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Paper

Optimization of Multi-Period Bilevel Supply Chain Planning for Single Supplier and Single Retailer under Demand Uncertainty*

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This paper presents the optimization of multi-period bilevel supply chain planning for single supplier and single retailer with leader-follower relationship under demand uncertainty. The multi-period production planning problem for the supplier and the retailer is formulated as the mixed integer nonlinear bilevel programming problem. In order to improve the total profit, the effects of the replacement of leader-follower relationship are investigated. The effectiveness of the replacement of leader-follower relationship is examined by some numerical experiments. A quantity discount contract with quadratic function is also proposed to maximize the total profit under demand uncertainty.

1. Introduction

Global supply chains are receiving much attention due to globalization of market places. In such situation, it is required to optimize total supply chain with cooperative or non-cooperative relationship among supply chain companies in order to maximize the total profit. However, it is difficult to coordinate the production planning between these companies because companies sometimes individually make decisions in order to maximize their own profit. The total profit in decentralized situation is inferior to the profit in centralized situation. It is important to consider the leader-follower relationship in the production planning because the organizations in global supply chain make their decisions under the leader-follower relationship in realistic situations. The game theoretical approach can be applied to find the equilibrium. The game which is conducted by a leader and a follower is called a Stackelberg game. In the Stackelberg game, the leader can predict the follower's optimal response when the leader determines its production planning. On the other hand, the follower can optimize its own sales planning under the leader's decisions. In [10], the multi-period production planning problem has been formulated as a nonlinear bilevel programming problem under uncertain demand. Several solution algorithms to find the equilibrium have been compared and the effectiveness of utilizing the Karush-Kuhn-Tucker (KKT) condition for reformulation of the bilevel programming problem into the single-level problem has been confirmed. Then, the replacement of leader-follower relationship in a multi-period supply chain planning has been proposed. However, the effectiveness of the replacement of leader and follower has not been examined in several cases and the introduction of optimal quantity discount policy has not been considered. In this paper, we investigate the effects of the replacement of leader-follower relationship in the bilevel supply chain planning under demand uncertainty. The purpose of the study is to propose the optimal policies in the bilevel production planning for single supplier and single retailer. The supplier is the leader at the beginning of the period, and then the leader is changed from the supplier to the retailer in the middle of periods in the multi-period bilevel production planning. This study examines the effectiveness of the replacement of the leader-follower relationship to the total profit in the decentralized problem and the effects of quantity discounts to improve the total profit from numerical experiments.

The paper is organized as follows. Section **2** introduces literature review. Section **3** states the problem description and the formulation of the replacement model of leader-follower relationship. Section **4** presents the optimization method of multi-period bilevel production planning under demand uncertainty. Section **5** describes the optimal quantity discount contract to improve the total profit of the bilevel production planning for single supplier and single retailer. In section **6**, the effects of the replacement of leader-follower relationship in the middle of periods are stud-

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ied. Then, the effectiveness of the quantity discount contract is examined to improve the total profit. The conclusion and future works are stated in Section **7**.

2. Literature Review

The general optimization algorithms for bilevel programming problems have been introduced [2]. Yue and You proposed an optimization approach for a multi-echelon supply chain in a Stackelberg game by using a piecewise linear approximation of nonconvex functions [11]. Qu et al. studied the optimal configuration of cluster supply chains [6]. They proposed the policies to convert the cluster supply chains into a reconfigurable model. Almehdawe studied the leader-follower relationship in a supply chain in which a manufacturer and multiple retailers make decisions [1]. They compared the total profit in the leader-manufacturer problem with the total profit in the leader-retailer problem. However, the replacement of the leader-follower relationship has not been studied so far.

The multi-period bilevel production planning is analyzed in order to respond to some changes such as demand uncertainty or inventory quantity through multi-period models. In some cases, it is considered that the leader-follower relation should be exchanged in the middle of periods [4]. There are some studies on nonlinear bilevel programming models for multi-period production planning under demand uncertainty. Zhang and Shi analyzed the retailer's multi-period production planning with a quantity discount contract under demand uncertainty [12]. Yoshida and Nishi addressed an analytical approach to the multi-period production planning for single supplier and single retailer under demand uncertainty [9]. The production planning problems were formulated in order to be analyzed theoretically. However, these models are not detailed for the realistic models where the optimal solution cannot be derived analytically, e.g. the setup costs cannot be considered. We extend the model into the mixed integer nonlinear bilevel programming model that can represent more realistic situations in this paper. Supply chain contracts are introduced into the bilevel production planning in order to improve the total profit. Tsai introduced an optimization approach to supply chain with quantity discount policy [7]. Yin et al. studied the bilevel production planning [8]. They also studied the optimal quantity discount contract in order to maximize the total profit in the Stackelberg game.

3. Problem Definition

We consider the multi-period production planning for single supplier and single retailer. In the model, the supplier and the retailer make their decisions according to the Stackelberg game. We assume two-echelon supply chain where there are a single supplier and a single retailer. The supplier produces the product items and the retailer deliveries the product items to customers. The supplier and the retailer make their decisions individually. The customer's demand is assumed to be uncertain, and the demand is given by a probability distribution. The supplier has enough raw materials and storage space to produce product items in order to satisfy the retailer's order quantity. The setup cost is incurred when the supplier produces different product item from that of a previous time period. The inventory holding cost is required when the supplier holds inventories due to over production. The supplier deliveries the product item to the retailer and the retailer deliveries the product item to the customer. The product items of the retailer is delivered to the customer at the end of a period. If the quantity of the product item delivered by the retailer is less than the realized demand, the retailer pays the penalty cost for inventory shortage. The quantity of shortage in a certain period is not the demand in the next period. The supplier and the retailer make their decisions under a price contract in which the supplier decides the wholesale price and the retailer decides the order quantity for the supplier's wholesale price. In the multi-period bilevel production planning, the single supplier and the single retailer maximize their own profit under the Stackelberg game where the supplier is the leader and the retailer is the follower. The supplier decides the wholesale price in order to maximize its profit. The retailer decides the order quantity with the supplier's wholesale price in order to maximize the profit under demand uncertainty. Customer's demand is represented by a normal distribution. The supplier's objective function is the wholesale revenue minus production cost minus inventory holding cost minus setup cost. The retailer's objective function is the sales revenue minus inventory holding cost minus the penalty cost of the inventory shortage minus wholesale cost.

The multi-period bilevel production planning problem where the supplier is a leader and the retailer is a follower is formulated as follows.

<u>Parameters</u>

c_{tj} : supplier's production cost of one unit of product item j in period t

D_{tj} : random variable of demand of product item j in period t

$f(D_{tj})$: probability density function followed by demand of product item j in period t

$\Phi(D_{tj})$: cumulative distribution followed by demand of product item j in period t

g_{tj} : retailer's penalty cost for inventory shortage of one unit of product item j in period t

h^r_{tj} : retailer's inventory holding cost of one unit of product item j in period t

h^s_{tj} : supplier's inventory holding cost of one unit of product item j in period t

J : number of product items

K_{tj} : ratio of the average demand of product item j to retailer's sales revenue in period t

M : positive large number

p_{tj} : retailer's unit sales revenue of one unit product

item j in period t
s_{tj} : setup cost of product item j in period t
T : total time horizon
U_t : number of product items which can be produced by the supplier in period t
$w^{\min}$, $w^{\max}$: minimum and maximum values of the supplier's wholesale price of one unit of product item j per period
Decision variables
H_{tj} : retailer's inventory shortage quantity of product item j in period t
I^r_{tj} : retailer's inventory quantity of product item j in period t
I^s_{tj} : supplier's inventory quantity of product item j in period t
O^s_{tj} : supplier's production quantity of product item j in period t
Q_{tj} : retailer's order quantity of product item j to the supplier in period t
R_{tj} : retailers sales revenue of product item j in period t
S_{tj} : retailer's delivery quantity of product item j to customer in period t
V_{tj} : retailer's inventory quantity of product item j in period t
w_{tj} : supplier's wholesale price per unit product item j in period t
x_{tj} : 0-1 variable which equals 1 if supplier does not produce product item j in period $t-1$ and produces product item j in period t, otherwise equals 0
y_{tj} : 0-1 variable which equals 1 if supplier produces product item j in period t, otherwise 0.

Supplier's decision problem

The supplier's profit maximization problem is formulated as follows.

$$\max\ J_s=\sum_{t=1}^{T}\sum_{j=1}^{J}(w_{tj}Q_{tj}-c_{tj}O_{tj}-h^s_{tj}I^s_{tj}-s_{tj}x_{tj}) \tag{1}$$

$$\text{s. t. } I^s_{tj}=I^s_{t-1j}+O_{tj}-Q_{tj},\qquad \forall t,\ \forall j \tag{2}$$

$$O_{tj}\ge Q_{tj}-I^s_{t-1j},\qquad \forall t,\ \forall j \tag{3}$$

$$\sum_{j=1}^{J}y_{tj}\le U_t,\qquad \forall t \tag{4}$$

$$x_{tj}\ge y_{tj}-y_{t-1j},\qquad \forall t,\ \forall j \tag{5}$$

$$O_{tj}\le My_{tj},\qquad \forall t,\ \forall j \tag{6}$$

$$w^{\min}_{tj}\le w_{tj}\le w^{\max}_{tj},\qquad \forall t,\ \forall j \tag{7}$$

$$O_{tj},\ I^s_{tj}\ge 0,\qquad \forall t,\ \forall j \tag{8}$$

$$x_{tj},\ y_{tj}\in\{0,\ 1\},\qquad \forall t,\ \forall j \tag{9}$$

$$\max\ J_r=\sum_{t=1}^{T}\sum_{j=1}^{J}E(R_{tj}-V_{tj}-H_{tj})-w_{tj}Q_{tj} \tag{10}$$

s. t.

$$I^r_{tj}=I^r_{0,j}+\sum_{t'=1}^{t}[Q_{t'j}-\min(D_{tj},S_{tj})],\ \forall t,\forall j \tag{11}$$

$$R_{tj}=p_{tj}\min(D_{tj},S_{tj}),\ \forall t,\forall j \tag{12}$$

$$V_{tj}=h^r_{tj}I^r_{tj},\ \forall t,\forall j \tag{13}$$

$$H_{tj}=g_{tj}\max(0,D_{tj}-S_{tj}),\forall t,\forall j \tag{14}$$

$$Q_{tj}+I^r_{t-1j}-S_{tj}\ge 0,\forall t,\forall j \tag{15}$$

$$I^r_{tj},\ Q_{tj},\ S_{tj}\ge 0,\qquad \forall t,\forall j \tag{16}$$

The supplier's decision variables are the quantity of inventories, production quantity, wholesale price, and 0-1 variable x_{tj} which represents the setup of product item j in period t. The supplier's objective function is the wholesale revenue to the retailer minus production cost minus inventory holding cost minus setup cost. The supplier's constraints include the inventory balancing constraints (2), the constraints on the lower value of the production quantity (3), the constraints on the number of product items to be produced (4), the setup constraints (5), the constraints on the production capacity of production quantity (6), and the lower and upper value of the wholesale price (7). The retailer's profit maximization problem is formulated as (10)-(16) where $E(X)$ is the expected value of the term X in the objective function due to demand uncertainty. The retailer's decision variables are the inventory quantity, the order quantity and the delivery quantity. The retailer's objective function is the sales revenue to the customer R_{tj} minus the inventory holding cost V_{tj} minus the penalty cost of the inventory shortage H_{tj} minus the wholesale cost $w_{tj}Q_{tj}$. The retailer's constraints include the inventory balancing constraint and the constraint on the upper value of the delivery quantity. In the retailer's profit maximization problem, the sales profit, the inventory holding cost and the penalty cost for the inventory shortage are expressed as the expected values due to demand uncertainty.

3.1 Replacement of Leader-follower Relationship

Most conventional study on the bilevel production planning assume that the configuration of the leader-follower relationship is fixed. In order to respond to demand uncertainty and cost changes caused by uncertain factors, we propose a model that can change the leader-follower relationship in which the supplier is the leader at the beginning of period and the leader is changed from the supplier to the retailer in the middle of periods. The multi-period bilevel production planning problem with (1)-(16) is the leader-supplier problem. In order to formulate the leader-follower replacement model, it is required to formulate the leader-retailer production planning problem. The leader-retailer problem is formulated by using the supplier's optimal response function to the retailer's decision making. However, the supplier's optimal response function of the problem with (1)-(16) is assumed to be constant because the supplier's objective function (1) is the monotone increasing function with the supplier's wholesale price regardless of the retailer's order quantity. In this section, the multi-period bilevel production planning is reformulated in order to obtain the supplier's optimal response function. In the revised multi-period bilevel production planning model, customer's demand is assumed to depend on the retailer's sales revenue p_{tj} of product item j in period t. The average of customer's demand μ_{tj} is reformulated as (17) with the retailer's sales revenue p_{tj}.

$$\mu_{tj} = K_{tj} {p_{tj}}^{-e_{tj}}, \qquad \forall t,\ \forall j \tag{17}$$

Both e_{tj} and K_{tj} are positive constants. In the leader-follower replacement model, the retailer's sales revenue p_{tj} of product item j in period t is the retailer's decision variable. In a realistic situation, the retailer's sales revenue p_{tj} is larger than the supplier's wholesale revenue w_{tj}. In order to realize $p_{tj} > w_{tj}$, the following equation (18) is introduced in the model.

$$p_{tj} = a_{tj} w_{tj}, \qquad \forall t, \forall j \tag{18}$$

a_{tj} is positive number which is larger than 1 ($a_{tj} > 1$). The lower and upper bounds of the supplier's wholesale revenue (7) are reformulated as (19) because the upper bound of the supplier's wholesale revenue is restricted by (17) and (18).

$$w_{tj} \geq c_{tj}, \qquad \forall t, \forall j \tag{19}$$

By introducing (17), (18) and (19) into the bilevel production planning problem in the previous section, the leader-supplier bilevel production planning problem (LSP) is formulated as follows.

Leader-supplier problem (LSP)

$$\begin{array}{lll} \text{(LSP)} & \max\ (1) & \\ & \text{s. t. } (2)-(9),\ (19) & \\ & \quad \max\ (10) & \\ & \quad \text{s. t. } (11)-(16),\ (17),\ (18) & \end{array}$$

In the leader-retailer problem, the supplier's profit maximization problem is considered as the retailer's constraints. The leader-retailer bilevel production planning (LRP) is formulated as follows.

Leader-retailer problem (LRP)

$$\begin{array}{lll} \text{(LRP)} & \max\ (10) & \\ & \text{s. t. } (11)-(16),\ (17),\ (18) & \\ & \quad \max\ (1) & \\ & \quad \text{s. t. } (2)-(9),\ (19) & \end{array}$$

4. Solution Approach

The multi-period bilevel production planning is formulated as a mixed integer nonlinear bilevel programming problem. However, it is difficult to solve the problem by commercial solvers. Therefore, an effective solution approach to derive a Stackelberg equilibrium is required.

4.1 Normalization of the Retailer's Decision Problem

The retailer's objective function is represented as the expected value due to demand uncertainty. However, it is difficult to solve the retailer's profit maximization problem by numerical computations. Petkov and Maranas addressed the normalization technique of the expectation of the costs under demand uncertainty [5]. The retailer's objective function is reformulated by the normalization of the probability distribution. The expected values are reformulated as follows.

$$\begin{aligned} R_{tj} &= p_{tj}\hat{D_{tj}} + p_{tj}\sigma_{tj}[-f(Y_{tj}) + (1-\Phi(Y_{tj}))Y_{tj}] \\ V_{tj} &= h^r_{tj}[I^r_{0j} + \sum_{t'=1}^{t} Q_{t'j} - \sum_{t'=1}^{t} \hat{D_{t'j}} \\ &\quad -\sigma_{t'j}\{-f(Y_{t'j}) + (1-\Phi(Y_{t'j}))Y_{t'j}\}] \\ H_{tj} &= -g_{tj}\sigma_{tj}[-f(Y_{tj}) + (1-\Phi(Y_{tj}))Y_{tj}] \end{aligned} \tag{20}$$

$f(D_{tj})$ and $\Phi(D_{tj})$ are the probability density function and the cumulative distribution function of the product item j in period t, respectively. $\hat{D_{tj}}$ is the average and σ_{tj} is the standard deviation of the customer's demand. The customer's demand is assumed to follow the normal distribution. $\Phi(D_{tj})$ and $f(D_{tj})$ are formulated by (21)-(23).

$$\Phi(Y_{tj}) = \int_{-\infty}^{Y_{tj}} f(z)dz \tag{21}$$

$$f(Y_{tj}) = \frac{1}{\sqrt{2\pi\sigma_{tj}^2}} \exp(-\frac{Y_{tj}^2}{2}) \tag{22}$$

$$Y_{tj} = \frac{S_{tj} - \hat{D_{tj}}}{\sigma_{tj}} \tag{23}$$

4.2 Reformulation of Bilevel Model into Single-level Model using the KKT Condition

The bilevel programming model is reformulated as a single-level programming model by the KKT condition of the lower-level problem. The KKT condition is the necessary condition which guarantees that the solution x is the local optimal solution. The Lagrangian function L is formulated as (24). J_r is the retailer's objective function. U and V are the number of equality constraints, inequality constraints in the retailer's profit maximization problem. $g^1_u(x) = 0$ $(1 \leq u \leq U)$ are the equality constraints, and $g^2_v(x) \leq 0$ and $(1 \leq v \leq V)$ are the inequality constraints. α_u and β_v are the dual variables related to these constraints.

$$L = J_r - \sum_{u=1}^{U} \alpha_u g^1_u(x) - \sum_{v=1}^{V} \beta_v g^2_v(x) \tag{24}$$

The KKT condition of the retailer's profit maximization problem is obtained. The mixed-integer bilevel programming problem is reformulated as the mixed-integer single level programming problem.

$$-h^r_{tj} - \alpha_{tj} + \beta_{t+1,j} = 0,\ \forall t, \forall j \tag{25}$$

$$-w_{tj} + \sum_{t'=t}^{T} \alpha_{t'j} + \beta_{tj} = 0,\ \forall t, \forall j \tag{26}$$

$$\frac{\partial R_{tj}}{\partial S_{tj}} - \frac{\partial H_{tj}}{\partial S_{tj}} - \beta_{tj} - \sum_{t'=t}^{T} \alpha_{t'j}[\sigma_{tj}(-\frac{\partial f(Y_{tj})}{\partial Y_{tj}}\frac{\partial Y_{tj}}{\partial S_{tj}} + (-\frac{\partial \Phi(Y_{tj})}{\partial Y_{tj}})Y_{tj} + (1-\Phi(Y_{tj}))\frac{\partial Y_{tj}}{\partial S_{tj}})] = 0,\ \forall t, \forall j \tag{27}$$

$$\beta_{tj}(S_{tj} - I^r_{t-1,j} - Q_{tj}) = 0,\ \forall t, \forall j \tag{28}$$

$$\beta_{tj} \geq 0,\ \forall t, \forall j \tag{29}$$

$$S_{tj} - I^r_{t-1j} - Q_{tj},\ \forall t, \forall j \tag{30}$$

$$\frac{\partial R_{tj}}{\partial S_{tj}} = p_{tj}\sigma_{tj}(-\frac{\partial f_{tj}}{\partial S_{tj}}\frac{\partial Y_{tj}}{\partial S_{tj}} - \frac{\partial f_{tj}}{\partial S_{tj}}Y_{yj} + (1-\Phi(Y_{tj})\frac{\partial Y_{tj}}{\partial S_{tj}}),$$

$$\frac{\partial Y_{tj}}{\partial S_{tj}} = \frac{1}{\sigma_{tj}},\ \frac{\partial f(Y_{tj})}{\partial S_{tj}} = -\frac{1}{\sqrt{2\pi\sigma^2_{tj}}}\exp(-\frac{Y^2_{tj}}{2})Y_{tj} \tag{31}$$

By applying the KKT condition to the leader-supplier problem, the bilevel programming problem is reformulated as the single level programming problem as follows.

$(LSP-KKT)$ max (1)
s. t. (2) − (9), (12) − (23), (25) − (31)
$(LRP-KKT)$ max (10)
s. t. (15), (20) − (23), (17) − (19)
KKT condition of the supplier's problem (See [10])

These problems are reformulated into single-level mixed integer nonlinear programming problems where bilinear terms such as $w_{tj}Q_{tj}$ are included. A general-purpose solver for solving mixed integer nonlinear programming problems is used to solve these problems.

5. Quantity Discount Contract to Improve the Total Profit of the Bilevel Production Planning

In the bilevel production planning, the total profit of the Stackelberg equilibrium is inferior to the optimal profit of the centralized problem because the supplier and the retailer make decisions according to their own preference. In this section, the quantity discount contract model in the bilevel production planning is proposed to improve the total profit.

5.1 Quantity Discount Contract with a Linear Function

If the quantity discount contract is considered in the bilevel planning problem, the supplier's wholesale revenue $w_{tj}Q_{tj}$ is formulated as the function of the retailer's order quantity Q_{tj}. A linear quantity discount model was introduced in [3]. The supplier's wholesale revenue is formulated as the linear function of the retailer's order quantity. Then, the customer's demand is assumed to follow the normal distribution. In this case, the supplier's wholesale price is formulated as (32).

$$w_{tj}Q_{tj} = W_{tj} - a_{tj}Q_{tj},\qquad \forall t, \forall j \tag{32}$$

where W_{tj} and a_{tj} are positive constants.

The multi-period production planning is formulated only as the LSP. In this case, the multi-period bilevel production planning with quantity discount is formulated as follows.

$$\max\ J_s = \sum_{t=1}^{T}\sum_{j=1}^{J}(w_1(Q_{tj}) - c_{tj}O_{tj} - h^s_{tj}I^s_{tj} - s_{tj}x_{tj})$$

s. t. (2) − (9)

$$\max\ J_r = \sum_{t=1}^{T}\sum_{j=1}^{J}E(R_{tj} - V_{tj} - H_{tj} - w_1(Q_{tj}))$$

s. t. (11), (15), (16), (20) − (23)

$$w_1(Q_{tj}) = W_{tj} - a_{tj}Q_{tj},\qquad \forall t,\ \forall j \tag{33}$$

Equation (26) in the KKT condition is reformulated as (34) by introducing (33).

$$-a_{tj} + \sum_{t'=t}^{T}\alpha_{t'j} + \beta_{tj} = 0,\qquad \forall t, \forall j \tag{34}$$

5.2 Quantity Discount Contract with a Quadratic Function

A quantity discount contract with a quadratic function is proposed in order to maximize the total profit. Q^c_{tj} is defined as the optimal solution of the retailer's order quantity of product item j in period t in the centralized problem. The supplier's wholesale revenue of product item j in period t, $w(Q_{tj})$ is minimized if $Q_{tj} = Q^c_{tj}$. We propose a quadratic function of the retailer's order quantity with respect to the wholesale revenue function. In this case, the supplier's wholesale revenue is formulated as (35). b_{tj} and c_{tj} are positive constants.

$$w_{tj}Q_{tj} = b_{tj}(Q_{tj} - Q^c_{tj})^2 + c_{tj},\qquad \forall t, \forall j \tag{35}$$

Equation (26) in the KKT condition is reformulated as (36) by introducing the quadratic quantity discount function of (35). This is included in the optimization model in the quantity discount bilevel planning model.

$$-2b_{tj}(Q_{tj} - Q^c_{tj}) + \sum_{t'=t}^{T}\alpha_{t'j} + \beta_{tj} = 0,\ \forall t, \forall j \tag{36}$$

6. Computational Experiments

Computational experiments are conducted to show the validity of the proposed models. The effects of the replacement of the leader-follower relationship in the multi-period bilevel production planning are studied in Section **6.1**. Then, the effects of the optimal quantity discounts are examined in Section **6.2**.

6.1 Effects of the Leader-follower Relationship

We study the effects of the bilevel production planning problem in which the leader-follower relationship can be changed dynamically. In the experiments, the end of periods is $T=2$ and the number of product items is $J=1$. Three cases of computational experiments are conducted.

Case 1: The supplier is the leader in all periods.
Case 2: The retailer is the leader in all periods.
Case 3: The leader is started from the supplier and the leader is changed from the supplier to the retailer at the start of period $t=2$.

In the Stackelberg game where the leader is changed in period $t=2$, the supplier is the leader in period $t=1$. The supplier makes a decision in order to maximize its own profit for whole periods. Then, the leader is changed from the supplier to the retailer, and the retailer makes decision as the leader in periods $t=2$. In this case, the decision variables of the supplier and the retailer in period $t=1$ are obtained and used in order to solve the Stackelberg game where the leader is the retailer in period $t=2$. The problems in Case 1, Case 2, and Case 3 were coded by mixed integer nonlinear programming problems. The experiment was performed by Intel (R) Core (TM) i7-2700 CPU 3.50GHz with General Algebraic Modeling System (GAMS). LINDOGLOBAL was used to solve the mixed integer nonlinear programming. The parameters were set as $a_{tj}=1.5$, $c^s_{t1}=100$, $g^r_{t1}=120$, $h^r_{t1}=20$, $h^s_{t1}=20$, $I^s_{0,1}=30$, $K_{t1}=8.5\times10^4$, $s_{t1}=1500$ and $\sigma_{t1}=20$. $I^s_{0,1}$ is the supplier's initial inventory quantity which are obtained by the optimization in the bilevel programming problem.

We investigated the effects of parameter e_{tj} on the replacement of leader-follower relationship because the parameter e_{tj} may influence on the selection of the optimal wholesale price and we have identified that other cost parameters do not have an impact on the replacement of leader-follower relationship in the problem.

Table 1 shows the comparison of the total profit among three cases when $e_{tj}=1.30$. The total profit in Case 2 is the largest in all cases. In the LRP, the wholesale revenue w_{tj} is the smallest in $t=1$ and $t=2$. Therefore, the sales revenue and the average of the customer's demand are the smallest in all cases because of (17) and (18). In Case 2, the average of the customer's demand becomes larger with the small wholesale price, and the total profit is maximized compared with Case 1 and Case 3. Table 2 shows the comparison of the total profit among three cases when $e_{tj}=1.40$ ($1.31\le e_{tj}\le 1.48$). The total profit in Case 3 is the largest in all cases. The wholesale revenue in period $t=1$ is equal to the wholesale revenue in period $t=1$ in the LSP and the wholesale revenue in period $t=2$ is larger than the wholesale price in period $t=2$ in Case 3. The retailer's wholesale cost is the same with the wholesale revenue multiplied by the order quantity. In the LRP of Case 2, the retailer who is the leader in period $t=1$ decides larger order quantity than that in Case 1 in order to enlarge the retailer's wholesale revenue. In Case 3, if the retailer decides the order quantity with larger wholesale revenue compared with that in LSP in Case 1, the average of the customer's demand is decreased compared with that in Case 1. Therefore, the retailer's profit is increased because the retailer can reduce the penalty for the inventory shortage. Table 3 shows the result of the comparison of the total profit among three cases in case $e_{tj}=1.50$ ($1.49\le e_{tj}\le 1.50$).

Table 1 Comparison of the total profit when $e_{tj}=1.30$

	Case 1	Case 2	Case 3
Supplier J_s	14933.04	1500.00	14966.17
Wholesales profit [-]	14955.02	17443.91	14988.14
Production cost [-]	0.00	14443.91	0.00
Inventory cost [-]	21.97	0.00	21.97
Setup cost [-]	0.00	1500.00	0.00
Retailer J_r	-904.74	12655.43	-876.13
Sales profit[-]	16040.63	31769.49	15994.83
Inventory cost [-]	456.74	1291.29	473.84
Opportunity loss cost [-]	1533.62	378.86	1408.98
Wholesales cost [-]	14955.02	17443.91	14988.14
Total Profit [-]	14028.29	14155.43	14090.04
Time [s]	41.73	77.11	42.31
derived decision variables			
w_{11}	502.32	100.00	502.32
w_{21}	398.08	265.33	428.23
Q_{11}	28.9	174.44	28.9
Q_{21}	1.1	0.0	1.1

Table 2 Comparison of the total profit when $e_{tj}=1.40$

	Case 1	Case 2	Case 3
Supplier J_s	9785.95	1500.00	9799.29
Wholesales profit [-]	9832.36	11504.59	9845.71
Production cost [-]	0.00	8504.59	0.00
Inventory cost [-]	46.42	0.00	46.42
Setup cost [-]	0.00	1500.00	0.00
Retailer J_r	-1652.54	5852.13	-1650.24
Sales profit[-]	10055.62	18910.31	10038.75
Inventory cost [-]	476.28	1058.1	482.15
Opportunity loss cost [-]	1399.52	495.5	1361.13
Wholesales cost [-]	9832.36	11504.59	9845.71
Total Profit [-]	8133.41	7352.12	8149.05
Time [s]	72.15	100.79	72.50
derived decision variables in			
w_{11}	333.56	100.00	333.56
w_{21}	258.37	200.73	264.12
Q_{11}	27.68	115.05	27.68
Q_{21}	2.32	0.00	2.32

Table 3 Comparison of the total profit when $e_{tj}=1.50$

	Case 1	Case 2	Case 3
Supplier J_s	6802.46	1500.00	6800.91
Wholesales profit [-]	6865.5	7732.26	6863.95
Production cost [-]	0.00	4732.26	0.00
Inventory cost [-]	63.04	0.00	63.04
Setup cost [-]	0.00	1500.00	0.00
Retailer J_r	-1973.29	2090.51	-1973.27
Sales profit[-]	6657.66	11320.77	6863.95
Inventory cost [-]	503.03	884.11	502.21
Opportunity loss cost [-]	1262.42	613.89	1267.13
Wholesales cost [-]	6865.5	7732.26	6863.95
Total Profit [-]	4829.17	3590.5	4827.64
Time [s]	59.12	100.79	59.58
derived decision variables			
w_{11}	234.72	100.00	234.72
w_{21}	178.84	156.92	178.34
Q_{11}	26.85	77.32	26.85
Q_{21}	3.15	0.00	3.15

The total profit in Case 1 is slightly larger than the total profit in Case 3. The increase of the wholesale revenue in period $t=2$ in the replacement of leader-follower relationship hardly improves the total profit and the total profit in the LSP is the largest. From our computational results, the total profit in the LRP (Case 2) is the largest if $e_{tj}=1.30$, the total profit in Case 3 is the largest if $1.31 \leq e_{tj} \leq 1.48$ and the total profit in the LSP is the largest if $1.49 \leq e_{tj} \leq 1.50$.

If e_{tj} is large, the impact of the mean of the customer's demand to the retailer's profit is small. In this case, the total profit depends on the costs such as the inventory holding cost or the penalty for inventory shortage. In case, $1.31 \leq e_{tj} \leq 1.48$, though the sales revenue is not so large, the penalty for inventory shortage is small in Case 3 compared with Case 1. Therefore, the total profit in the leader-follower replacement model is larger than the total profit in the LSP. On the other hand, in case $1.49 \leq e_{tj} \leq 1.50$, the retailer's total cost in the LSP is smaller than the retailer's total cost in the leader-follower replacement model because the supplier makes decision in each period. Therefore, the total profit in the LSP is larger than the total profit in the leader-follower replacement model.

6.2 Effects of the Quantity Discount Contract

The effects of introducing the quantity discount contract to the total profit in the decentralized model are examined from computational experiments. The experiments are conducted in four cases. In Case 1, the centralized problem is solved. In Case 2, the decentralized problem is solved without a quantity discount contract. In Case 3, the decentralized problem is solved with the wholesale price of the linear quantity discount. In Case 4, the decentralized problem is solved with the wholesale revenue with the quadratic quantity discount. The parameters of three instances are shown in Table 4. $a_{tj}=0.1$, $b_{tj}=10$, $c_{tj}=15000$, $U_{tj}=500$ and $W_{tj}=15000$. The number of product items is $J=3$.

Table 5 shows the comparison of the total profit among four cases. Time is the computation time with the maximum 3600 seconds. (3600) indicates the tentative results within 3600 seconds. From the results in Table 5, the total profit in the decentralized problem is improved by introducing the wholesale revenue of the linear function of the order quantity compared with that in the decentralized problem without contracts (Case 2). By introducing the wholesale revenue of the quadratic function of the order quantity, the total profit in the decentralized problem becomes much closer to the optimal profit in the centralized problem compared with the optimal profit introducing the wholesale revenue of the linear function. The results show that the total profit in the decentralized problem is improved by introducing the quantity discount contract. That is because both the production quantity and the delivery quantity increase in the decentralized problem by introducing the quantity discount contract compared with the production quantity and the delivery quantity in the decentralized problem without contracts. The wholesale revenue of the quadratic function is more effective to improve the total profit in the decentralized problem than the wholesale revenue of the linear function. That is because the wholesale revenue of the quadratic function is constructed with the optimal order quantity in the centralized problem. Therefore, the production quantity and the delivery quantity with the wholesale revenue of the quadratic function are much closer to those in the centralized problem.

Table 4 Parameters for each instance

Instance	item	c_{tj}	g_{tj}	h^r_{tj}	h^s_{tj}	p_{tj}	s_{tj}	μ_{tj}	σ_{tj}
1	1	93	119	12	12	318	1960	62	15
	2	119	106	16	16	288	2184	33	6
	3	84	123	19	19	296	2528	49	16
2	1	102	109	17	17	307	1634	66	13
	2	106	112	12	12	296	2363	41	3
	3	91	116	14	14	288	1603	65	14
3	1	99	108	19	19	303	1462	65	17
	2	109	134	20	20	282	1549	32	12
	3	97	127	15	15	313	2472	54	15

Table 5 Comparison of the total profit among four cases ($T=7,\ J=3$)

Parameter	Case	J_s	J_r	Total profit	Time[s]
Instance 1	Case 1	–	–	196760.00	7.02
	Case 2	268908.45	-99546.97	169361.48	(3600)
	Case 3	211696.42	-17707.31	193989.11	251.5
	Case 4	214418.03	-19181.95	195236.08	(3600)
Instance 2	Case 1	–	–	228789.54	8.87
	Case 2	320854.53	-117793.74	203060.79	(3600)
	Case 3	183222.11	35978.04	219200.15	1022.34
	Case 4	507816.81	-280696.66	227120.15	(3600)
Instance 3	Case 1	–	–	199354.52	4.12
	Case 2	259522.18	-105010.81	154511.37	(3600)
	Case 3	202982.43	-7359.26	195623.17	1022.34
	Case 4	521808.51	-323398.69	198409.82	(3600)

7. Conclusion and Future Works

In this paper, we have proposed an optimization method for multi-period bilevel production planning for single supplier and single retailer under demand uncertainty. In order to improve the total profit, the replacement of the leader-follower relationship is proposed. The effects of the replacement of the leader-follower relationship have been investigated. In this paper, we have shown that there is a case when the replacement of leader-follower relationship makes the profit better for the problem instance of $T=2$ only for the cases even when all the cost parameters are constant. There will be some cases where the replacement of leader-follower relationship makes the total profit better when the cost parameters are dynamically changed. The selection of the proper timing of replacement and the effects on the total planning horizon under dynamic change of cost parameters will be studied in future works. A quantity discount contract with quadratic function is also proposed in order to improve the total profit in the decentralized problem. By introducing the optimal quantity discount contract into the decentralized problem, the equilibrium solution in the decentralized problem is very close to the optimal solution in the centralized problem. Therefore, the total profit in the decentralized problem is much improved by introducing the optimal quantity discount contract.

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References

[1] E. Almehdawe and B. Mantin: Vendor managed inventory with a capacitated manufacturer and multiple retailers: retailer versus manufacturer leadership; *International Journal of Production Economics*, Vol. 219, pp. 9670–9680 (2013)

[2] J. F. Bard: *Practical Bilevel Optimization, Algorithms and Applications*, Kluwer Academic Publishers (1998)

[3] K. Kogan and C. S. Tapiero: *Supply Chain Games*, Springer (2007)

[4] T. Nishi and O. Yoshida: Optimization of bilevel supply chain under demand uncertainty; *Procedia CIRP*, Vol. 41, pp. 508–513 (2016)

[5] S. B. Petkov and C. D. Maranas: Multiperiod planning and scheduling of multiproduct batch plants under demand uncertainty; *Industrial and Engineering Chemistry Research*, Vol. 36, pp. 4864–4881 (1997)

[6] T. Qu, D. X. Nie, X. Chen, X. D. Chen, Q. Y. Dai and G. Q. Huang: Optimal configuration of cluster supply chains with augmented Lagrange coordination; *Computers and Industrial Engineering*, Vol. 84, pp. 43–55 (2015)

[7] J. F. Tsai: An optimization approach for supply chain management models with quantity discount policy; *European Journal of Operational Research*, Vol. 177, pp. 982–994 (2007)

[8] S. Yin, T. Nishi and I. E. Grossmann: Optimal quantity discount coordination for supply chain optimization with one manufacturer and multiple suppliers under demand uncertainty; *International Journal of Advanced Manufacturing Technology*, Vol. 76, No. 5, pp. 1173–1184 (2014)

[9] O. Yoshida and T. Nishi: Analysis of quantity discounts for multi-period production planning for single supplier and retailer under uncertain demand; *Proceedings of 2014 IEEE International Conference on Industrial Engineering and Engineering Management*, pp. 882–886 (2014)

[10] O. Yoshida and T. Nishi: Replacement of leader-follower relation in multi-period supply chain planning under demand uncertainty; *Proceedings of 2016 International Symposium on Flexible Automation*, pp. 192–197 (2016)

[11] D. Yue and F. You: Game-theoretic modeling and optimization of multi-echelon supply chain design and operation under Stackelberg game and market equilibrium; *Computers and Chemical Engineering*, Vol. 71, pp. 347–361 (2014)

[12] G. Zhang and J. Shi: Multi-period multi-product acquisition planning with supplier discounts; *Proceedings of 2009 CORS/INFORMS International Meeting* (2009)

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