Teaching Your PI Controller to Behave (Part IV)

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At the end of my last blog, we discussed the possibility of creating a single parameter that could automatically tune the PI coefficients for a velocity loop used in a motor speed control system. To develop such a parameter, let's review the open-loop transfer function for the entire velocity loop:

$$GH(s) = \frac{K\,K_c K_d\left(1+\frac{s}{K_d}\right)}{s^2\left(1+\frac{L}{K_a}s\right)(1+\tau s)} \qquad \text{Equ. 1}$$

where K is a coefficient that contains several terms related to the motor and load

Kc and Kd are the PI coefficients for the velocity loop

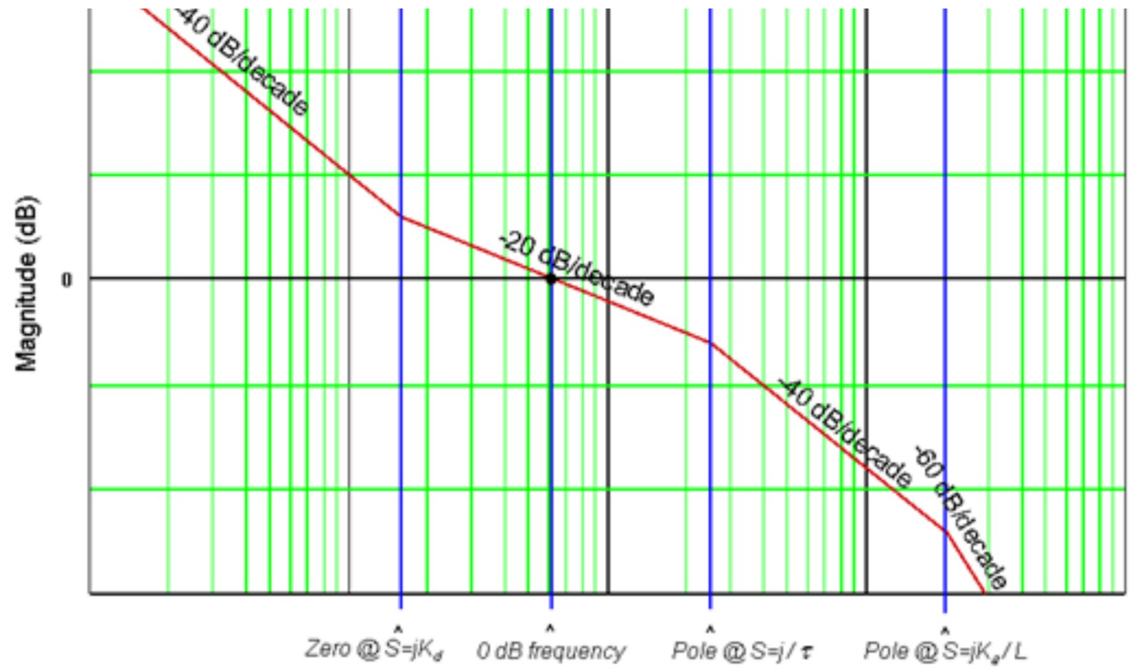
L is the motor inductance

Ka is one of the PI coefficients for the current loop

τ is the time constant of the velocity feedback filter

s is the Laplace frequency variable

Assuming that the zero dB frequency occurs somewhere between the zero at s = Kd and the two nonzero poles in the denominator of the expression, we should end up with a Bode plot that looks something like this:



The reason the shape of this curve is so important is because the phase shift at the 0 dB frequency determines the stability of the system. In general, in order to get a phase shift at 0 dB that leads to good stability, the magnitude response should cross 0 dB at a rate no steeper than -20 dB per decade.

As you can see from equation 1, we must solve for three unknowns (Ka, Kc, and Kd) which are affected by multiple system parameters. But instead of wading through pages and pages of esoteric equations, it's time to make another simplification. Let's assume that there is only *one* pole higher than the zero-dB frequency instead of two. This assumption could mean that you don't have a velocity filter in your system, OR that the velocity filter's pole is *way* higher than the current controller's pole, OR that the current controller's pole is *way* higher than the velocity filter's pole. For most systems, it is plausible to assume the latter scenario. So if we eliminate the effect of the current controller pole, we can rewrite the velocity open-loop transfer function as shown below:

$$GH(s) = \frac{K\,K_c K_d\left(1 + \frac{s}{K_d}\right)}{s^2\left(1 + \tau\,s\right)}$$

Equ. 2

For now, let's assume that the delta in frequency between the pole $1/\tau$ and the zero Kd is fixed. In order to achieve maximum phase margin (phase shift + 180°), the unity gain frequency should occur exactly half way in-between these two frequencies on a logarithmic scale. Translating from dB to a normal gain scale, this means the following is true:

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$$\omega_{unity\_gain} = \delta \cdot K_d$$ Equ. 3

and,

$$\frac{1}{\tau} = \delta \cdot \omega_{unity\_gain}$$ Equ. 4

Combining Equ. 3 and Equ.4 we can establish that:

$$\frac{1}{\tau} = \delta^2 \cdot K_d$$ Equ. 5

Solving for Kd,

$$K_d = \frac{1}{\delta^2 \tau}$$ Equ. 6

Where "δ"we will define as the "damping factor." If δ is increased, it forces the zero corner frequency (Kd) and the velocity filter pole ($1/\tau$) to be further apart. And the further apart they are, the phase margin is allowed to peak to a higher value in-between these frequencies. This improves stability but unfortunately reduces system bandwidth. If $\delta = 1$, then the zero corner frequency and the velocity filter pole are right on top of each other, resulting in pole/zero cancellation. In this case the system will be unstable. Theoretically, any value of $\delta > 1$ is stable since phase margin > 0. However, values of δ close to 1 are usually not practical as they result in severely underdamped performance.

I will talk more about δ later. But for now, let's turn our attention towards finding Kc. From Equ. 3 we see that the open-loop transfer function of the speed loop will be unity gain at a frequency equal to the zero frequency (Kd) multiplied by δ. In other words,

$$\left.\left|\frac{K\,K_c K_d\left(1+\dfrac{s}{K_d}\right)}{s^2\left(1+\dfrac{s}{\delta^2 K_d}\right)}\right|\right|_{s=j\delta K_d} = 1$$

Equ. 7

By performing the indicated substitution for “s” in Equ. 7, we obtain:

$$\left|\frac{K\,K_c K_d(1+j\delta)}{-\delta^2 K_d{}^2\left(1+\dfrac{j}{\delta}\right)}\right| = 1$$

Equ. 8

We can see that the expression within the magnitude brackets is a scalar term multiplied by a vector term. So we can pull the absolute value of the scalar term out of the brackets, resulting in the following expression:

$$\frac{K\,K_c K_d}{\delta^2 K_d{}^2}\left|\frac{(1+j\delta)}{\left(1+\dfrac{j}{\delta}\right)}\right| = 1$$

Equ. 9

It can be shown that the magnitude of the vector inside of the magnitude brackets is simply equal to δ. Performing this substitution and simplifying leads to the following equality:

$$\frac{K\,K_c}{\delta\,K_d} = 1$$

Equ. 10

Finally, we can solve for Kc:

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$$K_c = \frac{\delta K_d}{K} = \frac{1}{\delta K \tau}$$

Equ. 11

At this point, let's step back and try to see the forest for the trees. We have just designed a cascaded velocity controller for a motor which contains two separate PI controllers: one for the inner current loop and one for the outer velocity loop. In order to get pole/zero cancellation in the current loop, we chose Kb as follows:

$$K_b = \frac{R}{L}$$

Equ. 12

Next, we select a value for the damping factor (δ) which allows us to precisely quantify the tradeoff between velocity loop stability and bandwidth. Then it's a simple matter to calculate Kd and Kc:

$$K_d = \frac{1}{\delta^2 \tau}$$

Equ. 6

$$K_c = \frac{\delta K_d}{K} = \frac{1}{\delta K \tau}$$

Equ. 11

All that remains is the selection of Ka (the current controller bandwidth), which I will address in a later blog. The benefit of this design approach is that instead of trying to empirically tune four PI coefficients which have seemingly little correlation to system performance, all you need to do is select what damping factor you want for the velocity loop.

In my next blog, let's take a harder look at the damping factor and how it affects the performance of the velocity loop. Until then…

Keep Those Motors Spinning,

Dave

Joseph Lee90700 *over 10 years ago*

Your explanation is almost perfect. I enjoyed your prominent technical guide.

Dave (Wisconsin) Wilson *over 10 years ago*

Thanks Joseph. I consider this particular blog to be the heart of the entire series, and perhaps the most difficult one to understand due to the math associated with this technique. The fact that you followed it so well is a testimony to your technical prowess.

Best wishes for success of your own motor control projects!

-Dave

BING NIE *over 10 years ago*

detailed & accurate explanation

High Hopes *over 10 years ago*

i liked your explanation of shape of the curve and the slopes to determine system stability, in particularly "he magnitude response should cross 0 dB at a rate no steeper than -20 dB per decade." that tid-bit was new to me.

Dave (Wisconsin) Wilson *over 10 years ago*

Hello High Hopes,

Ha! I suspect the reason this may be new to you is because it's a very old technique. With all of the new-fangled computer analysis techniques available today, I doubt if they really emphasize the Bode Plot stability analysis technique anymore in control systems courses.

Bode Plots were developed as a graphical way to estimate system stability using semilog graph paper. My old file cabinets are full of them! It assumes that you are dealing with real poles and zeros that are reasonably well distributed along the damping axis (ie, no torsional resonances or other effects which would cause complex poles). Although it is the PHASE plot that really determines stability at the 0-dB frequency, there is a correlation between the phase plot and the gain plot. Whenever you encounter a pole, it causes a phase shift of -45 degrees/decade starting one decade before the pole frequency and ending one decade above the pole frequency. It also causes a -20dB/decade rolloff on the gain plot which starts at the pole frequency and continues for all higher frequencies. A "zero" has the opposite effect of creating a phase shift of PLUS 45 degrees/decade, and a 20dB/decade INCREASE on the gain plot.

If your gain plot crosses 0dB at -20dB/decade, it means that the number of lower frequency poles minus the number of lower frequency zeros equals 1. Translating this to the phase plot (assuming no other poles or zeros within +/- 1 decade), it means the phase shift will be -90 degrees (stable).

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However, if the gain plot is falling at -40 dB/decade under the same assumptions, it means the phase shift will be -180 degrees (unstable). Of course, if there are other poles or zeros within a +/-1 decade vicinity of 0dB, you have to hedge your guess of the true phase shift at 0dB based on how close they are. Older control systems engineers were really good at doing this. But with the advent of the modern digital computer, most engineers today just run an actual frequency plot to determine the exact gain and phase plots.

I'm glad to see you are progressing through my series! Please let me know if you have any other questions. :-)

-Dave

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