Considerations in simultaneous curve fitting for repeated height–diameter measurements

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The problem of fitting height–diameter curves for repeated measurements on growth plots is addressed. The context of the problem is fitting historical data with varying sampling protocols and varying measurement accuracy. A key consideration is obtaining good estimates of top height and top-height increment. A particular model and objective function for fitting are presented. The model has two parameters for each measurement and one common parameter; limited crossovers in the height–diameter curves for the various measurements are allowed. The objective function minimizes errors in predicted height and in predicted change in height. The programming is described, and the availability of code is announced. Examples show both the strengths and weaknesses of this approach.

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Le problème d'ajustement des courbes hauteur–diamètre pour des mesures répétées sur des parcelles arrivées à maturité est soulevé. Le contexte du problème est l'ajustement de données historiques avec des protocoles d'échantillonnage variés et une précision de mesures variable. Une considération clef est l'obtention de bonnes estimations de la hauteur de la cime et de son accroissement. Un modèle particulier et une fonction objective pour l'ajustement sont présentés. Le modèle a deux paramètres pour chaque mesure et un paramètre commun; un nombre limité de croisements dans les courbes hauteur-diamètre pour les différentes mesures est permis. La fonction objective minimise les erreurs pour la prédiction de la hauteur et des changements de hauteur. La programmation informatique est décrite et la disponibilité du programme source est annoncée. Des exemples illustrent à la fois les forces et les faiblesses de cette approche.

[Traduit par la rédaction]

Introduction

Growth plot data often include measured diameter at breast height (DBH) on all trees and measured height (h) on a subset of trees. The selection of height-sample trees often changes between measurements. Regression procedures to estimate height as a function of DBH and measurement number are commonly used in preparing growth and yield summaries. Procedures have been discussed by Curtis (1967), Hyink et al. (1988), and Omule and MacDonald (1991).

In the course of a growth-modelling project, we have recently fitted height–diameter curves for growth plots in stands of western hemlock (*Tsuga heterophylla* (Raf.) Sarg.). The data came from eight different agencies, and involved numerous sampling protocols. We devised criteria to determine whether a plot's height sample was adequate for fitting, developed a computer program to fit height–diameter curves, selected procedures to assess the acceptability of the resultant curves, and developed interpolation procedures for ages that lack height measurements. This article describes the fitting procedures in detail, and touches on the other steps involved in reaching acceptable height–diameter curves. Our purpose is to promote discussion of fitting criteria and methods; there is no claim or evidence that the procedures presented are "best" in any statistical sense.

Though top height is not involved in any of the proposed computations, it is a primary variable in the growth model being constructed. The Forest Productivity Councils of British Columbia (1990) defines top height in a single-species stand as the average height of the 100 trees/ha with the largest DBHs; a sampling protocol is being adopted that will address plot-size concerns discussed by Rennolls (1978) and Flewelling (1983). Unless all of the top height trees have been measured for height, the calculation of top height requires fitted height–diameter curves. The resultant estimates of top-height increment can be quite erratic; often the estimated 5-year increment in top height will double or triple from one 5-year period to the next. With the hemlock growth plot data, the erratic estimates of top-height increment could occur on plots where the remeasured height trees exhibited little fluctuation in growth rate; the evidence indicated that top height, had it been measured, would have progressed far more smoothly than did the estimated top height.

Model

The height–diameter model is

$$[1] \quad h_i(\mathrm{DBH}) = \mathrm{BH} + a_i \exp[b_i(\mathrm{DBH})^{-c}] \qquad \text{for DBH} > 0$$

where h_i(DBH) is the height–diameter function at measurement i, BH is breast height (1.3 m), a_i is a parameter controlling the asymptote for measurement i, b_i is a scale parameter for measurement i, and c is a shape parameter which remains constant for all measurements on a plot. Thus i indexes time, i = 1, 2, ..., m, and m is the total number of measurement occasions.

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Constraints on parameters are

[2] $a_1 > 0$

[3] $a_i \geq a_{i-1}$ for $i > 1$

[4] $b_i < 0$ for all i.

[5] $0.5 \leq c \leq 2.0$

[6] $h_i(\text{DBH}) \geq h_{i-1}(\text{DBH})$ for all DBH $\geq \text{DQ}_i$ and $i > 1$

where DQ_i is the quadratic mean DBH for measurement i. The above function constraint is equivalent to a parameter constraint:

[7] $b_i \geq b_{i-1} - (\log(a_i/a_{i-1}))(\text{DQ}_i)^c$ for $i > 1$

The effect of the constraints are that the asymptotes must increase between measurements [3], that the predicted heights must increase as DBH increases [4], and that crossovers can only occur below DQ. Crossovers imply that at small DBHs, a curve for one measurement predicts a lesser height than does the curve for some earlier measurement, with the opposite being true at large DBHs.

Objective function

Notation for data and predictions are given here. As before, the index i refers to measurement number or to the growth period starting with measurement i; m is the total number of measurements. The index j refers to tree number.

h_{ij}	Observed height
$\hat{h}_{ij}$	Predicted height from [1]
ϵ_{ij}	Error in predicted height: $h_{ij} - \hat{h}_{ij}$
δij	Change in height: $h_{(i+1)j} - h_{ij}$
e_{ij}	Error in change in height: $\delta_{ij} - \hat{\delta}_{ij} = (h_{(i+1)j} - h_{ij}) - (\hat{h}_{(i+1)j} - \hat{h}_{ij})$

The fitting objective is to minimize the function

[8] $$\Phi = \sum_{i=1}^{m} w_i \text{SSH}_i + \sum_{i=1}^{m-1} u_i \text{SSD}_i$$

where the first summation is over the measurements, and the second summation is over the periods. The sums of squares are defined as

[9] $$\text{SSH}_i = \sum (\epsilon_{ij})^2$$

[10] $$\text{SSD}_i = \sum (e_{ij})^2 \times \max\ (1, \text{DBH}_{ij}/\text{DQ}_i)$$

where the respective summations are over all the applicable measurement trees for the measurement or period. Thus SSH_i and SSD_i are sums of errors squared, in height and change in height respectively, at measurement or period i; the trees in [9] are those with measured heights at measurement i; the trees in [10] are those with measured heights at both measurements i and $i + 1$. The latter sum [10] is weighted by tree, within a period, using weights of 1 for large trees, and weights proportional to DBH for trees with DBH < DQ_i. The values w_i and u_i are weights applicable to measurements and periods, respectively.

Weighting is used because variances on some plots can differ by an order of magnitude between the first and last measurements. The weighting functions, w_i and u_i, are inverses of estimated variances. To obtain estimated variances, residuals from preliminary model fits are summarized. The model used here is [1], fit independently for each measurement, with $c = 1$, and without regard for the between-measurement constraints ([3] and [6]). Assuming that all errors are estimated about the preliminary model fits, the weights are assigned inversely proportional to the estimated variances:

[11] $$\hat{\sigma}_i^2 = \frac{\sum (\epsilon_{ij})^2}{(n_i - 2)}$$

[12] $$w_i = \frac{1}{\hat{\sigma}_i^2}$$

The procedure used to obtain u_i (increment error weighting term in [8]) also relies on the preliminary model fits. The error in change in height can be shown to be

[13] $$e_{ij} = \epsilon_{(i+1)j} - \epsilon_{ij}$$

The weight u_i is set to the inverse of the estimated variance of e_{ij}. The true variance is

[14] $$\text{var}(e_{ij}) = \sigma_{(i+1)}^2 + \sigma_i^2 - 2\text{cov}(\epsilon_{(i+1)j}, \epsilon_{ij})$$

The estimate of the variance is obtained from the above equation, using estimates of σ^2 from [11] and the following covariance estimate:

[15] $$\hat{\text{cov}} = \hat{\rho}[\hat{\sigma}_{(i+1)}^2 \times \hat{\sigma}_i^2]^{0.5}$$

with

[16] $$\hat{\rho} = \frac{\sum (\epsilon_{(i+1)j}\epsilon_{ij})}{\left\{\left[\sum (\epsilon_{(i+1)j})^2\right]\left[\sum (\epsilon_{ij})^2\right]\right\}^{0.5}}$$

where the three summations are over the trees with height data for both measurements i and $i+1$. The resultant estimate of ρ is constrained to the range 0–0.8. The effect of ρ can be easily explained for a plot with two measurements having equal estimated variances. If $\rho = 0$, then errors in change in height are given half the weight of errors in height. If $\rho = 0.5$, errors in height and change in height are given equal weights. If $\rho = 0.8$, errors in change in height are given 2.5 times the weight of errors in height. Thus the decision to limit the range of ρ can be seen as placing a limit on the relative weight given to growth estimation. In all cases, the numbers of measured trees for each measurement, and for the growth period, affect the relative importance assigned to the two terms in [8].

Once the weighting factors are determined for a plot (on the basis of the preliminary single-measurement fits), they are never changed; there is no attempt at iteratively reweighted least squares.

Implementation

A FORTRAN computer program[2] has been written to find estimates of the parameters (c, a_i, and b_i, $i = 1, 2, \ldots, m$) that produce a minimum value of Φ. This is not a trivial task. Briefly, the steps are:

(1) Calculate the preliminary model fits for a_i and b_i in [1], one measurement at a time, using nonlinear least squares; c is set to 1. The Levenberg–Marquardt method described by Press et al. (1986) is the fitting method; starting values are from a least squares fit applied to a linearized equa-

[2]FORTRAN code and brief documentation may be obtained from either of the authors. Optimization subroutines (Press et al. 1986) may be obtained at nominal cost from Cambridge University Press or may be found on a diskette distributed with some Microsoft FORTRAN compilers.

TABLE 1. Parameter estimates for first numeric example, by measurement (i): intermediate method and proposed method

	Intermediate method			Proposed method		
i	a_i	b_i	c	a_i	b_i	c
1	15.37	−0.021	0.8867	16.63	−1.101	0.9564
2	24.46	−4.618	0.8867	24.26	−5.388	0.9564
3	34.68	−8.449	0.8867	31.96	−9.037	0.9564
4	34.71	−8.096	0.8867	32.81	−9.005	0.9564
5	38.37	−9.061	0.8867	36.15	−10.05	0.9564

tion. The constraint on b_i [4], is programmed as $b_i \leq 0$ in this and subsequent optimization procedures; after the optimization process is complete, the solution is adjusted to enforce strict inequality.

(2) Calculate measurement weights, w_i, and period weights, u_i. On the basis of some arbitrary limits not described here, constrain the weights to prevent any one period from overwhelming the others.

(3) Optimize the coefficients with respect to the SSH component of Φ. An initial attempt at this involves specifying a set of trial values for c, applying the methods described in step 1 to each measurement at each value of c, and selecting the c value which yields the lowest value of $\Sigma\ w_i\text{SSH}_i$. If the solution after this initial attempt violates no constraints, the current step has been accomplished. If constraints on parameters have been violated, then a simplex method attributable to Nelder and Mead (1965) can perform the optimization on the $2m + 1$ parameters (a_i, b_i, and c). Our programming approach also interleaves the $2m + 1$ optimization with suboptimizations on pairs of adjacent measurements. The suboptimizations involve four parameters each, and often produce improvements that the larger optimization has not found.

(4) Optimize the coefficients with respect to Φ as defined in [8]. Use the result from step 3 as the starting point, the weights from step 2, and apply the simplex method. Interleave attempts of global optimization, suboptimization on a_i and b_i, and suboptimization on c.

Numerical examples

An example data set from Omule and MacDonald (1991) is used. That example has five measurements, 10–16 height trees per measurement; each remeasurement includes 5–10 trees from the previous measurement. The example includes all necessary data except DQ; for the five measurements, these DQ are 11.3, 13.6, 14.9, 16.0, and 17.0 cm. The plot size is such that top height is based on the five largest DBH trees in the plot; this set of five trees is included in the height sample at each measurement.

The proposed method and an intermediate method have both been applied to the data; the methods are identical except that the objective function for the intermediate method omits consideration of the increment errors (second summation in [8]). Fitted parameter values are in Table 1. The fitting process for the proposed model required the use of only one constraint: that controlling the crossover of measurements 2 and 3. A graph of the predicted curves for the proposed method is in Fig. 1. The graph has strong similarities to the independent fit reported by Omule and MacDonald (1991). Its most obvious failing is that the measurement 1 height–diameter curve could predict implausi-

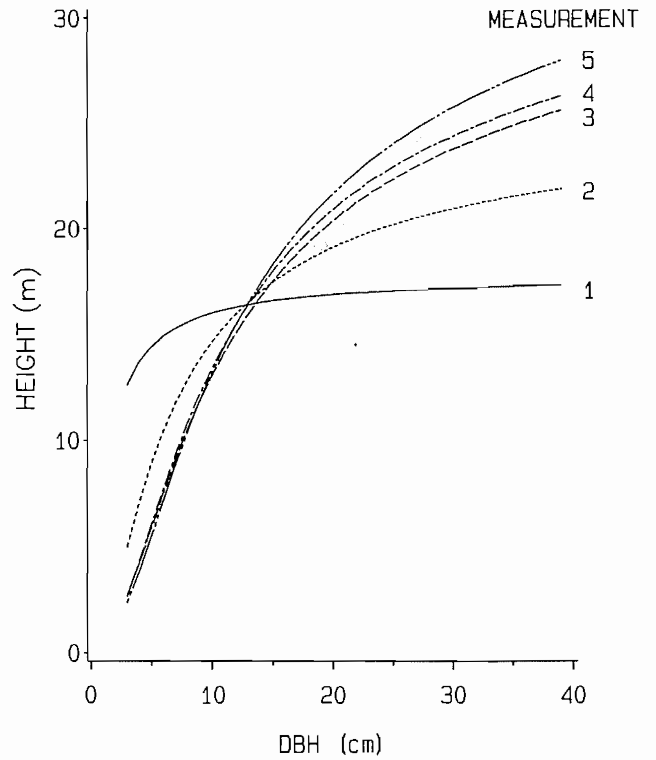

FIG. 1. Height–diameter predictions for proposed method; example 1.

ble heights for trees with small diameters. This happens because the measurement 1 data lack a positive correlation between height and DBH, and because the model does not force the height–DBH functions for all measurements to assume a common shape. This failing is addressed in the Discussion section.

The new methods (proposed and intermediate) are compared with the simultaneous fit by Omule and MacDonald (1991), which we refer to as the full-constraint method. The term simultaneous refers to the fact that the height–diameter functions are not fit one measurement at a time, but are fit for all measurements of a plot simultaneously. Hence the simultaneous label would also apply to the proposed method. The full-constraint method does not allow any crossovers, and is therefore somewhat less flexible than the proposed method. The other major difference is that Omule and MacDonald (1991) use the usual objective of minimizing total sums of error squared, where error refers to height prediction; this objective is similar to that used for the intermediate method. Results from all three methods are compared in Tables 2 through 4.

A second numerical example, using only the proposed method, offers a more forceful demonstration of a crossover. Data from two measurements 3 years apart on an after-thinning young stand with many measured trees are shown in Fig. 2; the modelled crossover has visible data support. The constraint preventing a crossover above the quadratic mean DBH does come into effect; without the constraint, the crossover would be at a DBH of 21 cm. Throughout the hemlock data set there are a large number of plots where crossovers appear to be indicated; this is particularly the case in young stands following a thinning.

A third example illustrates the effect of considering increment errors in the objective function. The intermediate

TABLE 2. Summary of mean heights and errors in heights for first numeric example, for all three fitting methods

			Full-constraint		Intermediate		Proposed	
Measurement	Tree count	Mean actual (m)	Mean error[a] (m)	MSE[b] (m^2)	Mean error (m)	MSE[c] (m^2)	Mean error (m)	MSE[c] (m^2)
1	10	16.6	−0.1	2.0	0.0	1.5	−0.2	1.6
2	9	19.1	1.0	2.6	0.1	1.2	−0.1	1.2
3	10	20.1	−0.4	1.0	−0.1	0.9	0.0	0.9
4	16	19.2	0.1	1.9	0.0	2.0	0.1	2.0
5	13	23.0	−0.5	2.3	0.0	2.3	0.1	2.4

[a]Errors are calculated as actual height minus predicted height (m).
[b]Mean squared errors (MSE) are calculated as the average value of the squared errors, divided by degrees of freedom = $n_i - 1 - 1/5$.
[c]Mean squared errors (MSE) are calculated as the average value of the squared errors, divided by degrees of freedom = $n_i - 2 - 1/5$.

TABLE 3. Summary of mean height changes and errors in change for first numeric example

			Full-constraint		Intermediate		Proposed	
Period i	Tree count	Mean actual (m)	Mean error[a] (m)	MSE[b] (m^2)	Mean error (m)	MSE[b] (m^2)	Mean error (m)	MSE[b] (m^2)
1	9	2.3	1.0	1.3	−0.1	0.30	−0.1	0.28
2	8	2.0	−1.7	2.9	−0.4	0.30	−0.1	0.11
3	5	1.0	0.4	0.2	−0.1	0.06	−0.2	0.08
4	12	1.4	−0.6	0.4	0.1	0.09	0.0	0.09

[a]Errors are calculated as actual change minus predicted change during the period.
[b]Mean squared errors (MSE) are clculated as the average value of the squared errors, without adjustment for degrees of freedom.

TABLE 4. Observed and predicted top height for first numerical example

Measurement	Actual	Full-constraint	Intermediate	Proposed
1	15.9	17.2	16.6	16.9
2	18.9	18.4	19.2	19.4
3	21.7	22.1	21.8	21.6
4	24.4	24.8	24.9	24.7
5	26.5	27.2	26.8	26.6

NOTE: Top height is calculated as the average measured or predicted height of the five largest DBH trees on the 0.05-ha plot

method (which does not specifically address increment) is applied to two measurements on an unthinned fertilized plot (Fig. 3). The proposed method is fit to the same data; Fig. 4 shows the results, and identifies which data points are from common trees. Both methods underestimate the growth of the remeasured trees, but the proposed method has a lesser mean error (Table 5).

Application

The proposed methodology was applied to data from over 1000 plots. The application was successful in that the program did not blow up! Prescreening included checks that at least six height trees were available on every measurement, and that at least one height tree qualified as a top height tree or was within 2 cm in DBH of qualifying. The resultant fits frequently utilized constraints. Nineteen percent of the plots required at least one asymptote constraint [3]; 0.3% of the plots required the slope constraint [4]; 8.5% of the plots required a crossover constraint [6]. Computer execution time is not excessive: the first numerical example requires about 60 s on a personal computer with an Intel 386/33 processor and a coprocessor.

Postscreening checks were made to identify unacceptable fits. This involved graphical checks of the fitted curves, and error analyses on heights and change in height. No plots were rejected on the basis of these checks. However, a small number of plots were conditionally accepted. That is, they were accepted with regard to top height and change in top height, but considered unreliable at small diameters. Figure 1 is an example of this; the source of the problem is that the height sample at the first measurement did not include any trees with DBH less than DQ.

Discussion

The first example raises questions on the appropriateness of fitting and checking criteria. In the previous section, it

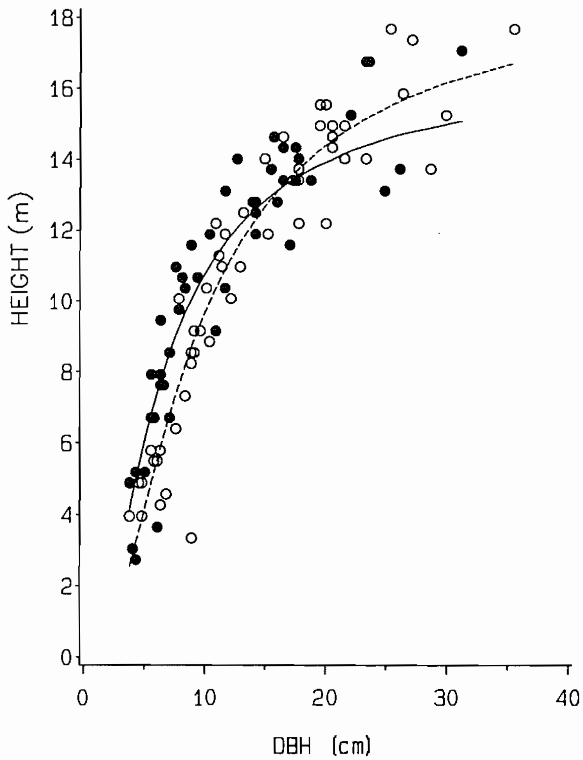

FIG. 2. Height–diameter data and predictions for proposed method; example 2. The solid symbols and solid line are for measurement 1; the open symbols and broken line are for measurement 2.

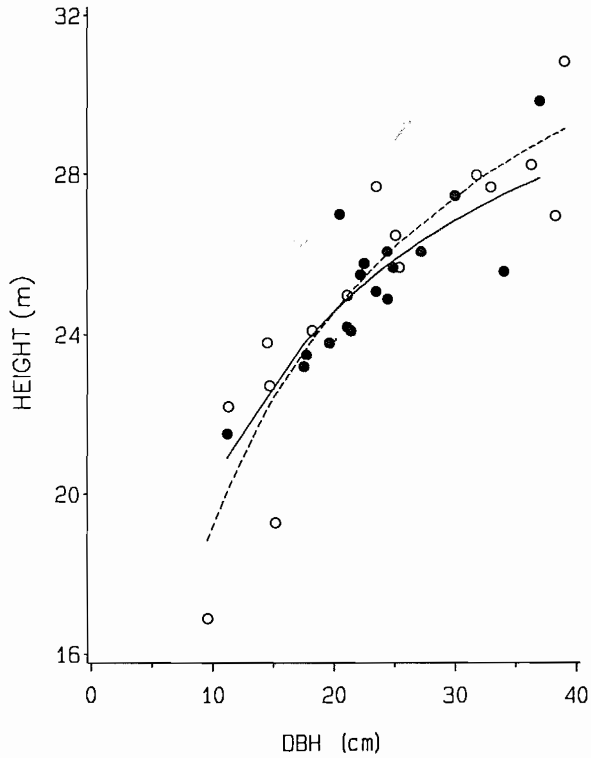

FIG. 3. Height–diameter data and predictions for intermediate method; example 3. The solid symbols and solid line are for measurement 1; the open symbols and broken line are for measurement 2.

was noted that the height–diameter function at measurement 1 would probably be unacceptable for trees with smaller diameters than found in the height–DBH data. Is the full-constraint model by Omule and MacDonald more acceptable? If very small trees exist in the plot, the full-constraint method would do a better job of estimating their heights. In contrast, the proposed model estimates that a tree with a DBH of 3 cm would have a height of 13 m; this prediction is absurd. The method could be modified to prevent such extreme predictions, and such modifications should be made if predictions for small trees are required. In this example, since all of the height-sample trees have DBH > DQ, we question whether it is appropriate to estimate heights for trees that are well beyond the range of the height data. Unless the height estimates are absolutely required, they should not be made. One approach to prevent absurd predictions is to apply species and region specific limits on the allowable range of heights for each DBH[3].

The error statistics comparing the three methods are of interest. The proposed method has negligible mean errors by measurement, and slightly better mean squared errors than the full-constraint method (Table 2). Errors in height growth for remeasured trees are also negligible (Table 3). And the errors in top height for the proposed method are marginally smaller than for the full-constraint method (Table 4). Overall, the proposed method is in closer agreement with the data than is the full-constraint method. Most of this improvement is due to the change in model form, and not to the change in objective function; Table 3 shows that the intermediate method has greatly reduced the mean errors in height increment; the additional improvement afforded by the proposed method is negligible.

The crossover question! Crossovers are exhibited to some degree in all four figures. Some plots with well-sampled, accurately measured heights can be expected to exhibit crossover. Accordingly, the proposed system allows for crossovers; this same decision was reached by Hyink et al. (1988). However, there are some drawbacks to this decision. For example, if a small tree exhibits no growth in DBH, its predicted change in height can be negative. The fault lies not in the prediction, but in the question. For a given diameter, the "best" estimate of height can be lower on a second occasion than on the first occasion. Let's change the question to that of predicting the original height and the change in height given successive measurements of DBH. Unless breakage is allowed for, the predicted changes should always be positive. This is a challenge which we are not currently addressing, but which could be critical for some applications.

The need for crossovers is not proven by Fig. 2 or by the improvement in the error statistics (Tables 2–4). An apparent crossover, such as that in Fig. 2, might be attributed to a sampling effect, coupled with the natural variation about the height–DBH curves. However, for these particular data, there is evidence that the modelled crossover is a fair representation of what is actually occurring on the plot. Over 80% of the trees have measured heights; of the 100 data points in Fig. 2, 90 are from 45 trees measured on both occasions. An examination of the data for remeasured trees with below-average DBH shows that the observed height increments are considerably less than would be predicted by applying the first height–diameter curve to the DBHs at

[3]Fred Martin at the Washington State Department of Natural Resources is developing such an approach, which he plans to publish.

TABLE 5. Mean heights (m) and height increments, actual and predicted, for the third numerical example

Trees	Method	Means h_1	Means h_2	Means $h_2 - h_1$	Mean errors h_1	Mean errors h_2	Mean errors $h_2 - h_1$
All[a]	Actual	25.26	25.05				
	Intermediate	25.26	25.05		0.00	0.00	
	Proposed	25.14	25.35		0.13	−0.29	
Remeasured[b]	Actual	24.97	26.30	1.33			
	Intermediate	25.14	25.65	0.51	−0.16	0.65	0.81
	Proposed	25.01	25.94	0.93	−0.03	0.36	0.39

[a] n = 17 trees on measurement 1, n = 15 on measurement 2.
[b] n = 8 remeasured trees.

both measurements; hence, the second height–diameter curve should be below the first at small DBHs. Our conclusion is that the predicted crossover provides a fair representation of what is occurring on this plot; and that a model that lacks a crossover ability would not do so. However, our decision to allow crossovers is pragmatic: the data are suggestive of crossovers, and allowing crossovers does not introduce any significant problems in our subsequent data analyses. On the other hand, we forced the asymptotes to increase with age [3] because failure to do so had led to some negative estimates of top height increment, with detrimental effects upon the stand growth model that we were attempting to fit.

Prior to including height growth in the objective function, we had many plots with quite irregular top-height increments. We concluded that a sizable number of irregularities were associated with sampling error, and that an objective function that placed some importance on achieving small residuals for change in height would produce more credible patterns of top-height increment. The third numerical example illustrates how the new objective function produces height–diameter curves with smaller errors for height increment. This new objective function did improve the regularity of estimated top-height increment for many plots. These improvements are judged to be worthwhile for the project at hand: fitting a growth model, rather than a yield model. An alternative strategy to smooth the estimated top-height increments is to require the asymptotes (a_i) to be a parametric function of age (Curtis 1967); this strategy was not adopted because the between-period differences in estimated top-height increment would be less influenced by the data, and would be controlled to a significant degree by the model formulation.

Two areas not yet discussed are extrapolation and interpolation on stand age. Some plots have insufficient or no height data at a particular measurement. If the age of that measurement lies between ages with adequate height data, interpolation should be considered. One way to interpolate is to let a_i vary linearly with age between two valid height measurements; b_i could also be made to vary linearly with age, but this strategy can lead to constraint violations and nonsmooth estimates of growth rate in top height. We are using an alternative strategy for adjusting b_i. We assume that some hypothetical tree might have DBH = DQ at all measurements with adequate height samples. Between such measurements, we assume that the tree's DBH varies linearly with age. At ages without adequate height samples, we calculate the value of b_i that would cause the tree's predicted height to vary linearly with age. This strategy seldom produces crossovers that would violate constraint [6]; in most cases where the constraint is violated, [7] can be converted to an equality, and used to find valid values of b_i. This interpolation procedure is one of many possible methods to obtain plausible interpolated values. We have not attempted to develop any procedures to extrapolate the height–diameter curves in age.

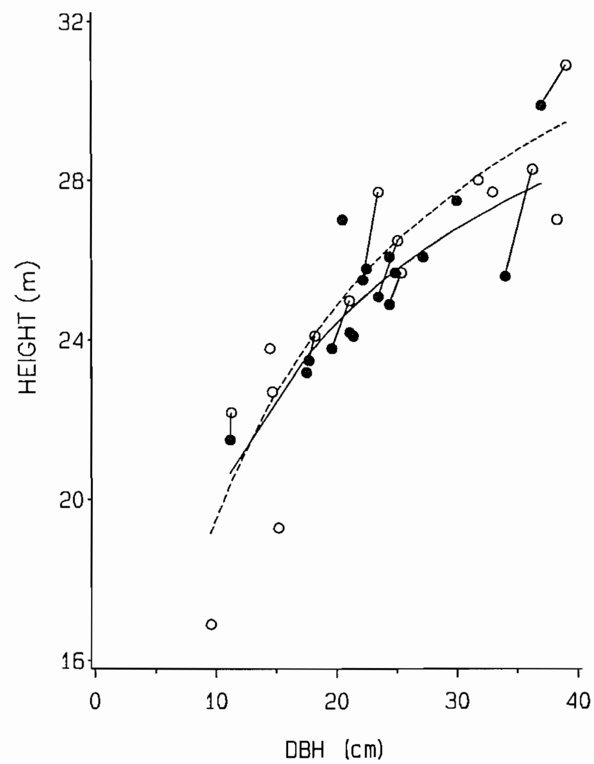

FIG. 4. Height–diameter data and predictions for the proposed method; example 3. The solid symbols and solid height–diameter curve are for measurement 1; the open symbols and broken line are for measurement 2. Short solid lines connect the two data points for each remeasured tree.

Finally, most benefits in using this objective function will be lost if there are few remeasured trees. Arabatzis and Burkhart (1992) show that if height–diameter curves are fit independently for each measurement, then deliberate resampling of trees may lead to a small loss of accuracy for mean height and volume at the later measurements. We would speculate that deliberate resampling of trees and simultaneous fits of equations using the objective function [8] may lead to superior estimates of change in height and volume. Furthermore, deliberate resampling promotes improved error detection and correction in the field.

Summary

A proposed methodology for simultaneously estimating several height–diameter curves has been presented. The novel feature is the objective function. This involves error terms in both height and change in height. Weighting within the objective functions is inversely proportional to variance; however some parts of the formulas are arbitrary. Though the resulting fits appear satisfactory, there is no assurance that the fits are "best" or even "good" in any statistical sense. The choice of model form, constraints and objective function should be pragmatic; a good combination of these elements will result in plot summary statistics which are satisfactory for a particular application.

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Arabatzis, A.A., and Burkhart, H.E. 1992. An evaluation of sampling methods and model forms for estimating height–diameter relationships in loblolly pine plantations. For. Sci. **38**: 192–198.

Curtis, R.O. 1967. Height–diameter and height–diameter–age equations for second-growth Douglas-fir. For. Sci. **13**: 365–375.

Flewelling, J.W. 1983. Estimation of future height growth in inventory stands. Southern Forestry Research, Weyerhaeuser Co., Hot Springs, Ark. Tech Rep. 050-0001-02/1.

Forest Productivity Councils of British Columbia. 1990. Minimum standards for the establishment and measurement of permanent sample plots in British Columbia. Inventory Branch, B.C. Ministry of Forests, Victoria.

Hyink, D.M., Scott, W., and Leon, R.M. 1988. Some important aspects in the development of a managed stand growth model for western hemlock. *In* Forest growth modelling and prediction. Vol. 1. *Edited by* A.R. Ek, S.R. Shifley, and T.E. Burk. USDA For. Serv. Gen. Tech. Rep. NC-120. pp. 9–21.

Omule, S.A.Y., and MacDonald, R.N. 1991. Simultaneous curve fitting for repeated height–diameter measurements. Can. J. For. Res. **21**: 1418–1422.

Nelder, J.A., and Mead, R. 1965. A simplex method for function minimization. Comput. J. **7**: 308–313.

Press, W.H., Flannery, B.P., Teukolsky, S.A., and Vetterling, W.T. 1986. Numerical recipes. Cambridge University Press, Cambridge, UK.

Rennolls, K. 1978 "Top height"; its definition and estimation. Commonw. For. Rev. **57**: 215–219.