**UNIVERSITY OF LAY ADVENTISTS OF KIGALI**

**FACULTY: COMPUTING AND INFORMATION SCIENCE**

**DEPARTMENT: IT**

**MASTER'S OF SCIENCE IN INFORMATION TECHNOLOGY (MIT)**

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**ASSIGNMENT OF GRADUATE MATHEMATICS FOR IT (MSIT6111)**

**Question 1**

**a)** By the bijection concept, where there is a one-to-one pairing between subsets with **a** and without **a,** they are equal in number

Let A= {a, b, c}

Subset S = {∅, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}

Subset S with a = {{a}, {a, b}, {a, c}, {a, b, c}}

Subset S without a (S\{a}) = {{∅}, {b}, {c}, {b, c}}

By Bijection, let Function **f** be from Subset P → Subset Q

$$F(S) = \boldsymbol{S} \cup \{\boldsymbol{a}\}$$

If you take a subset not containing **a** and add **a,** you get a subset that contains **a**

∅ → {a}

{b} → {a, b}

{c} → {a, c}

{b, c} → {a, b, c}

**b)** Prove $\sqrt{5}$is irrational by contradiction.

Assume $\sqrt{5} = \frac{p}{q}$ where $p, q$ are integers, $q \neq 0$, and fraction in lowest terms (gcd$(p, q) = 1$).

Square: $5 = \frac{p^2}{q^2}$,

So, $p^2 = 5q^2$.

5 divides $p^2$, so 5 divides $p$(since 5 prime).

Let $p = 5k$, where $k$ is an integer.

Then $(5k)^2 = 5q^2$

$25k^2 = 5q^2$

$q^2 = 5k^2$

5 divides $q^2$, so 5 divides $q$.

But then 5 divides both $p$ and $q$, contradicting the lowest terms.

Thus, $\sqrt{5}$ is irrational.

**QUESTION 2**

**a)** Relation on $\mathbb{Z}$: $m \sim n$ Iff $m - n$ divisible by 9 (same remainder mod 9).

Let a, b ∈ $\mathbb{Z}$ , aRb: a−b = 9 × $k$, k ∈ $\mathbb{Z}$

R= {(a, b) ∈ $\mathbb{Z} \times \mathbb{Z}$ : a−b = 9 × $k$, k ∈ $\mathbb{Z}$}

- **Reflexive**: $m - m = 0$, so it is divisible by 9.

i.e Let a ∈ $\mathbb{Z}$

a−a = 0, 0 = 9 × 0, 0 ∈ $\mathbb{Z}$, a−a = 0= 9× 0, 0 ∈ $\mathbb{Z}$ → $aRa$

Conclude it is Reflexive **aRa;** Since every integer is related to it itself

- **Symmetric**: If $m - n$ divisible by 9, then $n - m = -(m - n)$ divisible by 9.

Let a,b ∈ $\mathbb{Z}$ and assume aRb; a−b = 9 × k, k ∈ $\mathbb{Z}$, b−a = − (a−b), b−a = − (9 × k),

b−a = 9 × $(-k)$, −k ∈ $\mathbb{Z}$, b−a = 9 × $(-k)$ → $bRa$

To Conclude, it is Symmetric **bRa;** Because **a** is related to **b**, then **b** is related to **a**

- **Transitive**: If $m - n, n - p$divisible by 9, then $m - p = (m - n) + (n - p)$ divisible by 9.

Let a, b, c ∈ ℤ and assume aRb and bRc, aRb: a−b = 9 × k, k ∈ ℤ, bRc: b−$c$ = 9 × $m$, m ∈ ℤ

A−$c$ = (a−$b) + (b - c)$
A−$c$ = 9 × k + 9 × $m$
a−$c$ = 9 × $(k + m)$,
$k + m \in \mathbb{Z}$ → aRc

Conclude transitive **aRc**, because **a** is related to **b** and **b** is related to **c**, then **a** is related **c**

R is reflexive, symmetric and transitive hence R is an equivalent relation on ℤ

**b)** The relation R on ℤ is defined by: aRb: a−$b$ = 9 × $k$, k ∈ ℤ

b ∈ [a] ↔ a−$b$ = 9 × $k$, k ∈ ℤ

b = a +9 × $k$, k ∈ ℤ

Possible remainders when an integer is divided by 9 are: 0,1,2,3,4,5,6,7,8

[a] = {a +9$k$ : k ∈ ℤ}

Write each equivalence class

Using the formula a+9k:

[0] = {…, −18, −9, 0, 9, 18, …}

[1] = {…, −17, −8, 1, 10, 19, …}

[2] = {…, −16, −7, 2, 11, 20, …}

[3] = {…, −15, −6, 3, 12, 21, …}

[4] = {…, −14, −5, 4, 13, 22, …}

[5] = {…, −13, −4, 5, 14, 23, …}

[6] = {…, −12, −3, 6, 15, 24, …}

[7] = {…, −11, −2, 7, 16, 25, …}

[8] = {…, −10, −1, 8, 17, 26, …}

So, the quotient set is: ℤ $/R$ = {[0] , [1] , [2] , [3] , [4] , [5] , [6] , [7] , [8] }

**Question 3.**

A: 256 MB RAM, 32GB ROM, 8MP

B.288MB RAM, 64GB ROM, 4MP

C.128MB RAM 32GB ROM,5MP

If the statement meets the condition then it passes; otherwise it fails.

a) B most RAM ($288 > 256 > 128$). True

b) C more ROM than B ($32 < 64$, false) or a higher camera ($5 > 4$, true).

| C | B | C ∨ B |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

So, False ∨ true = true

**c)** B > A in RAM (true), ROM (true), and camera ($4 < 8$, false)

| P | Q | R | P^Q^R |
|---|---|---|---|
| T | T | T | T |
| T | T | F | F |
| T | F | T | F |
| T | F | F | F |
| F | T | T | F |
| F | T | F | F |
| F | F | T | F |
| F | F | F | F |

So, True ∧ true ∧ false = false.

d) If B > C in RAM and ROM (true and true = true), then higher camera (4 > 5? false).

| M | N | Y | M^N | M^N→Y |
|---|---|---|---|---|
| T | T | T | T | T |
| T | T | F | T | F |
| T | F | T | F | T |
| T | F | F | F | T |
| F | T | T | F | T |
| F | T | F | F | T |
| F | F | T | F | T |
| F | F | F | F | T |

**So,** True → false = false.

e)A > B RAM (256 > 288? false) iff B > A RAM (true).

| A | B | A↔B |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

So, False ↔ true = false.

**Question 4**

a)$\boldsymbol{X} \times Y \times Z$: Triples (company, origin city, destination city).

Ex: (RITCO, Kigali, Musanze) = RITCO route Kigali to Musanze.

(Volcano Express, Kigali, Huye) = volcano route Kigali to Huye

# Question 4

b) $B = \{3, 2, 1, 0\}$

$P(B) = 2^n = 16$

level 0 $\rightarrow \{\phi\}$

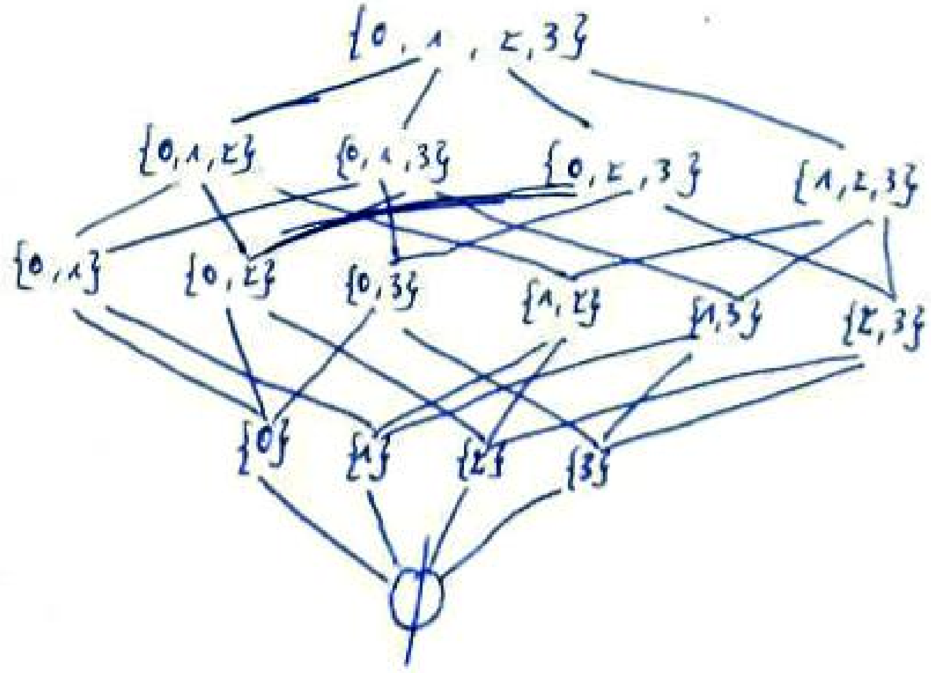
level 1 $\rightarrow \{\{0\}, \{1\}, \{2\}, \{3\}\}$

level 2 $\rightarrow \{\{0,1\}, \{0,2\}, \{0,3\}, \{1,2\}, \{1,3\}, \{2,3\}\}$

level 3 $\rightarrow \{\{0,1,2\}, \{0,1,3\}, \{0,2,3\}, \{1,2,3\}\}$

level 4 $\rightarrow \{0,1,2,3\}$

poset $(P(B), \subseteq)$, it is arranged from smallest to bottom and largest to top




~~Dual poset $(P(B), \supseteq)$, it~~

Dual poset $(\mathcal{P}(B), \supseteq)$