Programming, Problem Solving, and Algorithms

CPSC 203, 2024 W2
(January – April 2025)
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Lecture 12A

Slides from the Assigned Videos

Representing Sudoku

A *representation* of a system is a model of the system that is useful in analysis.

A *state space* is a collection of all possible configurations of a physical system.

Each configuration is described using its representation, and is called a *state*.

How would you represent the game of Sudoku?

2			
			3
	4	1	

State Space Graphs

Define a graph where the set of vertices is ______________________________.

And the set of edges consists of pairs (u,v) where ______________________________

____.

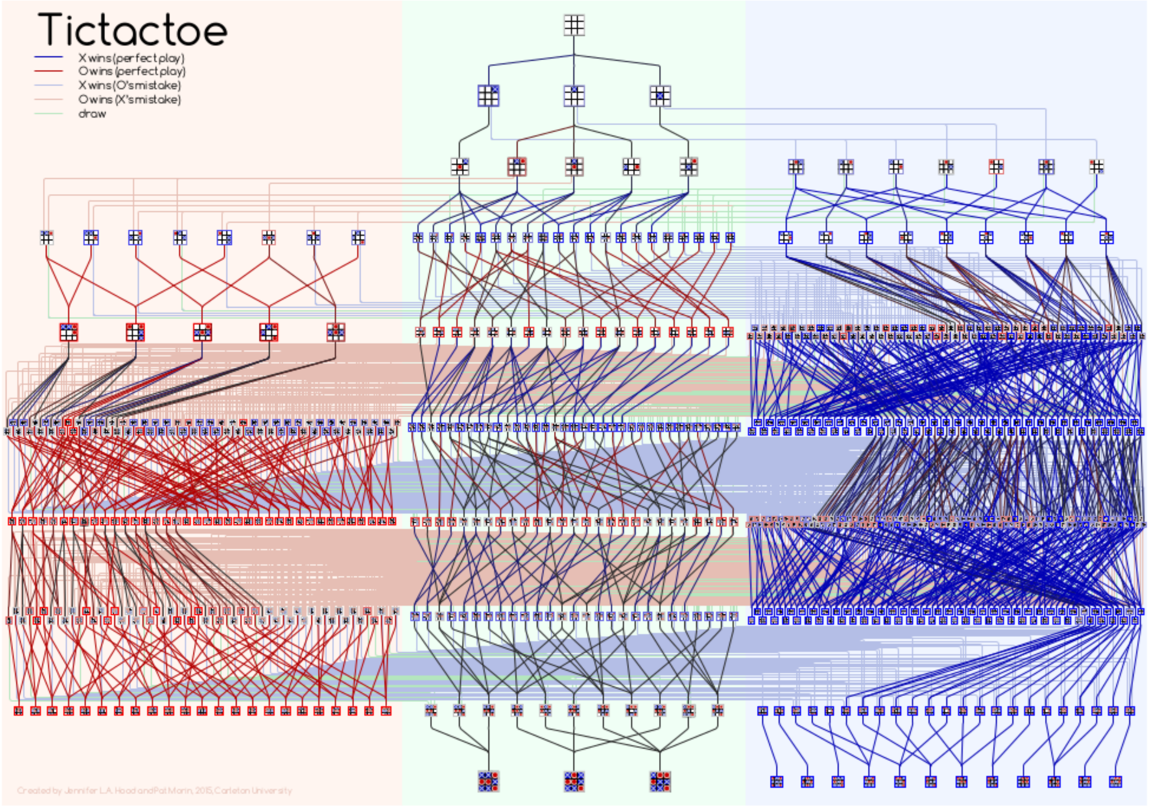
How many neighbors does this Sudoku puzzle state have?

2			
			3
	4	1	

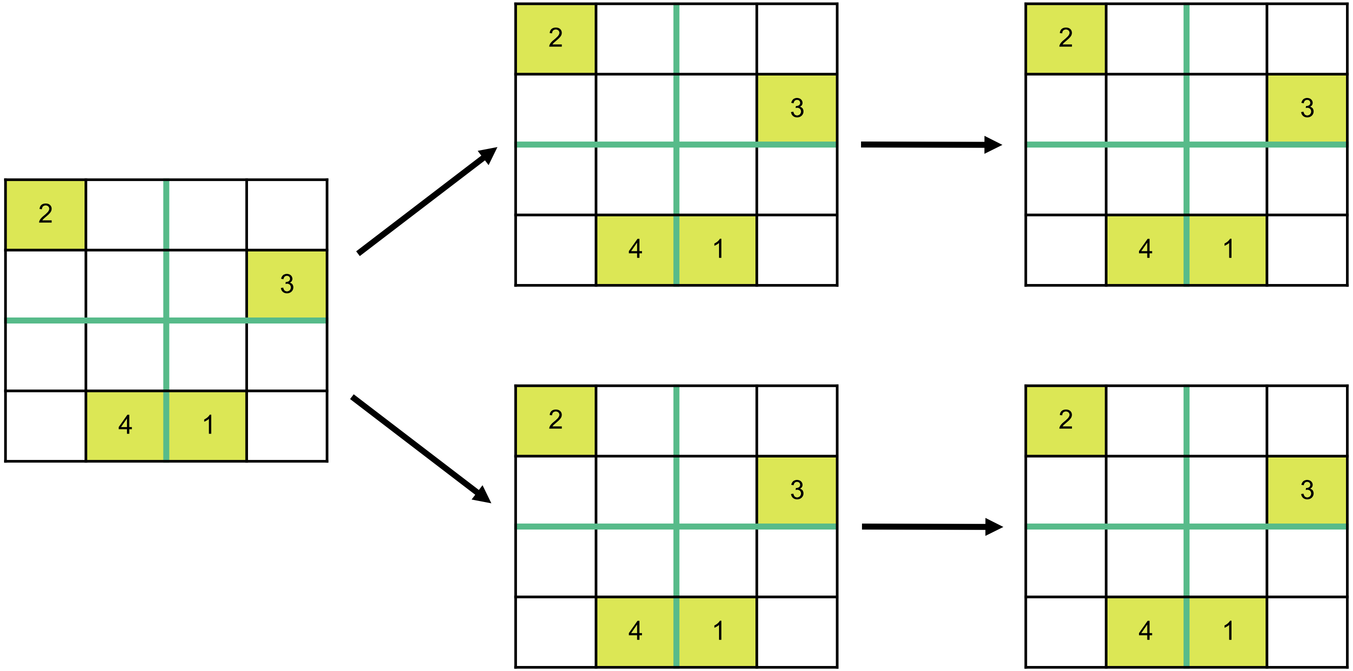
All neighbors:

Valid neighbors:

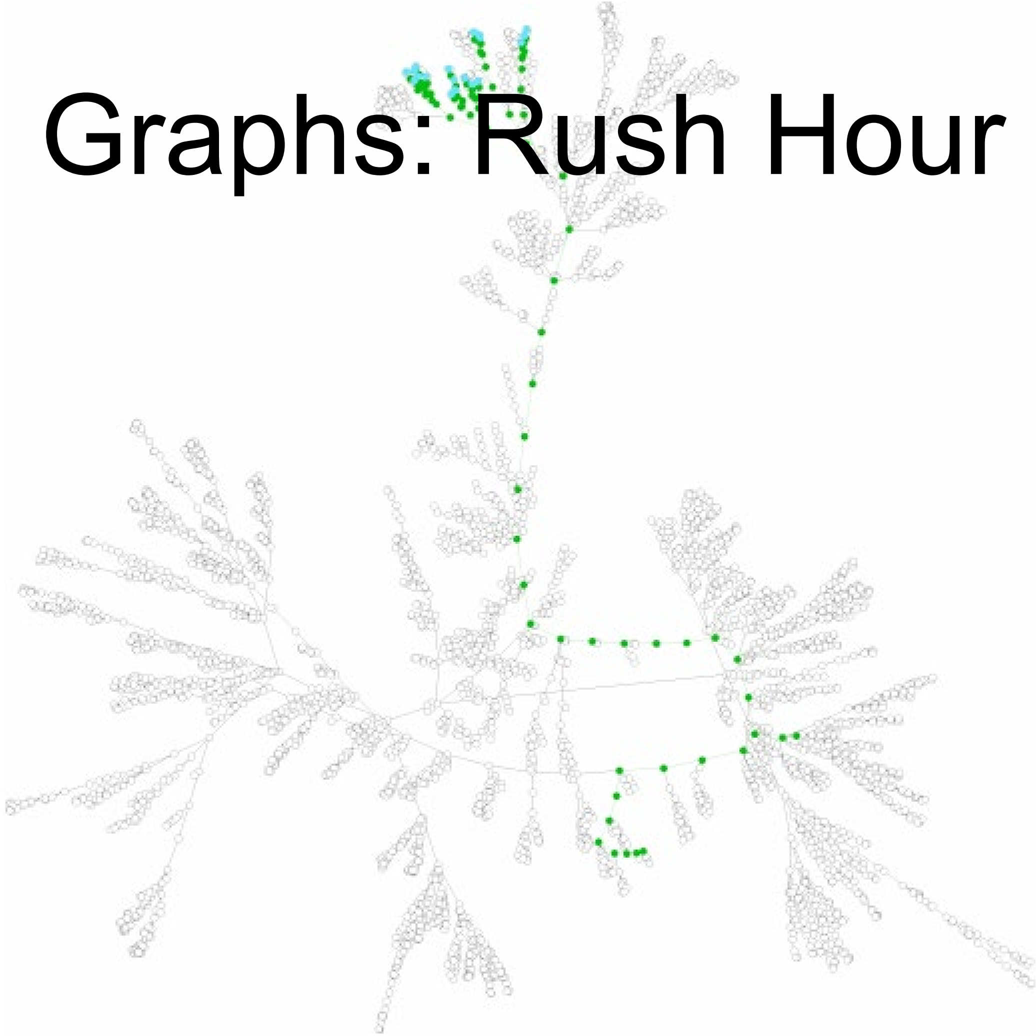
State Space Graphs - TicTacToe



Searching State Space Graphs



Graphs: Rush Hour



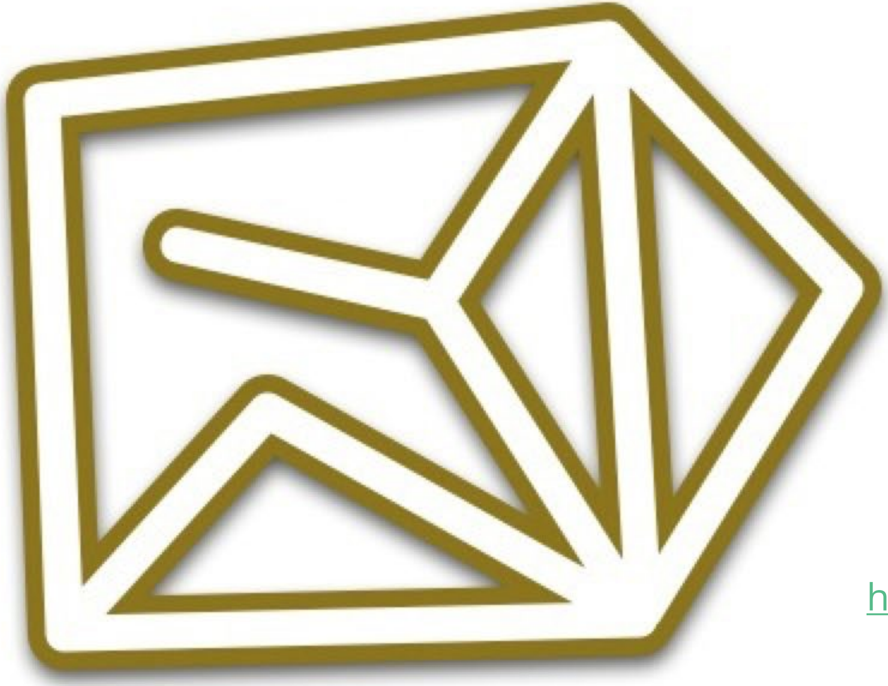
G = (V, E)

V: every vertex v is a board config

E: (u,v) means you can move from config u to config v

Path: sequence of vertices, connected by edges.

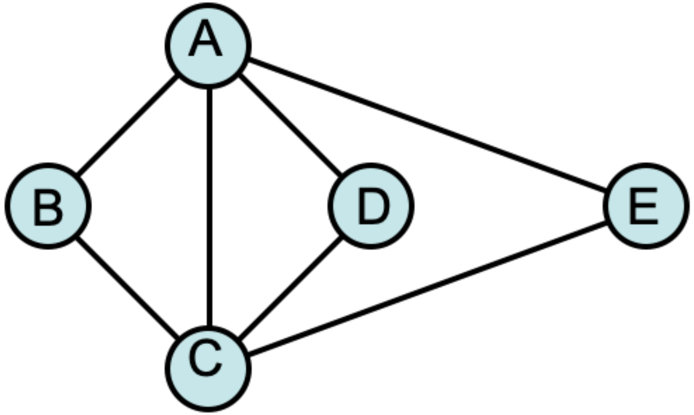
Depth First Search



Ariadne, Theseus, and the Minotaur

https://www.youtube.com/watch?v=8qrZ1clEp-Y

Depth First Search



Algorithm DFS(G,v)

Input: graph G and start vertex v

Output: labeling of the edges of G in the connected component of v as discovery edges and back edges

```
setLabel(v, VISITED)
For all w in G.adjacentVertices(v)
    if getLabel(w) = UNVISITED
      setLabel((v,w),DISCOVERY)
      DFS(G,w)
    else if getLabel((v,w)) = UNEXPLORED
      setLabel(e,BACK)
```

Remember Abstract Data Type: Stack?

Programmatic manifestation of a Pez dispenser.

ADT: Stack (the traditional operation names)

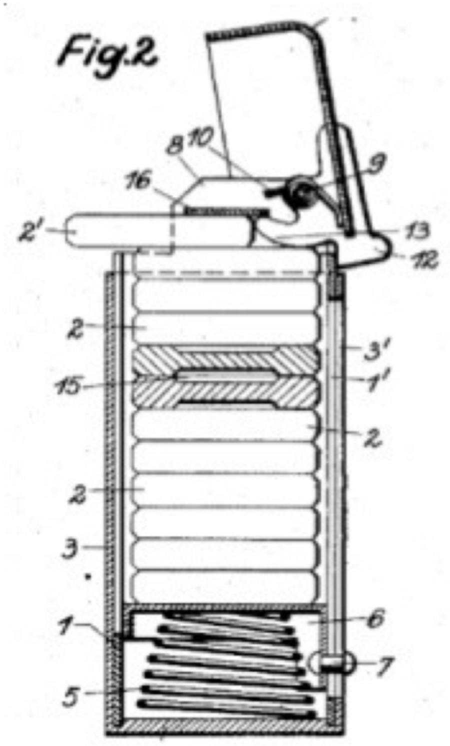
Insert -- push(data)

Remove -- pop() returns data

ADT: Deque (cuz python)

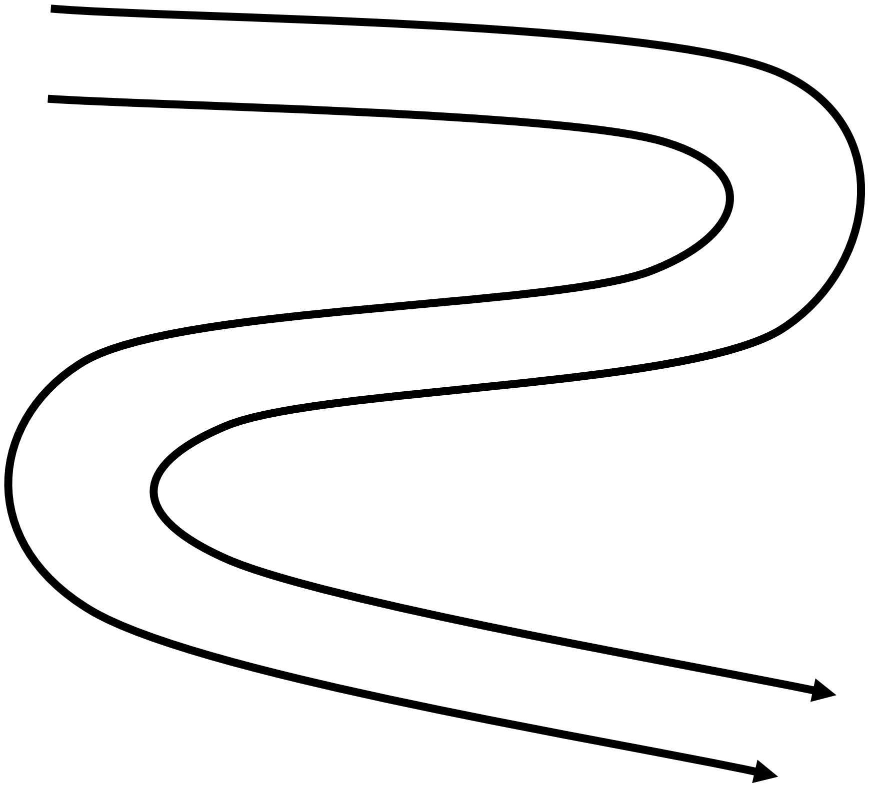
Insert -- append(data)

Remove -- pop()



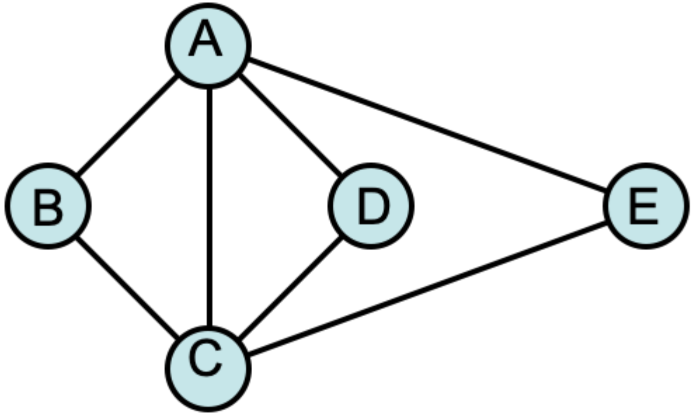
Remember: the Voronoi search algorithm

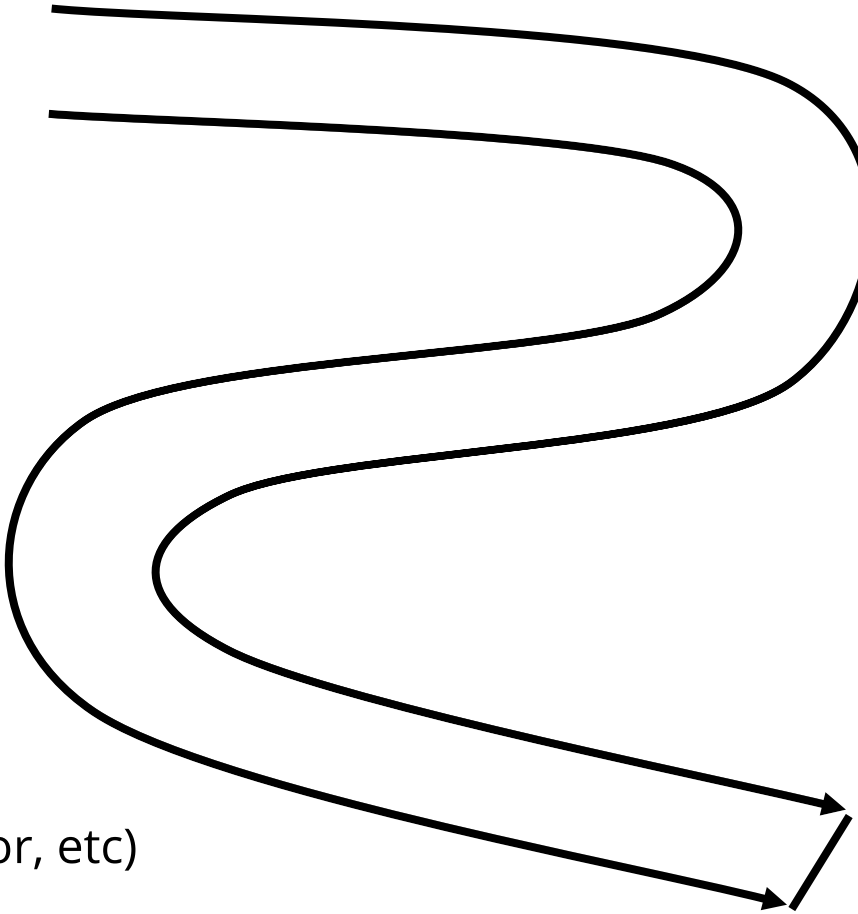
00	10	20	30	40	50	60	70	80	90
01	11	21	31	41	51	61	71	81	91
02	12	22	32	42	52	62	72	82	92
03	13	23	33	43	53	63	73	83	93
04	14	24	34	44	54	64	74	84	94
05	15	25	35	45	55	65	75	85	95



1) enqueue the center to start
2) while the queue is not empty:
   a) v = dequeue
   b) for each valid neighbor w, of v:
      i) color w
      ii) enqueue w

Iterative DFS





1) push the start
2) while the stack is not empty:
    a) v = pop()
    b) for each valid neighbor w, of v:
        i) process w (print, label, color, etc)
        ii) push(w)

Let’s Implement!

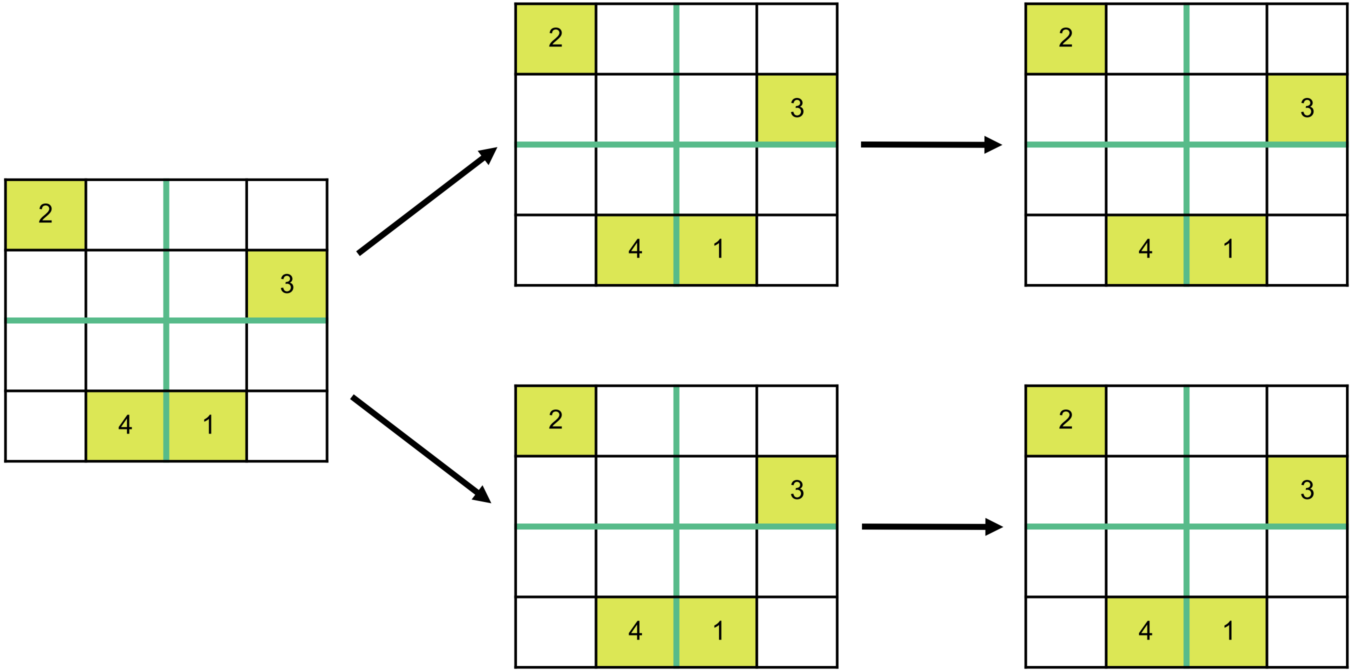
Overall Strategy:

Move forward through the states (board configurations) until you can’t go any farther.

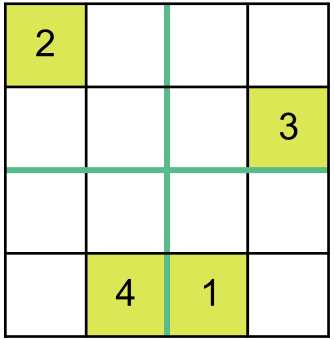
If the board is complete, you win!

If the board is not complete, then back up and try a new state in the most recent cell possible.

Searching State Space Graphs



Moving toward implementation:



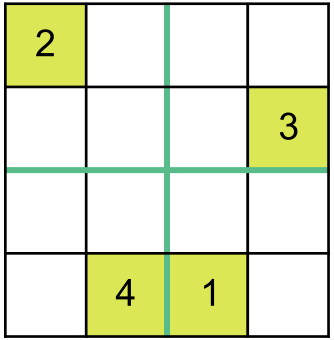
Need to be able to check whether a candidate entry is valid.

Suppose we have a variable `grid`, representing the board, and we want to place a value called `num`, in position `(x,y)`.

This code checks to see if num is valid. Which line checks row, which col?

```
if num in grid[x, :]:
    return False
elif num in grid[:, y]:
    return False
```

Moving toward implementation:



Need to be able to check whether a candidate entry is valid.

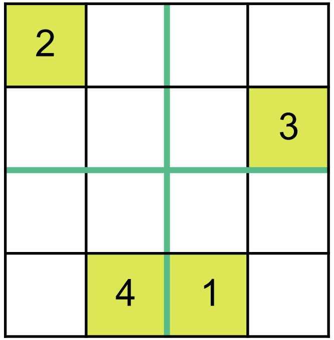
Suppose we have a variable `grid`, representing the board, and we want to place a value called `num`, in position `(x,y)`.

Region check?

EX: to query a region in a 2d numpy matrix, just define the bounds on the region and use `in`. In the above example, `2 in grid[0:2,0:2]` returns `True`.

New problem: define the region for given point `(x,y)` ?

New Problem:



Define the region for given point `(x,y)`

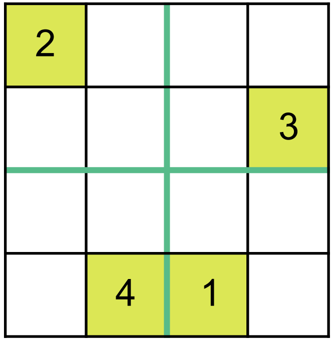
pt	region
(0,0)	[0:2,0:2]
(0,1)	
(1,0)	
(1,1)	

pt	region
(2,0)	[2:4,0:2]
(2,1)	
(3,0)	
(3,1)	

pt	region
(0,2)	[_:_,_:_]
(0,3)	
(1,2)	
(1,3)	

pt	region
(2,2)	[_:_,_:_]
(2,3)	
(3,2)	
(3,3)	

Now Generalize:



Goal: define the region for given point `(x, y)` in a r^2 x r^2 grid.

Here are some more examples:

(4, 8) in a 9x9 grid is in region [3:6, 6:9]

(22, 14) in a 25x25 grid is in region [20:25, 10:15]

(____,____) in a 100x100 grid is in region [____:____, ____:____]

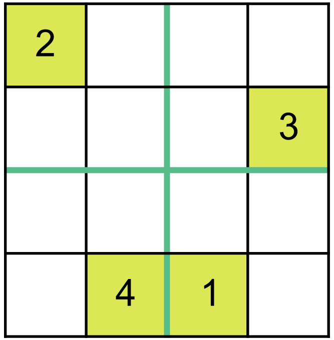
pt	region
(2,0)	[2:4,0:2]
(2,1)	
(3,0)	
(3,1)	

One last little thing:

We want to iterate over the 16 positions, but we need to refer to them by their `(x,y)` positions in the grid.

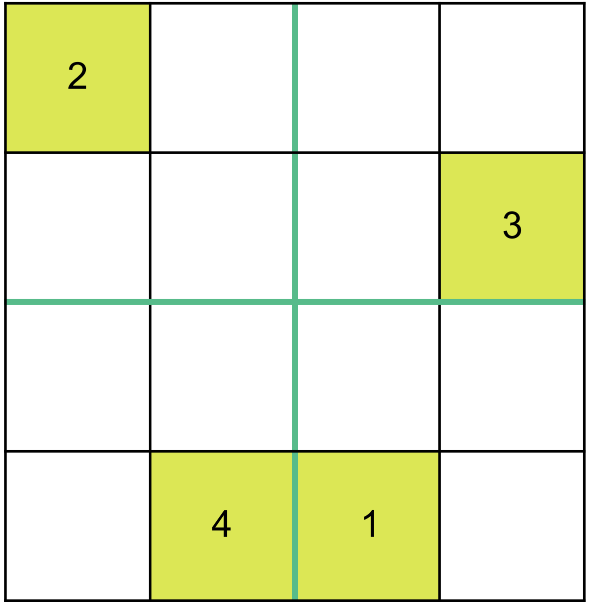
Write a function called `postup(p)` that takes a position `p` and returns `p`’s `(x,y)` coordinates in the grid.

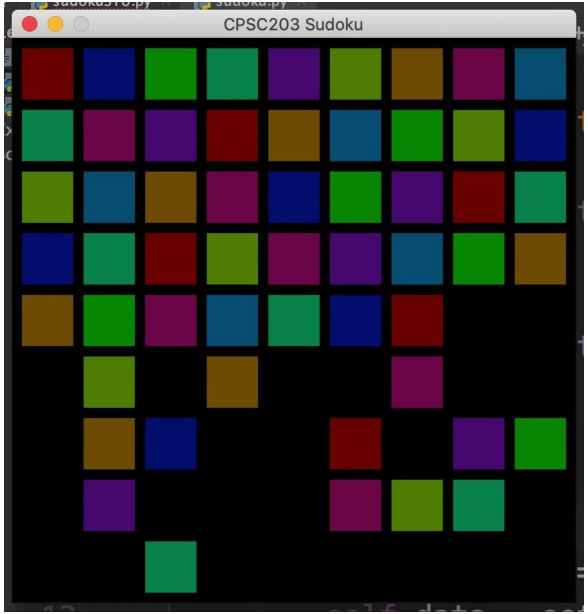
Note: The upper left corner is position 0 and has coordinates `(0,0)`.)

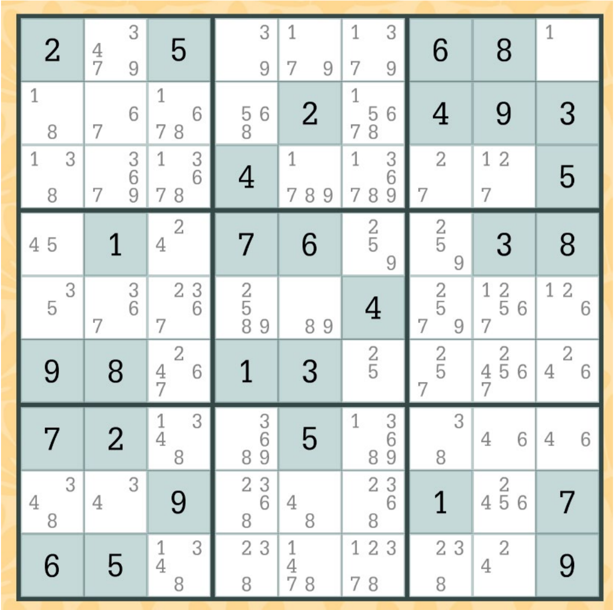


Sudoku, a closing thought...

Recall our algorithm for searching... could we be smarter?







Sudoku, another closing thought...

Is our solution to Sudoku tractable? (how fast does the state space grow, as we increase the board size?

3: 9x9

4: 16x16

5: 25x25

6: 36x36

Known to be NP-Complete --