

A comparison of estimation methods for fitting Weibull and Johnson's S_B distributions to mixed spruce–fir stands in northeastern North America

Lianjun Zhang, Kevin C. Packard, and Chuangmin Liu

Abstract: Four commonly used estimation methods were employed to fit the three-parameter Weibull and Johnson's S_B distributions to the tree diameter distributions of natural pure and mixed red spruce (*Picea rubens* Sarg.) – balsam fir (*Abies balsamea* (L.) Mill.) stands, respectively, in northeastern North America. The results indicated that the Weibull and the Johnson's S_B distributions were, in general, equally suitable for modeling the diameter frequency distributions of this forest type, but the relative performance directly depended on the estimation method used. In this study, the linear regression methods for Johnson's S_B were found to give the lowest mean Reynolds' error indices. The conditional maximum likelihood for Johnson's S_B and the maximum likelihood estimation for Weibull produced comparable results. However, moment- or mode-based methods were not well suited to the observed diameter distributions that were typically positively skewed, reverse-J, and mound shapes.

Résumé : Quatre méthodes courantes d'estimation ont été utilisées pour ajuster les fonctions à trois paramètres de Weibull et de Johnson S_B à la distribution diamétrale des peuplements naturels purs et mélangés d'épinette rouge (*Picea rubens* Sarg.) et de sapin baumier (*Abies balsamea* (L.) Mill.) du Nord-Est de l'Amérique du Nord. Les résultats indiquent que les deux fonctions sont en général aussi appropriées l'une que l'autre pour modéliser la distribution diamétrale de ces peuplements. Néanmoins, la performance relative des deux fonctions dépend directement de la méthode utilisée pour estimer leurs paramètres. La moyenne des indices d'erreur de Reynolds est la plus faible lorsque la régression linéaire est appliquée à la fonction de Johnson S_B . Lorsque la méthode du maximum de vraisemblance conditionnelle est appliquée à la fonction de Johnson S_B et que la méthode du maximum de vraisemblance est appliquée à la fonction de Weibull, les résultats sont comparables. Cependant, les méthodes d'estimation basées sur le moment ou sur le mode ne s'ajustent pas bien aux distributions diamétrales observées qui sont typiquement asymétriques à droite, en J inversé et en cloche.

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Introduction

Knowledge of the distribution of stand volume into specific size classes at a given point in time is valuable information to forest resource managers. One approach to facilitating these information needs is through diameter distribution models, which use a probability density function (PDF) to distribute a stand attribute across size classes such as diameter at breast height (DBH) or tree height. The probability density function is a continuous function defined by a

vector of parameters that does not, per se, have any direct biological meaning. Although a large number of PDFs have been used in modeling, such as the normal, beta, gamma, lognormal, Weibull, and Johnson systems (Hafley and Schreuder 1977), the primary reason for choosing one is mathematical (Shiver 1988). A desired PDF is able to take on a wide array of distribution shapes encountered in forest stands and has a cumulative density function (CDF) that can be easily integrated.

An array of diameter distribution models has been published in the literature. They are largely applied to single-species, even-aged stands or plantations such as southern pines in the U.S. Burkhart and Tham (1992) noted that relatively few growth and yield models exist for mixed-species stands. One example was the model developed for mixed Appalachian hardwoods (Bowling et al. 1989). Recent work by Maltamo (1997) with Scots pine (*Pinus sylvestris* L.) – Norway spruce (*Picea abies* (L.) Karst.) forests in Finland provides motivation that the complexity of mixed-species stands may suitably be described by diameter distribution models. However, in searching the literature, we found only

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L. Zhang¹ and C. Liu. Faculty of Forest and Natural Resources Management, State University of New York, College of Environmental Science and Forestry, One Forestry Drive, Syracuse, NY 13210, U.S.A.

K.C. Packard. USDA Forest Service, 165 S. Main Street, Cortland, NY 13045, U.S.A.

¹Corresponding author (e-mail: lizhang@esf.edu).

one published study (Lorimer and Krug 1983) that fit the Weibull function to the natural mixed red spruce (*Picea rubens* Sarg.) – balsam fir (*Abies balsamea* (L.) Mill.) forests of northern New England and the Maritime provinces of Canada. No study was found for this forest type using the Johnson system. In this study, we attempt (1) to test the adequacy of the Weibull and the Johnson's S_B functions for this forest type and (2) to test the adequacy of commonly used estimation techniques for each selected probability density function. The choice of these estimation methods was made to encompass modeling approaches frequently encountered in parameter prediction, parameter recovery, and percentile prediction models (Knoebel and Burkhart 1991).

Materials and methods

The data were selected from the database used for the development of FIBER 3.0 (Solomon et al. 1995). The geographical region covered northern New York, Vermont, New Hampshire, Maine, and the Canadian provinces of Nova Scotia and New Brunswick. The sample plots were established, fixed-area plots, ranging in size from 1/5 to 1/10 acre (0.08–0.04 ha). Measurements were taken repeatedly at 5-year intervals starting in 1955, with a maximum of five remeasurements available. However, the data from only the first measurement period were used for fitting the distributions.

This study was focused on the spruce–fir ecological habitat type, in which red spruce and balsam fir are dominant species, constituting 60% of the total plot basal area on average. A total of 138 plots were selected, ranging from pure balsam fir to pure red spruce to various mixtures of the two

species. A pure stand is defined as at least 75% of the total basal area consisting of a dominant species. A mixed stand is defined as at least 25% red spruce and 25% fir, and the sum of the basal area of spruce and fir must exceed 75%. All pure plots were screened to eliminate those with fewer than 20 trees of the dominant species or with a range of fewer than three 1-in. diameter classes (1 in. = 2.54 cm). Mixed plots were selected so that the plots contained at least 15 red spruce and 15 balsam fir, in addition to spruce and fir each having a range of diameters of least three 1-in. classes. Among the 138 plots, 39 were pure balsam fir plots, 50 were pure red spruce plots, and 49 were spruce–fir mixture plots. For each sample plot, a tree list was available, containing the species and DBH of each tree greater than 4.5 in. (11.4 cm), grouped into 1-in. diameter classes (tally data). During initial data screening, it was discovered for many plots that data were not collected for trees less than 6 in. (15.2 cm) DBH. As a result, the tree lists were truncated at 6 in. for all plots to ensure comparability. The average number of trees per plot was 41, ranging from 22 to 81 per plot. The overall mean of tree diameters was 7.6 in. (19.3 cm), with a range of 6 in. (15.2 cm) to 23 in. (58.4 cm). The structure of the diameter distributions in the selected plots showed the typical “reverse-J” shape characteristic. While some symmetric distributions were encountered, the majority of the distributions were strongly skewed to the right. No plots were left skewed (negative skewness). The average skewness of all plots was 1.4, ranging from 0.3 to 3.8, while the average kurtosis was 2.3, ranging from –1.2 to 17.9.

The probability density function, $f(x)$, for the three-parameter Weibull distribution is defined by Bailey and Dell (1973) as

$$[1] \quad f(x) = \left(\frac{\gamma}{\beta}\right) \left(\frac{x-\alpha}{\beta}\right)^{\gamma-1} \exp\left[-\left(\frac{x-\alpha}{\beta}\right)^{\gamma}\right], \quad \alpha \leq x \leq \infty, \quad \beta > 0, \quad \gamma > 0$$

where x is tree DBH and α , β , and γ are the location, scale, and shape parameters, respectively. The probability density function for the Johnson's S_B distribution is defined by Johnson (1949) as follows:

$$[2] \quad f(x) = \left(\frac{\delta}{\sqrt{2\pi}}\right) \left[\frac{\lambda}{(x-\epsilon)(\lambda+\epsilon-x)}\right] \exp\left\{-0.5\left[\gamma + \delta \ln\left(\frac{x-\epsilon}{\lambda+\epsilon-x}\right)\right]^2\right\}$$

$$\epsilon \leq x \leq \epsilon + \lambda, \quad -\infty < \epsilon < +\infty, \quad \lambda > 0, \quad \delta > 0, \quad -\infty < \gamma < +\infty$$

where x is tree DBH, ϵ is the location parameter, λ is the range parameter, and δ and γ are the shape parameters.

To fit the three-parameter Weibull distribution, we employed the following four estimation methods: (1) maximum likelihood estimation (MLE) (Bailey 1974; Schreuder et al. 1978; Gove and Fairweather 1989) with the boundary constraints $0 \leq \alpha \leq (D_{\min} - c)$, $0 \leq \beta \leq 80$, $0 \leq \gamma \leq 20$, where α , β , and γ are location, scale, and shape parameters of the Weibull distribution, respectively; D_{\min} is the minimum tree diameter; and c is a constant; (2) method of moments incor-

porating skewness (MOM1) (Cohen and Whitten 1982; Burk and Newberry 1984; Lindsay et al. 1996); (3) traditional method of moments (MOM2) (Frazier 1981; Burk and Newberry 1984; Shifley and Lentz 1985; Lindsay et al. 1996). The location parameter is assumed to equal $(D_{\min} - c)$; and (4) percentile-based estimation (PPM) (Dubey 1967; Zanakakis 1979). Different values (i.e., 0.5, 1.0, 1.5, and 2.0) were tried to choose a reasonable designation for c .

To estimate the four parameters of the Johnson's S_B distribution, the following four estimation methods were em-

Table 1. Reynolds' error index for comparing different values of the constant in the location parameter boundary ($D_{\min} - c$) for the estimation methods of the Weibull and Johnson's S_B distributions.

c	Estimation method				Weibull			
	Johnson's S_B							
	CML	KB	MODE	REG	MLE	MOM1	MOM2	PPM
0.5	68.9 (26.9)	91.4 (42.1)	207.7 (90.4)	65.0 (27.8)	75.5 (27.5)	107.9 (53.5)	98.2 (31.3)	84.1 (30.3)
1.0	98.5 (33.2)	122.8 (45.4)	166.2 (71.2)	94.8 (31.8)	97.4 (31.1)	107.9 (53.5)	127.8 (34.9)	92.1 (30.6)
1.5	120.5 (39.2)	147.8 (52.9)	154.6 (58.7)	115.3 (38.0)	133.7 (43.1)	107.9 (53.5)	253.7 (80.2)	130.1 (40.7)
2.0	134.9 (44.0)	164.5 (57.8)	157.4 (51.5)	128.2 (41.8)	143.5 (46.4)	107.9 (53.5)	243.6 (67.2)	137.8 (42.6)

Note: c, constant; CML, conditional maximum likelihood; KB, Knoebel–Burkhart; MODE, mode based; REG, linear regression; MLE, maximum likelihood; MOM1, method of moments incorporating skewness; MOM2, traditional method of moments; PPM, percentile based. Numbers in parentheses are standard deviations.

ployed: (1) conditional maximum likelihood estimation (CML) (Johnson 1949; Siekierski 1992); (2) Knoebel–Burkhart method (KB) (Knoebel and Burkhart 1991); (3) mode-based method (MODE) (Hafley and Buford 1985); and (4) linear regression method (REG) (Zhou and McTague 1996). Again, the location parameter ϵ is assumed to equal ($D_{\min} - c$), and different values (i.e., 0.5, 1.0, 1.5, and 2.0) were tried to choose a reasonable designation for c.

The Weibull and Johnson's S_B distributions were fit to the diameter data of each plot using SAS (SAS Institute Inc. 1999). Reynolds et al.'s (1988) error index was used to evaluate the model fitting, with the "best" method minimizing the sum of the absolute deviations from the predicted to the observed distributions. This error index is defined as follows:

$$[3] \quad e = N \sum_{j=1}^k \left| \int_{I_j} w(x) d\hat{F}(x) - \int_{I_j} w(x) dF(x) \right|$$

where x is DBH, $F(x)$ is the observed CDF of the tree diameter distribution, $\hat{F}(x)$ is the estimated CDF of the tree diameter distribution, $w(x)$ is a weight function of diameter, I_j is the j th diameter class ($j = 1, 2, \dots, k$), and N is the number of trees per unit area. In essence, the error index acts as the sum of the absolute deviations from the estimated and observed distributions, weighted by the function $w(x)$. Reynolds et al. (1988) recommend using a weighting function such as an individual tree volume equation that puts greater weight on larger diameter, more valuable trees. As tree height data are unavailable, the weighting index used here is simply $w(x) = 1$. Therefore, the error index actually becomes the sum of absolute differences between predicted (P_j) and observed (O_j) number of diameters in each diameter class j and can be expressed as

$$[4] \quad e = \sum_j^k |P_j - O_j|$$

For both Weibull and Johnson's S_B distributions, the P_j is calculated as

$$[5] \quad P_j = N \int_{L_j}^{U_j} f(x) dx$$

where U_j and L_j are the upper limit and lower limit of a specific diameter class j , respectively. Since the Johnson's S_B

Table 2. Average of the parameter estimates of the three-parameter Weibull for fitting the 138 spruce–fir plots.

Parameter	Estimation method			
	MLE	MOM1	MOM2	PPM
α	5.5 (0)	5.24 (0.45)	5.5 (0)	5.5 (0)
β	2.25 (0.73)	2.52 (1.10)	1.97 (0.78)	2.18 (0.82)
γ	1.32 (0.20)	1.41 (0.41)	0.91 (0.18)	1.27 (0.23)

Note: α , location; β , scale; γ , shape; MLE, maximum likelihood; MOM1, method of moments incorporating skewness; MOM2, traditional method of moments; PPM, percentile based. Numbers in parentheses are standard deviations.

Table 3. Average of the parameter estimates of the Johnson's S_B for fitting the 138 spruce–fir plots.

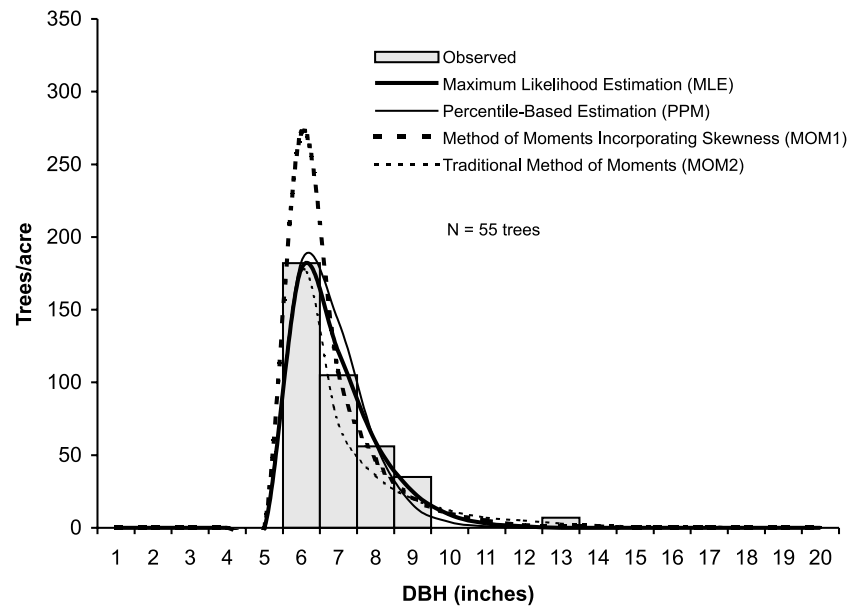
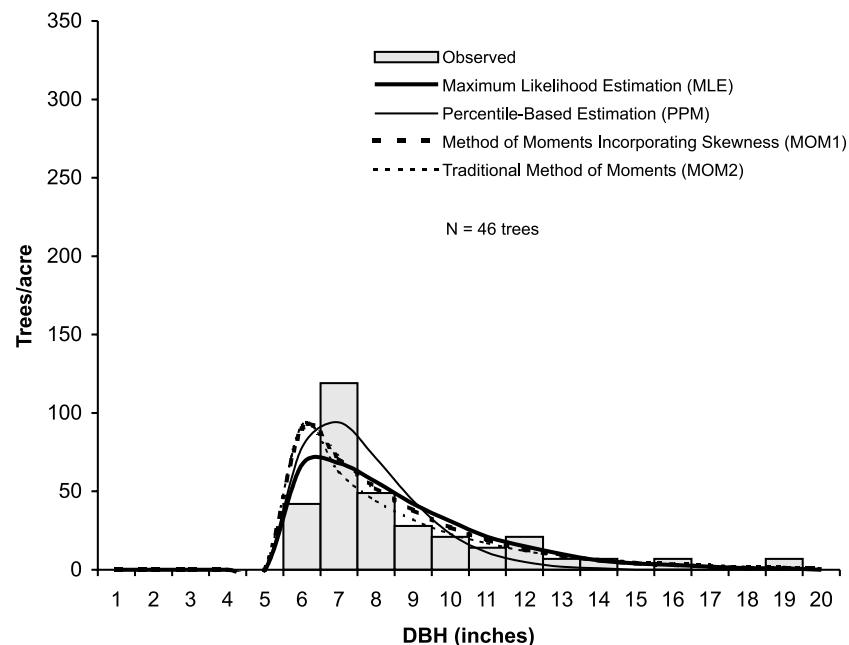
Parameter	Estimation method			
	CML	KB	MODE	REG
ϵ	5.50 (0)	5.50 (0)	5.50 (0)	5.50 (0)
λ	8.98 (3.03)	8.98 (3.03)	8.98 (3.03)	8.98 (3.03)
δ	0.87 (0.11)	0.87 (0.30)	1.31 (0.18)	0.77 (0.11)
γ	1.23 (0.34)	1.25 (0.46)	3.16 (0.89)	1.20 (0.39)

Note: ϵ , location; λ , range; δ and γ , shape; CML, conditional maximum likelihood; KB, Knoebel–Burkhart; MODE, mode based; REG, linear regression. Numbers in parentheses are standard deviations.

distribution does not have a closed CDF form, numerical integration was used to compute the probability (Newberry and Burk 1985).

Results and discussion

During the course of parameter estimation, the nonlinear programming procedure PROC NLP in SAS (SAS Institute Inc. 1999) was used to solve the parameter estimates given a set of boundary constraints. However, when the boundary constraint for the location parameter (i.e., α in Weibull and ϵ in Johnson's S_B) was set to the truncation point of 6 in. (15.24 cm), the estimation process failed to converge. Thus the location parameter was constrained to be no larger than the minimum diameter less a constant c . Different values (i.e., 0.5, 1.0, 1.5, and 2.0) were tried to choose a reasonable designation for the constant c . Table 1 indicated that in general, the estimation methods performed better in terms of the Reynolds' error index when c was set to 0.5 and worsened as the location parameter was constrained to be farther and far-

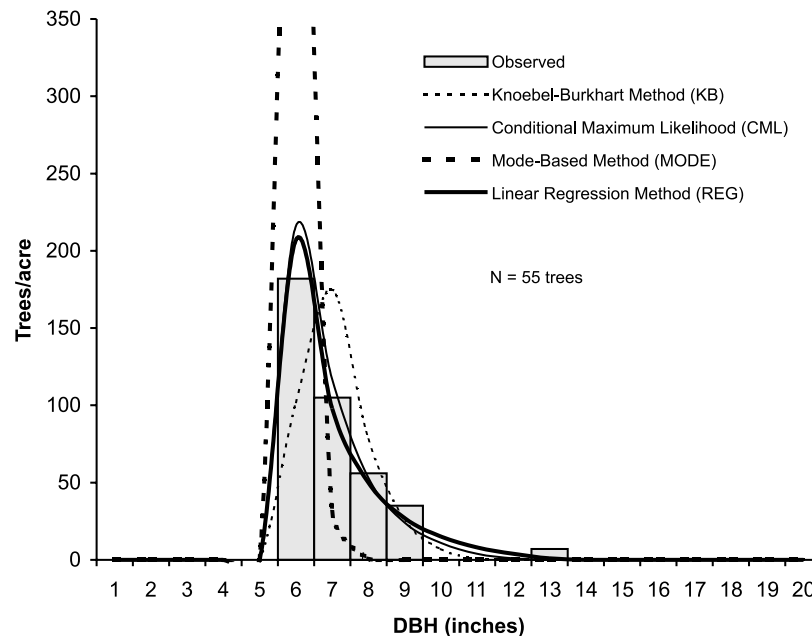
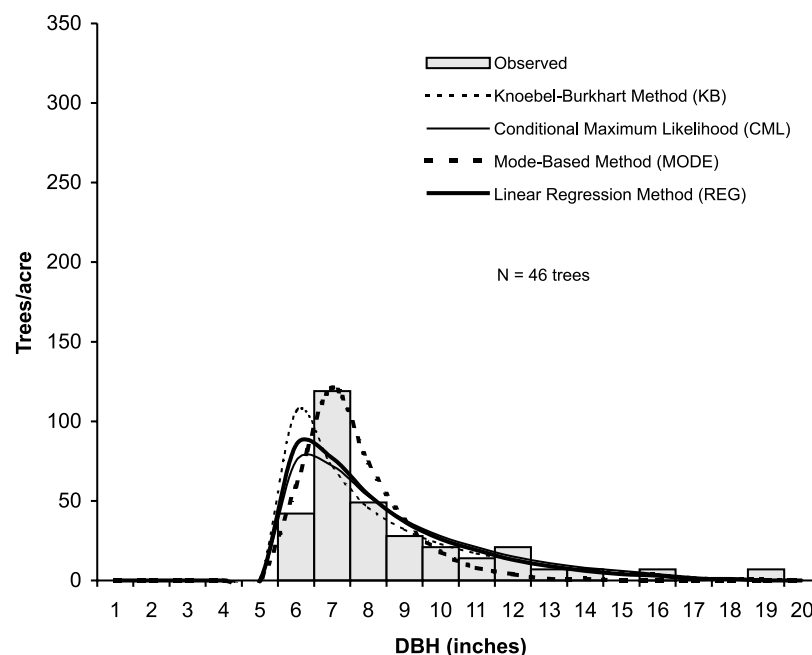
Fig. 1. Estimated Weibull distributions from the four methods for plot 41.**Fig. 2.** Estimated Weibull distributions from the four methods for plot 157.

ther away from the truncation point of 6 in. (15.24 cm). There was no practical difference between the Weibull and Johnson's S_B distributions. One exception was the MOM1 method for fitting the Weibull distribution. Since the three Weibull parameters were solved through the first three sample moments in the MOM1 method, the use of the minimum diameter and some arbitrary constant to set α was not relevant. Therefore, the location parameter was then constrained to be no larger than 5.5 in this study.

Of all methods, the REG procedure used with Johnson's S_B had the lowest mean error index at 65.0, followed by CML for the Johnson's S_B at 68.9, and MLE for the Weibull at 75.5 (Table 1, $c = 0.5$). The poor fit by MODE clearly stood out, with a mean error index of 207.7, over three times

as large as that for the REG method. In addition to differences in accuracy among the different estimation methods, Table 1 also revealed differences in precision of the estimation methods as well. In terms of the standard deviations of the Reynolds' error index, those methods with greater accuracy also had greater precision, with CML having the lowest standard deviation of 26.4, closely followed by MLE for the Weibull at 27.5 and REG for the Johnson's S_B at 27.8. The method having the highest variability by far was MODE for Johnson's S_B , with the standard deviation of the Reynolds' error index being 90.4.

The averages of the parameter estimates for each of these four methods were presented for the three-parameter Weibull distribution in Table 2. The results indicated that all four es-

Fig. 3. Estimated Johnson's S_B distributions from the four methods for plot 41.**Fig. 4.** Estimated Johnson's S_B distributions from the four methods for plot 157.

estimation methods provided comparably similar mean parameter estimates, with the location parameter near the diameter truncation point, the scale parameter in the range of 1.97–2.52, and the shape parameter in the range of 0.91–1.41. For PPM, some difficulty was encountered as well. Since the spruce–fir diameter distributions were strongly skewed to the right, a large proportion of the diameter distribution was located at the 6- and 7-in. (15.24- and 17.78-cm) classes. For nearly all plots, any percentile less than the first quartile (25%) took on the same value of 6 in., because the tree diameters were measured to the nearest inch (tally data). As a result, the percentile estimators recommended by Zanakos (1979) failed to solve. To avoid this problem, we set the lo-

cation parameter to the minimum diameter less 0.5, and the shape and scale parameters were solved using Zanakos' (1979) recommended estimators with the 17th, 63rd, and 97th percentiles.

In comparison, the estimation of the Johnson's S_B parameters was relatively free from such difficulties. The averages of the parameter estimates for all Johnson's S_B estimation methods were presented in Table 3. For estimating the location (ϵ) and range (λ) parameters, all methods resulted in the mean value for ϵ equal to 5.5 and for λ equal to 8.98. Differences among the parameter estimates for CML, KB, and REG were few. As evidenced in Table 3, the estimated shape parameters from these methods were very similar. In con-

trast, MODE, developed by Hafley and Buford (1985), gave much higher estimates for both shape parameters, with greater standard deviations in the estimates as well.

To provide a graphical comparison, Figs. 1–4 illustrate the estimation methods for two plots typical of the diameter distributions observed in the mixed spruce–fir stands. Figures 1 and 2 present the estimated Weibull distributions for plots 41 and 157, while Figs. 3 and 4 present the Johnson's S_B estimated distributions for the same plots, respectively. In Figs. 1 and 2, MLE appears to have the best fit to the data, whereas in Figs. 3 and 4, REG and CML appear to have the best fit. Perhaps most striking from these graphs is the difference between plots with the mode-based method for the Johnson's S_B distribution. When the mode and the minimum diameter have the same value, the mode-based method produces a very poor fit, but when the mode lies at a different diameter, the fit improves dramatically. Hafley and Buford (1985) point out, however, that their method should not be applied to conditions where the mode of the distribution lies at diameter extremes.

To assess the statistical significance of the Reynolds' error indices for the different estimation methods, we performed paired t tests, which compared the difference (pairing by plot) between the Reynolds' error indices for any two estimation methods. Table 4 presented the results of these paired t tests for each possible pair of estimation methods. To interpret the paired t statistics, we recognized that the difference between the error indices of two methods is calculated as $\text{method}_{\text{row}} - \text{method}_{\text{column}}$, where row and column refer to the position of the estimation methods being compared in Table 4. Calculated in this way, a paired t statistic greater than zero implies that $\text{method}_{\text{row}}$ has, on average, larger error indices than $\text{method}_{\text{column}}$ and, consequently, a poorer fit to the empirical distributions. Conversely, a negative paired t statistic implies that $\text{method}_{\text{row}}$ has a comparatively better fit than $\text{method}_{\text{column}}$. In this context, Table 4 showed that REG for the Johnson's S_B had significant lower error indices when compared with any other estimation method. For example, the comparison between CML and REG resulted in $t = 2.93$ ($p = 0.004$), and the comparison between REG and MLE resulted in $t = -6.27$ ($p < 0.001$). Clearly, MODE for the Johnson's S_B showed significantly larger error indices than any other method, with paired t statistics not less than 12.68.

As noted by Shiver (1988), for maximum likelihood, moment, and percentile estimation of Weibull parameters, it is desirable to have sample sizes of at least 50 to limit the prediction error for a particular diameter class to no more than 10%. Since the number of trees per plot was relatively low in the data (ranging from 22 to 81), it was concerned that some estimation methods may be sensitive to sample sizes. The mean of the Reynolds' error index was computed for three categories of sample sizes (i.e., trees per plot) as follows: <40 (80 plots), 40–60 (51 plots), and >60 (7 plots) (Table 5). Basically, the relative performance of the estimation methods was consistent across the three sample size groups.

Conclusion

Data used in this study represented the spruce–fir ecological habitat type in northeastern North America. The species

Table 4. Comparison of Reynolds' error index for the estimation methods of the Weibull and Johnson's S_B distributions by paired t test.

Estimation method									
Estimation method	Johnson's S_B								
	Weibull								
	CML	KB	MODE	REG	MLE	MOM1	MOM2	PPM	
	CML	—	−7.51 (<0.001)	−17.85 (<0.001)	2.93 (<0.004)	−9.81 (0.001)	−9.48 (<0.001)	−11.50 (<0.001)	−9.14 (<0.001)
	KB	—	—	−15.18 (<0.001)	8.73 (<0.001)	5.08 (<0.001)	−4.26 (<0.001)	−2.21 (0.028)	1.96 (0.052)
	MODE			—	17.95 (<0.001)	17.45 (<0.001)	12.68 (<0.001)	15.00 (<0.001)	16.11 (<0.001)
	REG			—	—	−6.27 (<0.001)	−10.06 (<0.001)	−16.02 (<0.001)	−10.12 (<0.001)
	MLE				—	—	−7.73 (<0.001)	−8.30 (<0.001)	−5.29 (<0.001)
	MOM1					—	—	2.70 (0.008)	5.39 (<0.001)
	MOM2							—	5.18 (<0.001)

Note: CML, conditional maximum likelihood; KB, Kneib–Burkhardt; MODE, mode based; REG, linear regression; MLE, maximum likelihood; MOM1, method of moments incorporating skewness; MOM2, traditional method of moments; PPM, percentile based. Numbers in parentheses are p values for the tests.

Table 5. Mean of Reynolds' error index across three sample size groups for the estimation methods of the Weibull and Johnson's S_B distributions.

No. of trees per plot	No. of plots	Estimation method							
		Johnson's S_B				Weibull			
		CML	KB	MODE	REG	MLE	MOM1	MOM2	PPM
<40	80	65.0	80.5	169.1	60.1	70.8	91.8	85.6	76.4
40–60	51	74.7	108.3	251.9	72.1	82.1	133.6	114.3	94.7
>60	7	70.0	93.0	327.3	68.4	81.9	104.1	123.6	101.3

Note: CML, conditional maximum likelihood; KB, Knoebel–Burkhart; MODE, mode based; REG, linear regression; MLE, maximum likelihood; MOM1, method of moments incorporating skewness; MOM2, traditional method of moments; PPM, percentile based.

composition of these stands ranged from pure balsam fir to pure red spruce to various mixtures of the two species. The observed diameter distributions of the whole plots were typically positively skewed, reverse-J, and mound shapes. This study found that the relative merits of the Weibull and Johnson's S_B distributions depended not on the inherent flexibility of the probability density functions themselves, but rather on the accuracy of the methods employed to estimate the functions' parameters. Those methods that best fit the observed frequency distributions used either the entire tree list to generate the PDF parameters (e.g., CML for the Johnson's S_B and MLE for the Weibull) or a number of percentiles (e.g., nine percentiles for REG method for the Johnson's S_B). In contrast, the methods that fit the poorest (namely MOM1 and MOM2 procedures for the Weibull and MODE method for the Johnson's S_B) used only three or four statistics to generate the parameters. As more information from the tree list was used to fit the Weibull and Johnson's S_B distributions, a more accurate representation of the diameter distribution was obtained. This study supported the finding by Zhou and McTague (1996) that the REG method was superior to the CML method. While not surprising, our results also supported the results by Nanang (1998) and Zarnoch and Dell (1985) that indicated MLE estimators as superior for use with the Weibull when compared with percentile estimators. In particular, Zarnoch and Dell (1985) indicated that percentile estimators have greater bias than MLE estimators. In contrast, Shiver (1988) argued that modified moment estimators proposed by Cohen and Whitten (1982) and percentile estimators were just as good at reproducing simulated diameter distributions. In conclusion, the Johnson's S_B distribution and the linear regression method for the parameter estimation are recommended for fitting the diameter distributions of mixed spruce–fir stands in north-eastern North America.

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