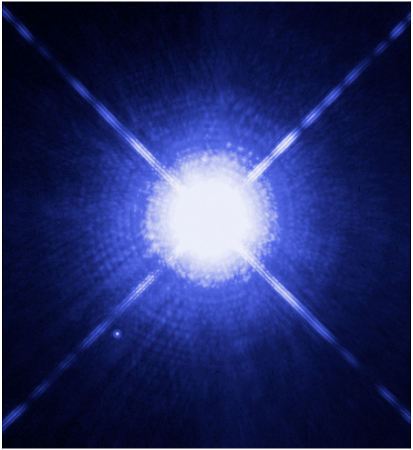
White Dwarfs

Ilaria Caiazzo
Jeremy Heyl

25 June 2018

What is a white dwarf?

- Sirius noticeably wobbles in its path.



HST

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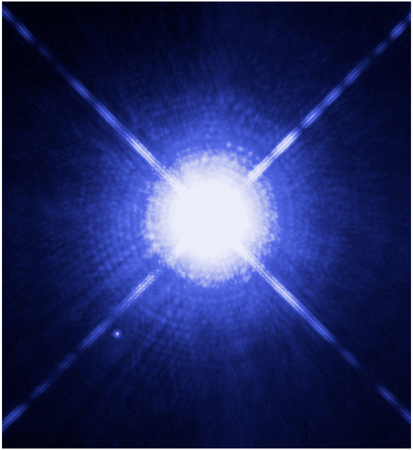

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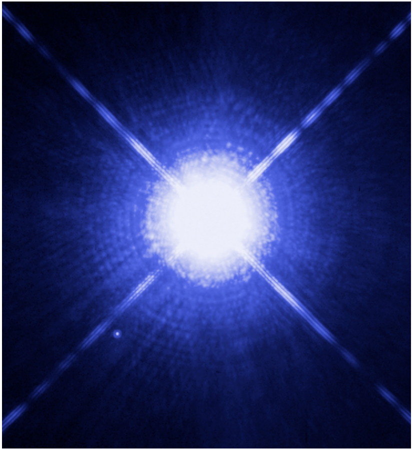

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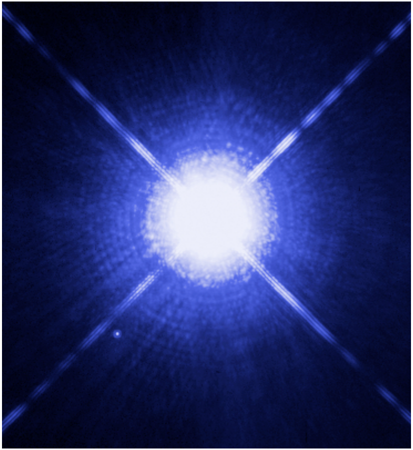

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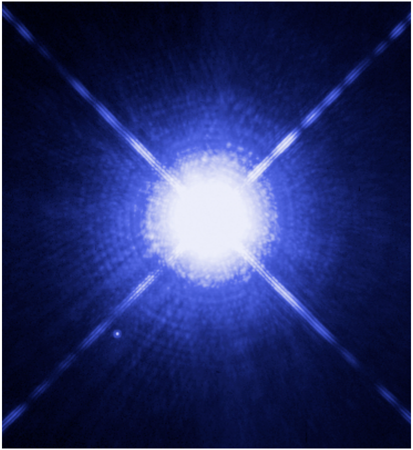

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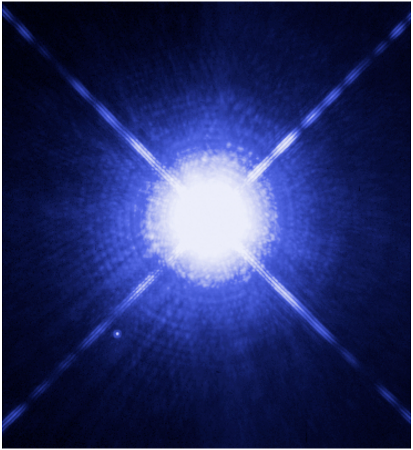
- The remnant of the effluence of a sun-like star.
- A giant diamond
- An accurate clock (Gyr)



HST

Structure

Older white dwarfs are supported by degeneracy pressure:

$$n = \int \frac{2}{h^3} f(\mathbf{x}, \mathbf{p}) d^3\mathbf{p} = \int_0^{p_F} \frac{g}{h^3} d^3\mathbf{p} = \frac{8\pi}{3h^3} p_F^3$$

Non-relativistic:

$$P = \frac{1}{3} \frac{2}{h^3} \int_0^{p_F} p v d^3\mathbf{p} = \frac{1}{3} \frac{2}{h^3} \int_0^{p_F} \frac{p^2}{m} d^3\mathbf{p} = \frac{8\pi}{15} \frac{p_F^5}{h^3 m_e}$$

Relativistic:

$$P = \frac{1}{3} \frac{2}{h^3} \int_0^{p_F} p v d^3\mathbf{p} = \frac{1}{3} \frac{2}{h^3} \int_0^{p_F} p c d^3\mathbf{p} = \frac{2\pi c}{3h^3} p_F^4$$

so $P = K\rho^\gamma$

Structure (Estimates)

We can approximate the equations of hydrostatic equilibrium as:

$$\rho \sim \frac{M}{R^3}, \frac{P}{R} \sim \frac{GM}{R^2}\rho$$

and

$$K\left(\frac{M}{R^3}\right)^{\gamma-1} \sim \frac{GM}{R}$$

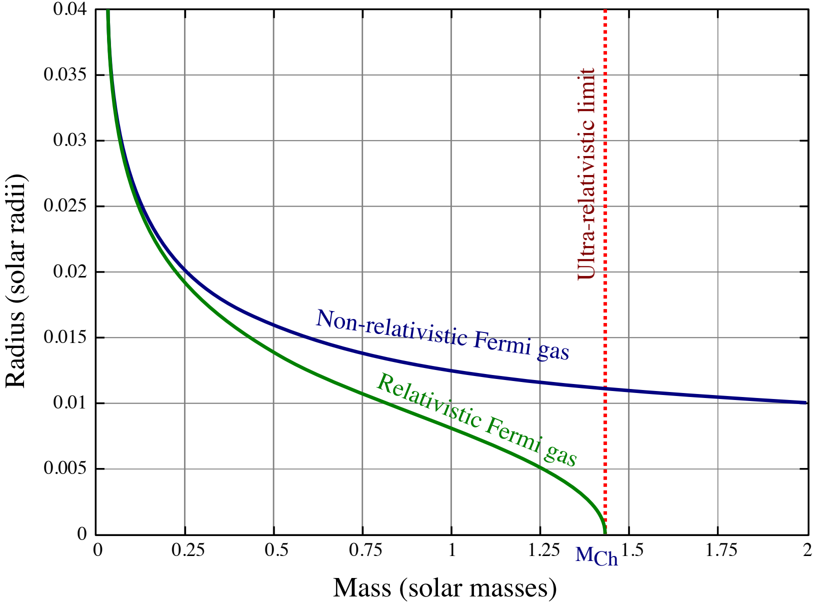
so

$$M^{\gamma-2}R^{4-3\gamma} \sim \frac{G}{K}.$$

and more precisely

$$M = 0.7011\left(\frac{R}{10^4\ \mathrm{km}}\right)^{-3}\left(\frac{2Z}{A}\right)^5 \mathrm{M}_\odot, M = 1.457\left(\frac{2Z}{A}\right)^2 \mathrm{M}_\odot$$

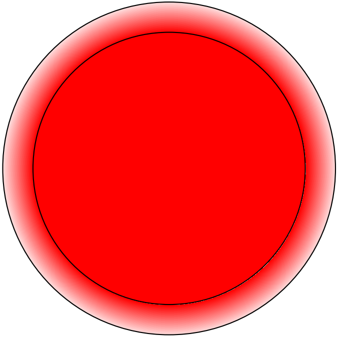
Structure



0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
Radius (solar radii)
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
Mass (solar masses)
Ultra-relativistic limit
Non-relativistic Fermi gas
Relativistic Fermi gas
MCh

Cooling

- Let's take the interior of the white dwarf to be isothermal with a constant heat capacity (liquid or gaseous).
- Let's assume that the temperature gradient from the core lies in a thin layer where photons carry the energy.



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Let's take the equation of radiative transfer

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$$L = c_V \frac{dT}{dt} \rightarrow t \propto T^{-5/2} \rightarrow T \propto t^{-2/5} \rightarrow L \propto t^{-7/5} \rightarrow T_{\text{eff}} \propto t^{-7/20}$$

Neutrino Cooling Theory

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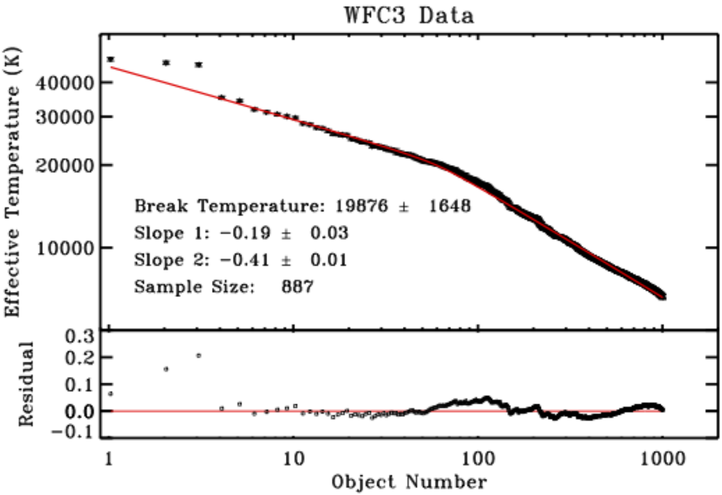
Cooling Summary

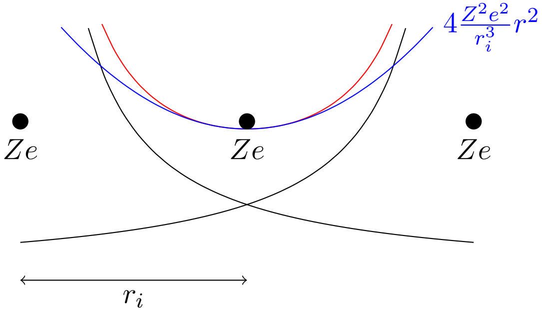
Regime	$T_{\rm core}$	Luminosity	$T_{\rm eff}$
Neutrino Cooling	$t^{-0.2}$	$t^{-0.7}$	$t^{-0.175}$
Photon Cooling	$t^{-0.4}$	$t^{-1.4}$	$t^{-0.35}$

Empirical Cooling Sequence



Cooling and Freezing

Let's imagine that the ions are in a lattice with spacing of r_i and focus on a particular ion:



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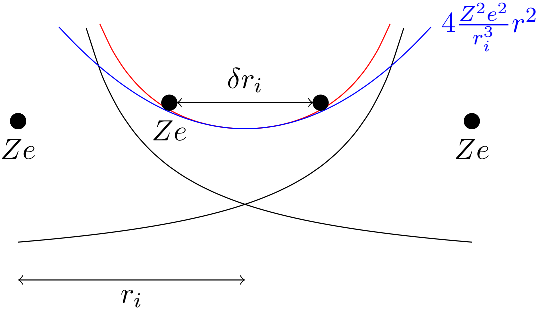


If the ion has a bit of thermal kinetic energy, $\frac{1}{2}kT$ it will move from its equilibrium position:

$$\left(\frac{r_i}{\delta r_i}\right)^2 \sim \frac{Z^2e^2}{r_i kT}$$

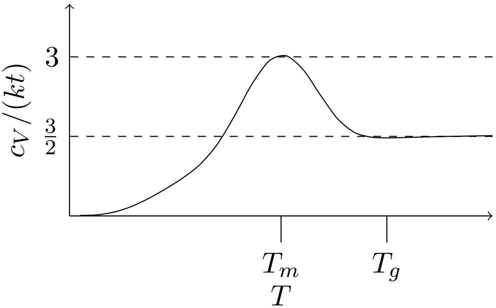
Cooling and Freezing

Let us define the ratio of the electrostatic energy to the thermal energy as

$$\Gamma \equiv \frac{(Ze)^2}{r_i k T} \text{ where } \frac{n_i 4\pi r_i^3}{3} = 1$$

If Γ is larger than about 180, the ions will essentially oscillate.

$$T_m \approx 2 \times 10^3 \left(\frac{\rho}{1 \text{ g cm}^{-3}} \right)^{1/3} Z^{5/3} \text{K}$$



Photon Cooling Theory — Frozen Ion Specific Heat

Let's take the equation of radiative transfer

$$F \propto \frac{3\sigma T^3}{\kappa}\frac{dT}{d\Sigma} \rightarrow \frac{F}{g_s} \propto \frac{T^{15/2}}{P}\frac{dT}{dP} \rightarrow P^2 L \propto T^{17/2}.$$

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Photon Cooling Theory — Electronic Specific Heat

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Let's take the equation of radiative transfer

$$F \propto \frac{3\sigma T^3}{\kappa}\frac{dT}{d\Sigma} \to \frac{F}{g_s} \propto \frac{T^{15/2}}{P}\frac{dT}{dP} \to P^2 L \propto T^{17/2}.$$

We take the opacity to be due to free-free absorption, $\kappa \propto \rho T^{-7/2} \propto P T^{-9/2}$ in the non-degenerate regime and to vanish in the degenerate gas. Now let's match to the degenerate regime.

$$P \propto \rho T \propto \rho^{5/3} \to T \propto \rho^{2/3} \to P \propto T^{5/2} \to L \propto T^{7/2}$$

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$$\to L \propto t^{-7/3} \to T_{\text{eff}} \propto t^{-7/12}$$

Cooling Summary

Regime	T_{core}	Luminosity	T_{eff}
Neutrino Cooling	$t^{-0.2}$	$t^{-0.7}$	$t^{-0.175}$
Photon Cooling	$t^{-0.4}$	$t^{-1.4}$	$t^{-0.35}$
Frozen Cooling	$t^{-0.6667}$	$t^{-2.3333}$	$t^{-0.5833}$

After the star is frozen, the lattice vibrations ($c_V \propto T^3$) first dominate the specific heat, but the cooling is rapid in this regime, and the electronic specific heat soon dominates ($c_V \propto T$).