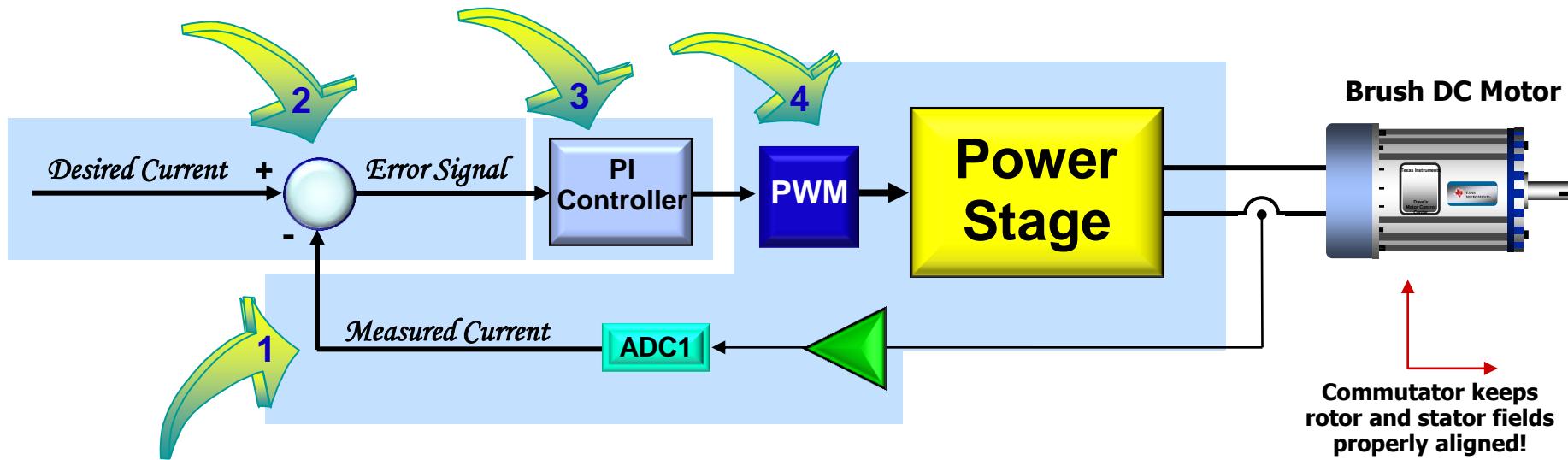
Interrupt:
Measure new rotor flux angle
Regulate current vector to be 90° wrt rotor flux
Exit ISR

⋮

Establishing Space Vector Conventions



How Do You Control Torque on a DC Motor?

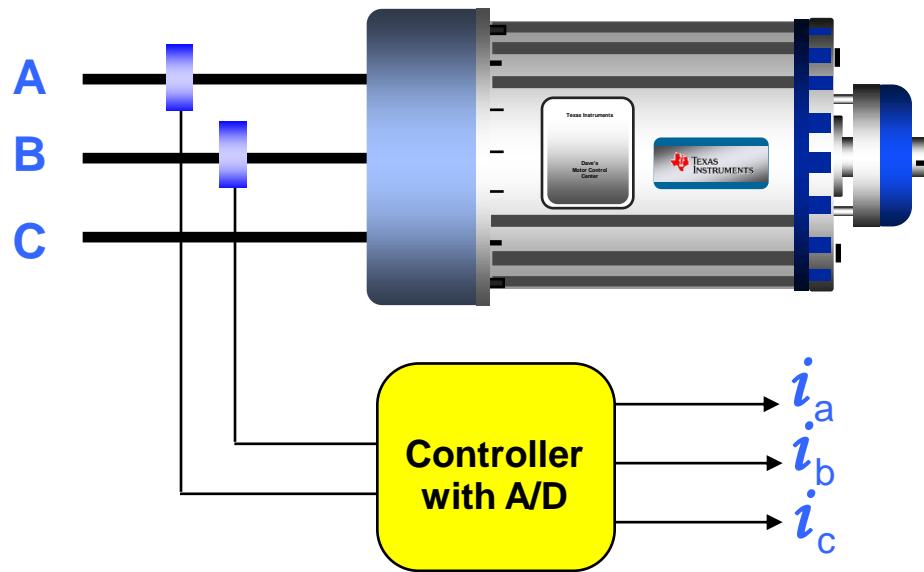


1. Measure current already flowing in the motor.
2. Compare the measured current with the desired current, and generate an error signal.
3. Amplify the error signal to generate a correction voltage.
4. Modulate the correction voltage onto the motor terminals.

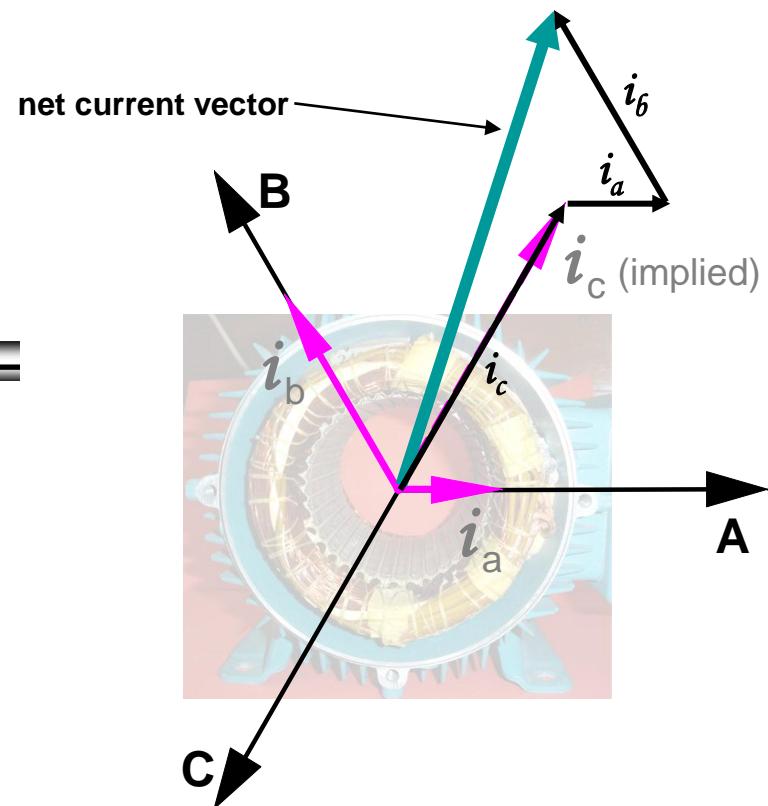
$$Torque = K_a i$$

1. Measure currents already flowing in the motor.

Only 2 phases are measured!
WHY?



A, B, and C axes are “fixed” with respect to the motor housing. This reference frame is also called the “stationary frame” or “stator frame”.



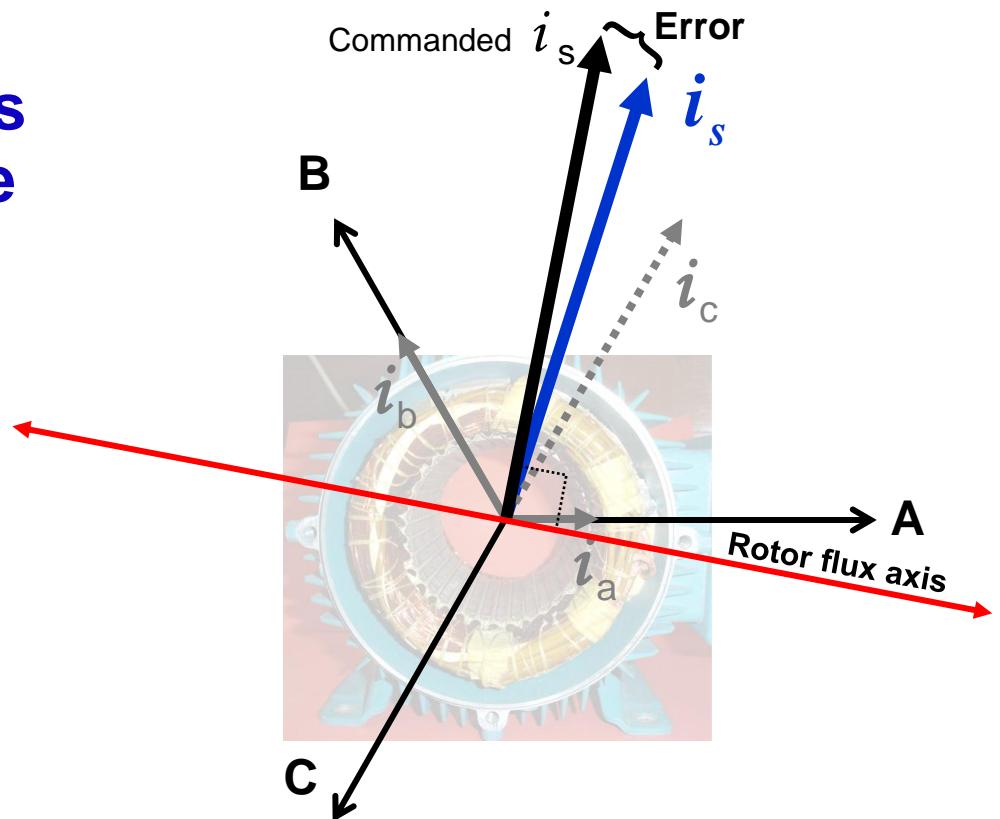
2. Compare the measured current (vector) with the desired current (vector), and generate error signals.

The desired phase currents can be calculated via these equations:

$$i_a = -I_m \sin(\theta_\lambda)$$

$$i_b = -I_m \sin(\theta_\lambda - 120^\circ)$$

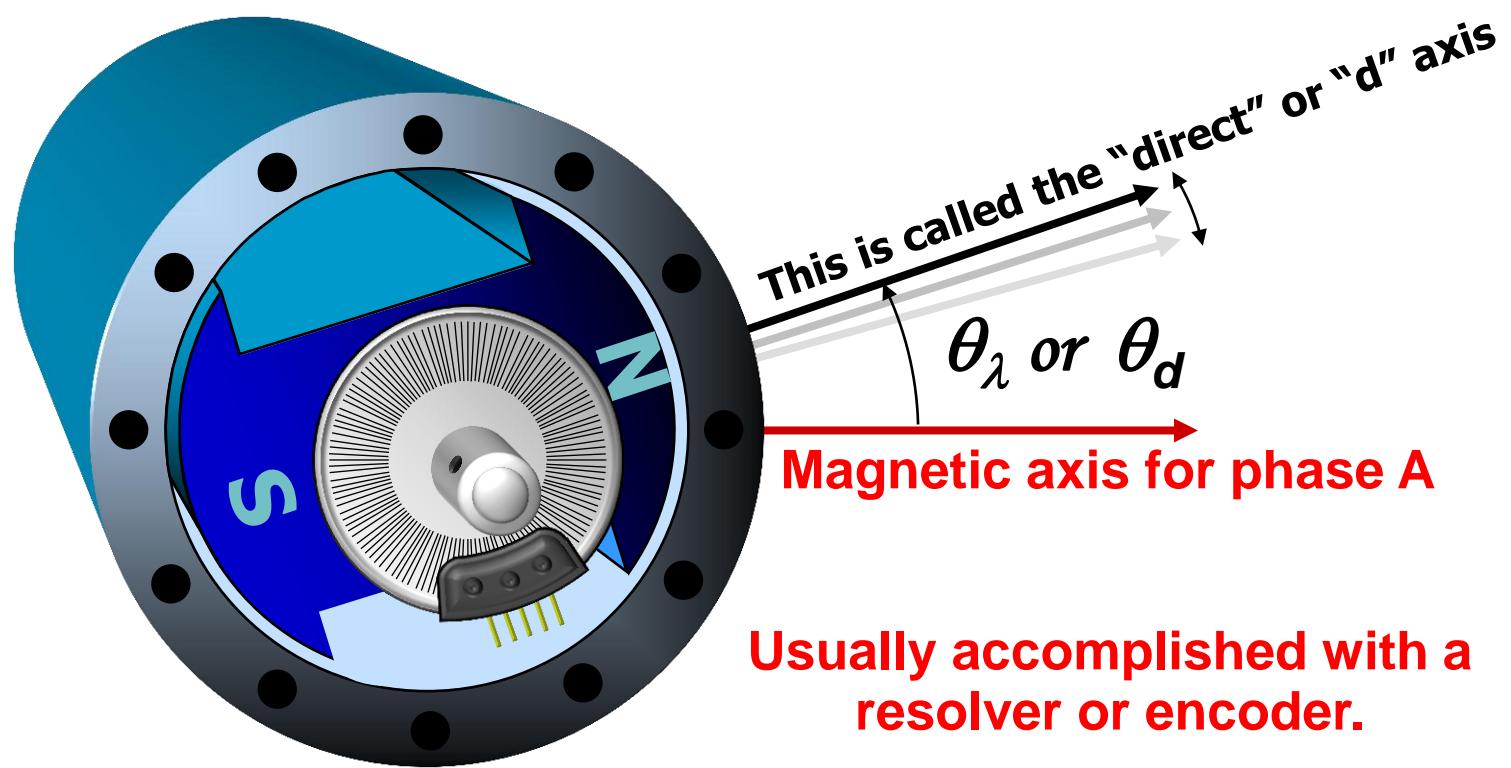
$$i_c = -I_m \sin(\theta_\lambda - 240^\circ)$$



I_m is proportional to motor torque
 θ_λ is the angle of the rotor flux

So how do we get the rotor flux angle?

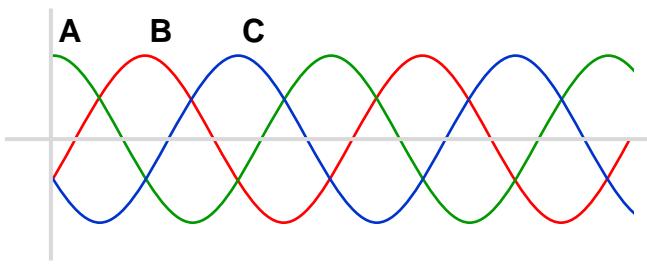
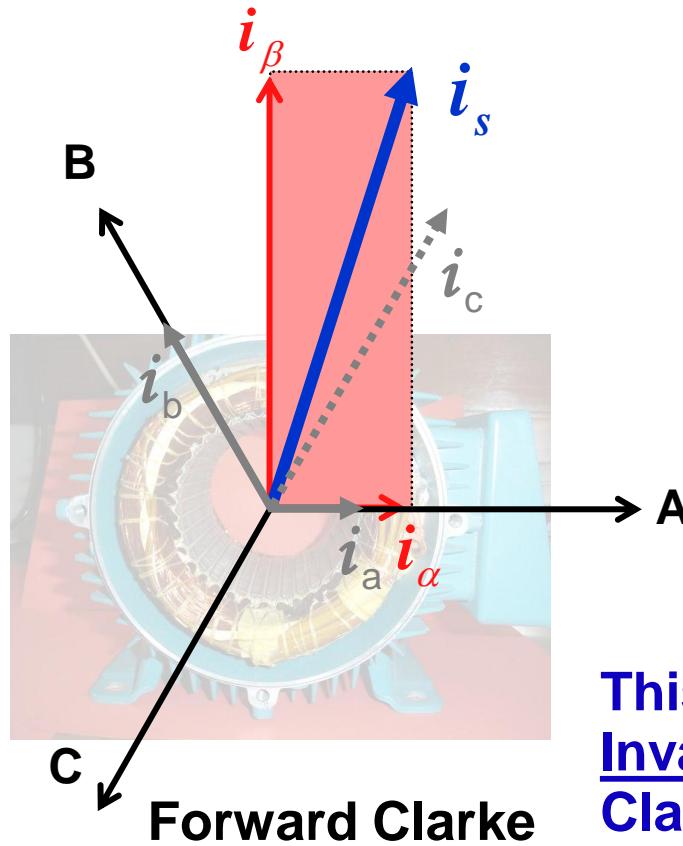
2. Compare the measured current (vector) with the desired current (vector), and generate error signals.



2. Compare the measured current (vector) with the desired current (vector), and generate error signals.

The CLARKE transform allows us to convert three vectors into two orthogonal vectors that produce the same net vector.

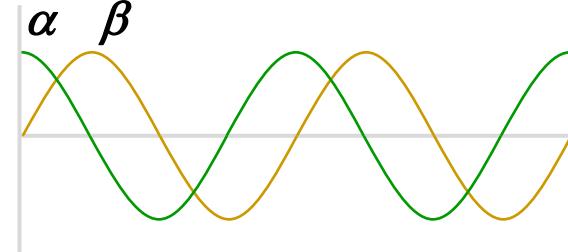
In other words, convert a 3-phase motor into a 2-phase motor.



$$\alpha = A$$
$$\beta = \frac{(B - C)}{\sqrt{3}}$$



This is the Amplitude Invariant form of the Clarke Transform



Stationary Frame Servo



Synchronous Frame Servo



Tracking a rotating reference signal
in the same rotating frame is easy!

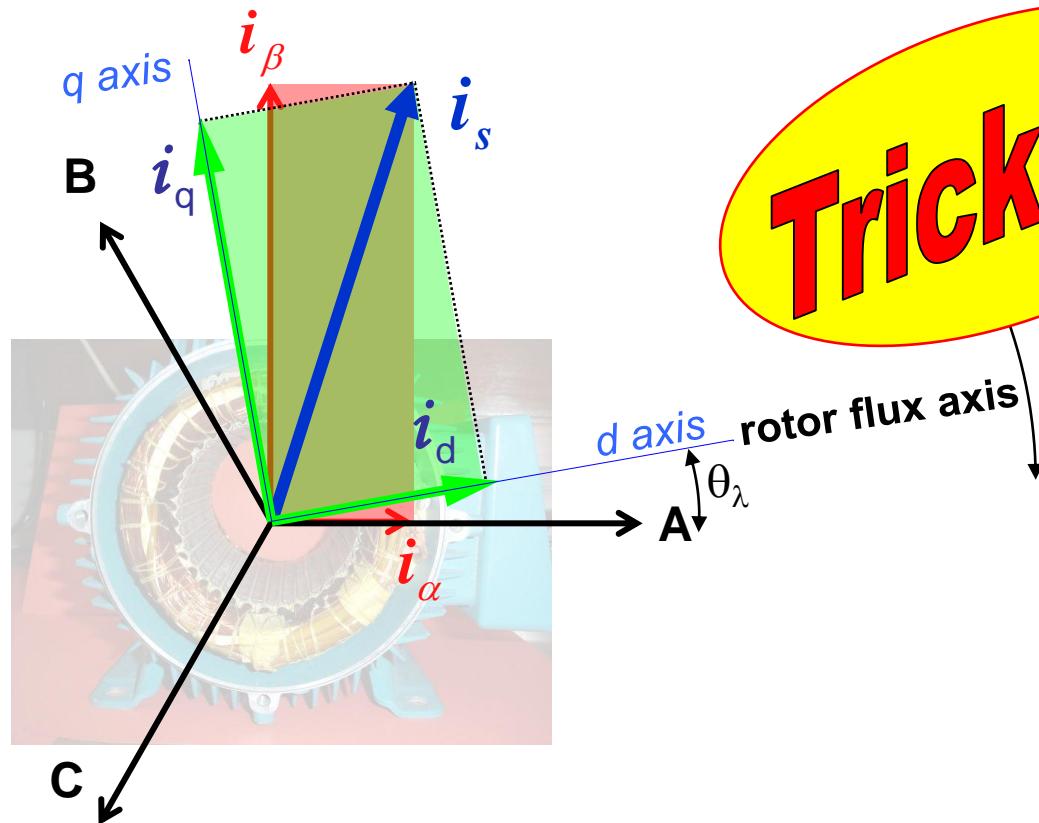
Take the “Coffee” Test



1. Give your coffee a good stir so that bubbles form on the surface.
2. Holding the cup as shown here, spin around in the same direction that you stirred the coffee.
3. As soon as you reach synchronous speed, the bubbles stop spinning.
Why?

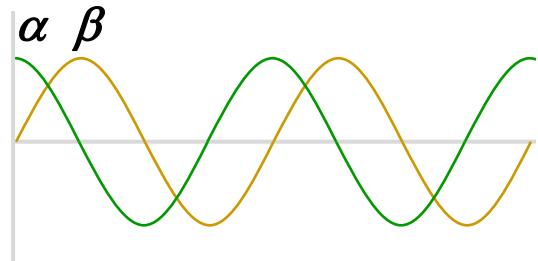
2. Compare the measured current (vector) with the desired current (vector), and generate error signals.

Jump up on the rotating reference frame, whose x-axis is the rotor flux axis.



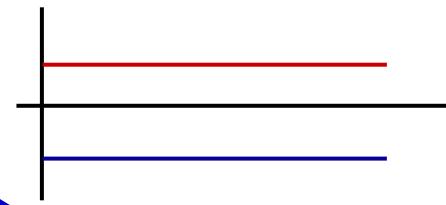
Trick #2

This is called the Park Transform



$$i_d = i_\alpha \cos \theta_\lambda + i_\beta \sin \theta_\lambda$$

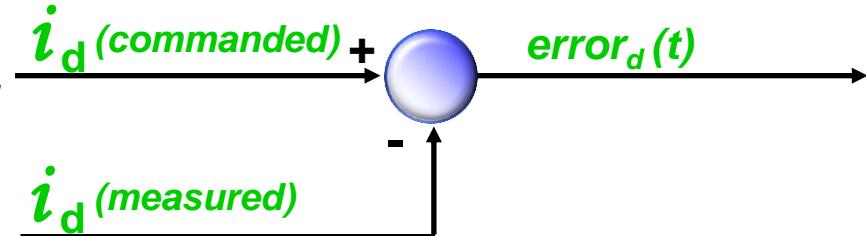
$$i_q = -i_\alpha \sin \theta_\lambda + i_\beta \cos \theta_\lambda$$



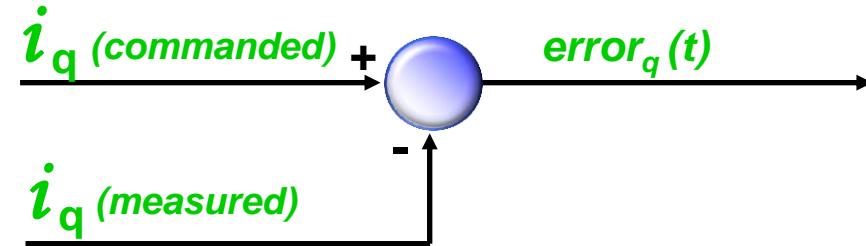
2. Compare the measured current (vector) with the desired current (vector), and generate error signals.

i_d and i_q are handled independently. Since the comparison is performed in the synchronous frame, motor AC frequency is not seen. Thus, they are DC quantities!

Under normal conditions, we have all the flux we need supplied by the permanent magnets on the rotor. So commanded i_d is set to zero.



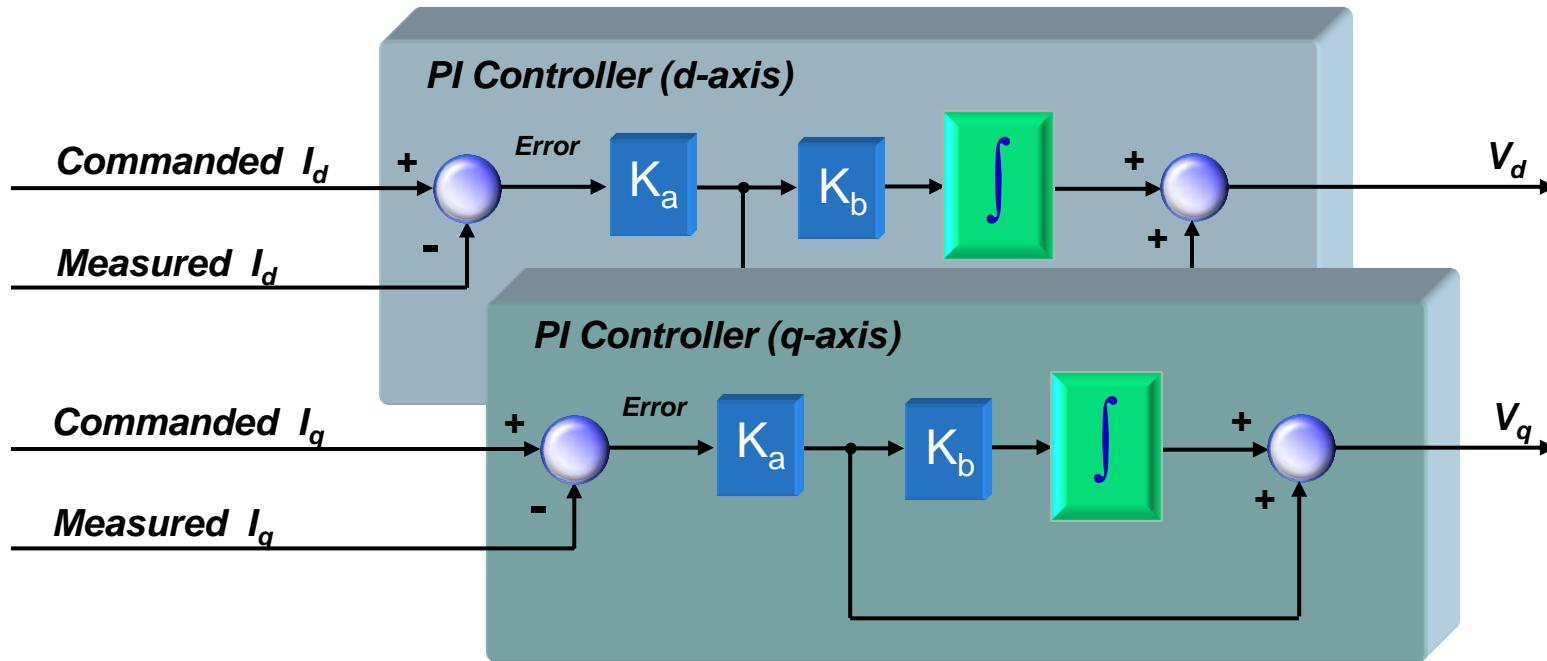
This is how much torque we want!



i_d can be used to weaken the field of the machine.

i_q controls the amount of torque generated by the motor

3. (Finally!) Amplify the error signals to generate correction voltages.

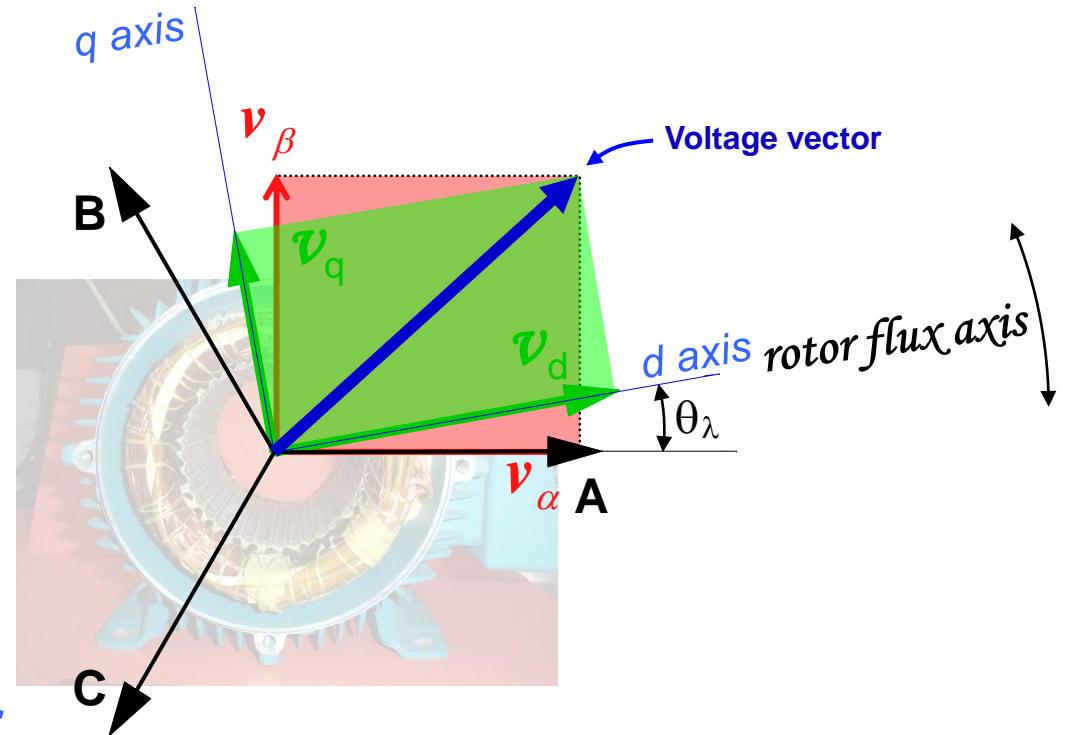


$$K_a = L \cdot \text{Current Bandwidth (rad/sec)}, K_b = R/L$$

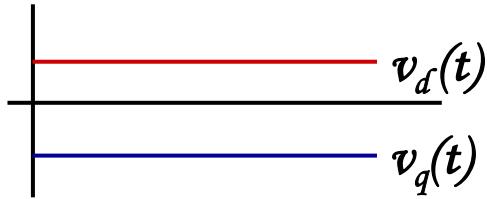
Motor	R _d	R _q	L _d	L _q
PMSM	R_s	R_s	L_s	L_s
ACIM	R_s	$R_s + R_r$	$L_s \left(1 - \frac{L_m^2}{L_r L_s}\right)$	$L_s \left(1 - \frac{L_m^2}{L_r L_s}\right)$
IPM	R_s	R_s	L_s_d	L_s_q

4. Modulate the correction voltages onto the motor terminals.

Before we can apply the voltages to the motor windings, we must first jump off of the rotating reference frame.

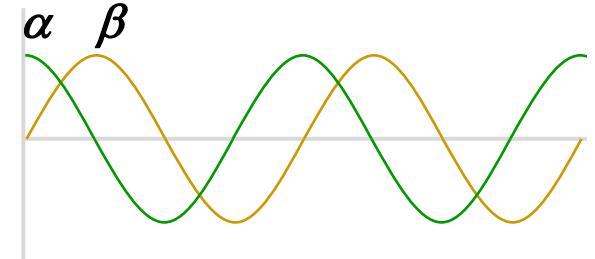


Part A. Transfer the voltage vectors back to the stationary rectangular coordinate system.



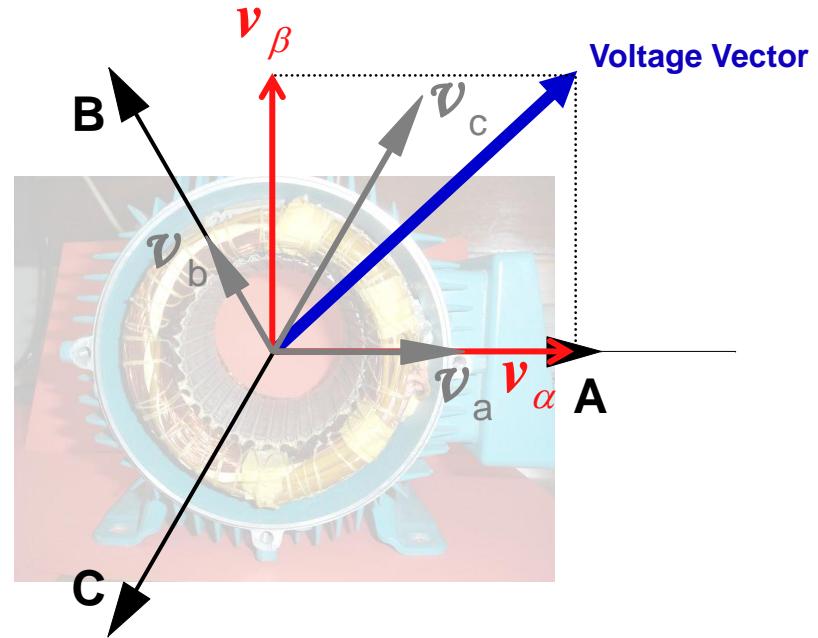
$$v_\alpha = v_d \cos \theta_\lambda - v_q \sin \theta_\lambda$$

$$v_\beta = v_d \sin \theta_\lambda + v_q \cos \theta_\lambda$$

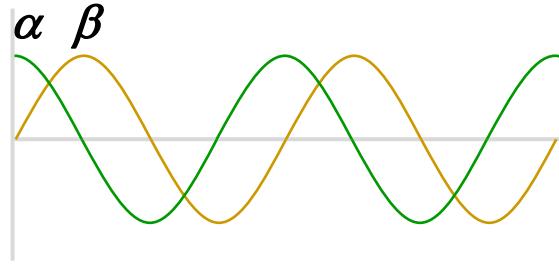


4. Modulate the correction voltages onto the motor terminals.

Part B. Next, we transform the voltage vectors from the rectangular coordinate system to three phase vectors.



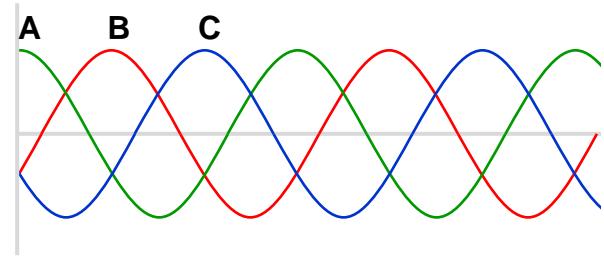
Reverse Clarke Transformation



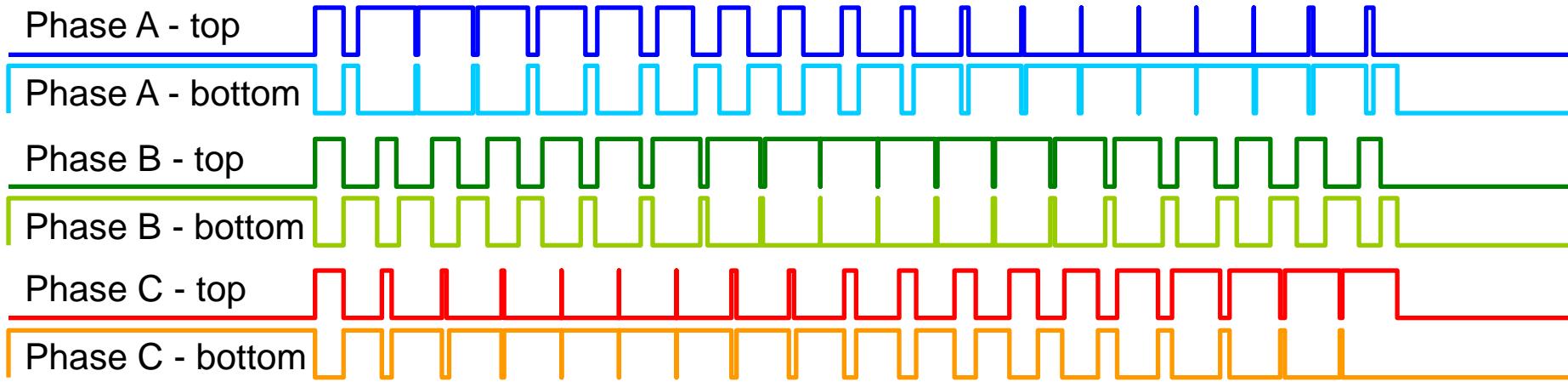
$$A = \alpha$$

$$B = -\frac{1}{2}\alpha + \frac{\sqrt{3}}{2}\beta$$

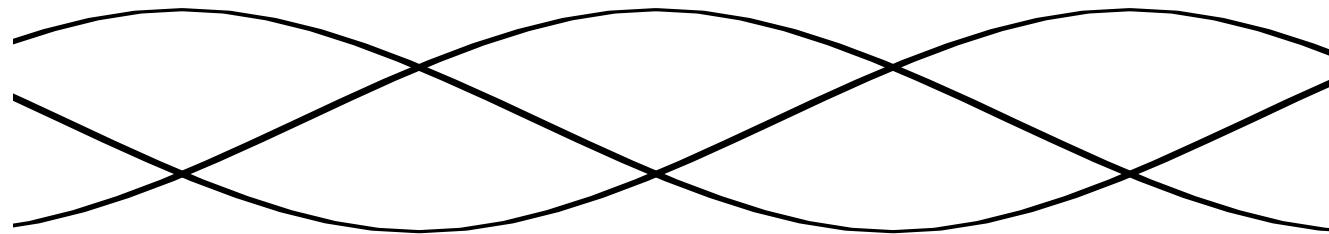
$$C = -\frac{1}{2}\alpha - \frac{\sqrt{3}}{2}\beta$$



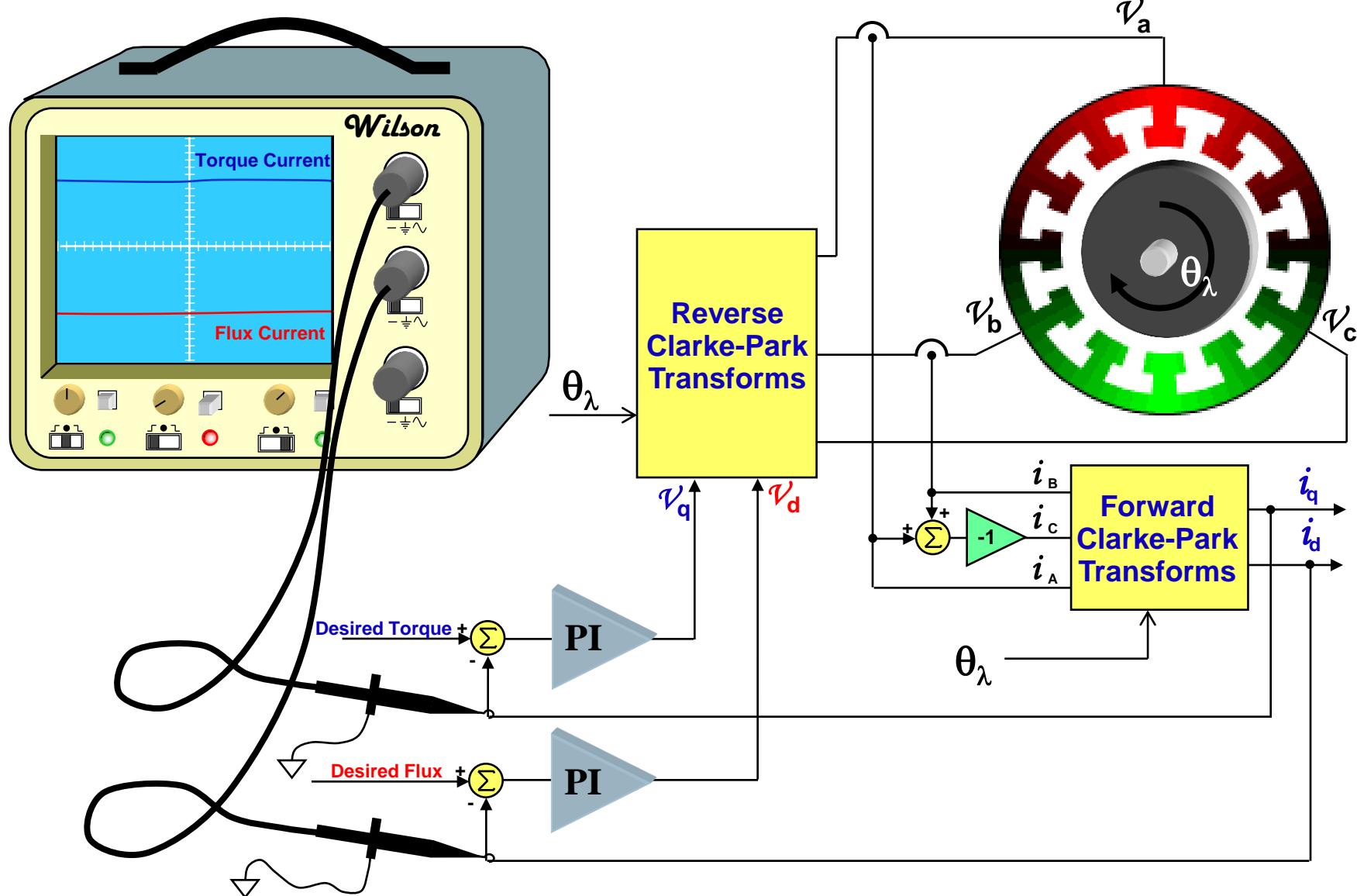
4. Modulate the correction voltages onto the motor terminals.



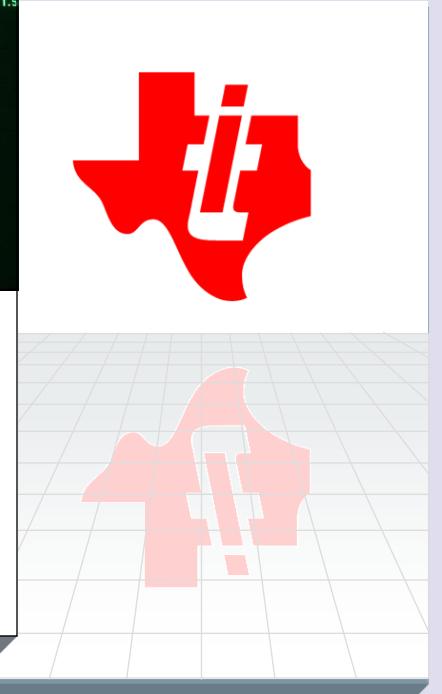
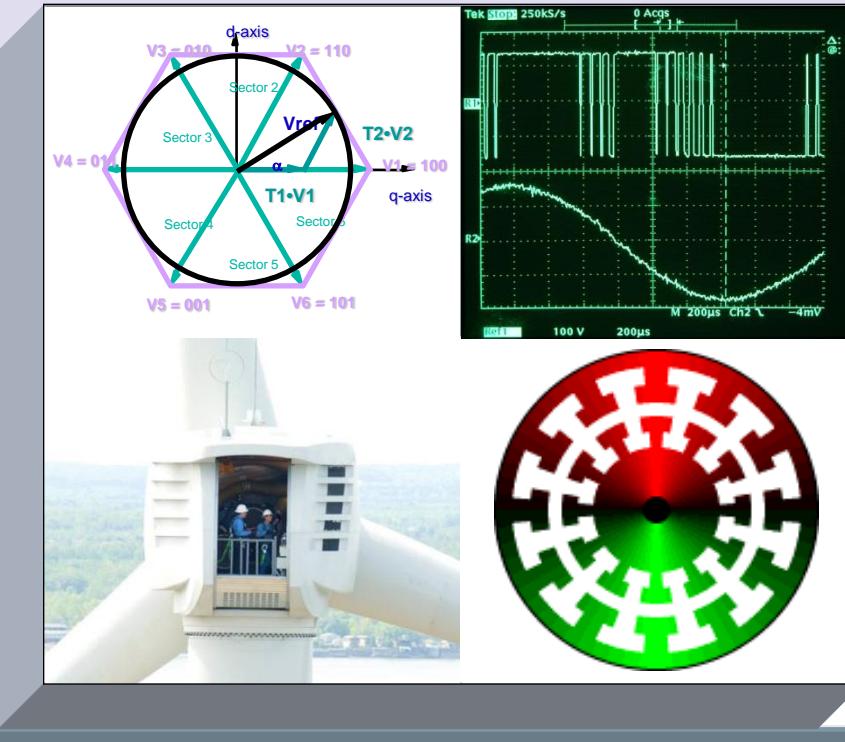
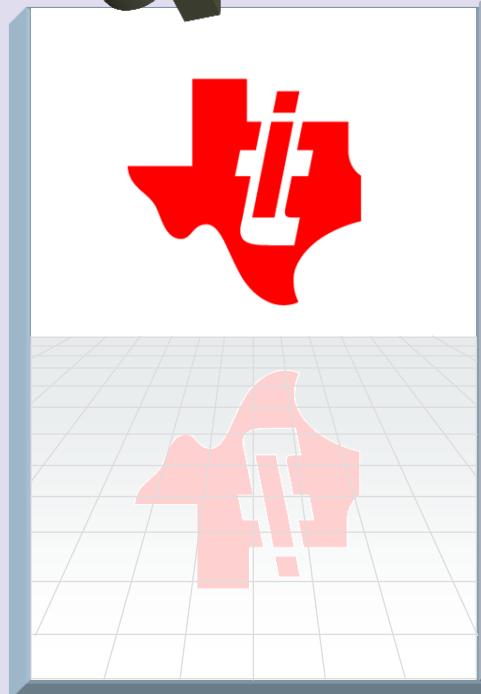
Over time, under steady-state conditions, the correction voltages v_a , v_b , and v_c will be sine waves phase shifted by 120° .



FOC in a Nutshell

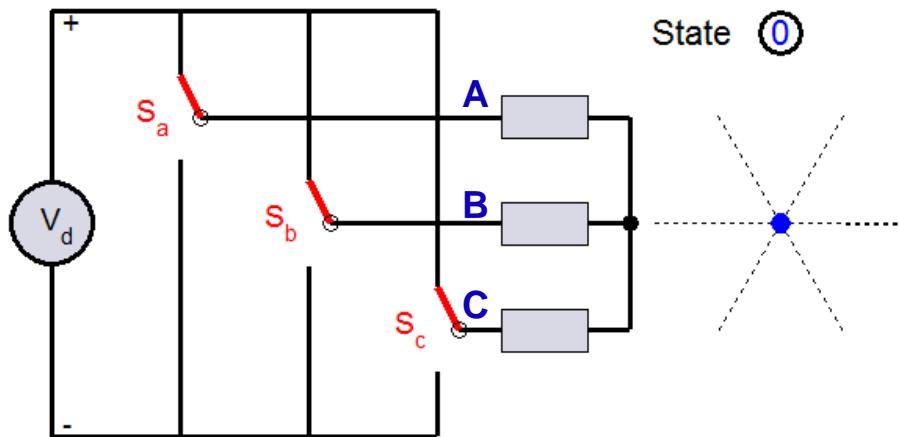


Space Vector Modulation

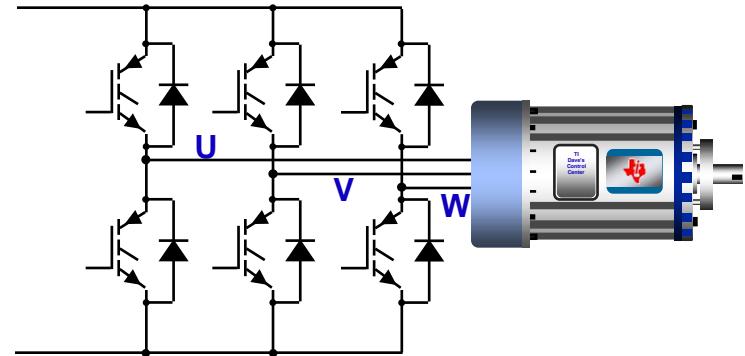


Space Vector Modulation

- PWMs treat each phase individually.
- SVM treats the inverter as ONE unit!!
 - ALL 6 switches affected.
- PWMs control the phase voltages.
 - 120° offsets between A, B, and C.
- SVM controls the Voltage Vector.

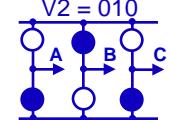
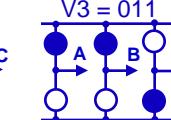
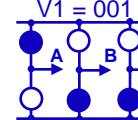


Source: Mahmoud Riaz, Sc.D., Professor of Electrical Engineering, Department of Electrical and Computer Engineering, University of Minnesota

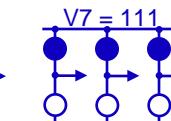
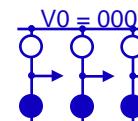


The inverter can be driven to 8 states.

- 6 voltage vectors

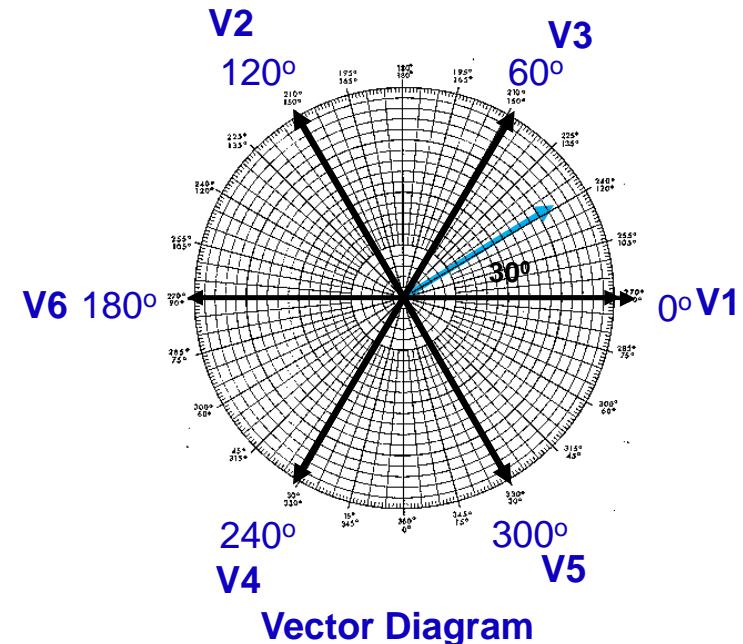
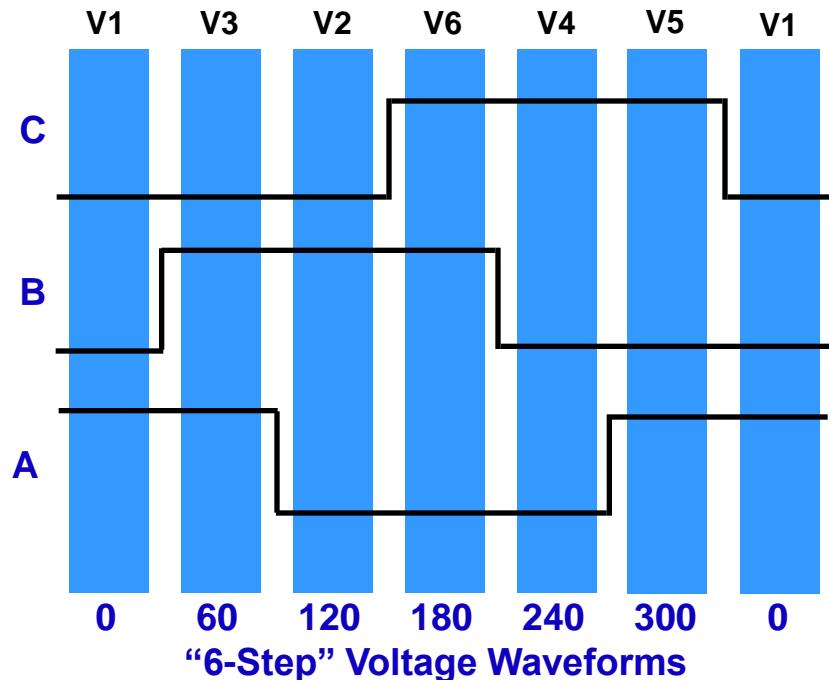


- 2 null vectors



1 = Top Switch is on
0 = Bottom Switch is on

Space Vector Modulation



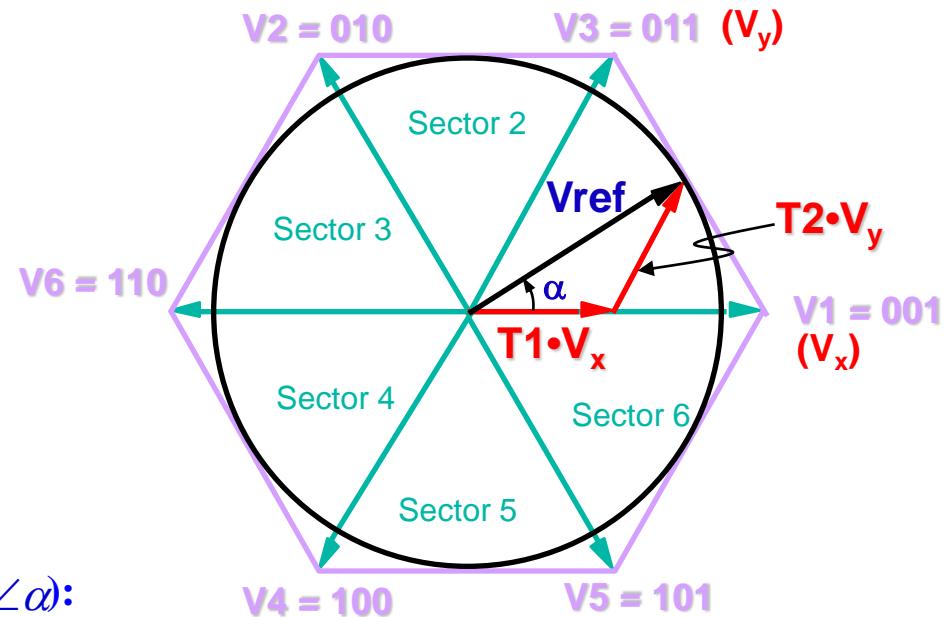
- ★ Output voltage vector created by repeatedly switching between adjacent vectors and the “null” vector (all three phases high or all three phases low).
- ★ Output angle θ determined by relative “on” time between two adjacent vectors.
- ★ Output magnitude determined by relative “on” time between two adjacent vectors **and** the null vector.

SVM Implementation

- V_{ref} , is created by the two adjacent state vectors V_x , V_y , and a null vector in a time averaging fashion:

$$V_{ref} = V_x \cdot T_1 + V_y \cdot T_2 + V_{null} \cdot T_0$$

Where:
 V_x = lowest angle voltage vector
 V_y = highest angle voltage vector



If V_{ref} is represented in POLAR notation ($m\angle\alpha$):

$$T_1 = T \cdot m \cdot \sin(60^\circ - \alpha)$$

$$T_2 = T \cdot m \cdot \sin(\alpha)$$

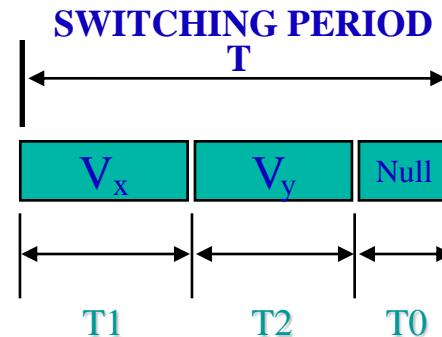
$$T_0 = T - T_1 - T_2$$

m = vector magnitude (0 to 1)

α = vector angle between V_{ref} and V_x (0 to 60°)

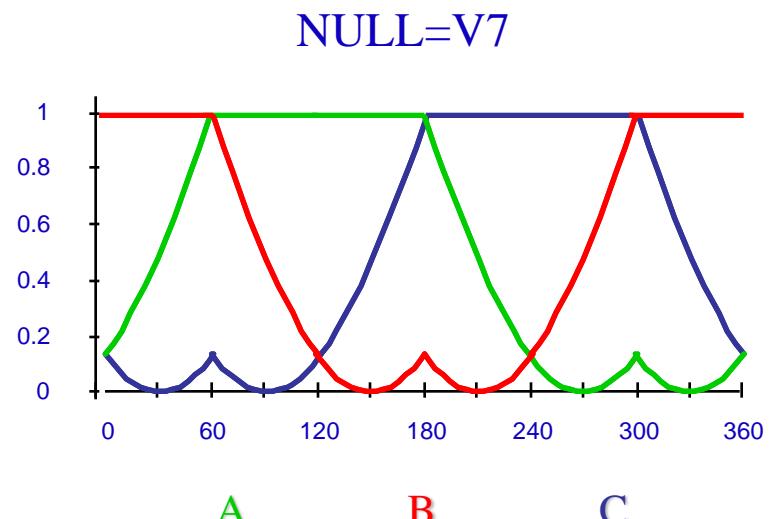
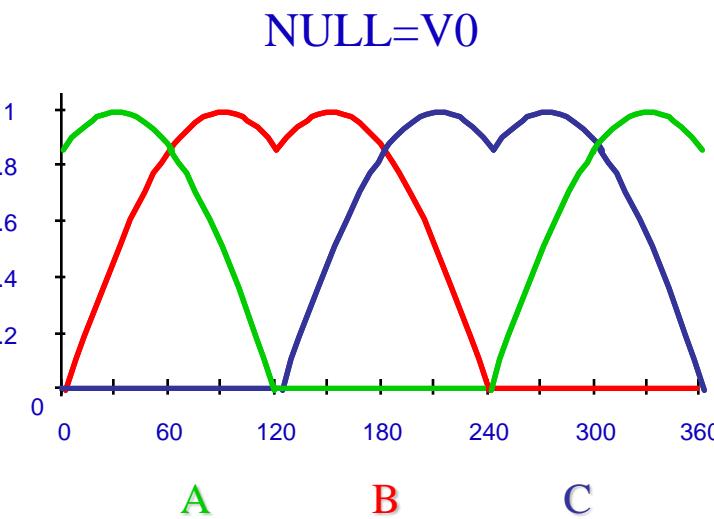
T = switching period

Example vector reconstruction in sector 1.



SVM Variations

- Choice of Null vectors effects SVM waveform creation and switching performance but motor still sees sinusoidal waveform at its phases.

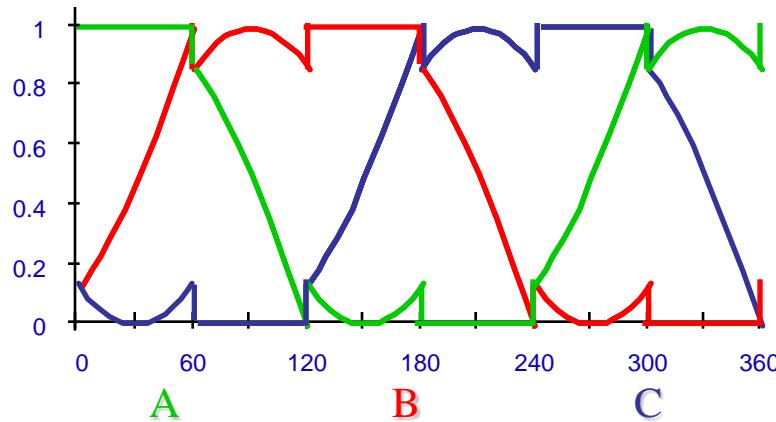


*Good choice when reduced switching losses
are desired in an inverter with
high-side bootstrap circuits.*

SVM Variations (continued)

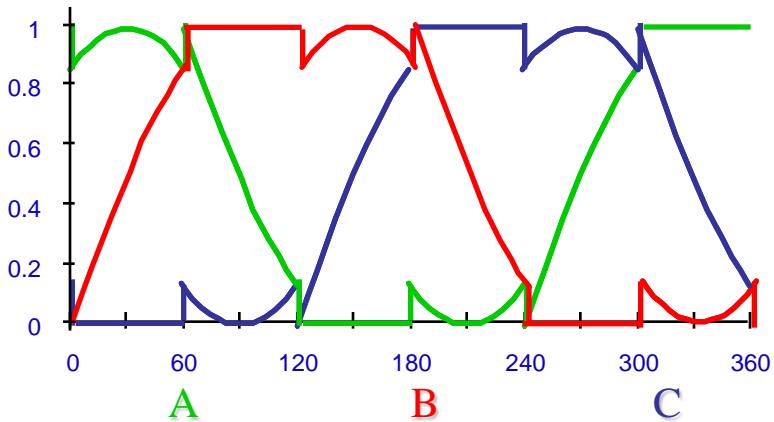
Null = V7 in sectors 1,3,5

Null = V0 in sectors 2,4,6



Null = V0 in sectors 1,3,5

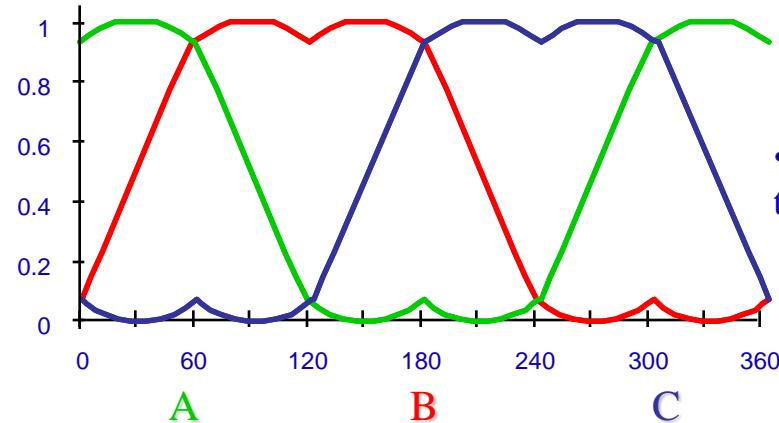
Null = V7 in sectors 2,4,6



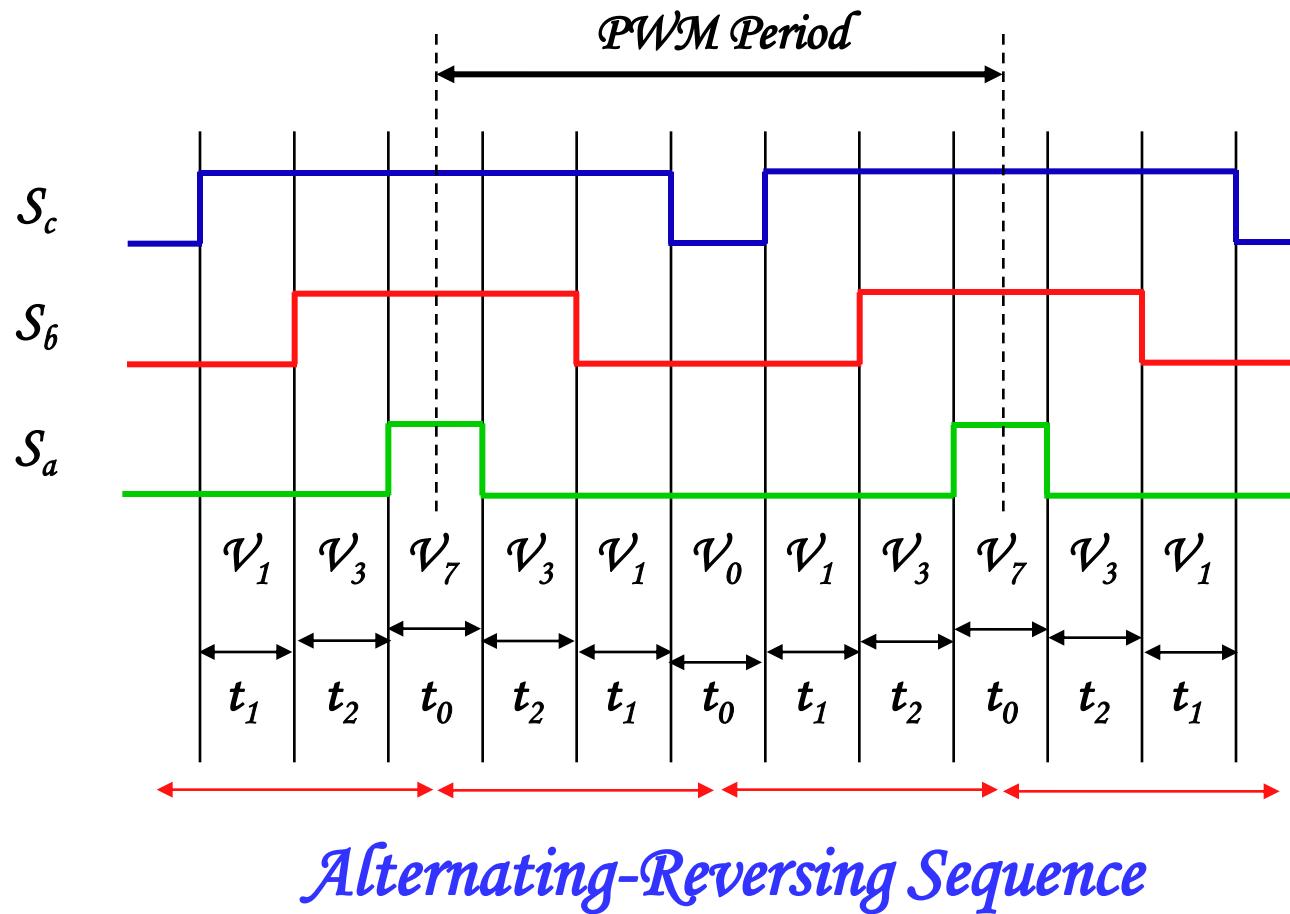
Null = alt-rev
(null alternates every sequence
and sequence reverses.)

Most popular form of SVM

- Similar waveforms to Third Harmonic



SVM Using Center-Aligned PWMs



Check out http://www.ipes.ethz.ch/ipes/Raumzeiger/e_RZ4.html for a neat SVM animation!

SVM with PWMs

Procedure for implementing SVM with Center-Aligned PWM Module

- 1 Check which sector the Vref vector is in. (5 compares)
- 2 Calculate T1, T2, and T0 from block below. (3 multiples)
- 3 Adjust PWMs High times based on table below. (1-3 additions)

$T_1 = T \cdot m \cdot \sin(60 - \alpha)$
 $T_2 = T \cdot m \cdot \sin(\alpha)$
 $T_0 = T - T_1 - T_2$

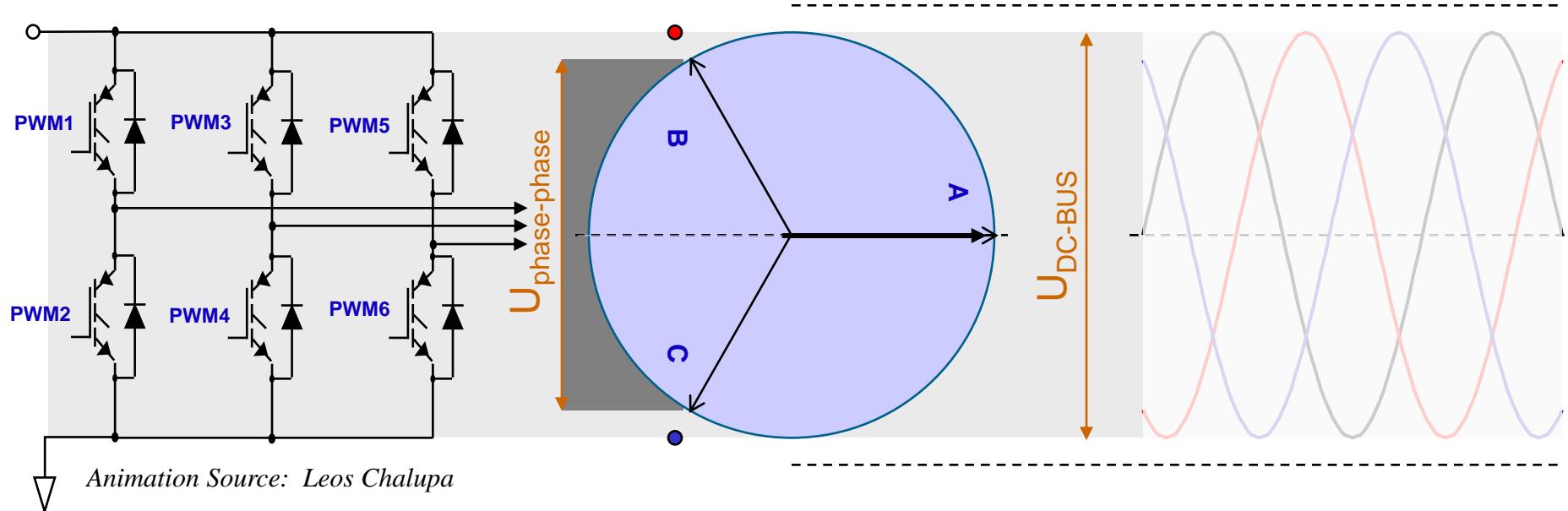
	Null = V0	Null = V7	V7 in 1,3,5 V0 in 2,4,6	V0 in 1,3,5 V7 in 2,4,6	Alternating Reversing Sequencing
<u>Sector 1</u>	$U = T_1 + T_2$ $V = T_2$ $W = 0$	$U = 100\%$ $V = T_0 + T_2$ $W = T_0$	$U = 100\%$ $V = T_0 + T_2$ $W = T_0$	$U = T_1 + T_2$ $V = T_2$ $W = 0$	$U = T_1 + T_2 + .5T_0$ $V = T_2 + .5T_0$ $W = .5T_0$
<u>Sector 2</u>	$U = T_1$ $V = T_1 + T_2$ $W = 0$	$U = T_0 + T_1$ $V = 100\%$ $W = T_0$	$U = T_1$ $V = T_1 + T_2$ $W = 0$	$U = T_0 + T_1$ $V = 100\%$ $W = T_0$	$U = T_1 + .5T_0$ $V = T_1 + T_2 + .5T_0$ $W = .5T_0$
<u>Sector 3</u>	$U = 0$ $V = T_1 + T_2$ $W = T_2$	$U = T_0$ $V = 100\%$ $W = T_0 + T_2$	$U = T_0$ $V = 100\%$ $W = T_0 + T_2$	$U = 0$ $V = T_1 + T_2$ $W = T_2$	$U = .5T_0$ $V = T_1 + T_2 + .5T_0$ $W = T_2 + .5T_0$
<u>Sector 4</u>	$U = 0$ $V = T_1$ $W = T_1 + T_2$	$U = T_0$ $V = T_0 + T_1$ $W = 100\%$	$U = 0$ $V = T_1$ $W = T_1 + T_2$	$U = T_0$ $V = T_0 + T_1$ $W = 100\%$	$U = .5T_0$ $V = T_1 + .5T_0$ $W = T_1 + T_2 + .5T_0$
<u>Sector 5</u>	$U = T_2$ $V = 0$ $W = T_1 + T_2$	$U = T_0 + T_2$ $V = T_0$ $W = 100\%$	$U = T_0 + T_2$ $V = T_0$ $W = 100\%$	$U = T_2$ $V = 0$ $W = T_1 + T_2$	$U = T_2 + .5T_0$ $V = .5T_0$ $W = T_1 + T_2 + .5T_0$
<u>Sector 6</u>	$U = T_1 + T_2$ $V = 0$ $W = T_1$	$U = 100\%$ $V = T_0$ $W = T_0 + T_1$	$U = T_1 + T_2$ $V = 0$ $W = T_1$	$U = 100\%$ $V = T_0$ $W = T_0 + T_1$	$U = T_1 + T_2 + .5T_0$ $V = .5T_0$ $W = T_1 + .5T_0$

Note: All calculations referenced to top switch

Source: *Understanding Space Vector Modulation*,
by Peter Pinewski, EDN Products Edition, March 7, 1996

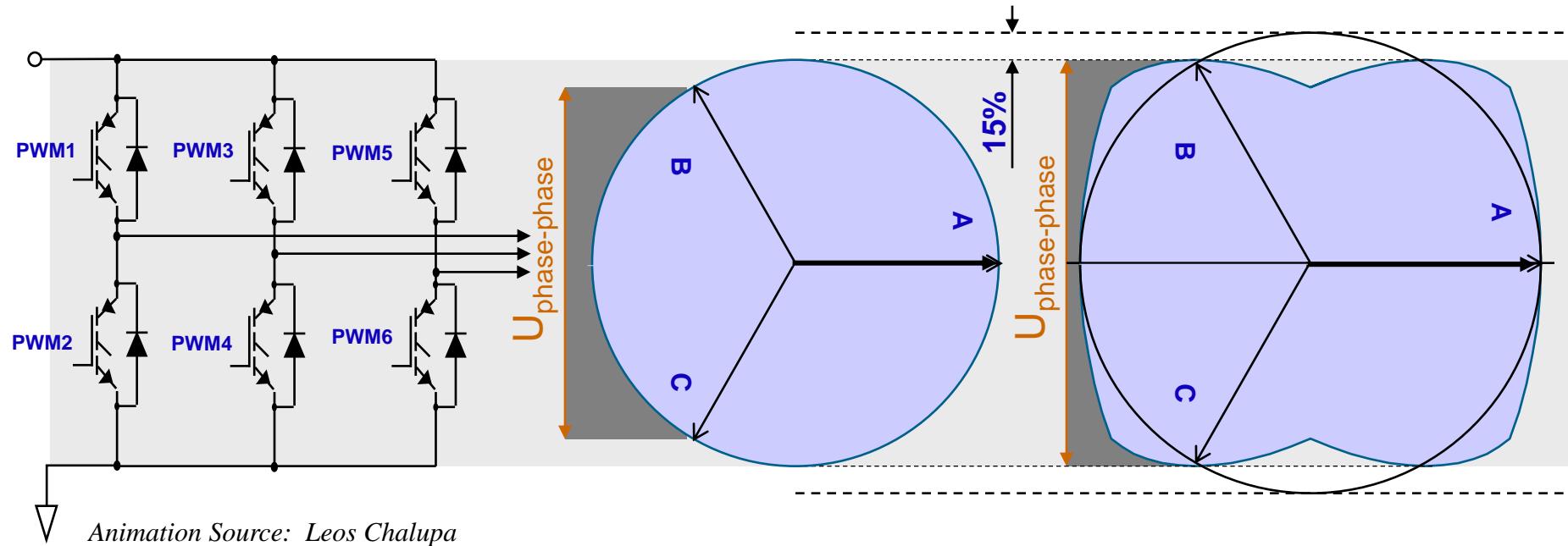
Sinusoidal Modulation - Limited Amplitude

- In sinusoidal modulation the amplitude is limited to half of the DC-bus voltage.
- The phase to phase voltage is then lower than the DC-bus voltage (although such voltage can be generated between the terminals).



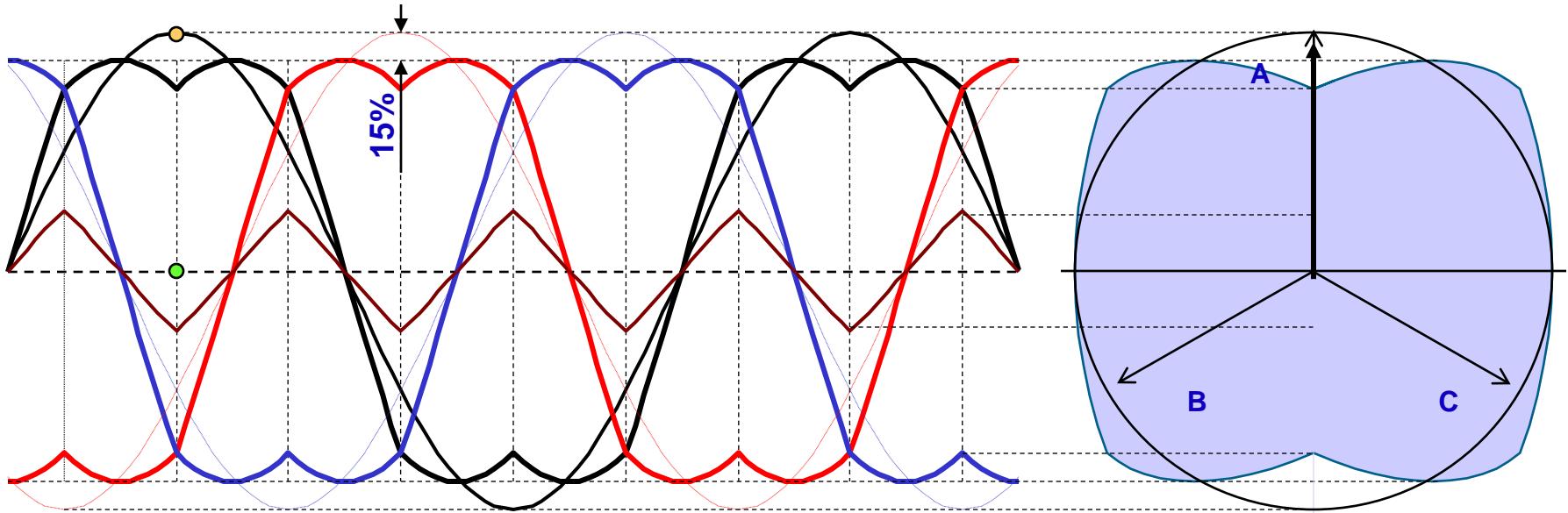
Full Phase-to-Phase Voltage Generation

- Full phase-to-phase voltage can be generated by continuously shifting the 3-phase voltage system.
- The amplitude of the first harmonic can be then increased by 15.5%.



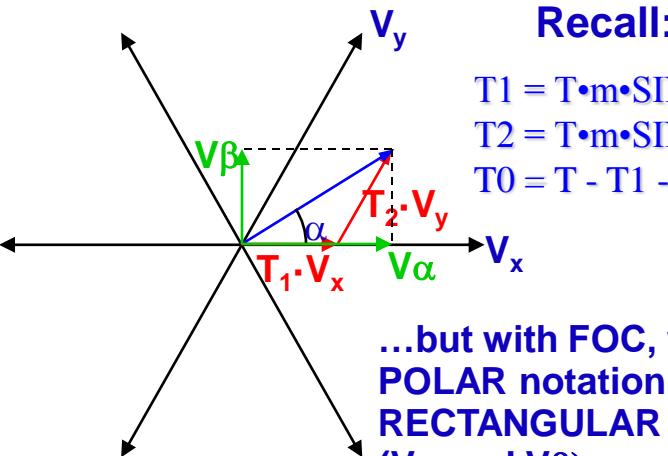
How to Increase Modulation Index

- Modulation index is increased by adding the “shifting” voltage u_0 to first harmonic.
- “Shifting” voltage u_0 must be the same for all three phases, thus it can only contain 3^r harmonics!



Animation Source: Leos Chalupa

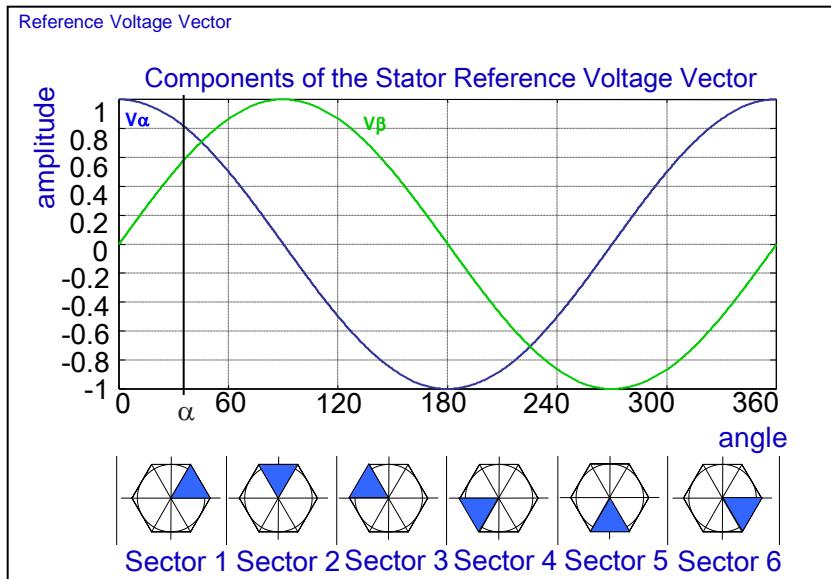
SVM with Field Oriented Systems



Recall:

$$\begin{aligned}T_1 &= T \cdot m \cdot \sin(60 - \alpha) \\T_2 &= T \cdot m \cdot \sin(\alpha) \\T_0 &= T - T_1 - T_2\end{aligned}$$

...but with FOC, we don't have POLAR notation. We have RECTANGULAR notation (V_α and V_β).



We could calculate "m" and " α :

$$m = \sqrt{(V_\alpha)^2 + (V_\beta)^2}$$

$$\alpha = \tan^{-1} \frac{V_\beta}{V_\alpha}$$

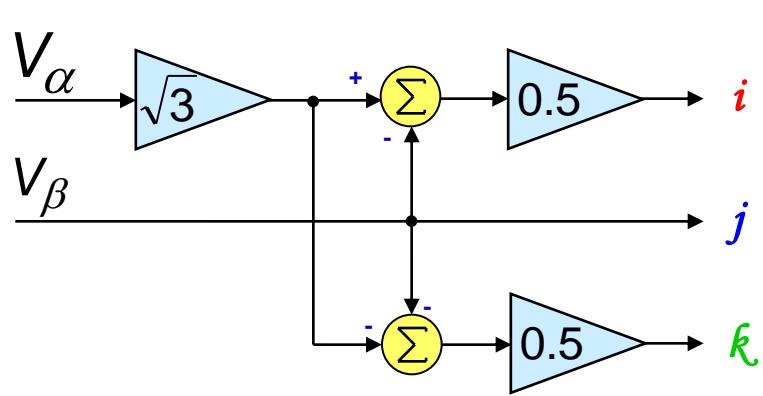


Or...

We could use a simple procedure designed specifically for RECTANGULAR notation:

1. Perform modified inverse Clarke-transform on the voltage vector
2. Calculate which sector the voltage vector is in
3. Determine T_1 , T_2 , and T_0
4. Directly calculate the PWM register values

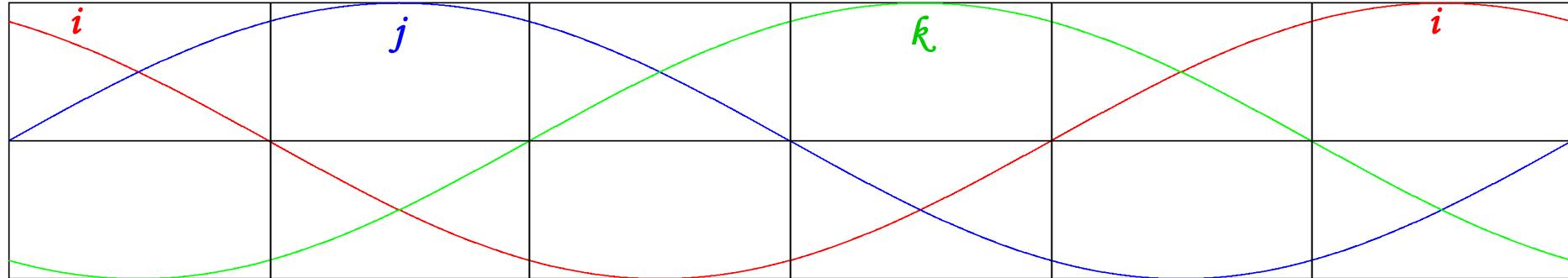
1. Perform Modified Inverse Clarke Transform



$$i = \frac{\sqrt{3}}{2} V_\alpha - \frac{V_\beta}{2}$$

$$j = V_\beta$$

$$k = -\frac{\sqrt{3}}{2} V_\alpha - \frac{V_\beta}{2}$$



2. Identify the correct sector based on i, j, and k variables

a. Calculate the following expression:

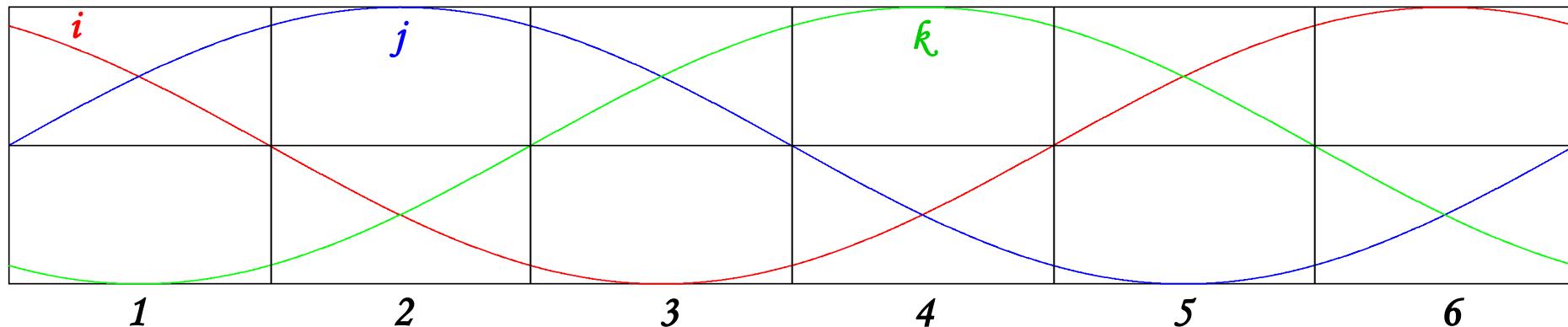
$$N = \text{sign}(i) + 2 \text{ sign}(j) + 4 \text{ sign}(k)$$

where $\text{sign}(+) = 1$; $\text{sign}(-) = 0$

b. Use look-up table below to determine the sector from the value of N

N =	1	2	3	4	5	6
Sector =	6	2	1	4	5	3

Alternatively, you can determine the sector by using IF THEN statements to check the polarities of i, j, and k.



3. Based on the sector, determine T1, T2, and T0

Sector→ Bounded by:	1 (U ₀ -U ₆₀)	2 (U ₆₀ -U ₁₂₀)	3 (U ₁₂₀ -U ₁₈₀)	4 (U ₁₈₀ -U ₂₄₀)	5 (U ₂₄₀ -U ₃₀₀)	6 (U ₃₀₀ -U ₀)
T1 [†]	<i>i</i>	- <i>k</i>	<i>j</i>	- <i>i</i>	<i>k</i>	- <i>j</i>
T2 [†]	<i>j</i>	- <i>i</i>	<i>k</i>	- <i>j</i>	<i>i</i>	- <i>k</i>

$$T0 = 1 - T1 - T2$$

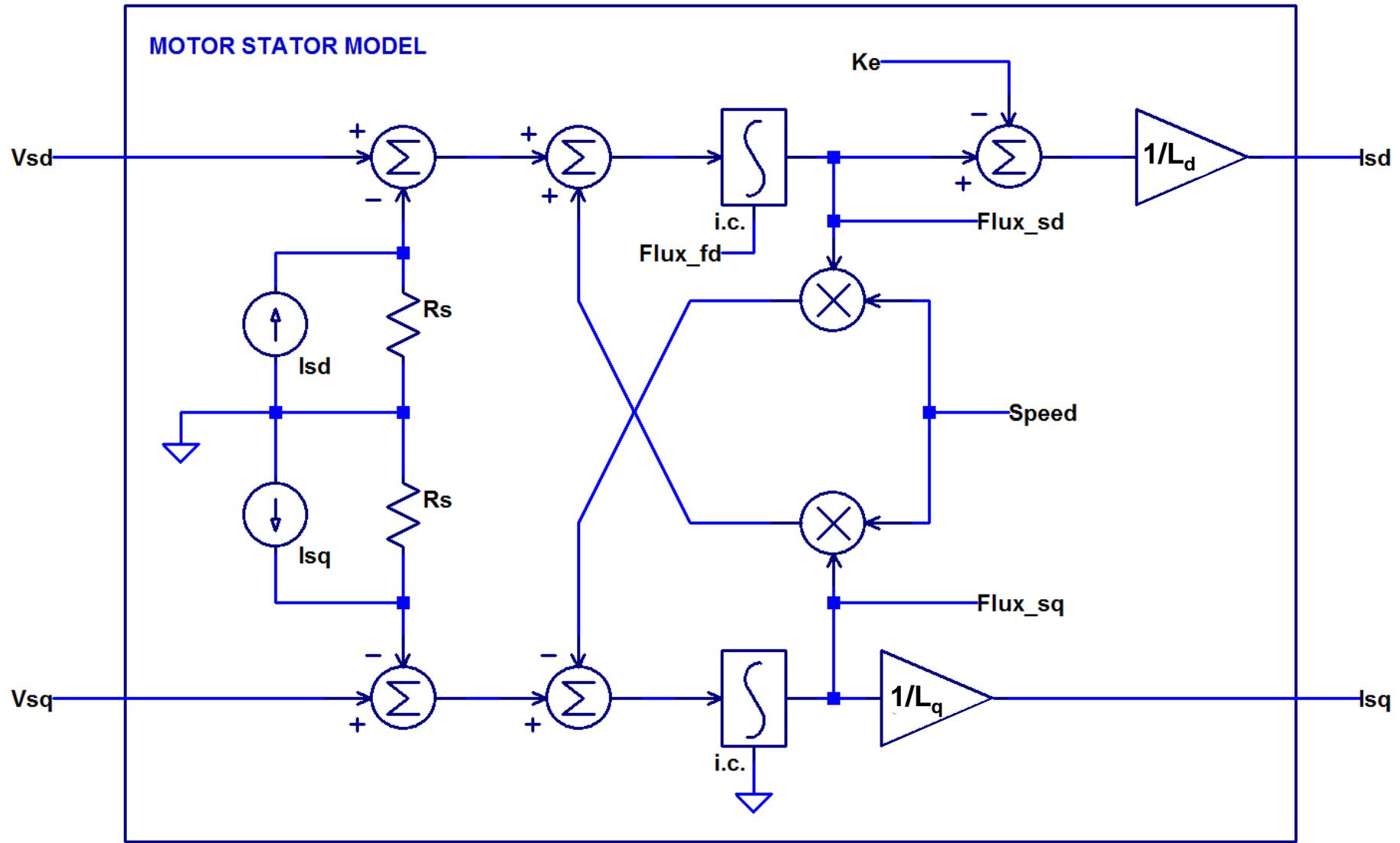
4. Load the PWM registers based on this table

	Null = V0	Null = V7	V7 in 1,3,5 V0 in 2,4,6	V0 in 1,3,5 V7 in 2,4,6	Alternating Reversing Sequencing
<u>Sector 1</u>	U = T1+T2 V = T2 W = 0	U = 100% V = T0+T2 W = T0	U = 100% V = T0+T2 W = T0	U = T1+T2 V = T2 W = 0	U = T1+T2+.5T0 V = T2+.5T0 W = .5T0
<u>Sector 2</u>	U = T1 V = T1+T2 W = 0	U = T0+T1 V = 100% W = T0	U = T1 V = T1+T2 W = 0	U = T0+T1 V = 100% W = T0	U = T1+.5T0 V = T1+T2+.5T0 W = .5T0
<u>Sector 3</u>	U = 0 V = T1+T2 W = T2	U = T0 V = 100% W = T0+T2	U = T0 V = 100% W = T0+T2	U = 0 V = T1+T2 W = T2	U = .5T0 V = T1+T2+.5T0 W = T2+.5T0
<u>Sector 4</u>	U = 0 V = T1 W = T1+T2	U = T0 V = T0+T1 W = 100%	U = 0 V = T1 W = T1+T2	U = T0 V = T0+T1 W = 100%	U = .5T0 V = T1+.5T0 W = T1+T2+.5T0
<u>Sector 5</u>	U = T2 V = 0 W = T1+T2	U = T0+T2 V = T0 W = 100%	U = T0+T2 V = T0 W = 100%	U = T2 V = 0 W = T1+T2	U = T2+.5T0 V = .5T0 W = T1+T2+.5T0
<u>Sector 6</u>	U = T1+T2 V = 0 W = T1	U = 100% V = T0 W = T0+T1	U = T1+T2 V = 0 W = T1	U = 100% V = T0 W = T0+T1	U = T1+T2+.5T0 V = .5T0 W = T1+.5T0

Note: All calculations referenced to top switch

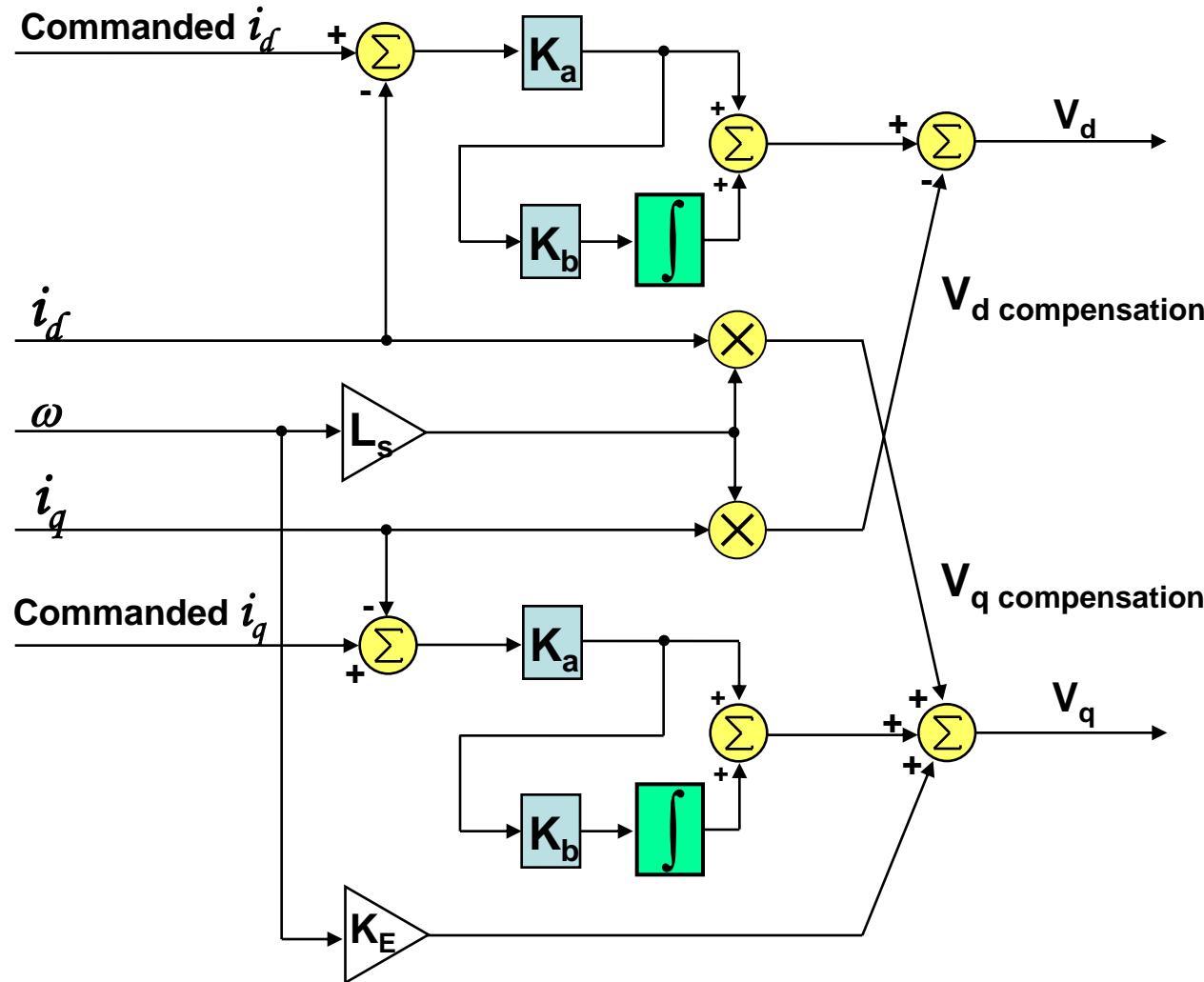
Source: *Understanding Space Vector Modulation*,
by Peter Pinewski, EDN Products Edition, March 7, 1996

Axis Coupling



Current Regulator Decoupling

Permanent Magnet Motors



Too Much Flux???

The larger the flux, the higher the $d\lambda/dt$ is for a given speed.

$d\lambda/dt$ is voltage, so the back-EMF is higher for a given speed.

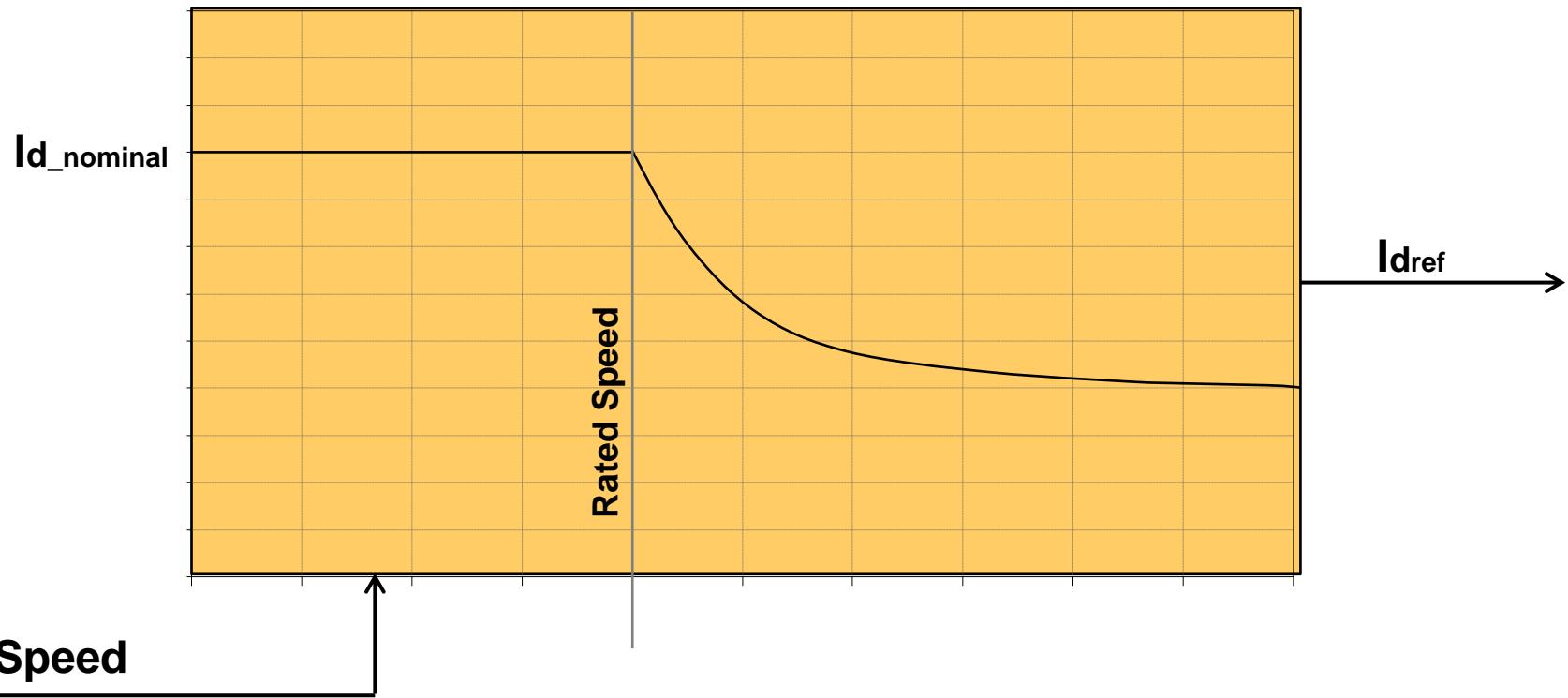
When the back-EMF amplitude reaches the bus voltage, current goes to zero.

When current goes to zero, torque goes to zero.

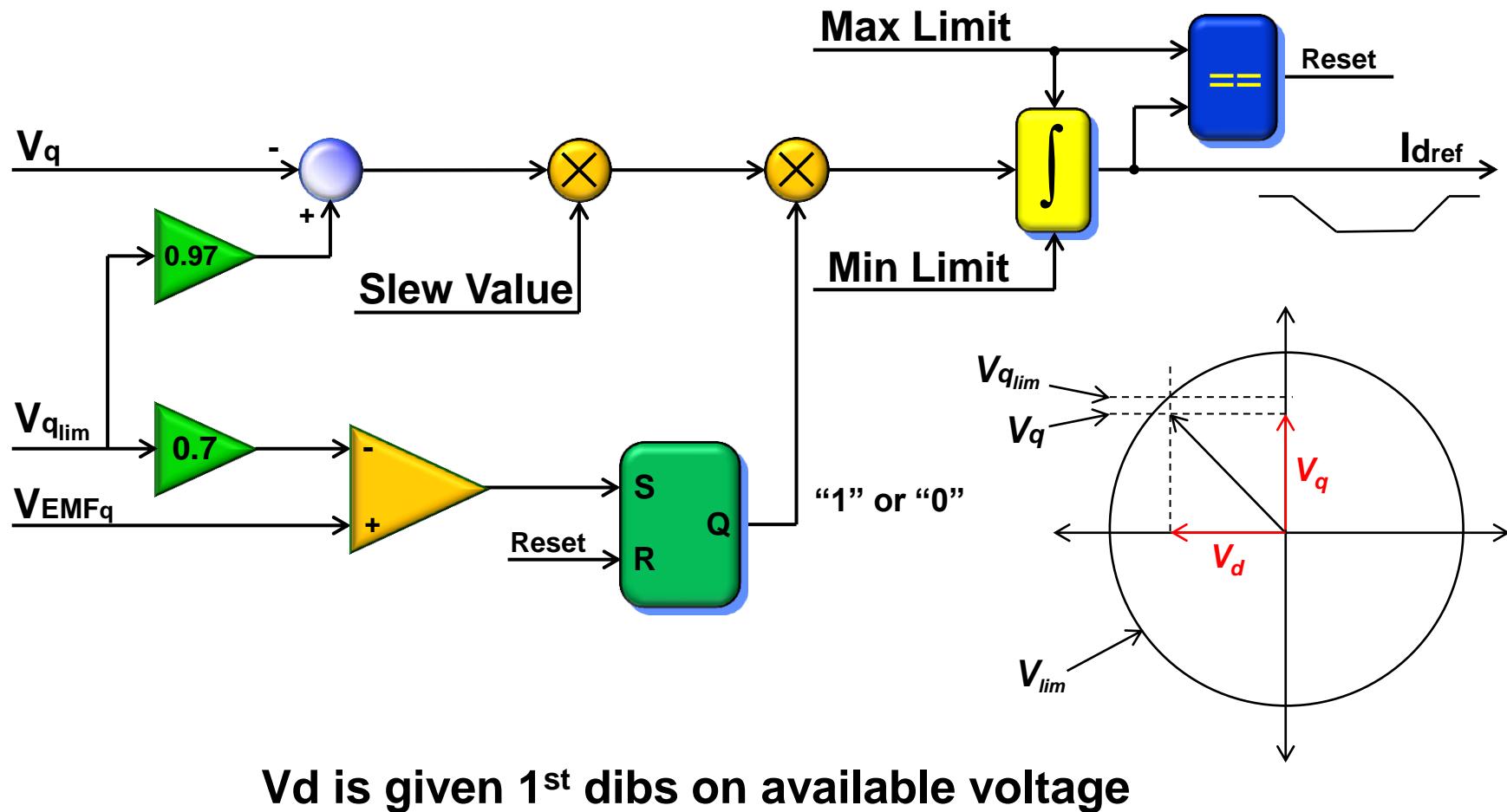
Therefore, the motor cannot go any faster...all because of too much flux.



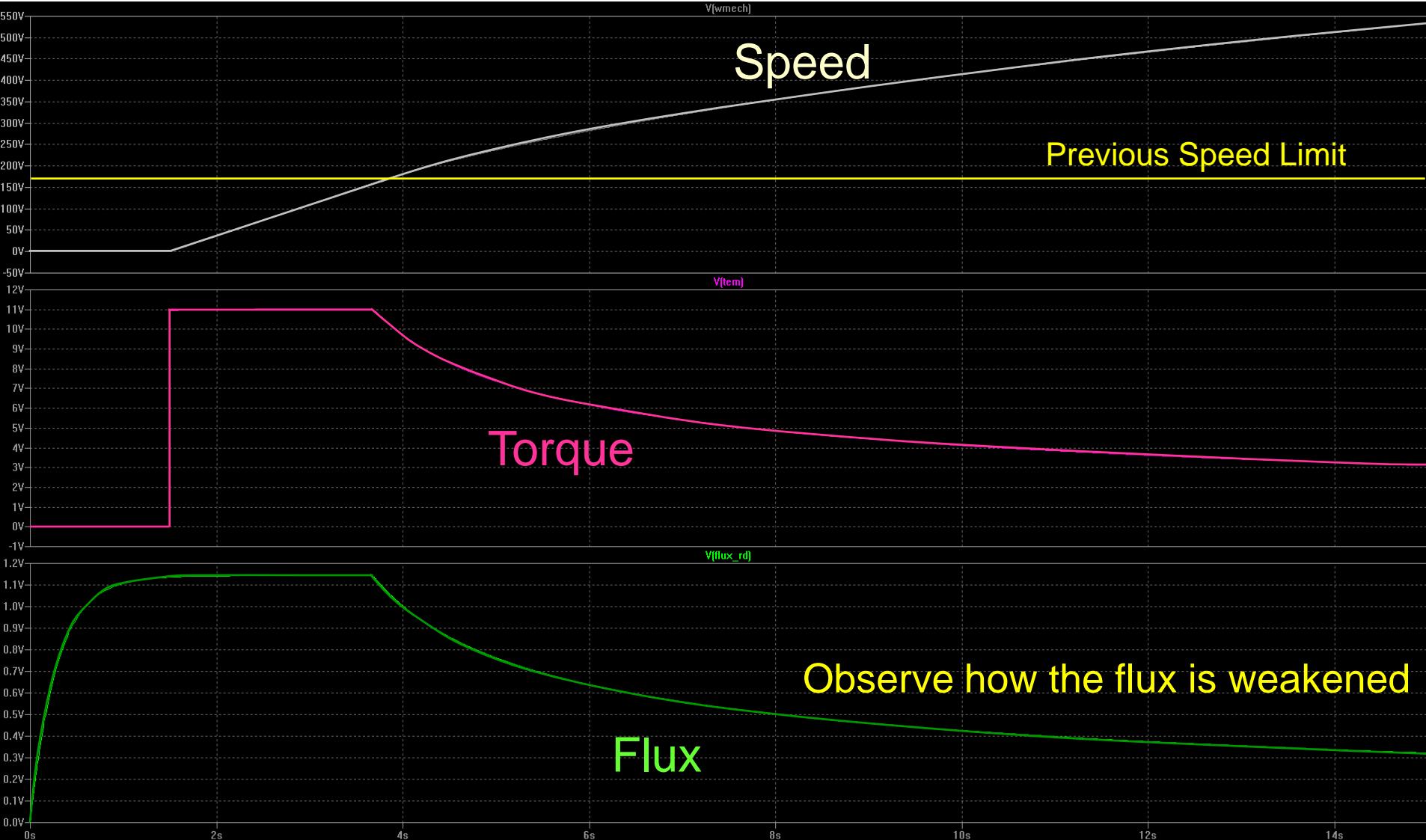
Weakening the Field...LUT



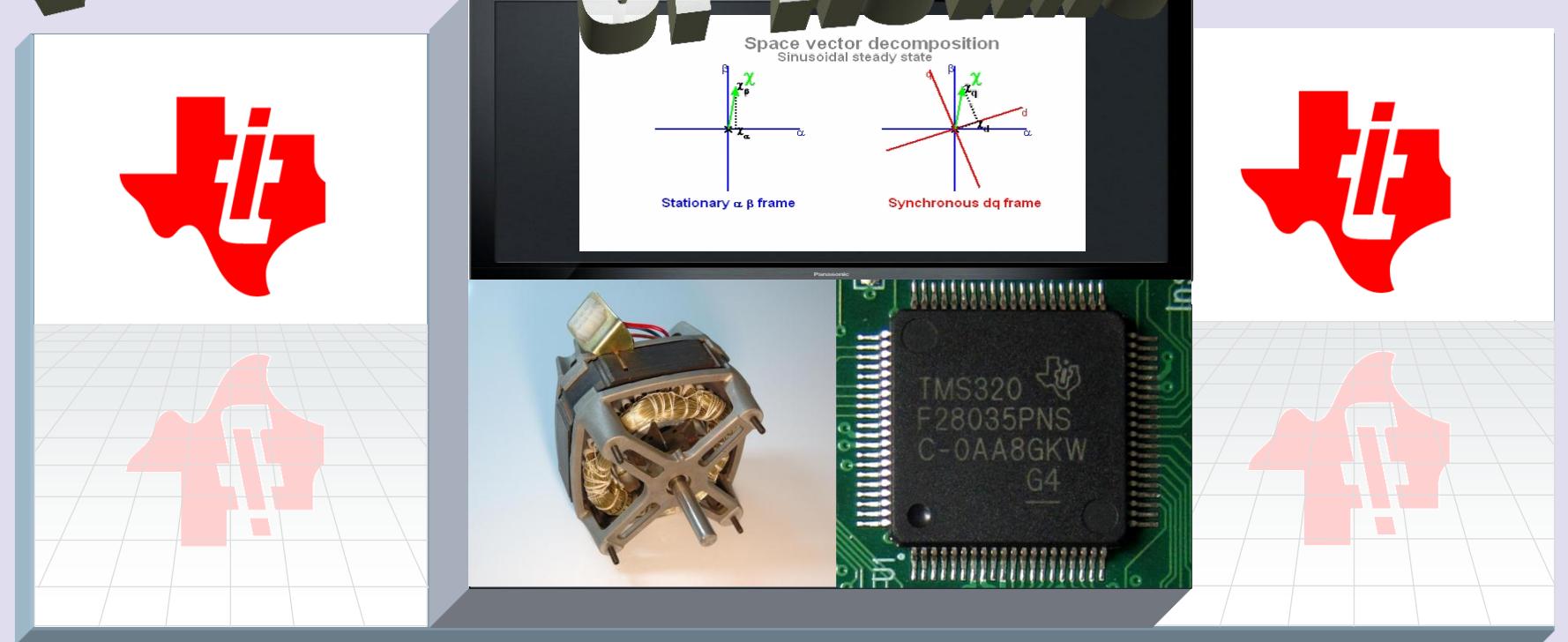
Weakening the Field...Voltage Limit



Less Flux Means More Speed

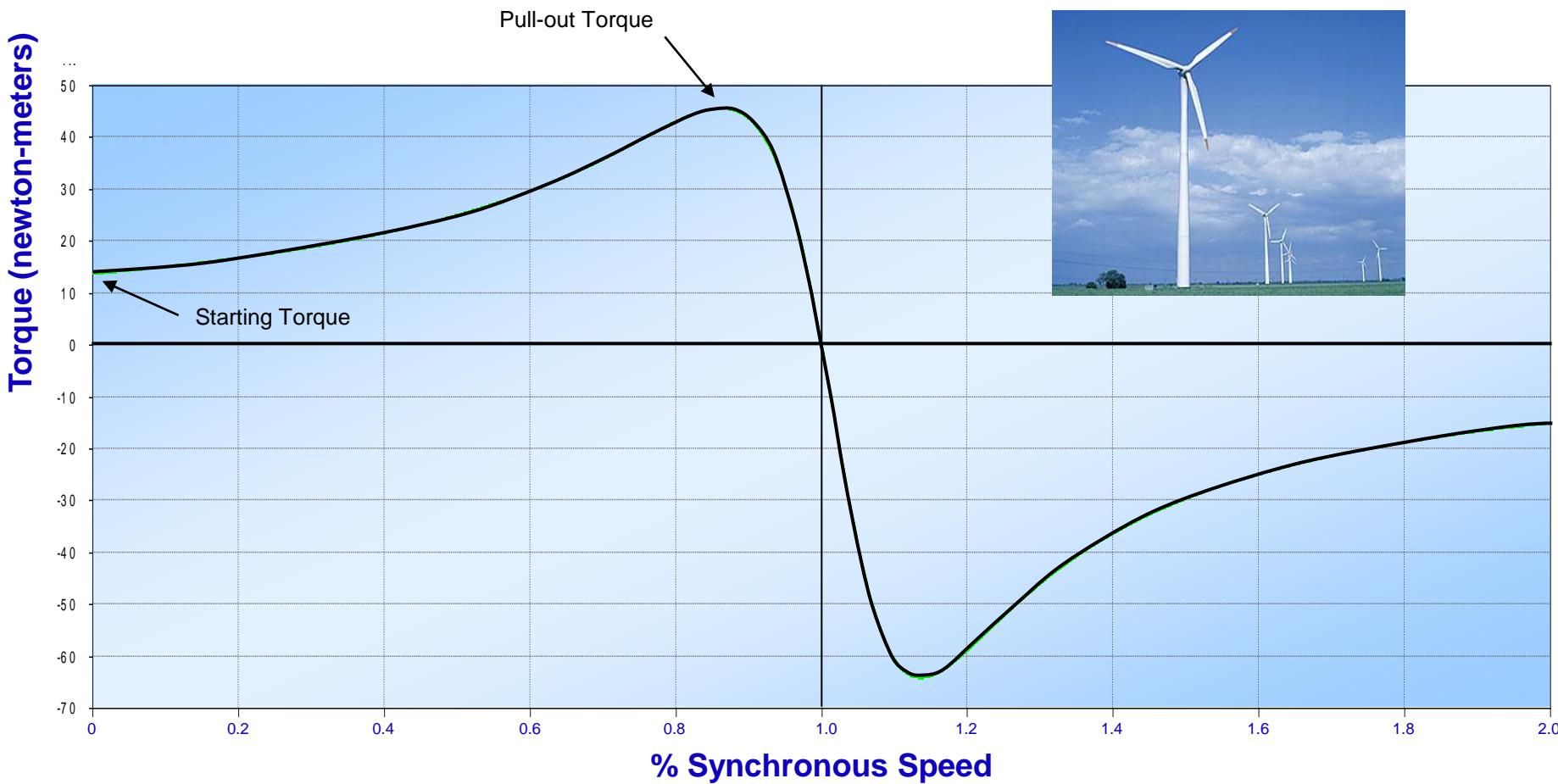


Field Oriented Control of ACIMs

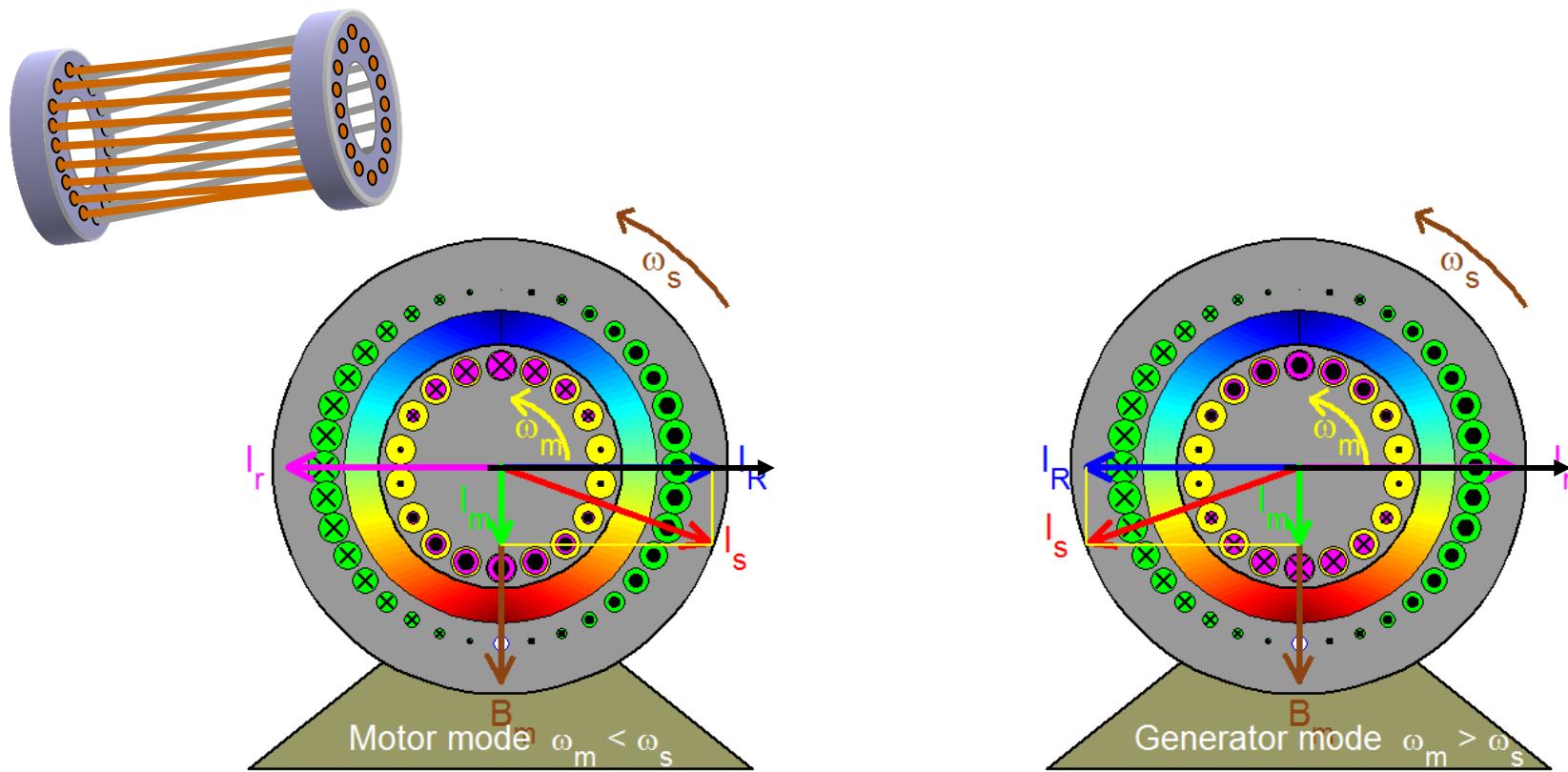


Dave Wilson

Speed-Torque Performance of Induction Motors

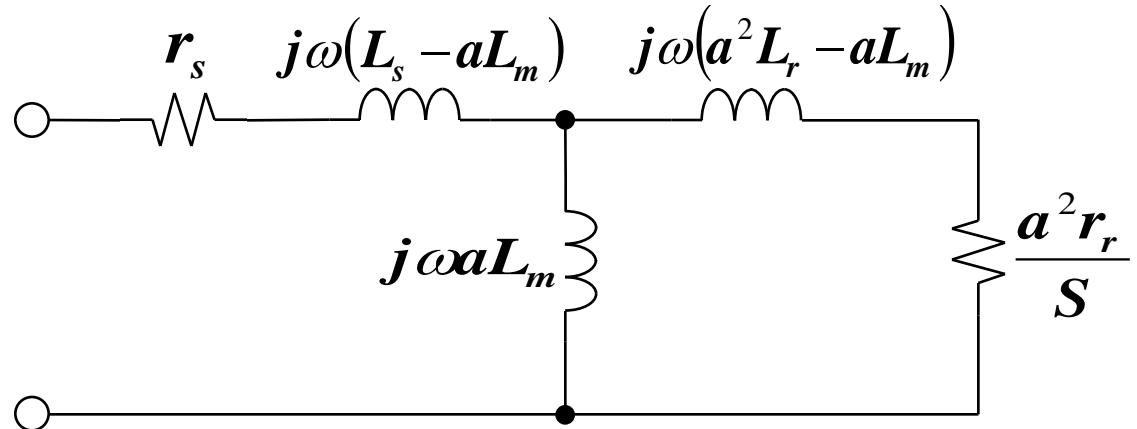
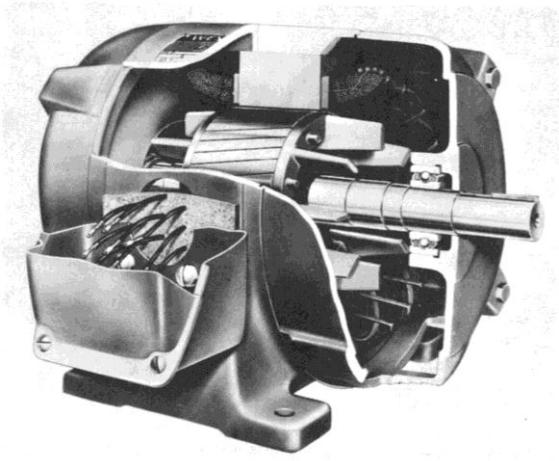


Stator and Rotor Fields



Source: Mahmoud Riaz, Sc.D., Professor of Electrical Engineering, Department of Electrical and Computer Engineering, University of Minnesota

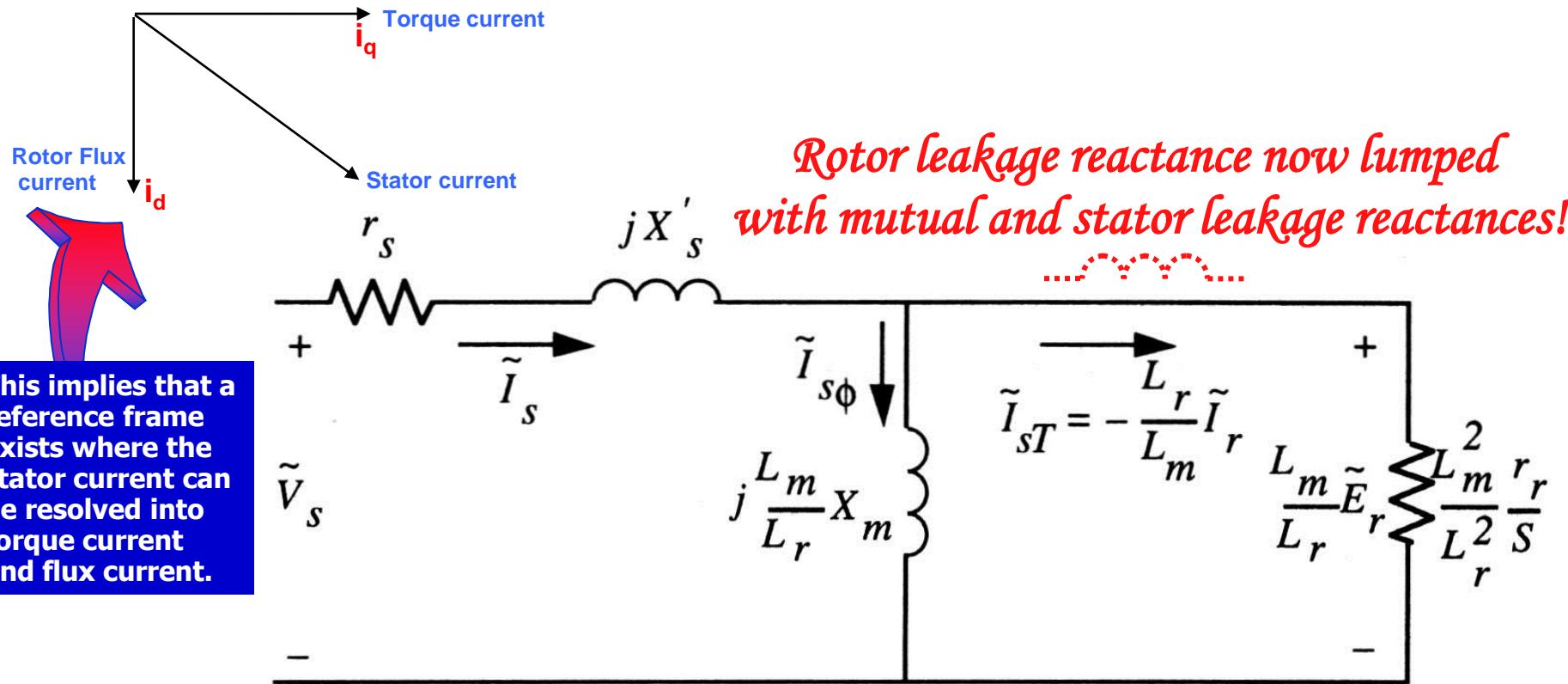
ACIM Circuit Representation with Arbitrary Turns Ratio “a”



General equivalent circuit showing arbitrary value of referral ratio “a” (a=1 corresponds to a turns ratio of N_s/N_r , which yields the conventional circuit.)

If the actual value of rotor current isn't required, “a” can be any value except zero or infinity, resulting in an infinite number of possible circuits!

ACIM Circuit Representation with Turns Ratio $a=L_m/L_r$



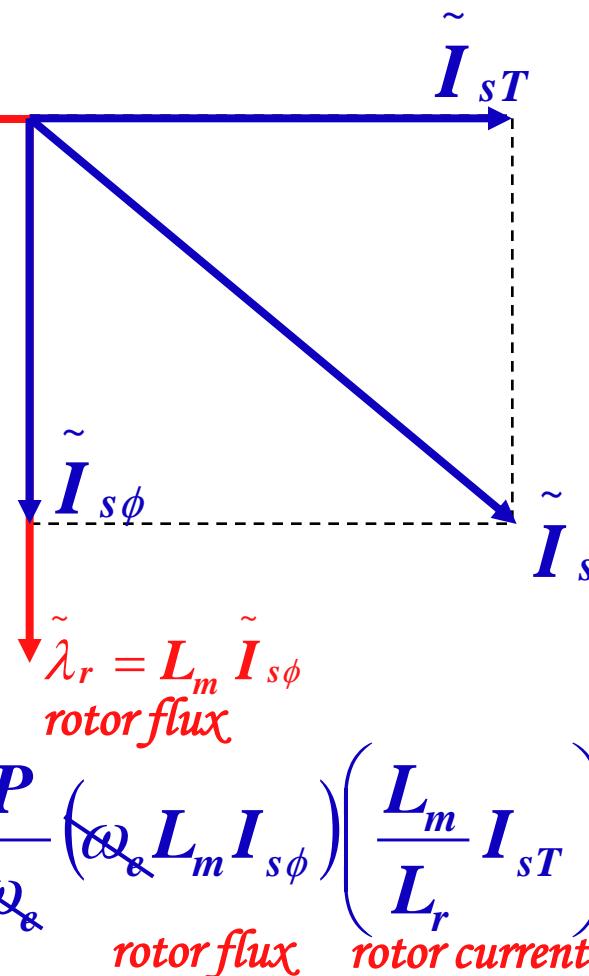
Equivalent circuit showing torque component (I_{sT}) and rotor flux component ($I_{s\phi}$) of stator current

Source: *Vector Control and Dynamics of AC Drives*, by Don Novotny and Tom Lipo, Oxford University Press, 2000

Torque Production in an ACIM

$\tilde{I}_r = -\frac{L_m}{L_r} \tilde{I}_{sT}$
rotor current

One expression for torque is the cross product between the rotor current and the rotor flux.



Stator current is resolved into flux producing and torque producing components.

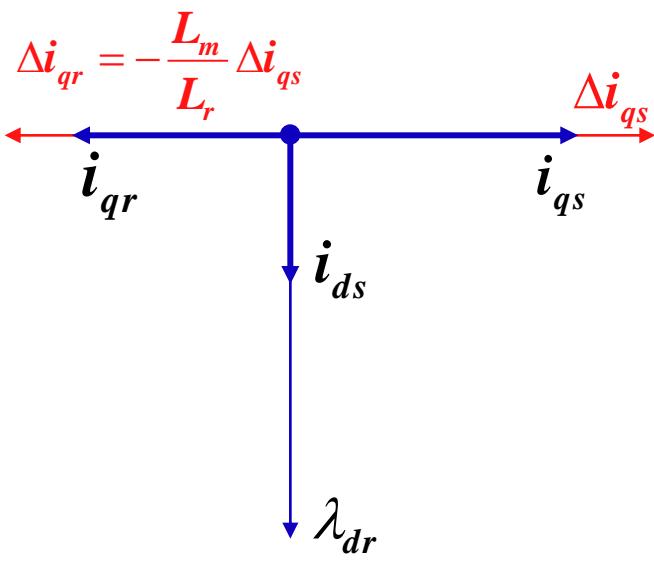
Rotor flux is NOT fixed w.r.t. rotor. It is “asynchronous” to rotor position.

$$T_e = 3 \frac{P}{2} \frac{E_r I_r}{2\omega_e} = \frac{3}{4} P \left(\cancel{\frac{1}{2}} L_m \tilde{I}_{s\phi} \right) \left(\frac{L_m}{L_r} \tilde{I}_{sT} \right) = \frac{3}{4} P \frac{L_m^2}{L_r} \tilde{I}_{s\phi} \tilde{I}_{sT}$$

P is the number of poles

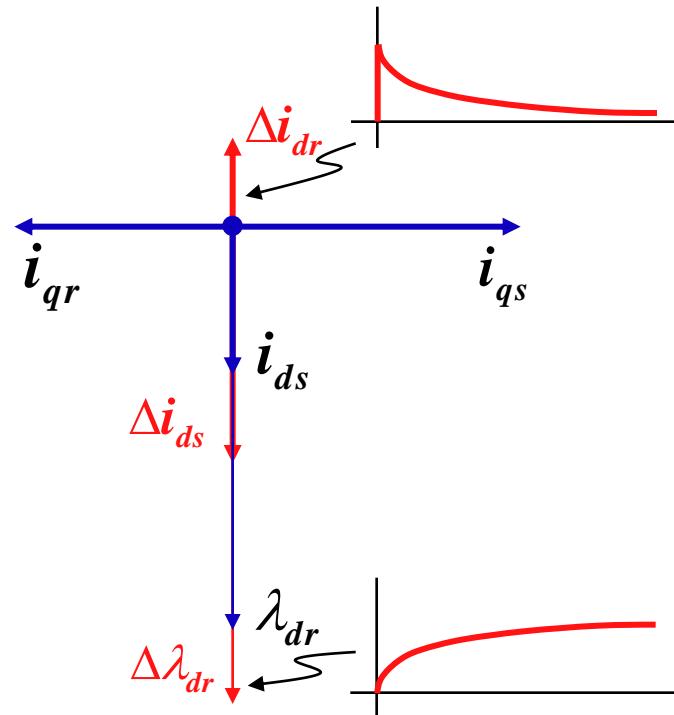
Dynamic Response of Rotor-Referenced FOC

Step change in i_{qs}



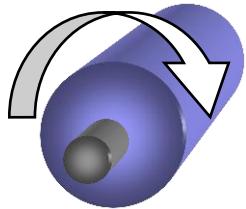
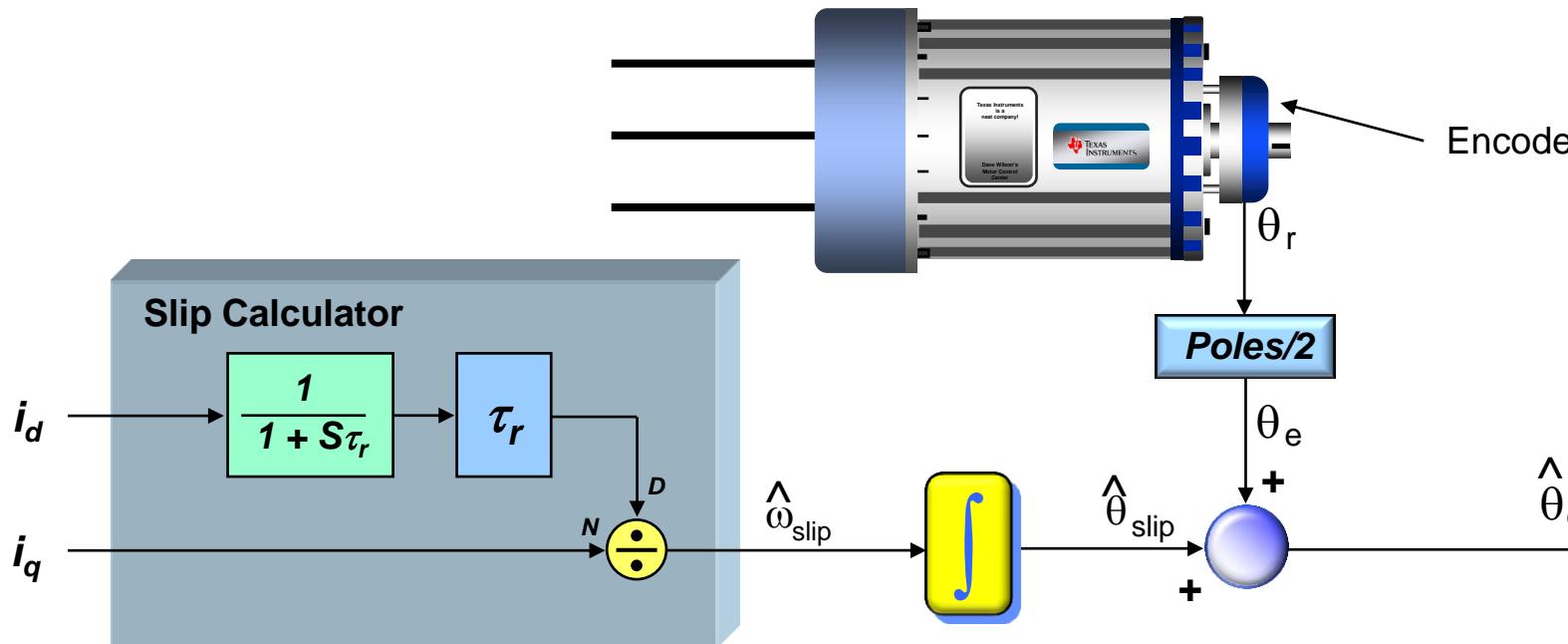
Rotor current (and thus torque) are
instantaneously changed.

Step change in i_{ds}

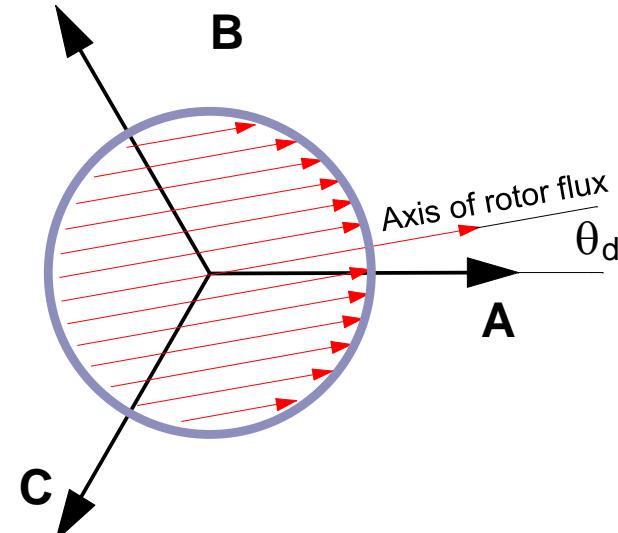


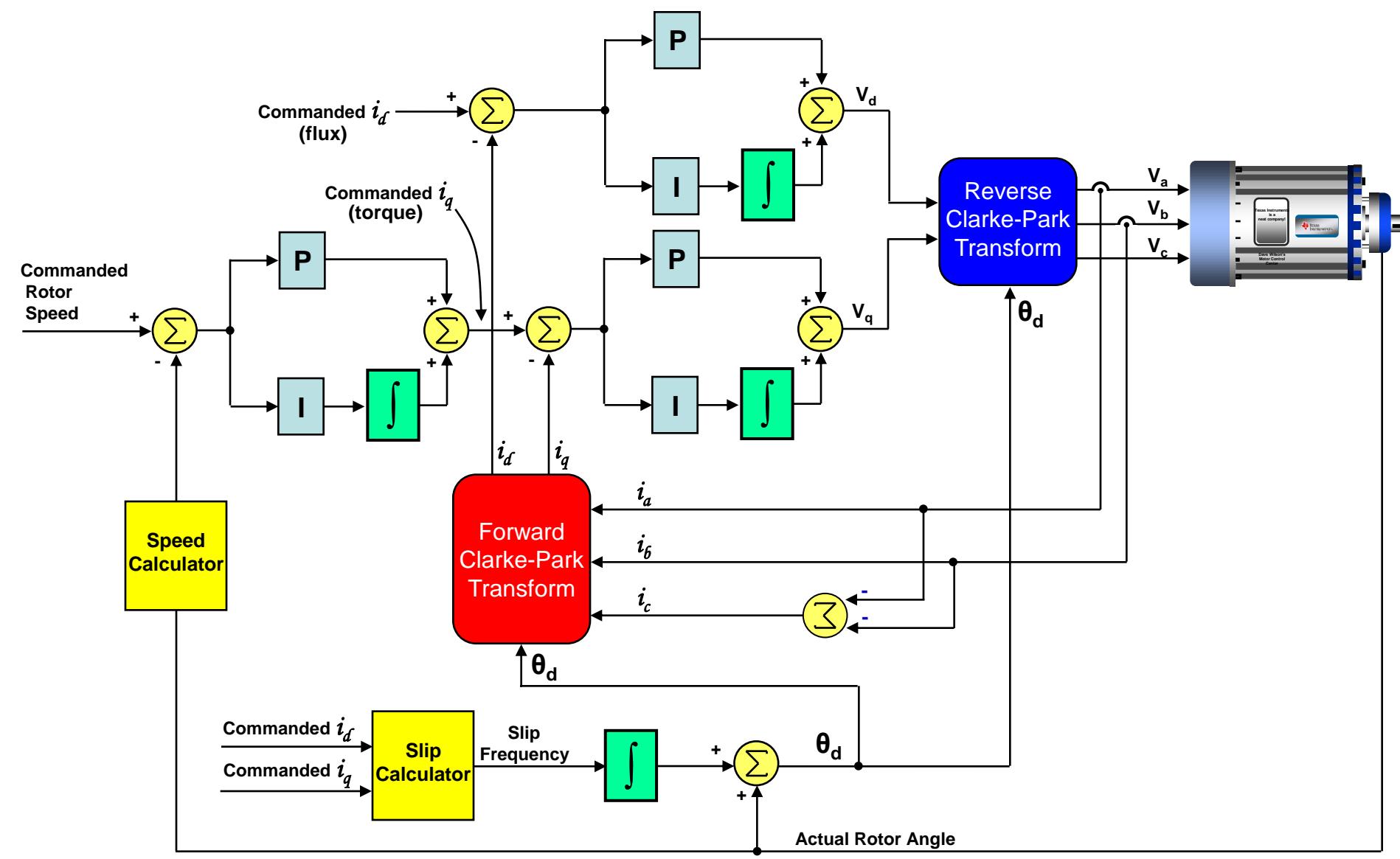
Rotor flux (and thus torque) are
gradually changed.

ACIM Slip Frequency Calculation



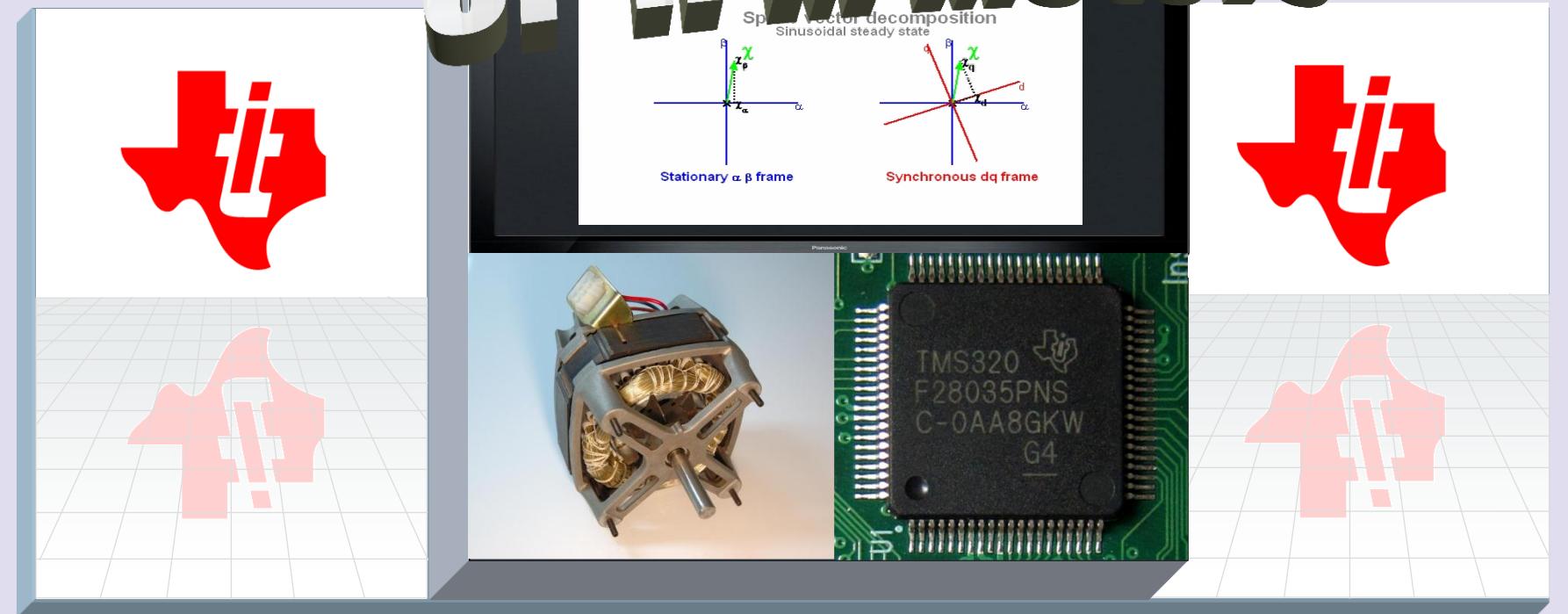
Recall that the rotor flux sweeps across the surface of the rotor at a speed equal to the slip frequency, i.e., it is “asynchronous” to the rotor.





Control Diagram of an Induction Motor Speed Controller Utilizing Field Oriented Control.

Field Oriented Control of IPM Motors

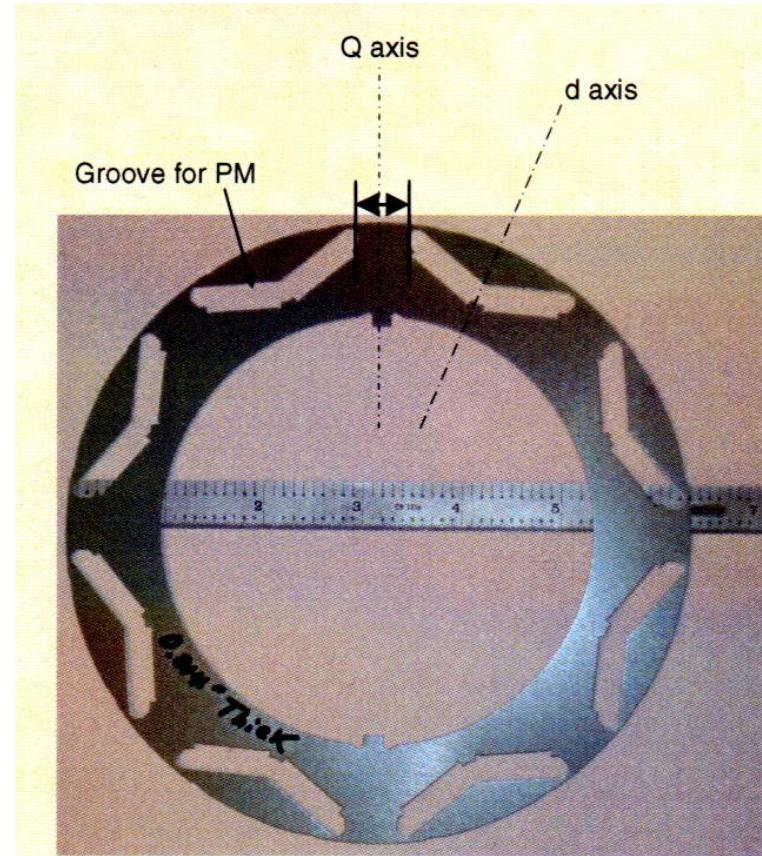


Dave Wilson

Buried Magnets Create NEW Torque

Buried rotor magnets produce different inductances on the *d-q* axes.

This results in a NEW torque component proportional to the difference in these inductances.



Prius 2004 rotor punchings

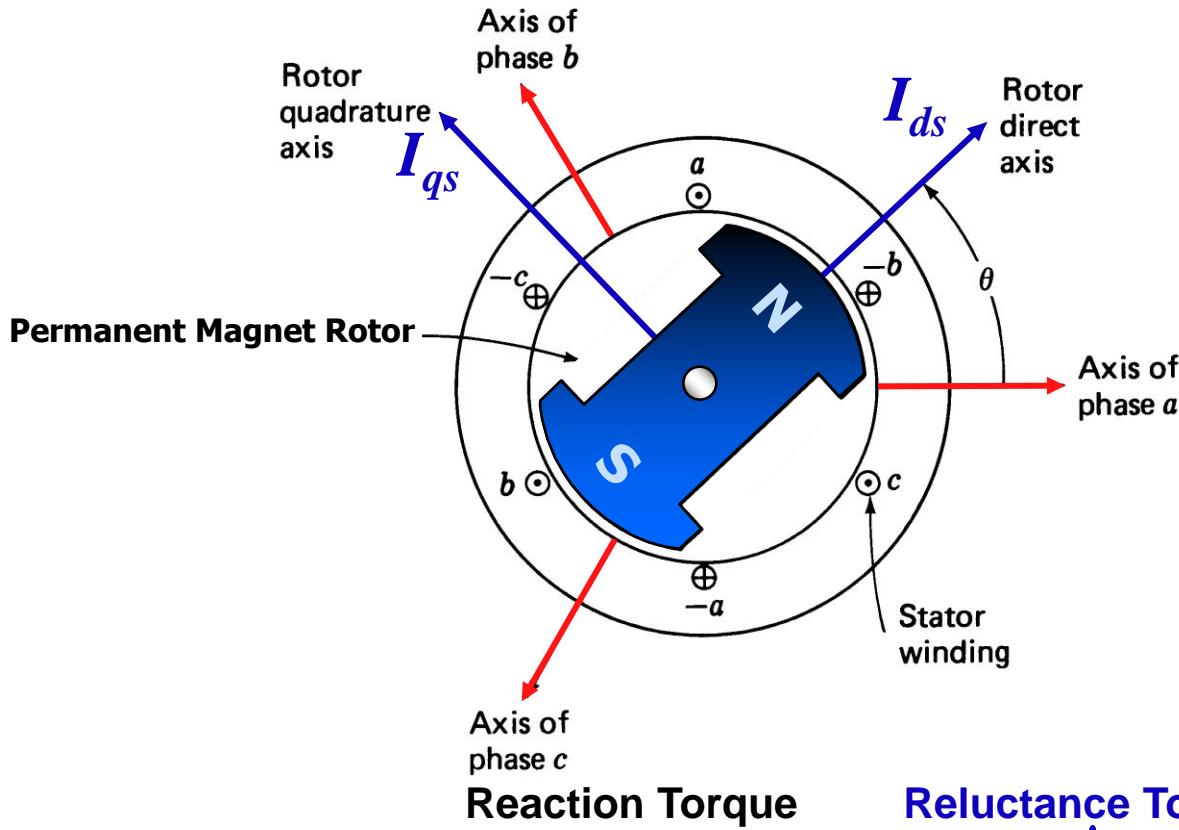
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U. S. DEPARTMENT OF ENERGY

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UT-BATTELLE

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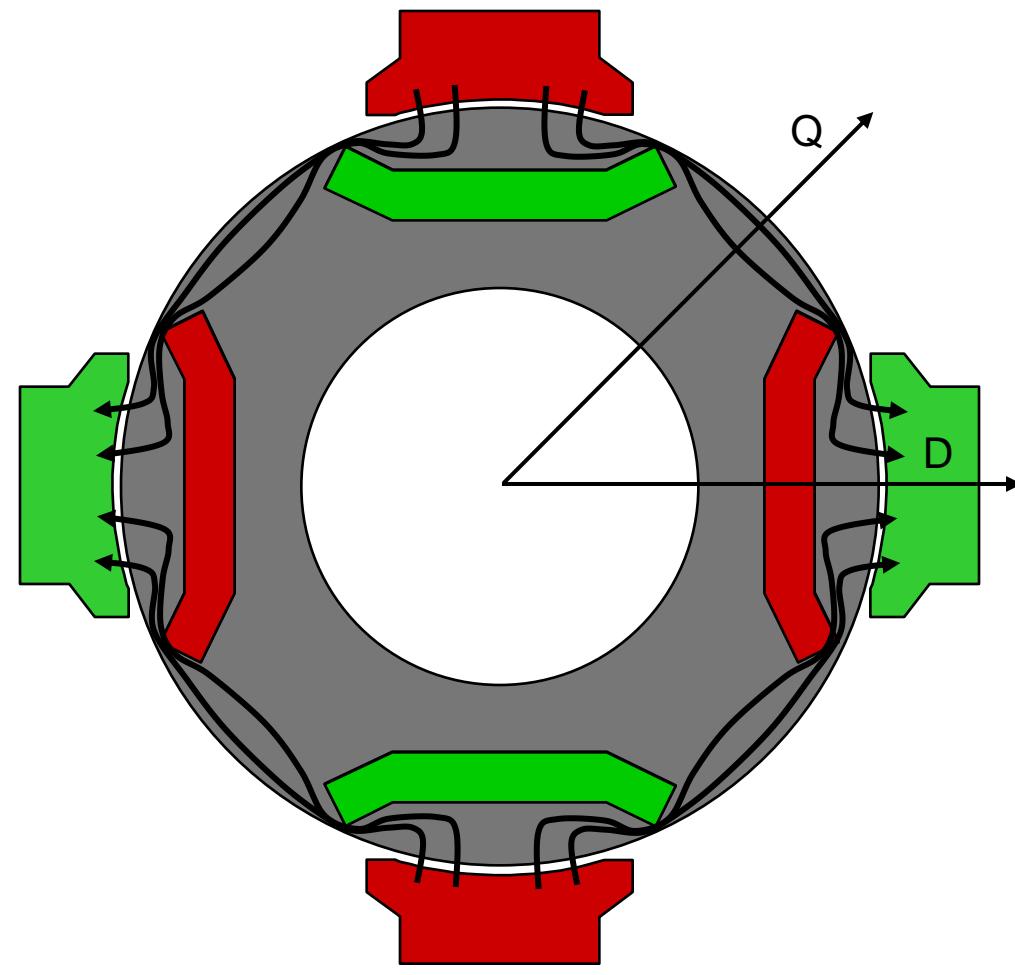
Total Motor Torque



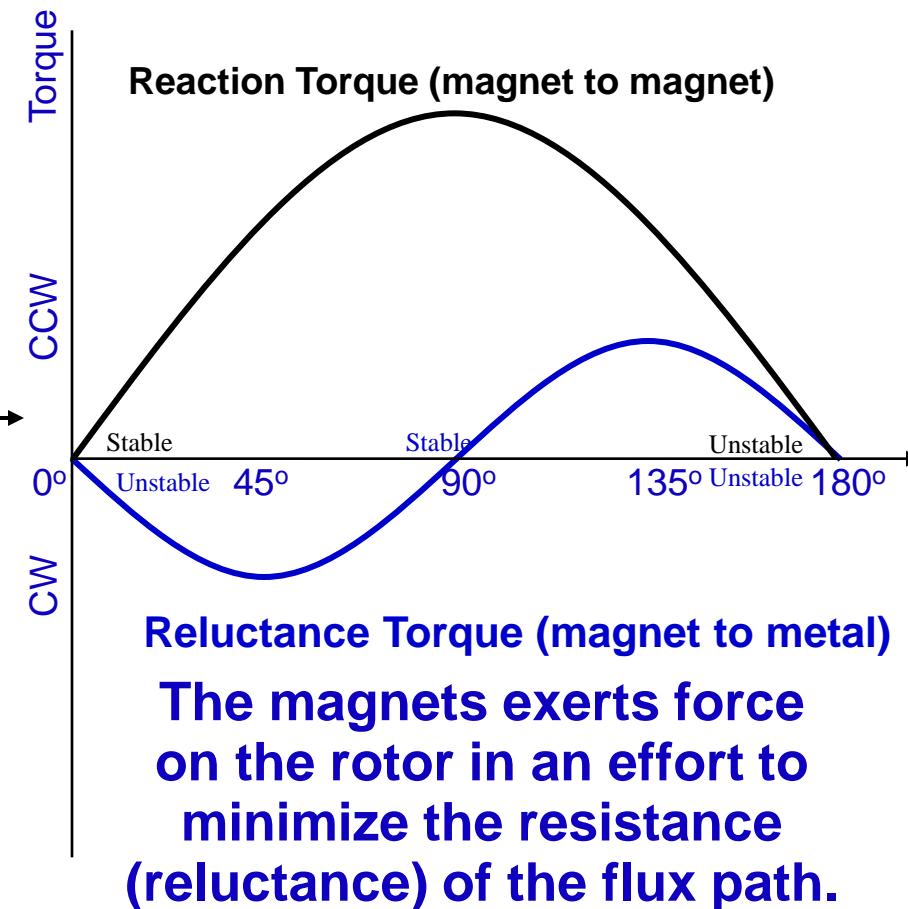
$$Torque = \frac{3}{2} \frac{P}{2} \left[\lambda_{dr} I_{qs} + (L_{ds} - L_{qs}) I_{ds} I_{qs} \right]^+$$

[†] *Torque expression based on amplitude invariant form of Clarke Transform.*

Torque vs. Angle

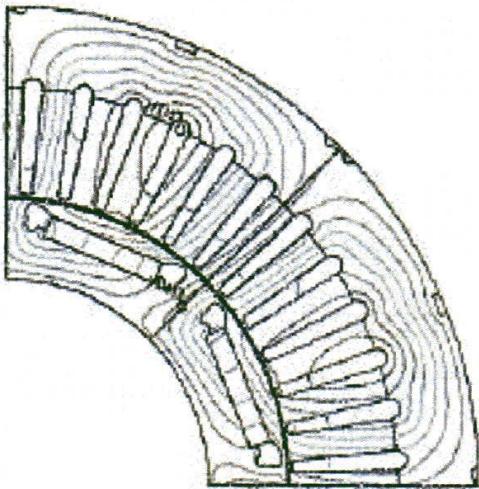


Locked Rotor

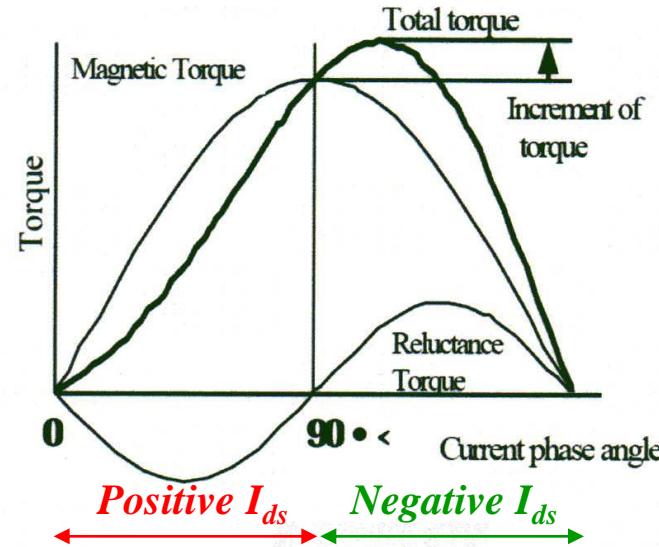


Effect of Saliency on Optimum Torque Angle

Additional Reluctance Torque of Toyota/Prius Hybrid THS II Motor



Torque Simulation by Electromagnetic Analysis



Torque-Current Phase Angle Characteristic

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U. S. DEPARTMENT OF ENERGY

Ref: Development of Electric Motors for the TOYOTA Hybrid Vehicle "PRIUS"
Kazuaki Shingo, Kaoru Kubo, Toshiaki Katsu, Yuji Hata
TOYOTA MOTOR CORPORATION



MTPA Control of IPM Motors

